Generating Initial Data for Interacting Compact Binaries

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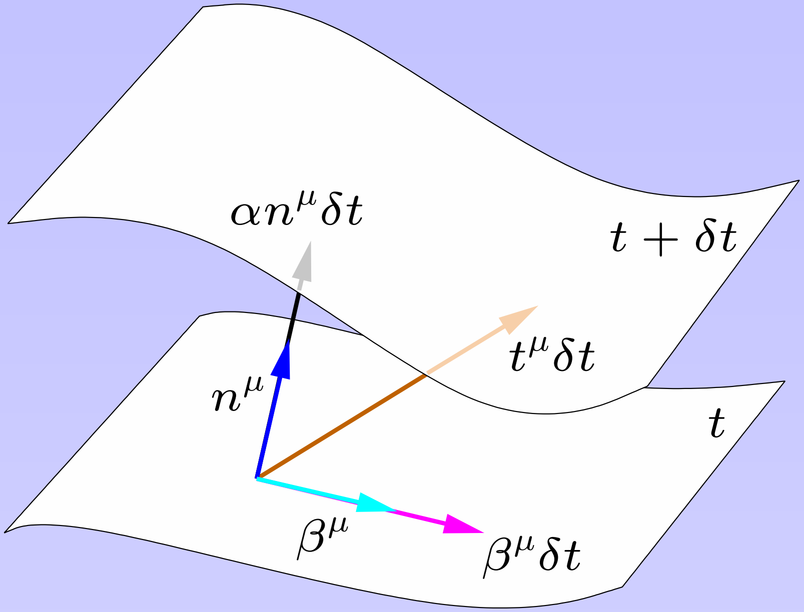
Abstract

We will look at some of the general formalisms used to construct initial data and review the approaches that have been used in generating black-hole and neutron-star initial data. We will conclude with a brief look at some recent work in generating quasi-equilibrium black-hole binary initial data.

A related review article is online at Living Review in Relativity[18]

The $3+1$ Decomposition



Lapse : α Spatial metric : γ_{ij}

Shift vector : β^i Extrinsic Curvature : K_{ij}

Time vector : $t^\mu = \alpha n^\mu + \beta^\mu$

$$\mathrm{d}s^2 = -\alpha^2 \mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i \mathrm{d}t)(\mathrm{d}x^j + \beta^j \mathrm{d}t)$$

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \qquad K_{\mu\nu} = -\tfrac{1}{2}\gamma^\alpha_\mu \gamma^\beta_\nu \mathcal{L}_n g_{\alpha\beta}$$

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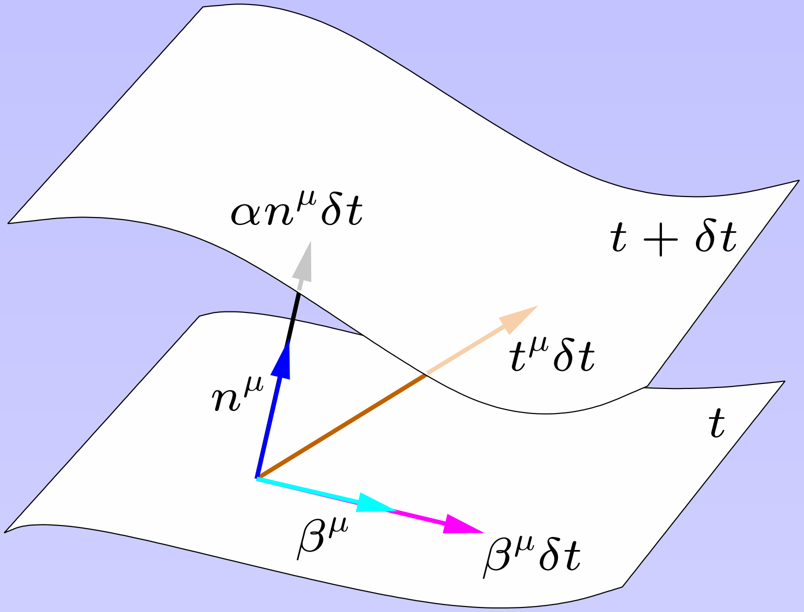

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Constraint equations

$$\bar{R} + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$\bar{\nabla}_j\Big(K^{ij} - \gamma^{ij}K\Big) = 8\pi j^i$$

$$S_{\mu\nu} \equiv \gamma^\alpha_\mu \gamma^\beta_\nu T_{\alpha\beta}$$

$$j_\mu \equiv -\gamma^\nu_\mu n^\alpha T_{\nu\alpha}$$

$$\rho \equiv n^\mu n^\nu T_{\mu\nu}$$

$$T_{\mu\nu} = S_{\mu\nu} + 2n_{(\mu}j_{\nu)} + n_\mu n_\nu \rho$$

Evolution equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i$$

$$\partial_t K_{ij} = -\bar{\nabla}_i \bar{\nabla}_j \alpha + \alpha\Big[\bar{R}_{ij} - 2K_{i\ell}K^\ell_j + KK_{ij} - 8\pi S_{ij} + 4\pi\gamma_{ij}(S-\rho)\Big] + \beta^\ell \bar{\nabla}_\ell K_{ij} + K_{i\ell}\bar{\nabla}_j \beta^\ell + K_{j\ell}\bar{\nabla}_i \beta^\ell$$

Generalized Conformal/TT Initial-Data Decomposition[46]

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad : \quad K^{ij} = \psi^{-10}\tilde{A}^{ij} + \tfrac{1}{3}\psi^{-4}\tilde{\gamma}^{ij}K \quad : \quad \tilde{A}^{ij} \equiv \frac{1}{\tilde{\sigma}}(\tilde{\mathbb{L}}V)^{ij} + \tilde{M}^{ij}$$

$$\tilde{\sigma} \equiv \psi^{-6}\sigma \quad : \quad \tilde{\nabla}^2\psi - \tfrac{1}{8}\psi\tilde{R} - \tfrac{1}{12}\psi^5 K^2 + \tfrac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} = -2\pi\psi^5\rho$$

$$\tilde{\nabla}_j(\tilde{\mathbb{L}}V)^{ij} - (\tilde{\mathbb{L}}V)^{ij}\tilde{\nabla}_j \ln\tilde{\sigma} - \tfrac{2}{3}\tilde{\sigma}\psi^6\tilde{\nabla}^i K = -\tilde{\sigma}\tilde{\nabla}_j\tilde{M}^{ij} + 8\pi\tilde{\sigma}\psi^{10}j^i$$

$\tilde{M}^{ij}$ is symmetric-tracefree, but not divergenceless. The variable V^i incorporates the solution of the constraints and the decomposition of $\tilde{M}^{ij}$ into $\tilde{M}^{ij}_{TT}$.

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$\tilde{\sigma} \Rightarrow 2\tilde{\alpha}$

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Conformal Thin Sandwich(TS)[59] $\qquad [\tilde{u}^{ij} \equiv -\partial_t\tilde{\gamma}^{ij}]$

$$\gamma_{ij} = \psi^4\tilde{\gamma}_{ij} \quad : \quad K^{ij} = \psi^{-10}\tilde{A}^{ij} + \tfrac{1}{3}\psi^{-4}\tilde{\gamma}^{ij}K \quad : \quad \tilde{A}^{ij} \equiv \frac{1}{2\tilde{\alpha}}\left((\tilde{\mathbb{L}}\beta)^{ij} - \tilde{u}^{ij}\right)$$

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$$\tilde{\nabla}^2(\psi^7\tilde{\alpha}) - \tfrac{1}{8}\psi^7\tilde{\alpha}\tilde{R} - \tfrac{5}{12}\psi^{11}\tilde{\alpha}K^2 - \tfrac{7}{8}\psi^{-1}\tilde{\alpha}\tilde{A}_{ij}\tilde{A}^{ij} - \psi^5\beta^i\tilde{\nabla}_iK = -2\pi\psi^{11}\tilde{\alpha}K(\rho + 2S) - \psi^5\partial_tK$$

α and β^i are the lapse and shift. $\tilde{u}^{ij}$ is symmetric-tracefree.

Degrees of Freedom

Kinematical variables

- Lapse α : 1 degree of freedom
- Shift β^i : 3 degrees of freedom

Initial-data variables

- Metric γ_{ij} : 6 degrees of freedom
- Extrinsic curvature K_{ij} : 6 degrees of freedom

Decomposition of initial-data variables

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \begin{cases} \psi & : \quad \text{1 constrained DOF} \\ \tilde{\gamma}_{ij} & : \quad \left\langle \begin{matrix} \text{3 spatial gauge DOF} \\ \text{2 dynamical DOF} \end{matrix} \right\rbrace \textit{Freely Specifiable} \end{cases}$$

$$K^{ij} = \psi^{-10}\left[\frac{1}{\tilde{\sigma}}(\tilde{\mathbb{L}}V)^{ij} + \tilde{M}^{ij}\right] + \tfrac{1}{3}\psi^{-4}\tilde{\gamma}^{ij}K \begin{cases} V^i & : \left\langle \begin{matrix} (\tilde{\mathbb{L}}V)^{ij} \equiv \tilde{\nabla}^i V^j + \tilde{\nabla}^i V^j - \frac{1}{3}\tilde{\gamma}^{ij}\tilde{\nabla}_k V^k \\ \text{3 constrained DOF} \end{matrix} \right. \\ \tilde{M}^{ij} & : \left\langle \begin{matrix} \tilde{\nabla}_j \tilde{M}^{ij} = \tilde{\nabla}_j\left(\frac{1}{\tilde{\sigma}}(\tilde{\mathbb{L}}X)^{ij}\right) \\ \text{2 dynamical DOF} \end{matrix} \right. \\ K & : \text{1 temporal gauge DOF} \\ \tilde{\sigma} & : \textit{Defn. of TT decomp.} \end{cases} \Bigg\rbrace \textit{Freely Specifiable}$$

“Traditional” Black-Hole Data

Conformal flatness and maximal slicing

$$\left.\begin{array}{rcl}\tilde{\gamma}_{ij} &=& f_{ij}\ \textit{(flat)}\\ \tilde{M}^{ij} &=& 0\\ K &=& 0\\ \tilde{\sigma} &=& 1\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\tilde{\nabla}_i(\tilde{\mathbb{L}}V)^{ij} = 0 \quad \Rightarrow \\ \tilde{\nabla}^2\psi + \frac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} = 0\end{array}\right.$$

Bowen-York solution[11]
Analytic solutions for $\tilde{A}^{ij}$

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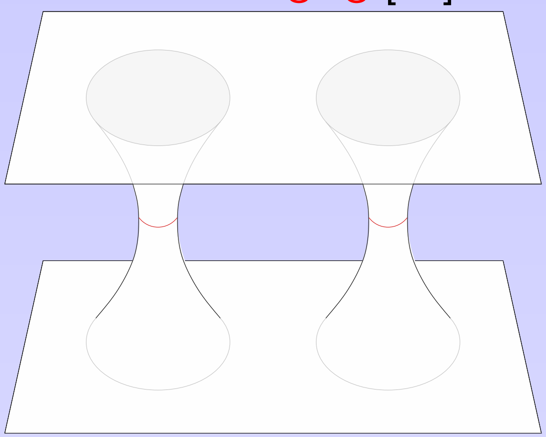
Analytic solutions for $\tilde{A}^{ij}$

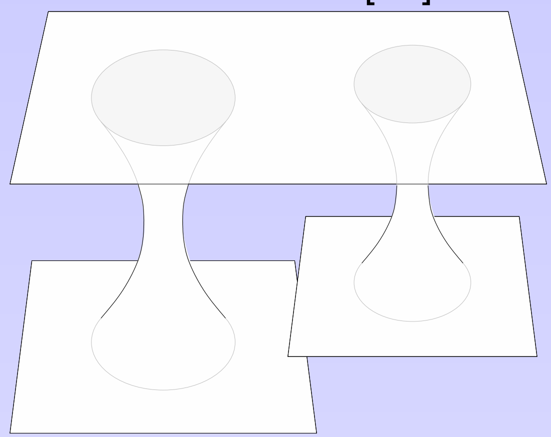
Three general solution schemes

Conformal Imaging-[21]



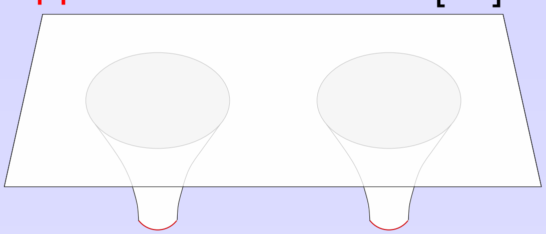
Inversion symmetry inner-BC

Puncture Method-[12]



No inner-BC: singular behavior factored out

Apparent Horizon BC-[51]



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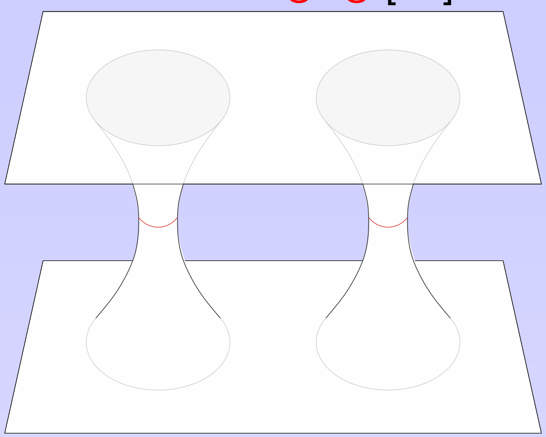

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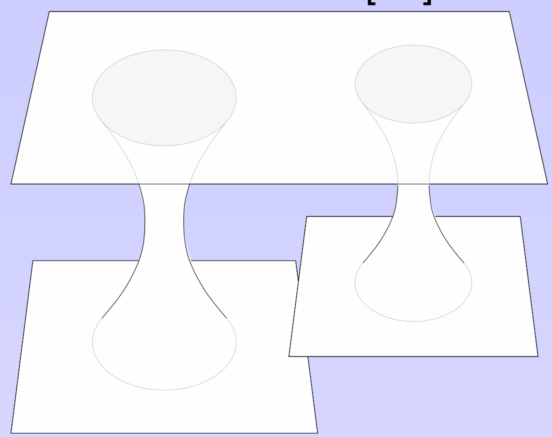

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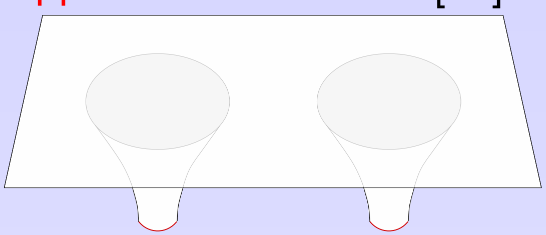

Apparent horizon inner-BC

All methods can produce very general configurations of multiple black holes, but are fundamentally limited by choices for $\tilde{\gamma}_{ij}$ and Bowen-York $\tilde{A}^{ij}$.

Early work with BH and NS ID

- The "traditional" BH initial data approach was motivated by *computational convenience*, not by any strong physical arguments. Research focused on:
  - methods for solving the Hamiltonian constraint for one or two holes. [60, 15, 16, 21, 1, 37, 45, 27]
  - understanding the physical content (and limitations) of initial data containing one or two holes.[60, 25, 16, 20]
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- Boosted Kerr — Use a superposition of analytic black hole metrics and extrinsic curvatures as trial data and solve the constraints to correct.[43, 42]
  - This approach lacks well motivated boundary conditions
- Single, stationary neutron stars have been studied extensively. The matter is assumed to be in hydrostatic equilibrium. Research has focused on:
  - various numerical methods.[10, 14, 13, 38, 22, 24, 48, 9, 32, 5, 7]
  - rigid and differential rotation.[56, 39, 22]
  - studies with various equations of state.[29, 23]

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  - The Bowen-York analytic extrinsic curvature does not model the physics of binary inspiral well.[36, 44, 26]
  - Boosted Kerr extrinsic curvature does not do significantly better than the Bowen-York extrinsic curvature.[44]

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⋆ Quasi-equilibrium — For sufficiently separated binaries, the timescale for orbital decay is much larger than the orbital period.

  - Binaries in quasi-circular orbits allow for the notion of an approximate hellical Killing vector.

Compact Binary ID

- NS binaries were explored first. Research has focused on:
  - various numerical methods.[57, 58, 6, 3, 34, 35]
  - corotating (tidally locked) binaries.[6, 3, 4, 55, 52, 40, 55, 49]
  - irrotational binaries.[50, 47, 31, 8, 41, 52, 53, 49]
  - no conformal flatness assumption.[54]

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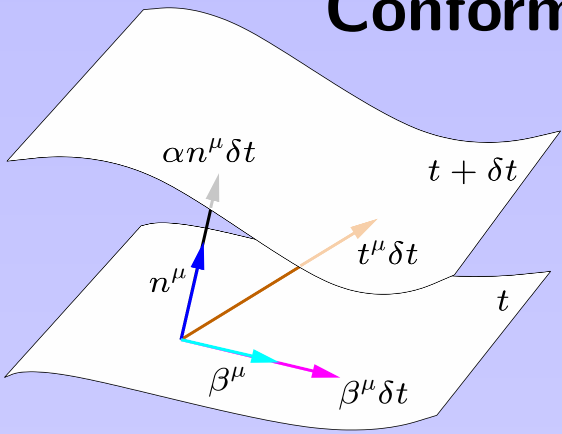
⋆ For black holes, use the principle of *quasi-equilibrium* to derive boundary conditions for the constrained data.[19, 28]

Conformal Thin-Sandwich Decomposition



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Momentum Const. $\tilde{\nabla}_j(\tilde{\mathbb{L}}\beta)^{ij} - (\tilde{\mathbb{L}}\beta)^{ij}\tilde{\nabla}_j\tilde{\alpha} = \frac{4}{3}\tilde{\alpha}\psi^6\tilde{\nabla}^i K + \tilde{\alpha}\tilde{\nabla}_j\left(\frac{1}{\tilde{\alpha}}\tilde{u}^{ij}\right) + 16\pi\tilde{\alpha}\psi^{10} j^i$

Const. Tr(K) eqn. $\tilde{\nabla}^2(\psi^7\tilde{\alpha}) - (\psi^7\tilde{\alpha})\left[\frac{1}{8}\psi\tilde{R} + \frac{5}{12}\psi^5 K^2 + \frac{7}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} - \psi^5\beta^i\tilde{\nabla}_i K\right]$

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$$\tilde{A}^{ij} \equiv \frac{1}{2\tilde{\alpha}}\left[(\tilde{\mathbb{L}}\beta)^{ij} - \tilde{u}^{ij}\right]$$

Constrained vars : ψ and β^i

Freely specified : $\tilde{\gamma}_{ij}$, $\tilde{u}^{ij}$, K, and $\partial_t K$

$\tilde{u}^{ij}$ and β^i have a simple physical interpretation, unlike $\tilde{M}^{ij}$ and V^i.

Quasi-equilibrium $\Rightarrow \begin{cases} \tilde{u}^{ij} = -\partial_t\tilde{\gamma}^{ij} = 0 \\ \partial_t K = 0 \end{cases}$

Equations of Quasi-Equilibrium

Ham. & Mom. const. eqns., & Const Tr(K) eqn. from Conf. TS $+\ \tilde{u}^{ij} = \partial_t K = 0$ $\Rightarrow$ **Eqns. of Quasi-Equilibrium**

With $\tilde{\gamma}_{ij} = f_{ij}$ and $K = 0$, these equations have been widely used to construct binary neutron star initial data[3, 40, 8, 52].

Binary neutron star initial data require:

- boundary conditions at infinity compatible with asymptotic flatness and corotation.

$$\psi|_{\mathrm{r}\to\infty} = 1 \quad \beta^i|_{\mathrm{r}\to\infty} = \Omega \left(\frac{\partial}{\partial\phi}\right)^i \quad \alpha|_{\mathrm{r}\to\infty} = 1$$

- compatible solution of the equations of hydrostatic equilibrium. $(\Rightarrow \Omega)$

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- compatible solution of the equations of hydrostatic equilibrium. $(\Rightarrow \Omega)$

Binary black hole initial data require:

- a means for choosing the angular velocity of the orbit Ω.

⋆ *with excision*, inner boundary conditions are needed for ψ, β^i, and $\tilde{\alpha}$.

Gourgoulhon, Grandclément, & Bonazzola[33, 36]: Black-hole binaries with $\tilde{\gamma}_{ij} = f_{ij}$ & $K = 0$, "inversion-symmetry", and "Killing-horizon" conditions on the excision boundaries.

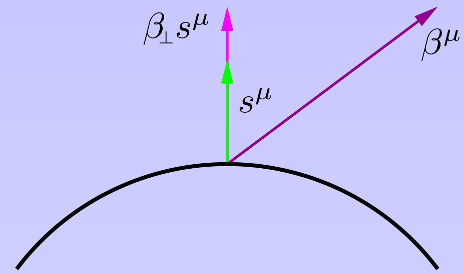
"Solutions" require *constraint violating* regularity condition imposed on inner boundaries!

AH and QE Conditions on the Inner Boundary

The quasi-equilibrium inner boundary conditions start with the following assumptions:

1. The inner boundary $\mathcal{S}$ is a (MOTS):
   marginally outer-trapped surface

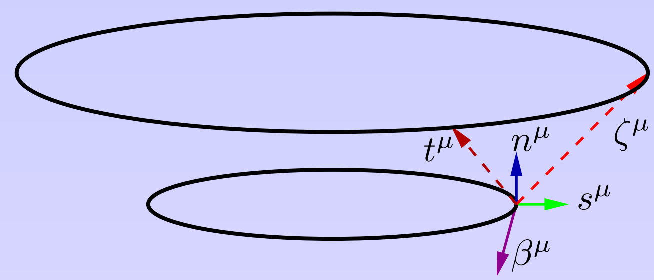
   $\rightarrow \quad \theta = 0$



2. The inner boundary $\mathcal{S}$ doesn't move:
   $\rightarrow \quad \mathcal{L}_\zeta \tau = 0$ and $D_i \mathcal{L}_\zeta \tau \equiv h^j_i \bar{\nabla}_j \mathcal{L}_\zeta \tau = 0$

$$t^\mu = \alpha n^\mu + \beta^\mu \qquad \zeta^\mu \equiv \alpha n^\mu + \beta_\perp s^\mu$$

$$\beta_\perp \equiv \beta^i s_i$$



3. The inner boundary $\mathcal{S}$ remains a MOTS:
   $\rightarrow \quad \mathcal{L}_\zeta \theta = 0$

4. The horizons are in quasi-equilibrium:
   $\rightarrow \quad \sigma_{ij} = 0$ and no matter is on $\mathcal{S}$

AH/Quasi-Equilibrium Boundary Conditions

$$\theta = \frac{\psi^{-2}}{\sqrt{2}}\left[\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j + 4\tilde{s}^k\tilde{\nabla}_k\ln\psi - \psi^2 J\right]$$

$$\begin{aligned}
\mathcal{L}_\zeta\theta \quad &= \quad -\tfrac{1}{\sqrt{2}}\left[\theta(\theta + \tfrac{1}{2}\acute{\theta} + \tfrac{1}{\sqrt{2}}K) + \mathcal{E}\right](\beta_\perp + \alpha) \\
&\quad\; -\tfrac{1}{\sqrt{2}}\left[\theta(\tfrac{1}{2}\theta - \tfrac{1}{2}\acute{\theta} + \tfrac{1}{\sqrt{2}}K) + \mathcal{D} + 8\pi T_{\mu\nu}k^\mu\acute{k}^\nu\right](\beta_\perp - \alpha) \\
&\quad\; + \theta s^i\bar{\nabla}_i\alpha
\end{aligned}$$

$$\begin{aligned}
\mathcal{D} \quad &\equiv \quad h^{ij}(D_i + J_i)(D_j + J_j) - \tfrac{1}{2}{}^2R \\
\mathcal{E} \quad &\equiv \quad \sigma_{ij}\sigma^{ij} + 8\pi T_{\mu\nu}k^\mu k^\nu
\end{aligned}$$

$$\begin{aligned}
\sigma_{ij} \quad &= \quad \frac{1}{\sqrt{2}}(H_{ij} - \tfrac{1}{2}h_{ij}H)\left(1 - \frac{\beta_\perp}{\alpha}\right) \\
&\quad\; - \frac{1}{\sqrt{2}}\frac{\psi^4}{\alpha}\left\{\tilde{h}_{k(i}\tilde{D}_{j)}\beta_\parallel{}^k - \tfrac{1}{2}\tilde{h}_{ij}\tilde{D}_k\beta_\parallel{}^k - \tfrac{1}{2}[\tilde{h}_{ik}\tilde{h}_{j\ell}\tilde{u}^{k\ell} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{h}_{k\ell}\tilde{u}^{k\ell}]\right\}
\end{aligned}$$

AH/Quasi-Equilibrium Boundary Conditions

$$\theta = \frac{\psi^{-2}}{\sqrt{2}}\left[\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j + 4\tilde{s}^k\tilde{\nabla}_k \ln\psi - \psi^2 J\right]$$

$$\begin{aligned}
\mathcal{L}_\zeta\theta \quad &= \quad -\tfrac{1}{\sqrt{2}}\left[\theta(\theta + \tfrac{1}{2}\acute{\theta} + \tfrac{1}{\sqrt{2}}K) + \mathcal{E}\right](\beta_\perp + \alpha) \\
&\quad\; -\tfrac{1}{\sqrt{2}}\left[\theta(\tfrac{1}{2}\theta - \tfrac{1}{2}\acute{\theta} + \tfrac{1}{\sqrt{2}}K) + \mathcal{D} + 8\pi T_{\mu\nu}k^\mu \acute{k}^\nu\right](\beta_\perp - \alpha) \\
&\quad\; + \theta s^i \bar{\nabla}_i \alpha
\end{aligned}$$

$$\begin{aligned}
\mathcal{D} \quad &\equiv \quad h^{ij}(D_i + J_i)(D_j + J_j) - \tfrac{1}{2}{}^2R \\
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\sigma_{ij} \quad &= \quad \frac{1}{\sqrt{2}}(H_{ij} - \tfrac{1}{2}h_{ij}H)\left(1 - \frac{\beta_\perp}{\alpha}\right) \\
&\quad - \frac{1}{\sqrt{2}}\frac{\psi^4}{\alpha}\left\{\tilde{h}_{k(i}\tilde{D}_{j)}\beta_\parallel{}^k - \tfrac{1}{2}\tilde{h}_{ij}\tilde{D}_k\beta_\parallel{}^k - \tfrac{1}{2}[\tilde{h}_{ik}\tilde{h}_{j\ell}\tilde{u}^{k\ell} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{h}_{k\ell}\tilde{u}^{k\ell}]\right\}
\end{aligned}$$

$$\begin{aligned}
\tilde{s}^k\tilde{\nabla}_k \ln\psi \;&=\; -\tfrac{1}{4}(\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j - \psi^2 J) \\
\beta^i \;&=\; \alpha\psi^{-2}\tilde{s}^i + \beta^i_\parallel \\
0 \;&=\; \tilde{D}^{(i}\beta_\parallel{}^{j)} - \tfrac{1}{2}\tilde{h}^{ij}\tilde{D}_k\beta_\parallel{}^k
\end{aligned}$$

Summary of BH QE Formalism

$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$ $\quad K^{ij} = \psi^{-10}\tilde{A}^{ij} + \frac{1}{3}\gamma^{ij} K$ $\quad \tilde{A}^{ij} = \frac{\psi^6}{2\alpha}(\tilde{\mathbb{L}}\beta)^{ij}$ $\quad \partial_t \tilde{\gamma}_{ij} = 0$

$$\begin{aligned}
\tilde{\nabla}^2\psi - \tfrac{1}{8}\psi\tilde{R} - \tfrac{1}{12}\psi^5 K^2 + \tfrac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} &= 0 \\
\tilde{\nabla}_j(\tilde{\mathbb{L}}\beta)^{ij} - (\tilde{\mathbb{L}}\beta)^{ij}\tilde{\nabla}_j \ln \alpha\psi^{-6} &= \tfrac{4}{3}\alpha\tilde{\nabla}^i K \\
\tilde{\nabla}^2(\alpha\psi) - (\alpha\psi)\Big[\tfrac{1}{8}\tilde{R} + \tfrac{5}{12}\psi^4 K^2 + \tfrac{7}{8}\psi^{-8}A_{ij}A^{ij}\Big] &= \psi^5\beta^i\tilde{\nabla}_i K
\end{aligned}$$

$\partial_t K = 0$

$$\tilde{s}^k\tilde{\nabla}_k \ln\psi|_{\mathcal{S}} = -\tfrac{1}{4}(\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j - \psi^2 J)|_{\mathcal{S}}$$

$\theta = 0$

$$\beta^i|_{\mathcal{S}} = \begin{cases} \alpha\psi^{-2}\tilde{s}^i|_{\mathcal{S}} & \text{corotation} \\ \alpha\psi^{-2}\tilde{s}^i|_{\mathcal{S}} + \Omega\xi^i|_{\mathcal{S}} & \text{irrotation} \end{cases}$$

$\mathcal{L}_\zeta \theta = 0$

$\sigma_{ij} = 0$

$\alpha|_{\mathcal{S}} =$ unspecified by QE !?

$$\begin{aligned}
\psi|_{\mathrm{r}\to\infty} &= 1 \\
\beta^i|_{\mathrm{r}\to\infty} &= \Omega\left(\frac{\partial}{\partial\phi}\right)^i \\
\alpha|_{\mathrm{r}\to\infty} &= 1
\end{aligned}$$

The only remaining freedom in the system is the choice of the lapse boundary condition, the initial spatial and temporal gauge, and the initial dynamical("wave") content found in $\alpha|_{\mathcal{S}}$, $\tilde{\gamma}_{ij}$ and K.

New Results

with H. Pfeiffer

Corotation

$\tilde{\gamma}_{ij} = f_{ij}$: Maximal Slicing:

- $\frac{\partial(\alpha\psi)}{\partial r} = 0$
- $\alpha\psi = \frac{1}{2}$
- $\frac{\partial(\alpha\psi)}{\partial r} = \frac{\alpha\psi}{2r}$

$\tilde{\gamma}_{ij} = f_{ij}$: Eddington-Finkelstein Slicing:

- $\frac{\partial(\alpha\psi)}{\partial r} = 0$
- $\alpha\psi = \frac{1}{2}$
- $\frac{\partial(\alpha\psi)}{\partial r} = \alpha\psi$

Compared with

- Effective-One-Body PN[26]
- Inversion-Symmetric HKV[36]

Irrotation

$\tilde{\gamma}_{ij} = f_{ij}$: Maximal Slicing:

- $\frac{\partial(\alpha\psi)}{\partial r} = 0$
- $\alpha\psi = \frac{1}{2}$
- $\frac{\partial(\alpha\psi)}{\partial r} = \frac{\alpha\psi}{2r}$

$\tilde{\gamma}_{ij} = f_{ij}$: Eddington-Finkelstein Slicing:

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- $\alpha\psi = \frac{1}{2}$
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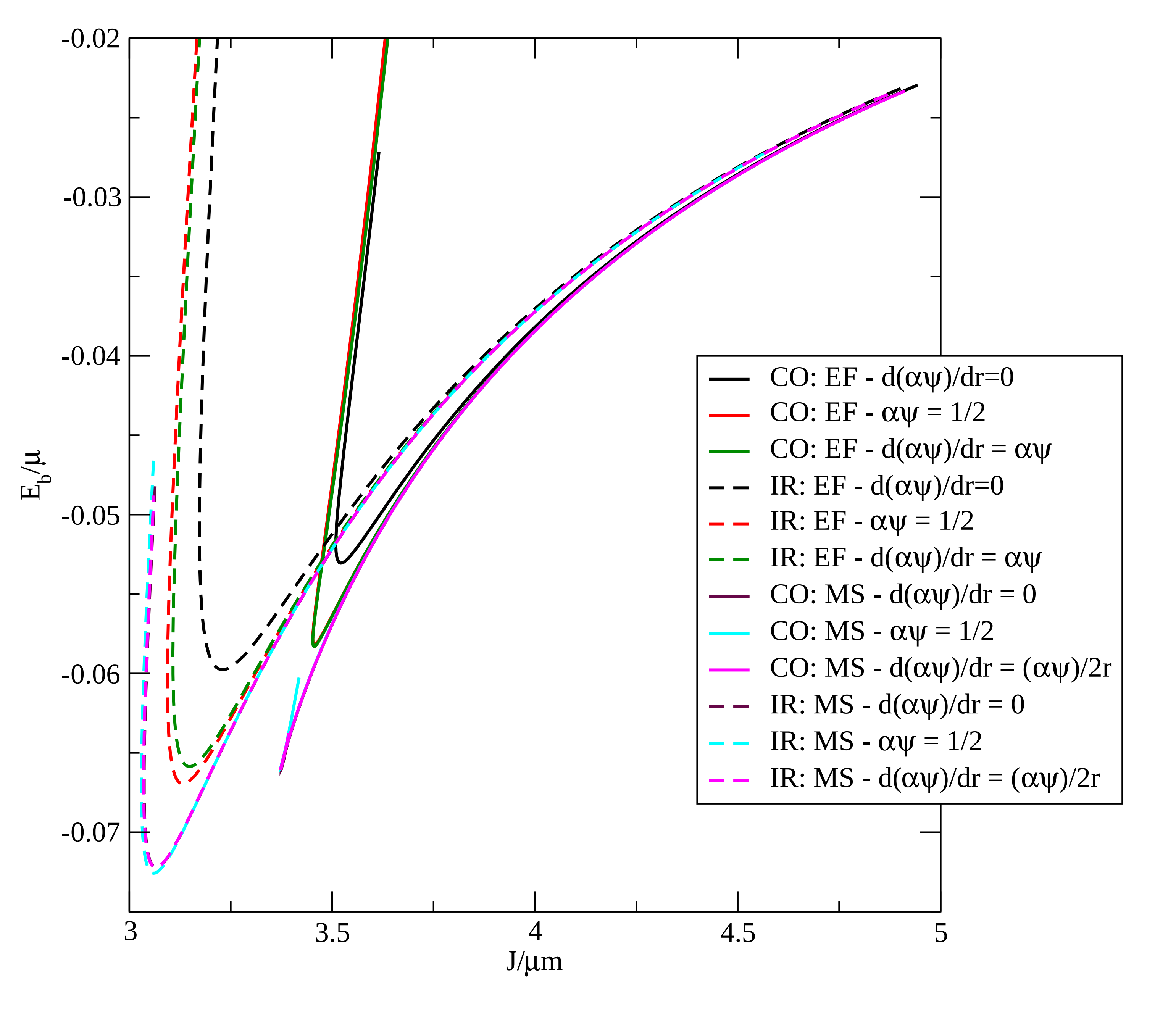
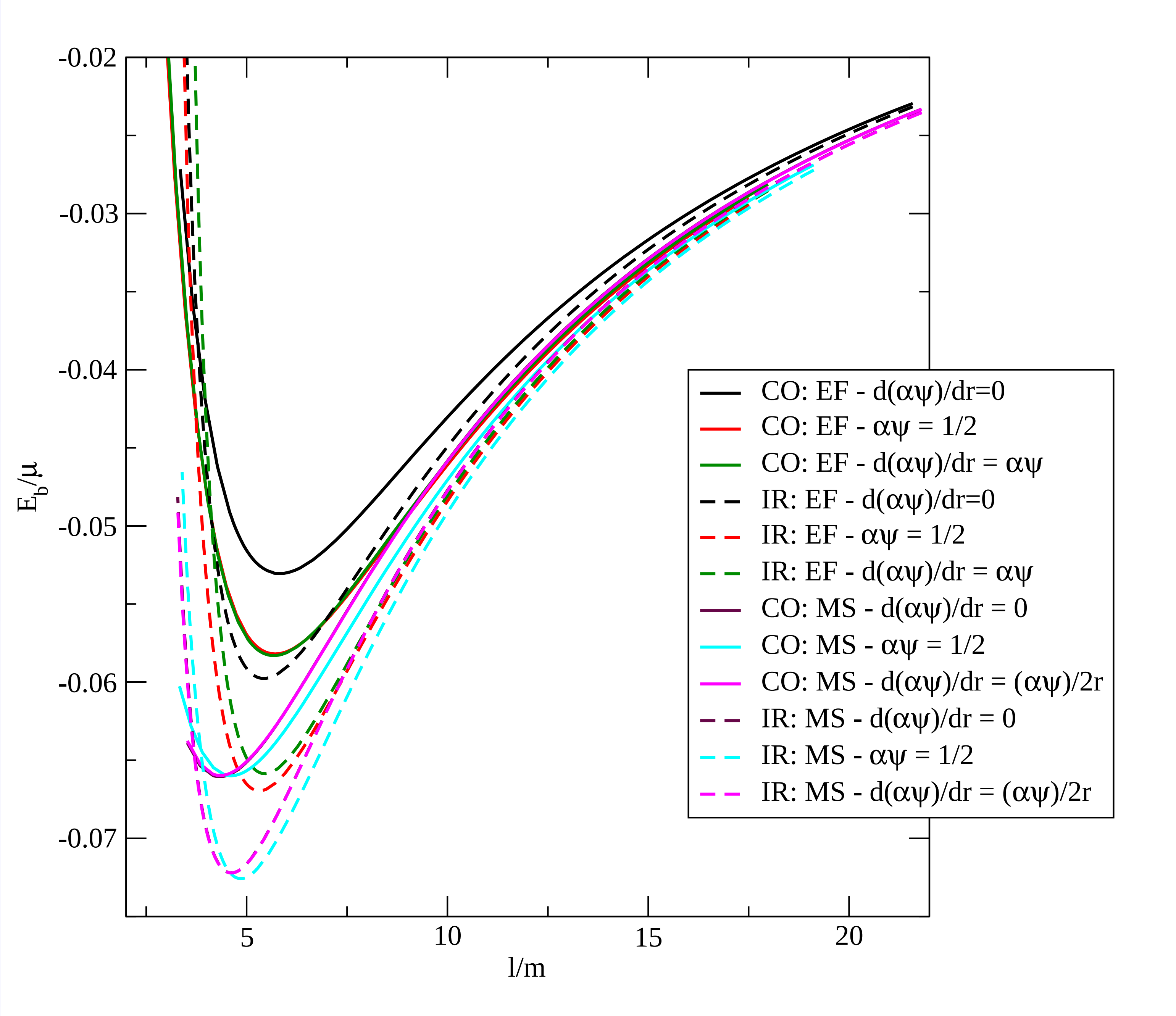
Compared with

- Effective-One-Body PN[26]
- Conformal Imaging[17]
- Puncture Method[2]

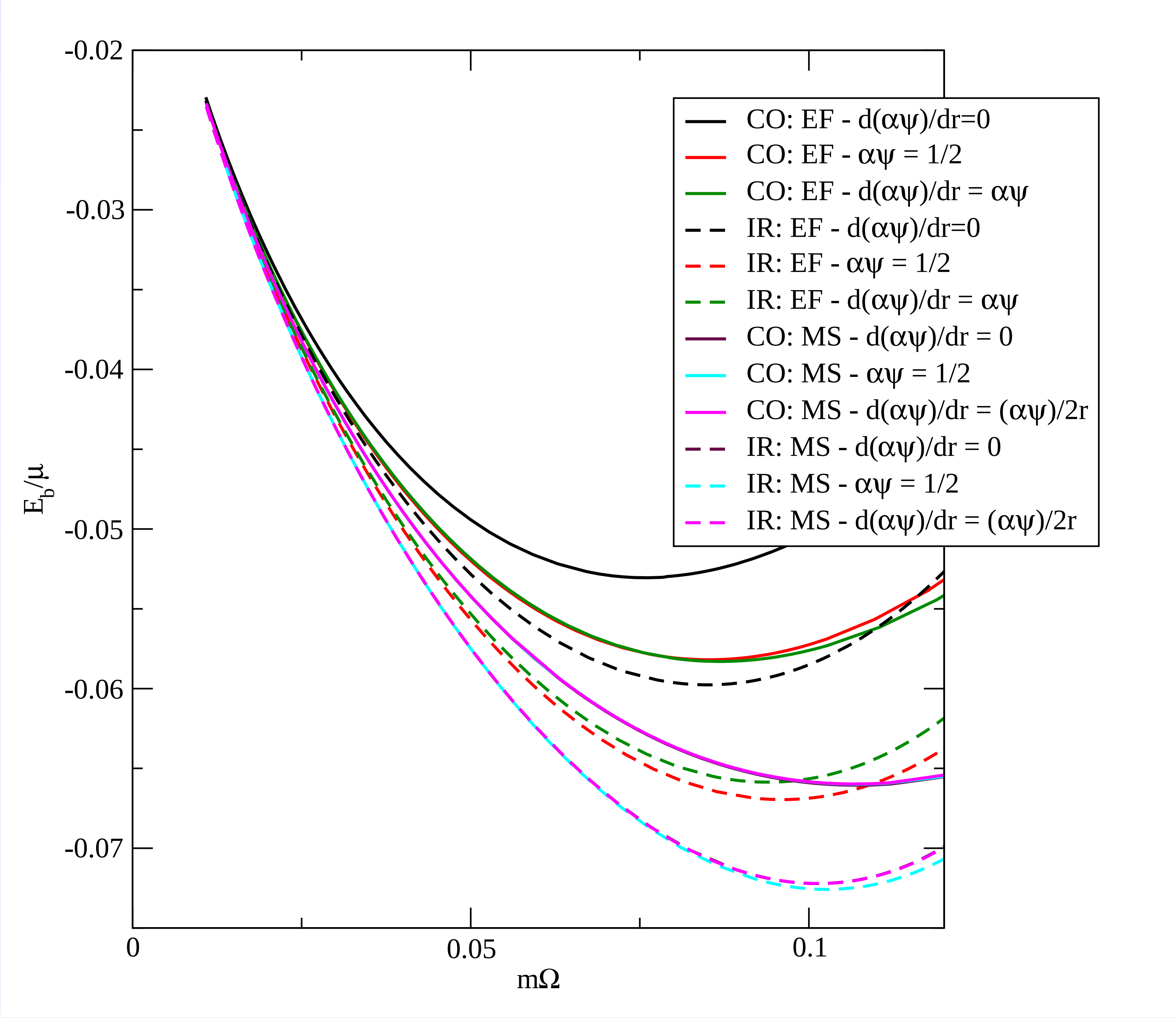
Results: E_b/μ vs $J/\mu m$



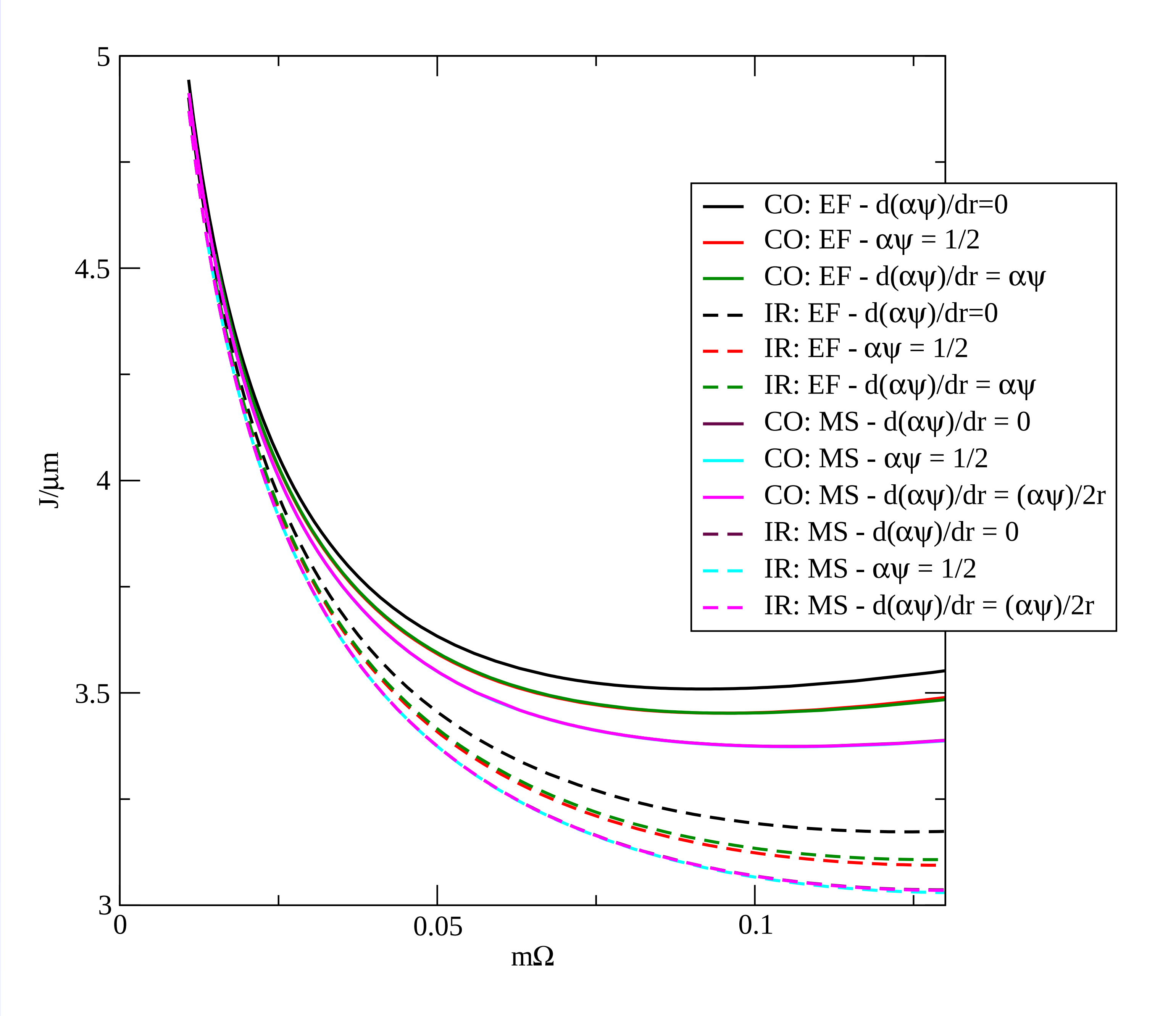
Results: E_b/μ vs ℓ/m



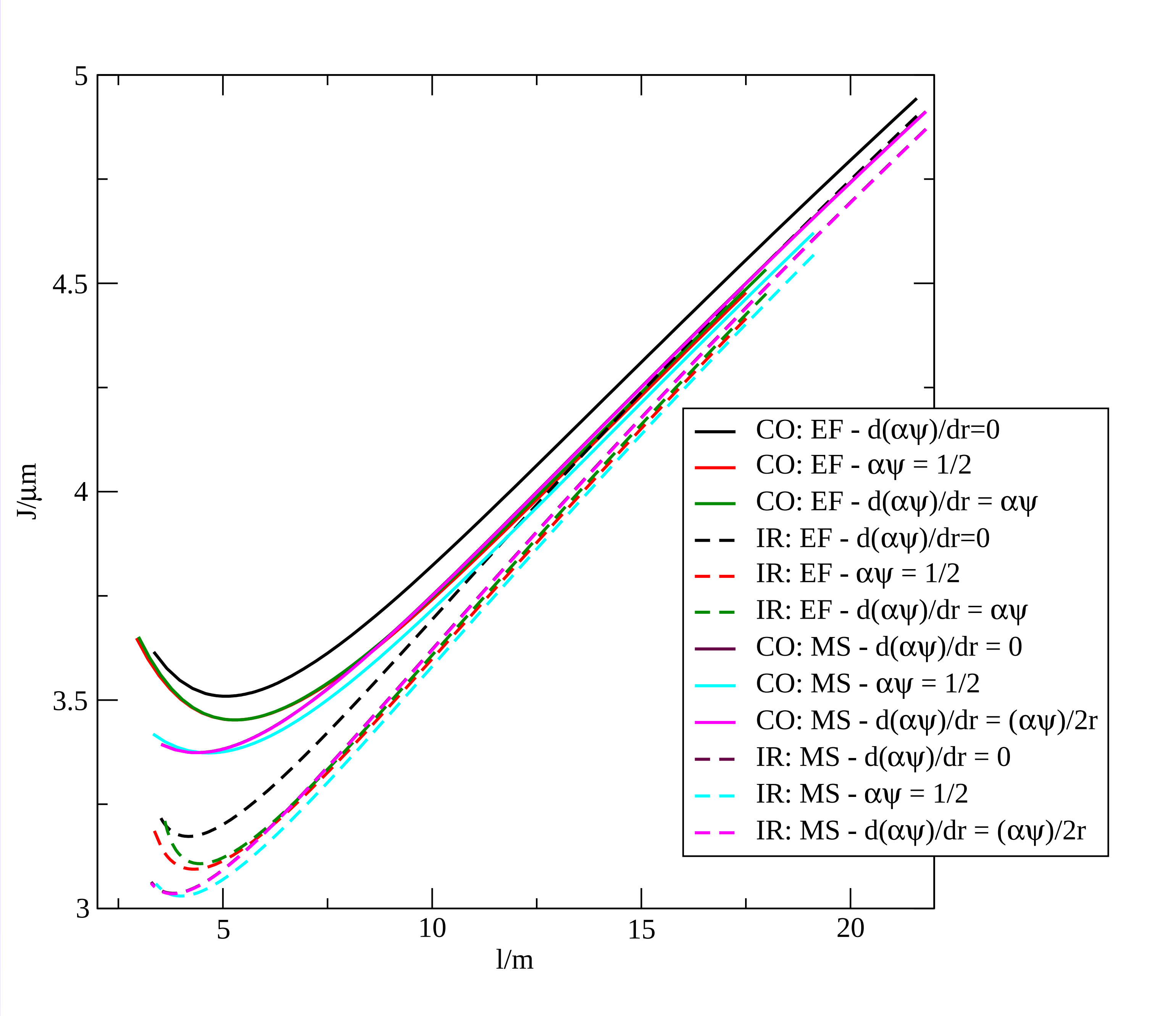
Results: E_b/μ vs $m\Omega$



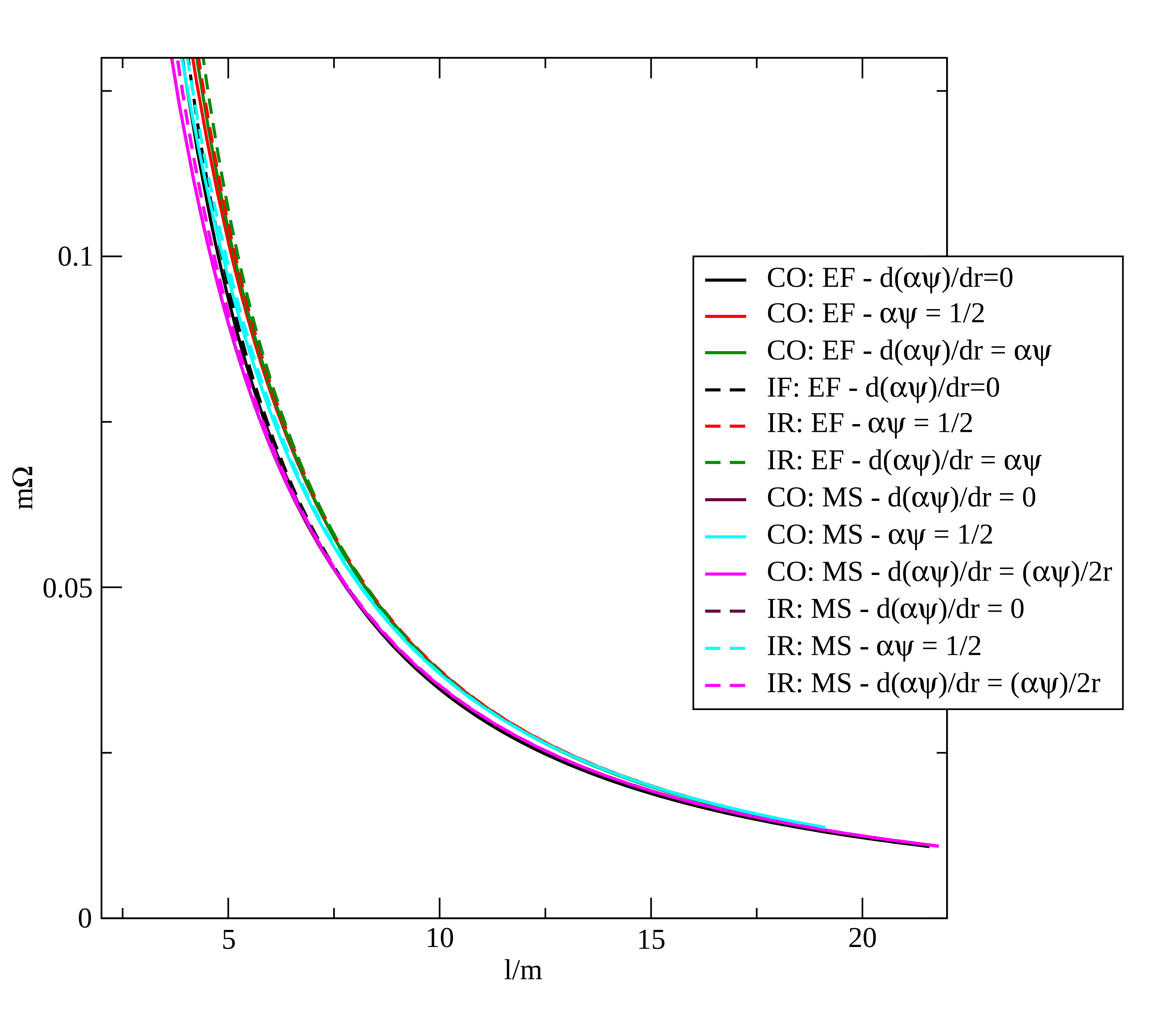
Results: $J/\mu m$ vs $m\Omega$



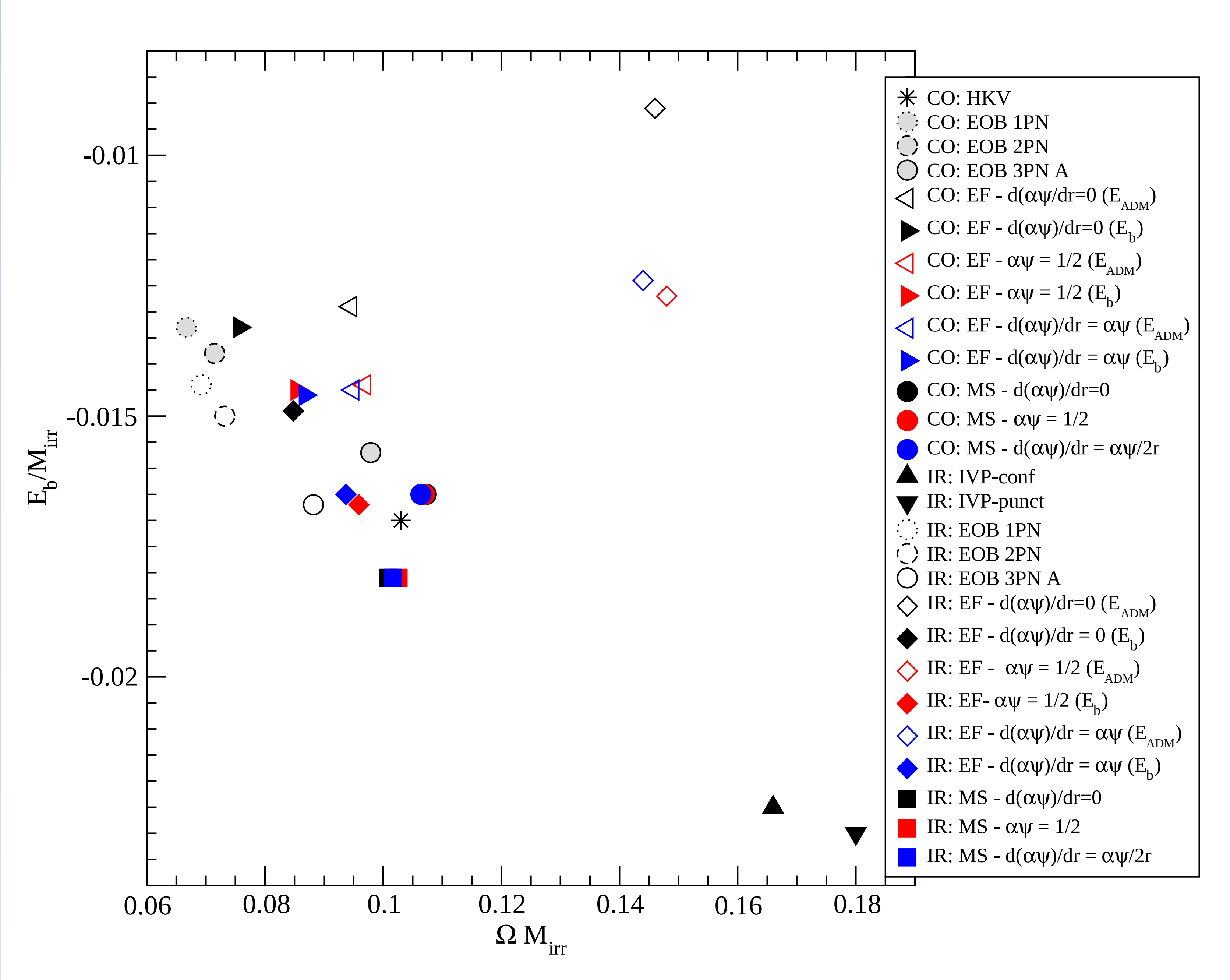
Results: $J/\mu m$ vs ℓ/m



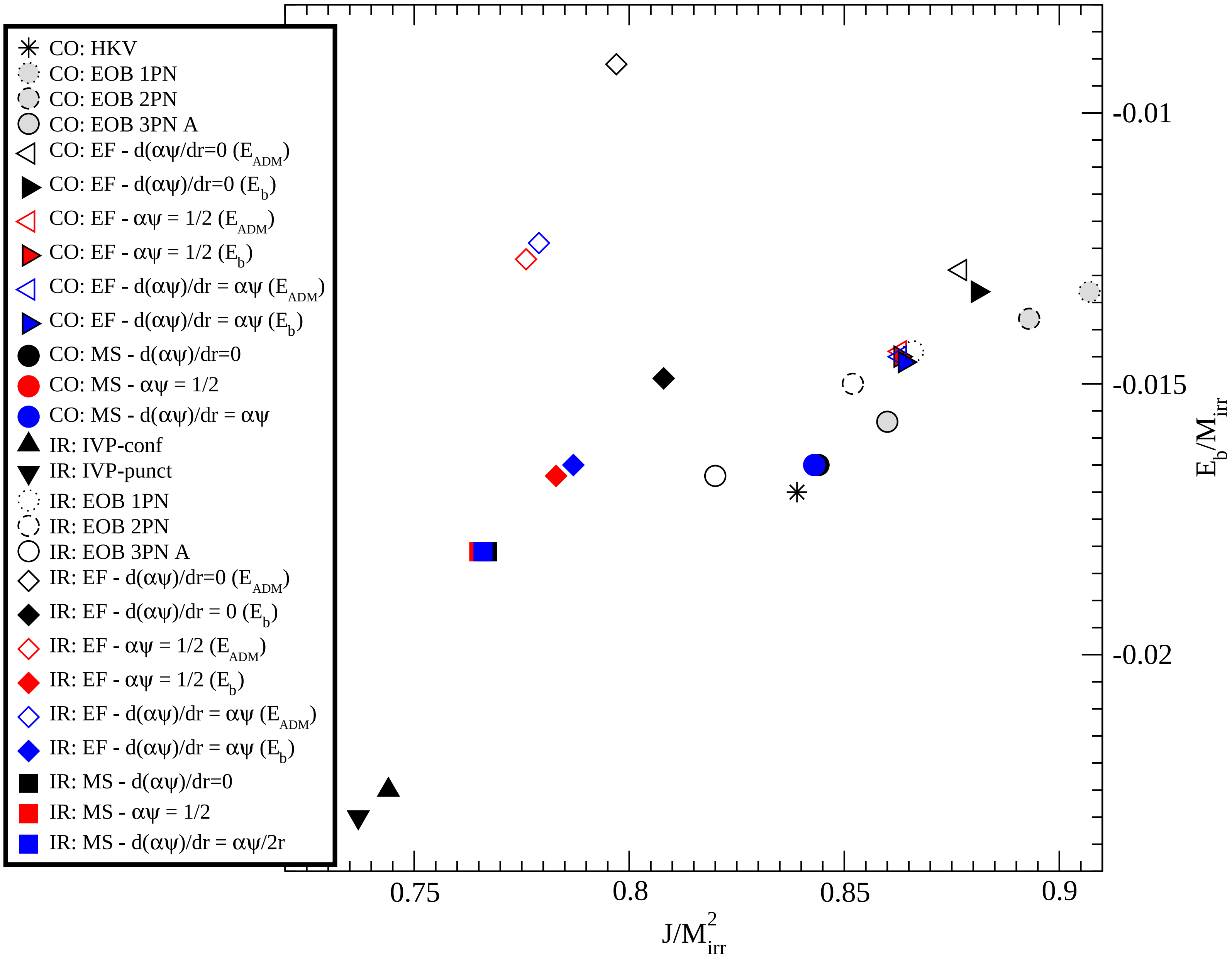
Results: $m\Omega$ vs ℓ/m



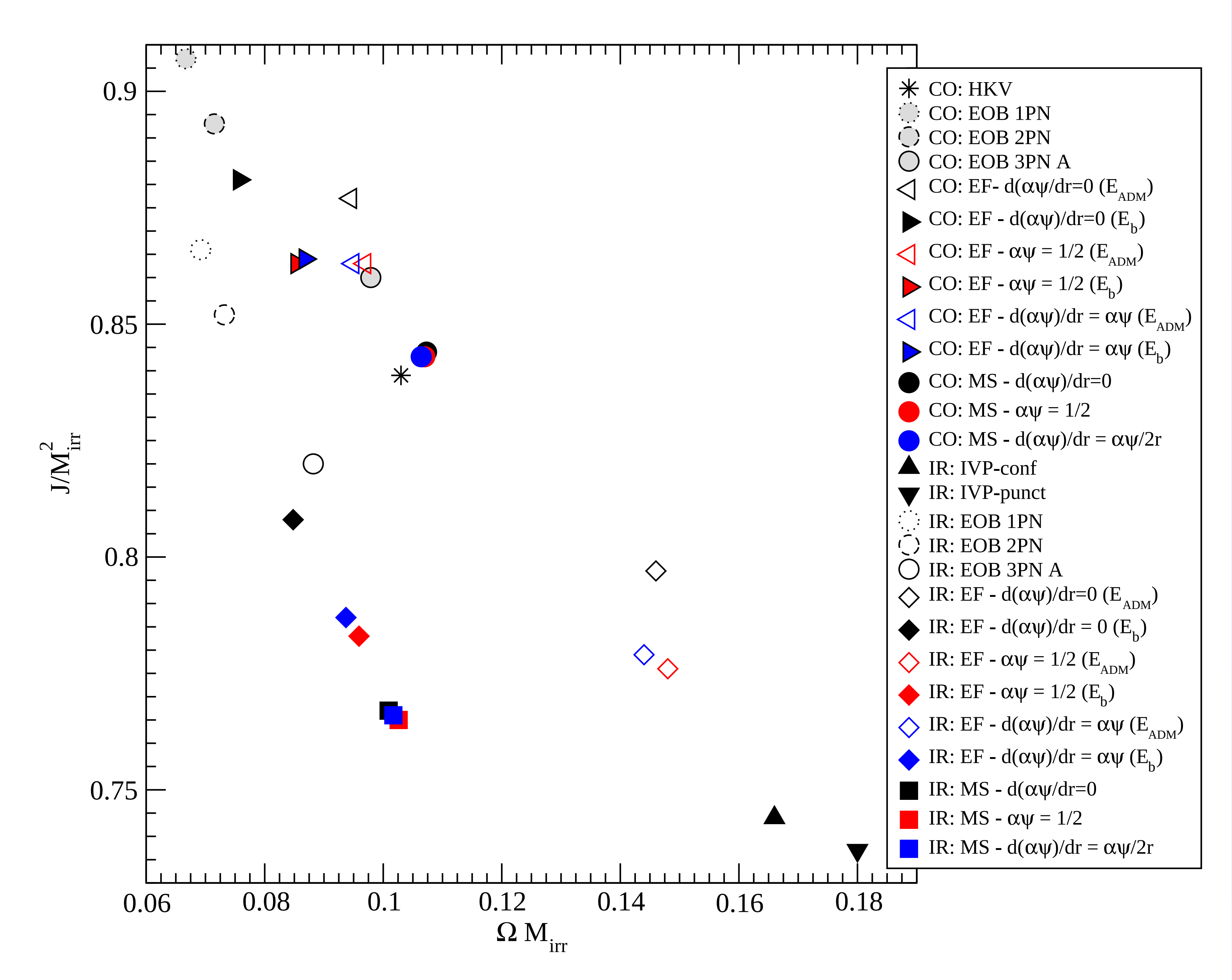
Results: ISCO — E_b/M_{irr} vs ΩM_{irr}



Results: ISCO — E_b/M_{irr} vs J/M_{irr}^2



Results: ISCO — $J/M_{\rm irr}^2$ vs $\Omega M_{\rm irr}$



Future Issues

- How do we choose $\tilde{\gamma}_{ij}$?[55, 44, 54]
  - Can we use post-Newtonian metrics to improve the choice of $\tilde{\gamma}_{ij}$?
  - Can we use a perturbative evolution to iteratively improve the choice of $\tilde{\gamma}_{ij}$?

Future Issues

- How do we choose $\tilde{\gamma}_{ij}$?[55, 44, 54]
  - Can we use post-Newtonian metrics to improve the choice of $\tilde{\gamma}_{ij}$?
  - Can we use a perturbative evolution to iteratively improve the choice of $\tilde{\gamma}_{ij}$?
- How do we incorporate non-vanishing radial motion for close binaries?
- What can we do for elliptical orbits?
- . . . ?

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