
How spin, charge and superconducting orders intertwine in the cuprates

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E. Berg, E. Fradkin, E.-A. Kim, S. Kivelson, V. Oganesyan, J. M. Tranquada, and S.-C. Zhang, *Phys. Rev. Lett.* **99**, 127003 (2007).

E. Berg, E. Fradkin and S. Kivelson, *The striped superconductor*, *Phys. Rev. B* **79**, 064515 (2009); arXiv:0810.1564.

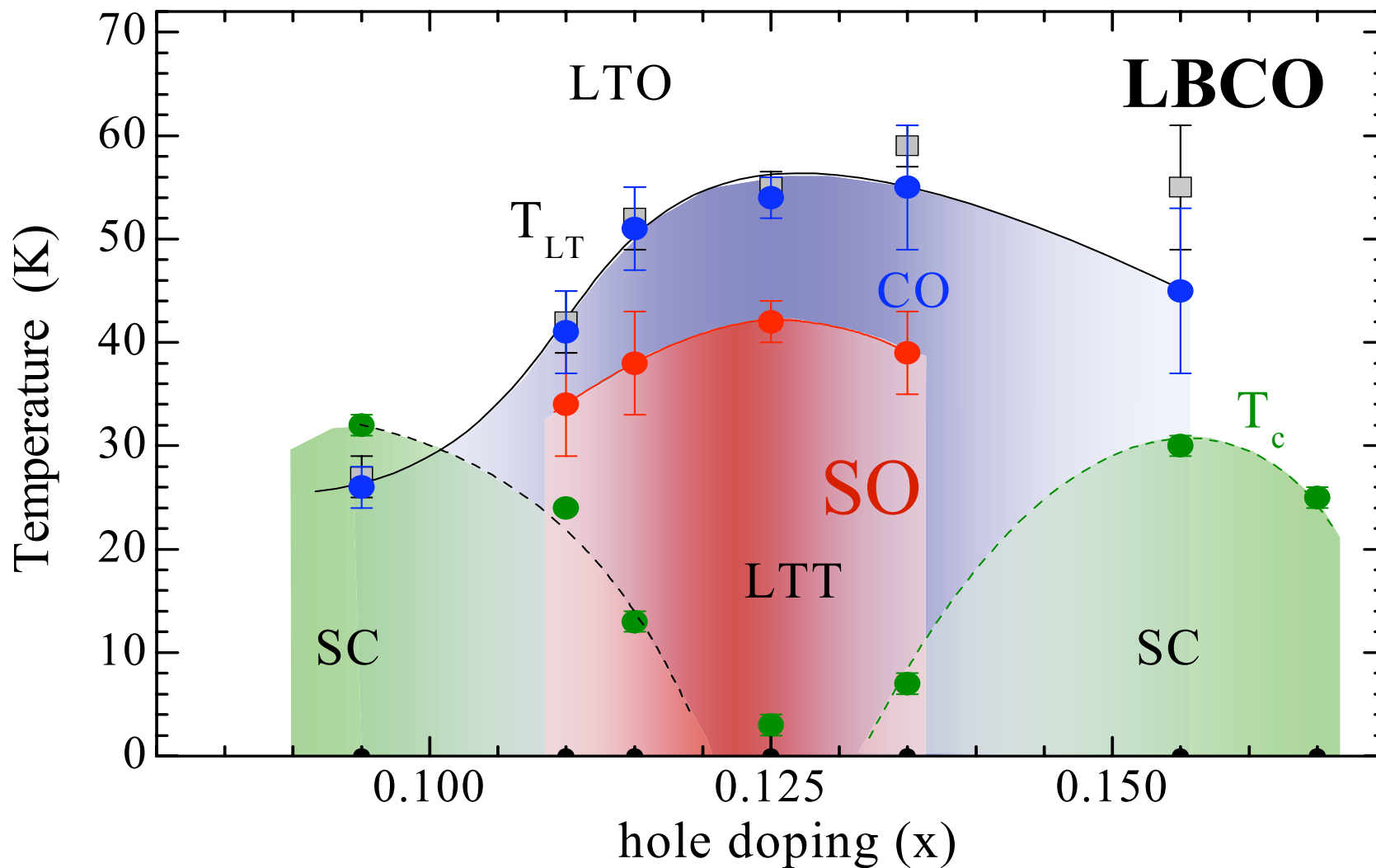
E. Berg, E. Fradkin, S. Kivelson, and J. Tranquada, *Striped Superconductors: How the cuprates intertwine spin, charge, and superconducting orders*, to appear in *New J. Phys.*, arXiv:0901.4826 (2009).

E. Berg, E. Fradkin, and S. Kivelson, *Charge $4e$ superconductivity from pair density wave order in certain high temperature superconductors*, to appear in *Nature Physics*; arXiv:0904.1230 (2009).

Outlook

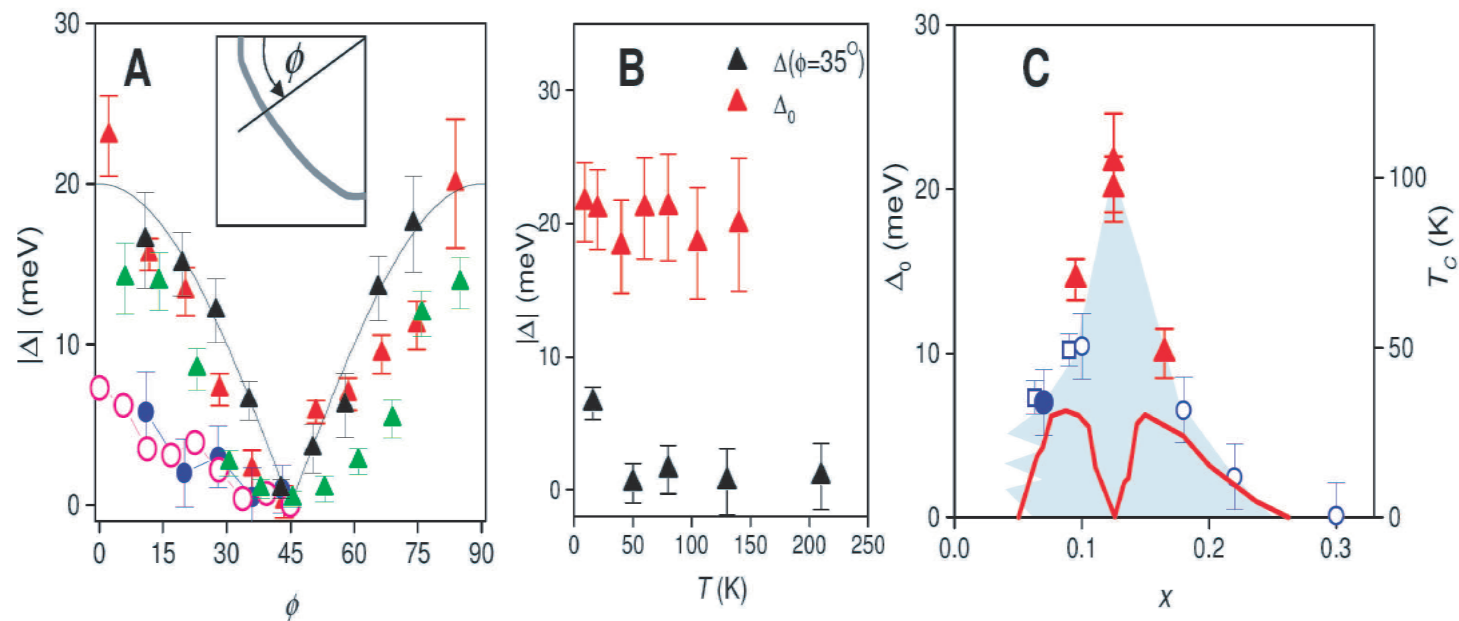
- LBCO is known to exhibit low energy stripe fluctuations in its superconducting state
- It has a very low T_c near $x=1/8$ where it shows static stripe order in the LTT crystal structure
- Experimental evidence for superconducting layer decoupling in LBCO at $x=1/8$
- Layer decoupling, long range charge and spin stripe order and superconductivity: a novel striped superconducting state, a Pair Density Wave, in which charge, spin, and superconducting orders are intertwined!

Phase Diagram of LBCO



M. Hücker et al (2009)

ARPES and the 1/8 anomaly: Optimal Inhomogeneity in LBCO

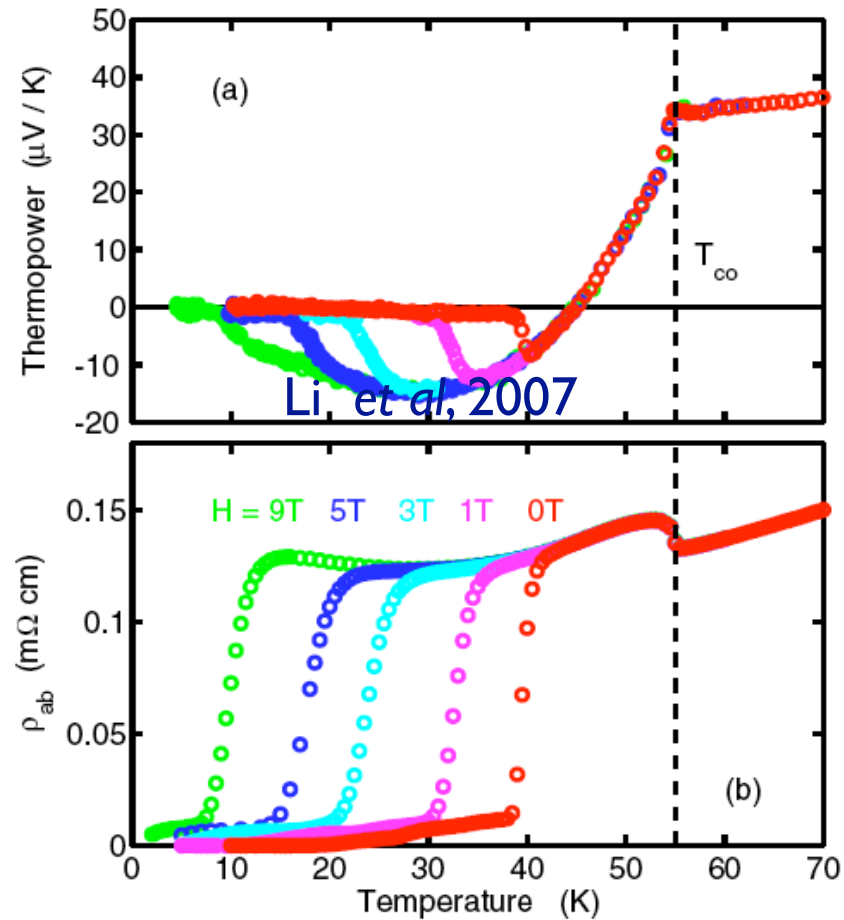


T.Valla *et al* (BNL) (2005), He *et al* (Stanford)(2008)

Li *et al* (2007): Dynamical Layer Decoupling in LBCO

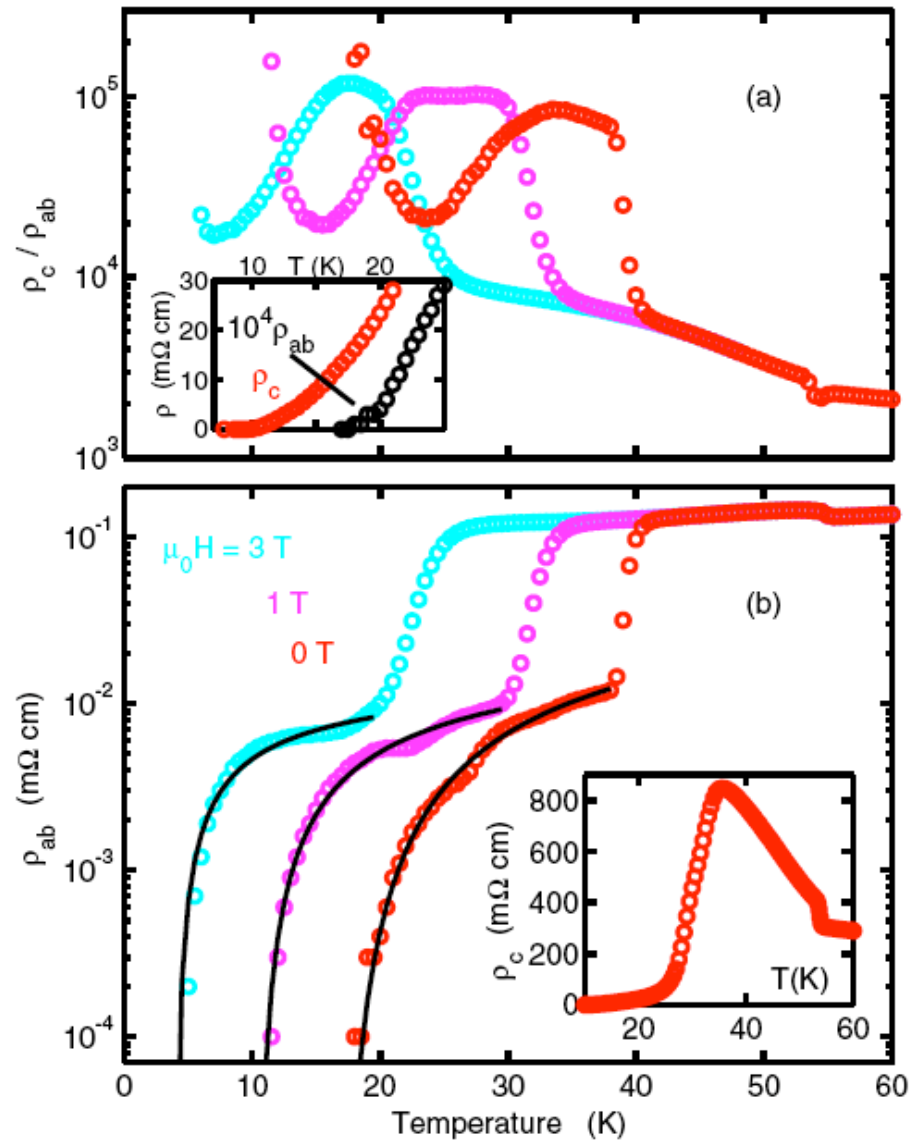
- ARPES: anti-nodal d-wave SC gap is large and unsuppressed at 1/8
- Static charge stripe order for $T < T_{\text{charge}} = 54 \text{ K}$
- Static Stripe Spin order $T < T_{\text{spin}} = 42 \text{ K}$
- ρ_{ab} drops rapidly to zero from T_{spin} to T_{KT}
- ρ_{ab} shows KT behavior for $T_{\text{spin}} > T > T_{\text{KT}}$
- $\rho_{\text{c}} \uparrow$ as $T \downarrow$ for $T > T^{**} \approx 35 \text{ K}$
- $\rho_{\text{c}} \rightarrow 0$ as $T \rightarrow T_{3\text{D}} = 10 \text{ K}$
- $\rho_{\text{c}} / \rho_{\text{ab}} \rightarrow \infty$ for $T_{\text{KT}} > T > T_{3\text{D}}$
- Meissner state only below $T_{\text{c}} = 4 \text{ K}$

Transport Below The Charge Order Transition



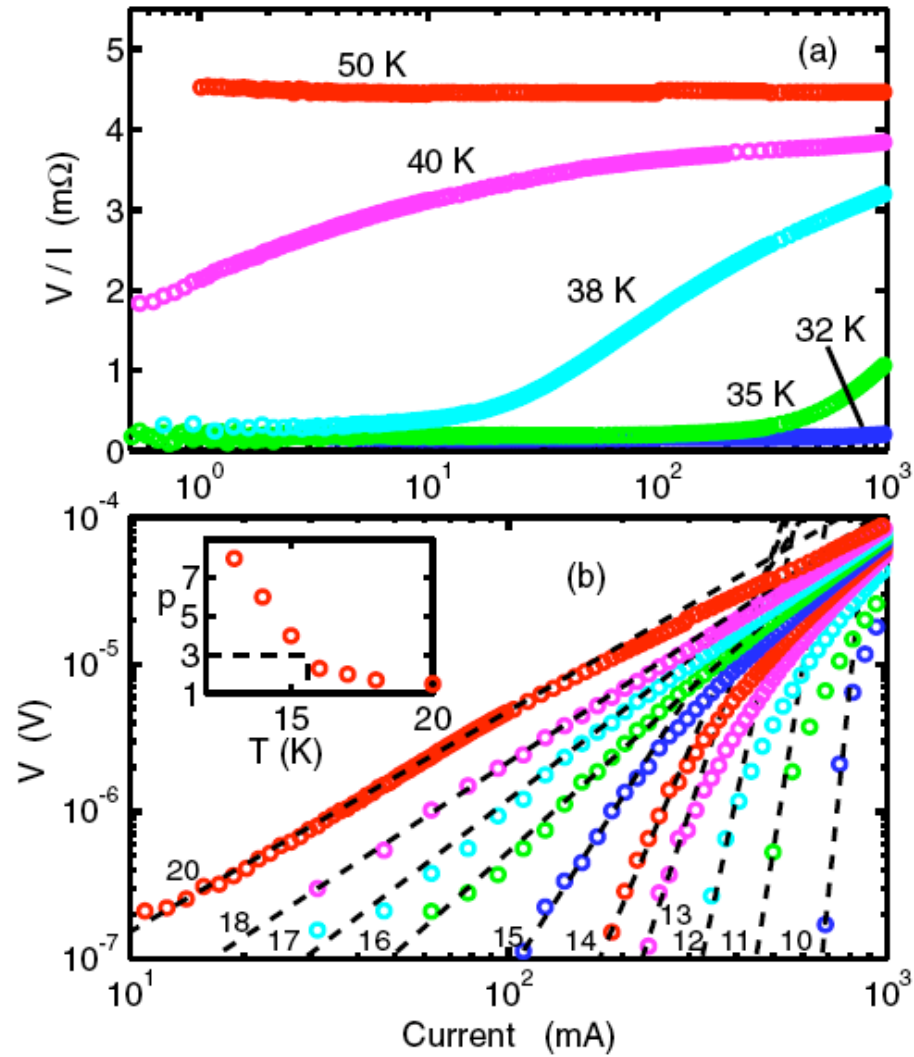
Li et al, 2007

Anisotropic Transport



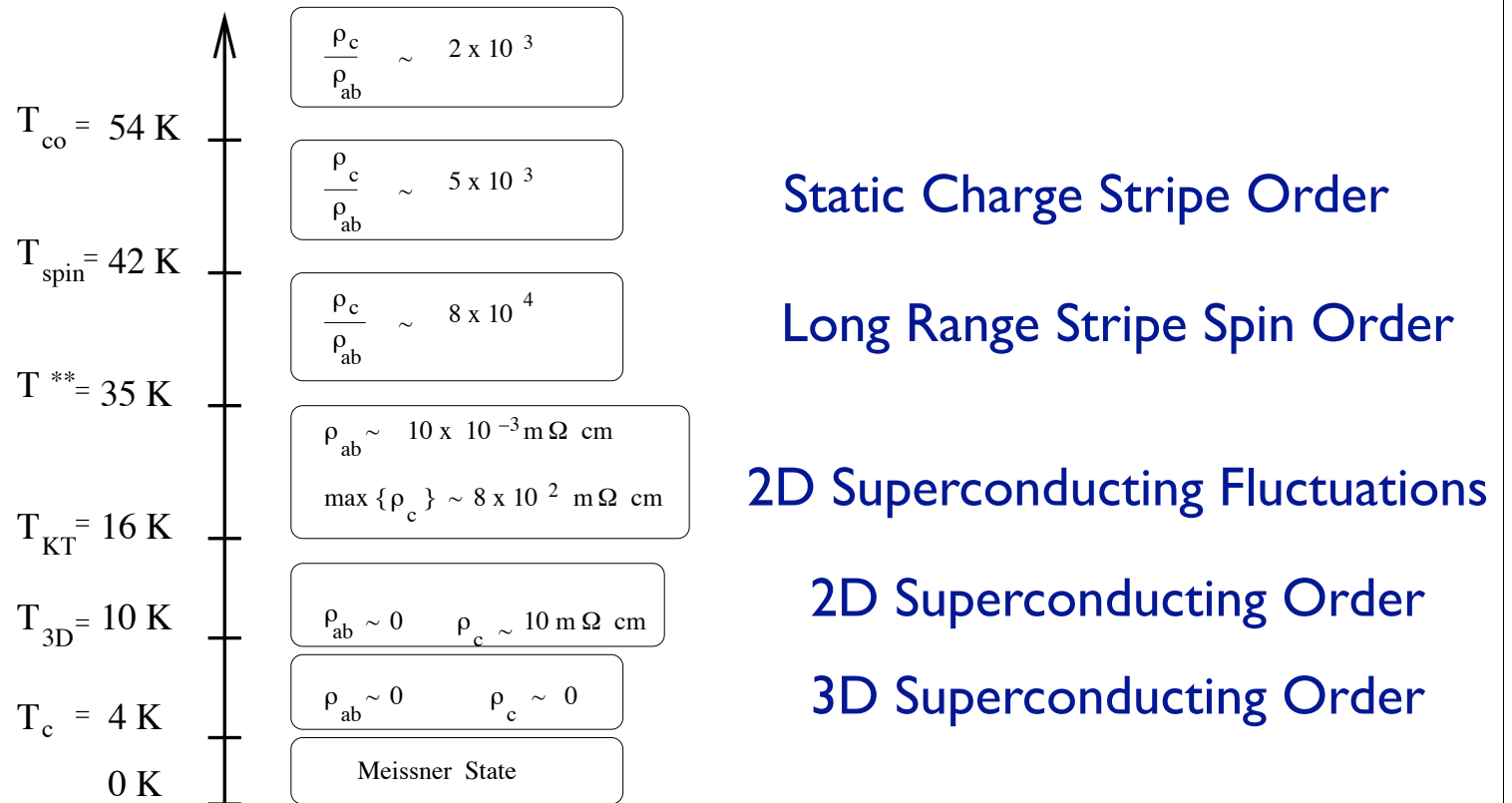
Li *et al*, 2007

The 2D Resistive State and 2D Superconductivity



Li *et al*, 2007

A hierarchy of ordering scales in $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ at $x=1/8$

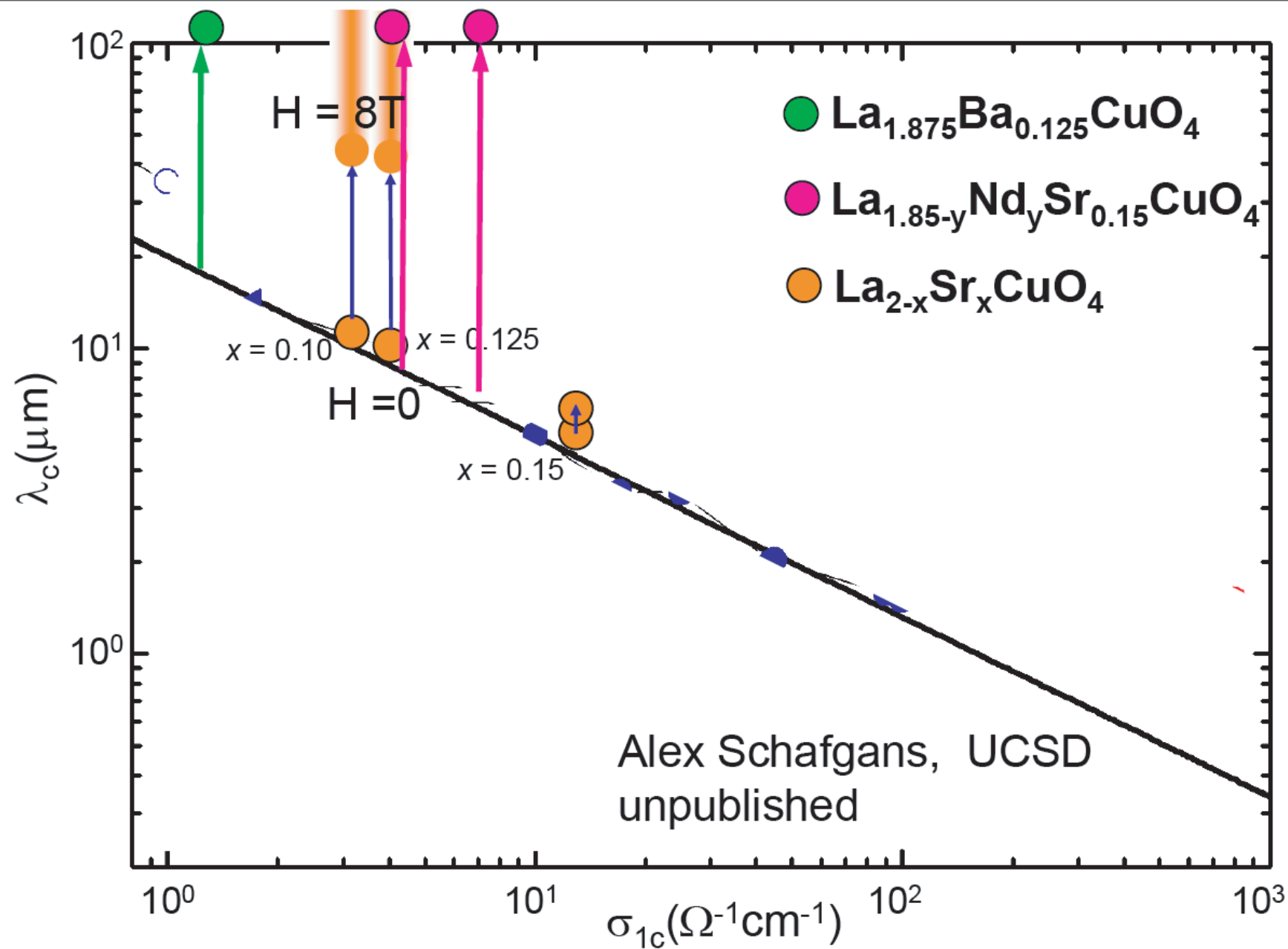


Li, Hücker, Gu, Tsvetlik and Tranquada, PRL 99, 067001 (2007)

Connection with “Fluctuating” Stripe Phases in LSCO

- SC phase of LSCO has “fluctuating” stripes: inelastic neutron scattering detects low energy incommensurate spin fluctuations (Tranquada and coworkers, 1998)
- Upon doping with low amounts of Zn ($< 1\%$) the stripe state becomes static and sharp
- In a magnetic field $\sim 8\text{T}$ LSCO has static spin stripe order (B. Lake and coworkers, 2002)
- In the same range of magnetic fields, Josephson resonance experiments (Basov *et al*, 2008) also detect dynamical layer decoupling in LSCO
- LNSCO also displays similar temperature-dependent anisotropic transport (Ding *et al*, 2008)

• Dynamical Layer Decoupling in the Cuprates



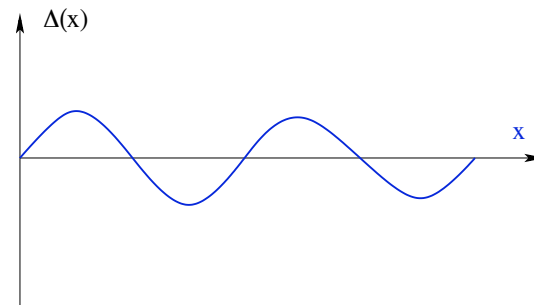
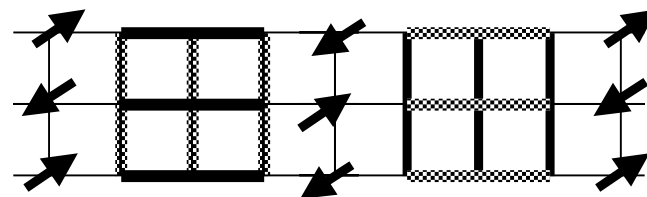
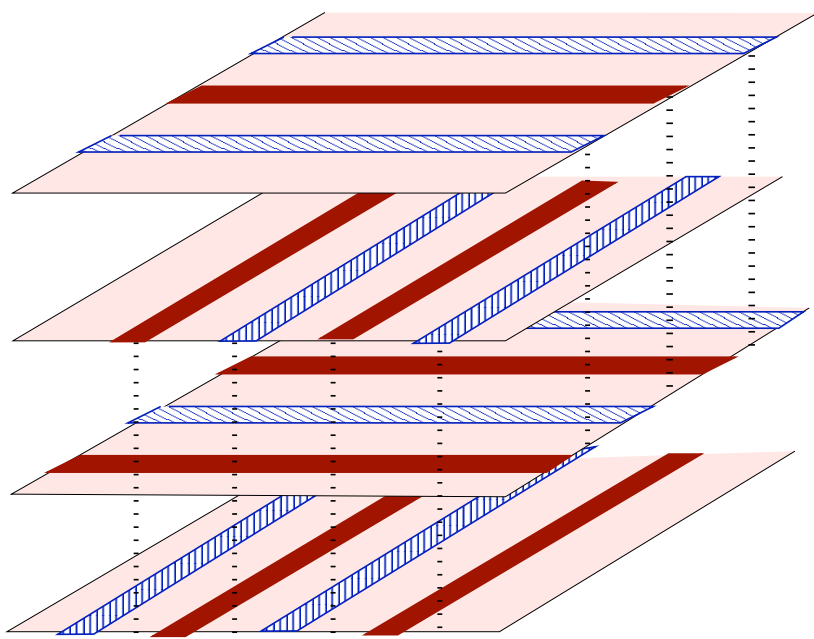
How Do We Understand This Remarkable Effects?

- Broad temperature range, $T_{3D} < T < T_{2D}$ with 2D superconductivity but not in 3D, as if there is not interlayer Josephson coupling
- In this regime there is both striped charge and spin order
- This can only happen if there is a special symmetry of the superconductor in the striped state that leads to an almost complete cancellation of the c-axis Josephson coupling.

A Striped Textured Superconducting Phase

- The stripe state in the LTT crystal structure has two planes in the unit cell.
- Stripes in the 2nd neighbor planes are shifted by half a period to minimize the Coulomb interaction: 4 planes per unit cell
- The AFM spin order suffers a π phase shift across the charge stripe which has period 4
- We propose that the superconducting order is also striped and also suffers a π phase shift.
- The superconductivity resides in the spin gap regions and there is a π phase shift in the SC order across the AFM regions

Period 4 Striped Superconducting State



- This state has intertwined striped charge, spin and superconducting orders.
- A state of this type was found in variational Monte Carlo (Ogata *et al* 2004) and MFT (Poilblanc *et al* 2007)

How does this state solve the puzzle?

- If this order is perfect, the Josephson coupling between neighboring planes cancels exactly due to the symmetry of the periodic array of π textures
- The Josephson couplings J_1 and J_2 between planes two and three layers apart also cancel by symmetry.
- The first non-vanishing coupling J_3 occurs at four spacings. It is quite small and it is responsible for the non-zero but very low T_c
- Defects and/or discommensurations gives rise to small Josephson coupling J_0 neighboring planes

Are there other interactions?

- It is possible to have an inter-plane biquadratic coupling involving the product SC of the order parameters between neighboring planes $\Delta_1 \Delta_2$ and the product of spin stripe order parameters also on neighboring planes $\mathbf{M}_1 \cdot \mathbf{M}_2$
- However in the LTT structure $\mathbf{M}_1 \cdot \mathbf{M}_2=0$ and there is no such coupling
- In a large enough perpendicular magnetic field it is possible (spin flop transition) to induce such a term and hence an effective Josephson coupling.
- Thus in this state there should be a strong suppression of the 3D SC T_c but not of the 2D SC T_c

Away from $x=1/8$

- Away from $x=1/8$ there is no perfect commensuration
- Discommensurations are defects that induce a finite Josephson coupling between neighboring planes $J_1 \sim |x-1/8|^2$, leading to an increase of the 3D SC T_c away from $1/8$
- Similar effects arise from disorder which also lead to a rise in the 3D SC T_c

Landau-Ginzburg Theory of the striped SC: Order Parameters

- Striped SC: $\Delta(\mathbf{r}) = \Delta_{\mathbf{Q}}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \Delta_{-\mathbf{Q}}(\mathbf{r}) e^{-i\mathbf{Q}\cdot\mathbf{r}}$, complex charge $2e$ singlet pair condensate with wave vector, (*i.e.* an FFLO type state at zero magnetic field)
- Nematic: detects breaking of rotational symmetry: N , a real neutral pseudo-scalar order parameter
- Charge stripe: $\rho_{\mathbf{K}}$, unidirectional charge stripe with wave vector \mathbf{K}
- Spin stripe order parameter: $\mathbf{S}_{\mathbf{Q}}$, a neutral complex spin vector order parameter, $\mathbf{K} = 2\mathbf{Q}$

Rotations by $\pi/2$

- The nematic order parameter changes sign: $N \rightarrow -N$
- The CDW ordering wave vector rotates: $\rho_{\mathbf{k}} \rightarrow \rho_{\mathbf{k}'}$
- The SDW ordering wave vector rotates: $\mathbf{S}_{\mathbf{Q}} \rightarrow \mathbf{S}_{\mathbf{Q}'}$
- The striped SC (s or d wave): $\Delta_{\pm\mathbf{Q}} \rightarrow \pm \Delta_{\pm\mathbf{Q}}$

Translations

- Under a translation by \mathbf{r}
 - $N \rightarrow N$, $\rho_{\mathbf{k}} \rightarrow e^{i\mathbf{k}\cdot\mathbf{r}} \rho_{\mathbf{k}}$, $\mathbf{S}_{\mathbf{Q}} \rightarrow e^{i\mathbf{Q}\cdot\mathbf{r}} \mathbf{S}_{\mathbf{Q}}$

Ginzburg-Landau Free Energy Functional

- $F = F_2 + F_3 + F_4 + \dots$
- The quadratic and quartic terms are standard
- $F_3 = \gamma_s \rho_{\mathbf{k}}^* \mathbf{S}_{\mathbf{Q}} \cdot \mathbf{S}_{\mathbf{Q}} + \pi/2 \text{ rotation} + \text{c.c.}$
+ $\gamma_{\Delta} \rho_{\mathbf{k}}^* \Delta_{-\mathbf{Q}}^* \Delta_{\mathbf{Q}} + \pi/2 \text{ rotation} + \text{c.c.}$
+ $g_{\Delta} N (\Delta_{\mathbf{Q}}^* \Delta_{\mathbf{Q}} + \Delta_{-\mathbf{Q}}^* \Delta_{-\mathbf{Q}} - \pi/2 \text{ rotation}) + \text{c.c.}$
+ $g_s N (\mathbf{S}_{\mathbf{Q}}^* \cdot \mathbf{S}_{\mathbf{Q}} - \pi/2 \text{ rotation})$
+ $g_c N (\rho_{\mathbf{k}}^* \rho_{\mathbf{k}} - \pi/2 \text{ rotation})$

Some Consequences of the GL theory

- The symmetry of the term coupling charge and spin order parameters requires the condition $\mathbf{K} = 2\mathbf{Q}$
- Striped SC order implies charge stripe order with $1/2$ the period, and of nematic order
- Charge stripe order with wave vector $2\mathbf{Q}$ and/or nematic order favors stripe superconducting order which may or may not occur depending on the coefficients in the quadratic part

Charge 4e SC order

- Coupling to a charge 4e SC order parameter Δ_4
- $F'_3 = g_4 [\Delta_4^* (\Delta_{\mathbf{Q}} \Delta_{-\mathbf{Q}} + \text{rotation}) + \text{c.c.}]$
- Striped SC order (PDW) \Rightarrow uniform charge 4e SC order!
- Since the coupling is independent of θ_- , the charge 4e SC order is unaffected by the Bragg glass of the pinned CDW
- The half vortices of θ_+ are the fundamental $hc/4e$ vortices of the charge 4e SC.

Coexisting uniform and striped SC order

- PDW order $\Delta_{\mathbf{Q}}$ and uniform SC order Δ_0
- $F_{3,u} = Y_{\Delta} \Delta_0^* \rho_{\mathbf{Q}} \Delta_{-\mathbf{Q}} + \rho_{-\mathbf{Q}} \Delta_{\mathbf{Q}} + g_{\rho} \rho_{-2\mathbf{Q}} \rho_{\mathbf{Q}}^2 + \text{rotation} + \text{c.c.}$
- If $\Delta_0 \neq 0$ and $\Delta_{\mathbf{Q}} \neq 0 \Rightarrow$ there is a $\rho_{\mathbf{Q}}$ component of the charge order!
- The small uniform component Δ_0 removes the sensitivity to quenched disorder of the PDW state

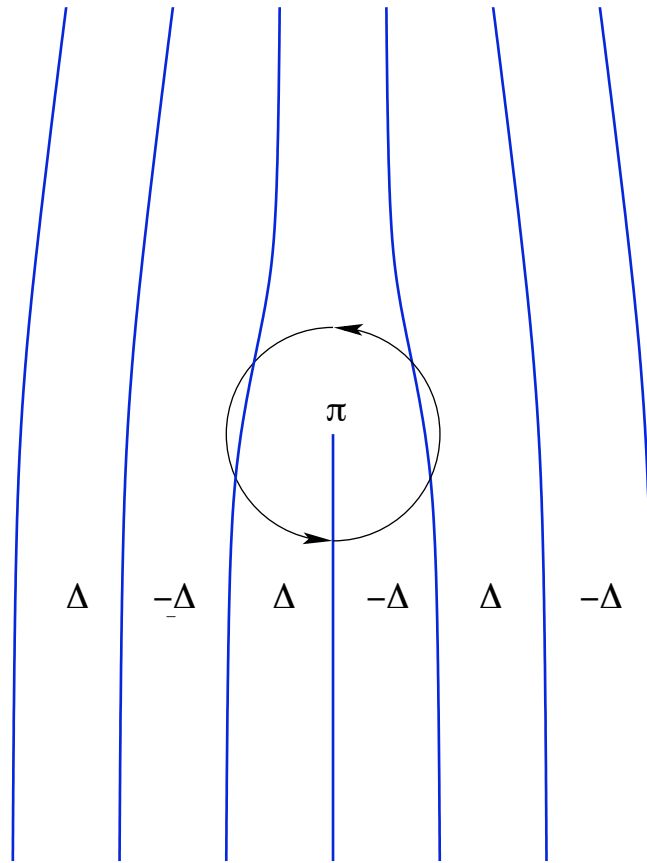
Topological Excitations of the Striped SC

- $\rho(r) = |\rho_{\mathbf{k}}| \cos [K r + \Phi(r)]$
- $\Delta(r) = |\Delta_{\mathbf{Q}}| \exp[i \mathbf{Q} \cdot \mathbf{r} + i \theta_{\mathbf{Q}}(r)] + |\Delta_{-\mathbf{Q}}| \exp[-i \mathbf{Q} \cdot \mathbf{r} + i \theta_{-\mathbf{Q}}(r)]$
- $F_{3,\gamma} = 2\gamma_{\Delta} |\rho_{\mathbf{k}} \Delta_{\mathbf{Q}} \Delta_{-\mathbf{Q}}| \cos[2 \theta_{-}(r) - \Phi(r)]$
- $\theta_{\pm\mathbf{Q}}(r) = [\theta_{+}(r) \pm \theta_{-}(r)]/2$
- $\theta_{\pm\mathbf{Q}}$ single valued mod $2\pi \Rightarrow \theta_{\pm}$ defined mod π
- ϕ and θ_{-} are locked \Rightarrow topological defects of ϕ and θ_{+}

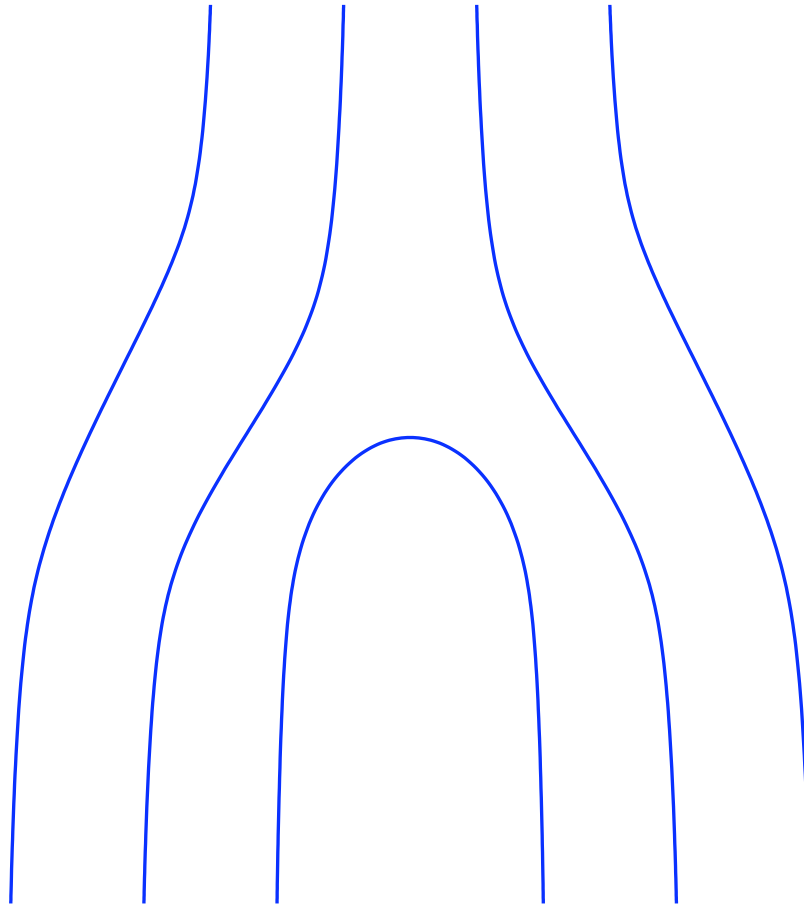
Topological Excitations of the Striped SC

- SC vortex with $\Delta\theta_+ = 2\pi$ and $\Delta\phi=0$
- Bound state of a $1/2$ vortex and a dislocation
 $\Delta\theta_+ = \pi, \Delta\phi= 2\pi$
- Double dislocation, $\Delta\theta_+ = 0, \Delta\phi= 4\pi$
- All three topological defects have logarithmic interactions

Half-vortex and a Dislocation



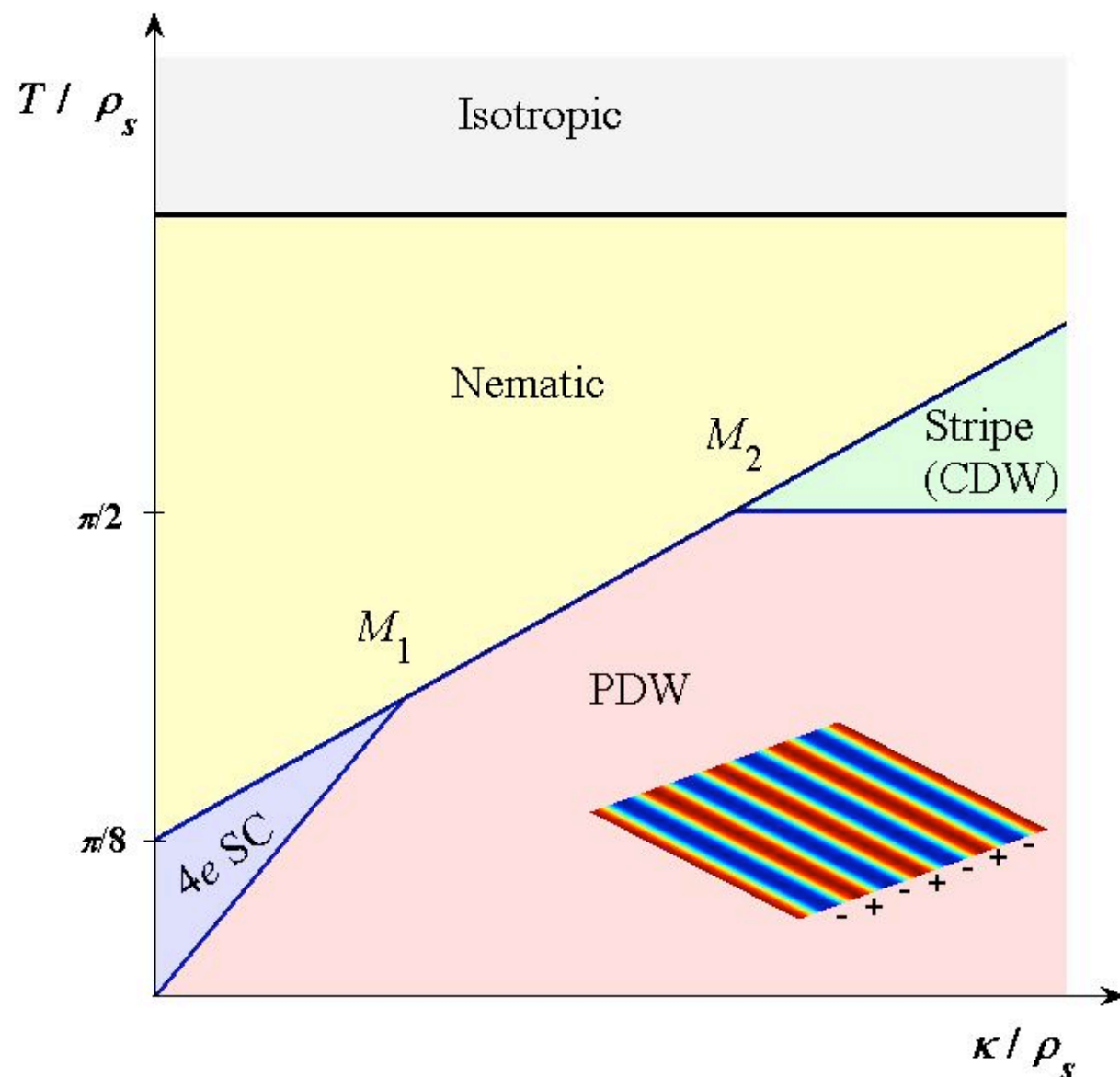
Double Dislocation



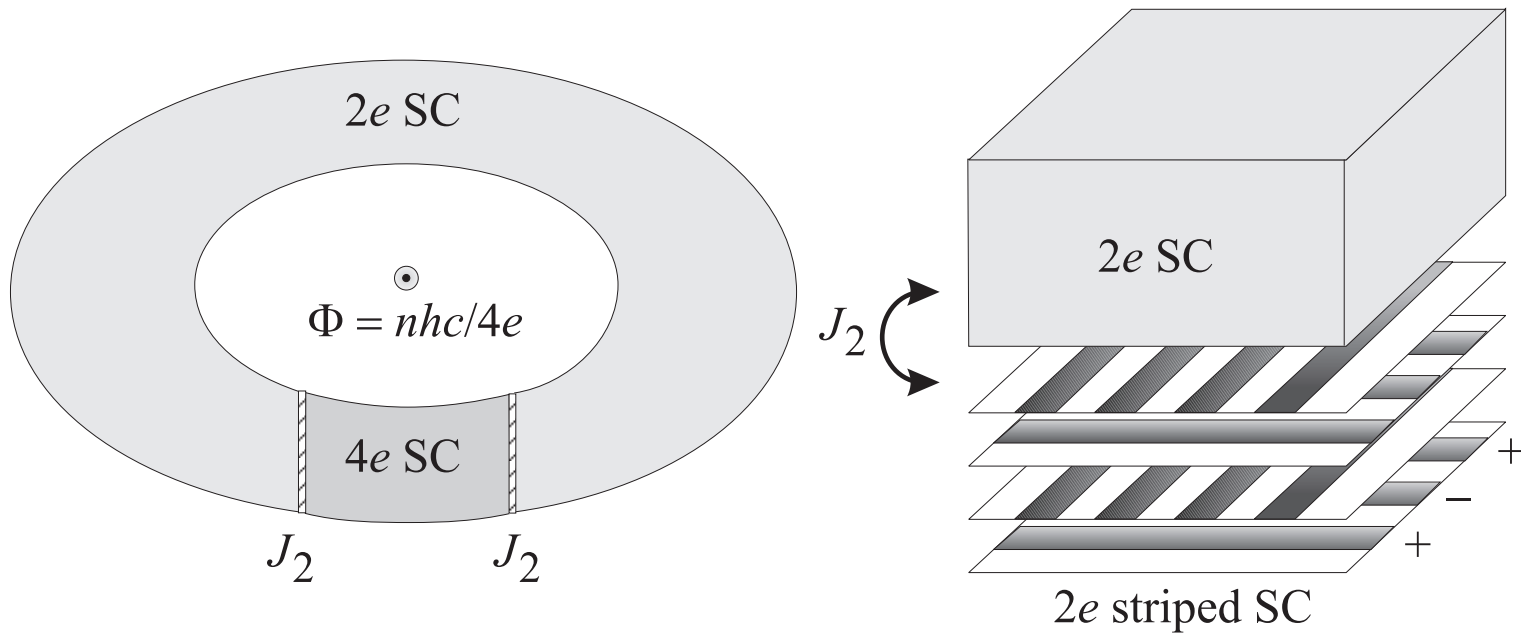
Thermal melting of the PDW state

- Three paths for thermal melting of the PDW state
- Three types of topological excitations: (1,0) (SC vortex), (0,1) (double dislocation), ($\pm 1/2, \pm 1/2$) (1/2 vortex, single dislocation bound pair)
- Scaling dimensions: $\Delta_{p,q} = \pi(\rho_{sc} p^2 + K_{cdw} q^2)/T = 2$ (for marginality)
- Phases: PDW, Charge 4e SC, CDW, and normal (Ising nematic)

Schematic Phase Diagram



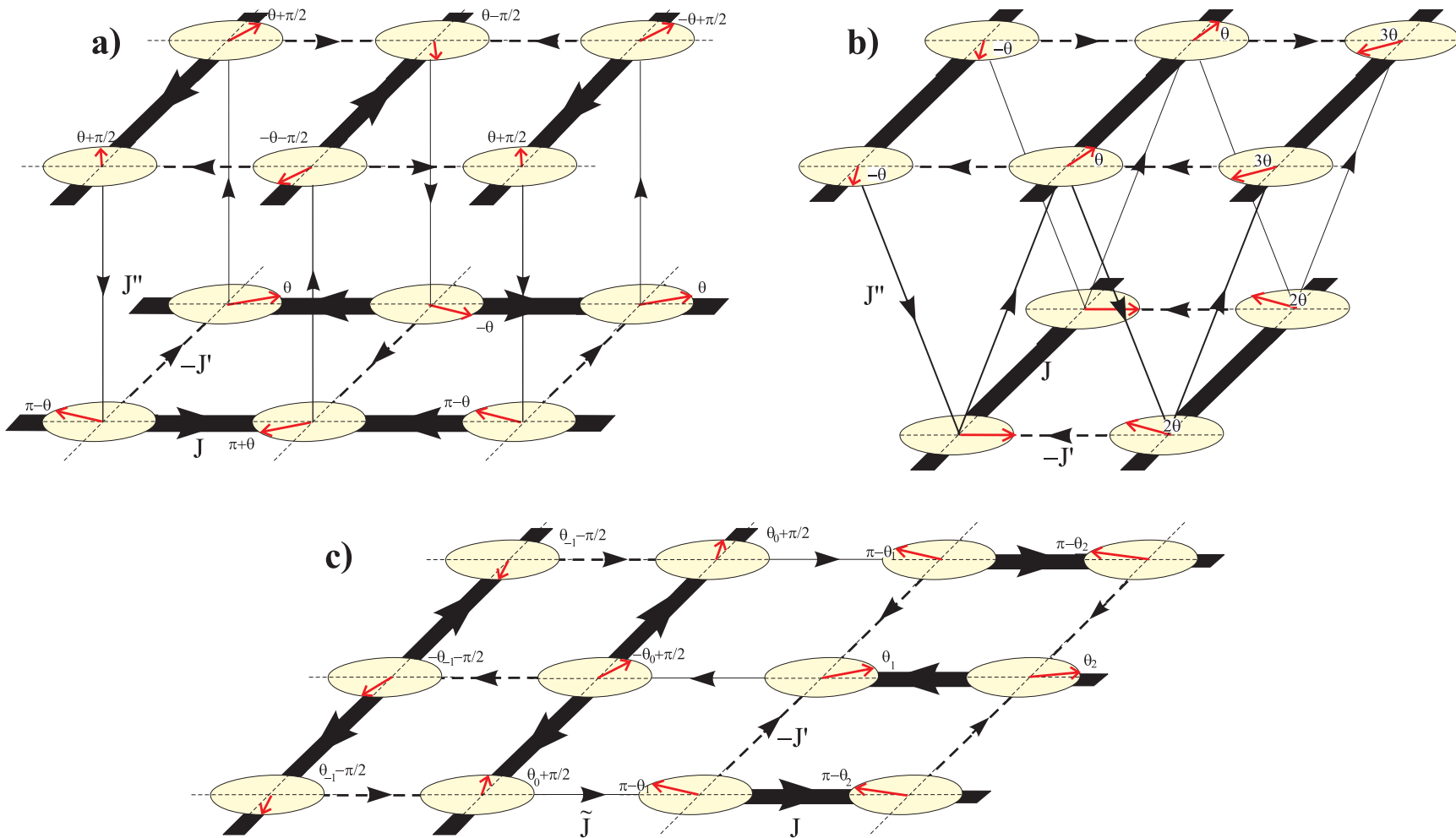
Phase Sensitive Experiments



$$I = J_2 \sin(2\Delta\theta)$$

Non-collinear Order and Time Reversal Symmetry Breaking

- PDW order in the planes leads to frustration of the inter-plane Josephson coupling
- We will regard the SC phase as an XY “pseudo-spin” and we get non-collinear order
- Non-collinear order \Leftrightarrow circulating currents \Rightarrow Time Reversal symmetry breaking effects
- The resulting non-collinear order depends on the lattice, and it is different for the LTT structure of LBCO than for the orthorhombic (chain) structure of YBCO
- Twin boundaries (domain walls) and edges also lead to non-collinear order
- The period of the Josephson coupling between a uniform and a striped SC is π



Non-collinear order and circulating currents for (a) the LTT structure (LBCO), (b) the orthorhombic (chain) structure of YBCO, and (c) an in-plane domain wall ("twin boundary").

Effects of Disorder

- The striped SC order is very sensitive to disorder: disorder \Rightarrow pinned charge density wave \Rightarrow coupling to the phase of the striped SC \Rightarrow SC “gauge” glass with zero resistance but no Meissner effect in 3D
- Disorder induces dislocation defects in the stripe order
- Due to the coupling between stripe order and SC, $\pm\pi$ flux vortices are induced at the dislocation core.
- Strict layer decoupling only allows for a magnetic coupling between randomly distributed $\pm\pi$ flux vortices
- Novel glassy physics and “fractional” flux

Conclusions

- LBCO at $x=1/8$ is a 2D HTSC with striped PDW d-wave superconductivity and very low 3D critical temperature
- It has a high pairing scale and an optimal degree of inhomogeneity
- Charge order is part of the SC order and both orders are *intertwined* rather than competing
- We introduced a striped SC, a Pair Density Wave, whose symmetries can explain the experiments
- Natural charge $4e$ SC order and $hc/4e$ flux quantization

Conclusions, cont'd

- If the striped SC competes with the uniform d-wave SC in general both orders are present
- This leads to a uniform nodal d-wave SC at low energies and a striped SC at high energies, “two gaps”?
- An external magnetic field suppresses the uniform d-wave SC and can allow a striped SC to develop
- How pervasive is PDW order in the cuprates?