ENERGETICS OF SENSORY TRANSDUCTION

Vision:
500-nm photon
$E \approx 240 \text{ kJ} \cdot \text{mol}^{-1}$
$\approx 400 \text{ zJ} \text{ per molecule}$
($\approx 100 \cdot kT$)

Hearing:
$\pm 0.3$-nm vibration
$\Delta G \approx 0.3 \text{ kJ} \cdot \text{mol}^{-1}$ per cycle
$\approx 0.5 \text{ zJ} \text{ per molecule per cycle}$
($\approx 0.1 \cdot kT$ per cycle)
Acoustic stimulation

Vibration measurement by laser interferometer
THE ACTIVE PROCESS OF HAIR CELLS

Amplification

Frequency tuning

Compressive nonlinearity

Spontaneous otoacoustic emission

Stimulus frequency (logarithmic scale)

Basilar-membrane response (logarithmic scale)

Stimulus frequency (logarithmic scale)

Basilar-membrane response (logarithmic scale)

Sound power density (logarithmic scale)

350 X

100 X

10 X
MANIFESTATIONS OF THE EAR’S ACTIVE PROCESS
BY INDIVIDUAL HAIR BUNDLES
F = K_{SF}(X_B - X)
At any time,

\[ F_{\text{EXTERNAL}} + F_{\text{INERTIAL}} + F_{\text{DRAG}} + F_{\text{ELASTIC}} + F_{\text{ACTIVE}} = 0 \]

\[ F_{\text{ACTIVE}} = - K_{\text{SF}}(X_B - X) - (m_{\text{HB}} + m_{\text{SF}}) \cdot \frac{d^2X}{dt^2} - (\xi_{\text{HB}} + \xi_{\text{SF}}) \cdot \frac{dX}{dt} - K_{\text{HB}}X \]

Over an average cycle,

\[ \overline{W}_{\text{EXTERNAL}} + \overline{W}_{\text{INERTIAL}} + \overline{W}_{\text{DRAG}} + \overline{W}_{\text{ELASTIC}} + \overline{W}_{\text{ACTIVE}} = 0 \]

\[ \overline{W}_{\text{ACTIVE}} = - K_{\text{SF}} \int (X_B - X) \cdot dX - (\xi_{\text{HB}} + \xi_{\text{SF}}) \int \frac{dX}{dt} \cdot dX \]
Fiber-base motion

Fiber-tip motion

Drag force

Drag power

Work done on bundle by drag (-39 zJ)
Fiber-base motion

Fiber-tip motion

Fiber force

Fiber power

Work done by fiber on bundle: -40 zJ

Time (ms)
Work done on bundle by drag (-39 zJ)

Work done on bundle by fiber (-40 zJ)

$\bar{W}_{ACTIVE} = -\bar{W}_{DRAG} - \bar{W}_{FIBER}$

$\bar{W}_{ACTIVE} = +79 \text{ zJ}$
Outset of experiment

Work done on bundle by drag

\[ \vec{W}_{ACTIVE} = - \vec{W}_{DRAG} - \vec{W}_{FIBER} = +48 \text{ zJ} \]

10 nm

After hair-bundle fatigue

Work done on bundle by fiber

\[ \vec{W}_{ACTIVE} = - \vec{W}_{DRAG} - \vec{W}_{FIBER} = -1 \text{ zJ} \]

1 pN
DYNAMICAL DESCRIPTION OF HAIR–BUNDLE MOTION AND ADAPTATION

\[ \dot{x} = \left( \frac{1}{\xi_{HB} + \xi_{EXT}} \right) \left[ A(x-a) - B(x-a)^3 - (\kappa_{SP} + \kappa_{EXT}) x + F_{EXT} \right] \]

Hair-bundle velocity

\[ \dot{a} = \frac{1}{\tau} (C x - a) \]

Adaptation rate

Adaptation time constant
Equation for a generic Hopf bifurcation with stimulation by a force $F \text{e}^{i\omega t}$:

$$\frac{dz}{dt} = (\mu + i\omega_0)z - |z|^2z + F \text{e}^{i\omega t}$$

- $z$, displacement (complex variable: $z = x + iy$)
- $\omega_0$, natural (characteristic) frequency
- $\mu$, control parameter

During stimulation near bifurcation ($\mu \approx 0$) and near resonance ($\omega \approx \omega_0$),
response $R \approx F^{1/3}$, amplification with compressive nonlinearity
sensitivity $S = \frac{R}{F} \approx F^{-2/3}$

In the absence of stimulation,
- $\mu < 0$,
  $z = 0$, a quiescent system
- $\mu > 0$,
  $z = \sqrt{|\mu|} \text{e}^{i\omega_0 t} = \sqrt{|\mu|} [\cos(\omega_0 t) + i \cdot \sin(\omega_0 t)]$, a stable limit cycle
ADVANTAGES OF OPERATION NEAR A HOPF BIFURCATION

Principal virtues
- amplification: 100-fold to 1000-fold gain in humans
- frequency tuning: 0.2% frequency discrimination in humans
- compressive nonlinearity: 1,000,000-fold amplitude range in humans

Simplicity
- two-dimensional: minimal requirement for resonant tuning and oscillation
- codimension-1: adjustment of only one parameter required to traverse bifurcation
- local: bifurcation about a fixed point

Entrainment
- easy phase-locking to periodic stimuli

Frequency constancy
- frequency of resonant responsiveness near that of spontaneous oscillation

Epiphenomena
- combination tones: generation of responses at nonharmonic frequencies, e.g., $2f_1 - f_2$
- spontaneous otoacoustic emissions (SOAEs): unprovoked production of one or more pure tones

Defining characteristics of the cochlear active process
THE MECHANICAL LOADS OF HAIR BUNDLES

Cupular organs
lateral-line organs and
semicircular canals

Otolith organs
utricles and saccules

Tectorial organs
cochleas and basilar
papillae

Zebrafish’s lateral-line organ
Bullfrog’s sacculus
Mouse’s cochlea
DYNAMICAL DESCRIPTION OF HAIR–BUNDLE MOTION AND ADAPTATION

Hair-bundle velocity

\[ \dot{x} = \left( \frac{1}{\xi_{HB} + \xi_{EXT}} \right) \left[ A(x - a) - B(x - a)^3 - (\kappa_{SP} + \kappa_{EXT})x + F_{EXT} \right] \]

Adaptation rate

\[ \dot{a} = \frac{1}{\tau} (Cx - a) \]
EXPERIMENTAL STATE DIAGRAM

Amplitude (nm)

Constant force (pN)

Total elastic load (\(\mu\text{N} \cdot \text{m}^{-1}\))

Frequency (Hz)

Constant force (pN)

Total elastic load (\(\mu\text{N} \cdot \text{m}^{-1}\))
HOMEOSTATIC REGULATION OF HAIR – BUNDLE SENSITIVITY

Homeostasis off

Homeostasis on

Force

Stiffness

Force

Stiffness
ADVANTAGES OF HAIR CELLS FOR BIOPHYSICAL INVESTIGATION

Stereotyped cells with high reproducibility

Experimental accessibility

Ability to measure and control conjugate variables
displacement and force of hair bundle
voltage and current across cellular membrane

Few relevant degrees of freedom
hair-bundle motion along axis of symmetry
state of adaptation

Evolutionary pressures on system
threshold near level of thermal noise
energetic efficiency