

# Interplay between charge, pairing (and orbital current) modulations in some t-J models

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Didier Poilblanc\*



Manuela Capello, Marcin Raczkowski

R. Frésard and A. Oleś

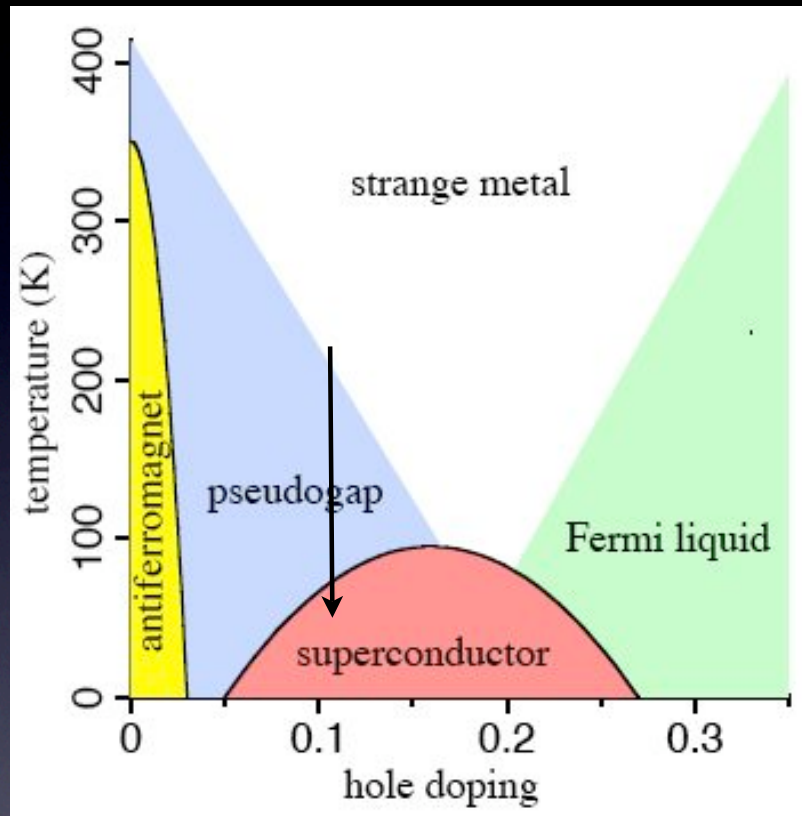
*\*CNRS and Université Paul Sabatier, Toulouse (France)*

# Outline

- The basics of RVB and the pseudo-gap
- Observation of charge order in High-T<sub>c</sub> superconducting materials
- Results on superconducting RVB hole stripes
- Results for a single Zinc impurity
- Open issue: competition with orbital current states



# High-T<sub>c</sub> Phase diagram

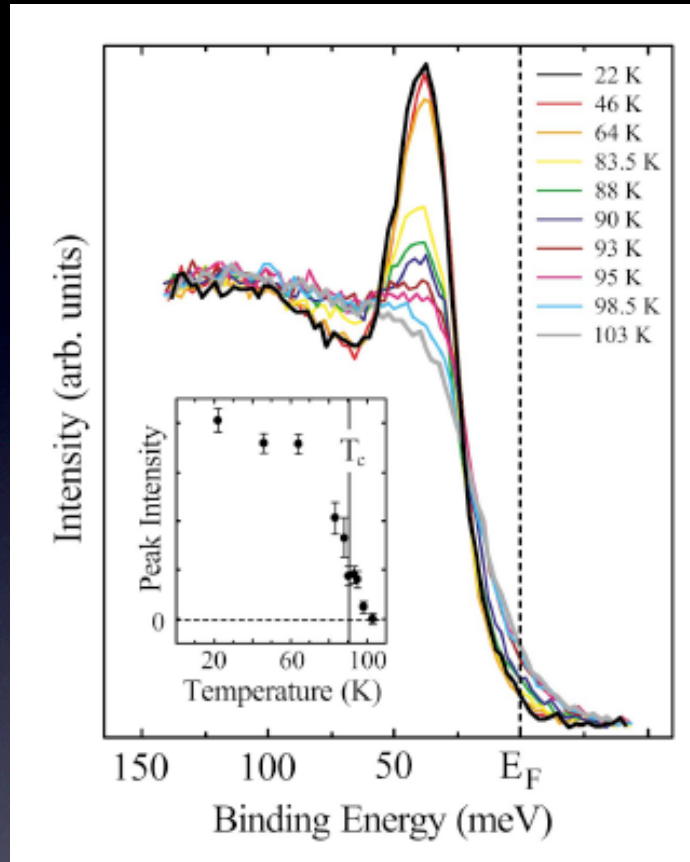


\*RVB is a simple appealing theory for pseudo gap

\* Can we also describe inhomogeneous states with the RVB framework ?

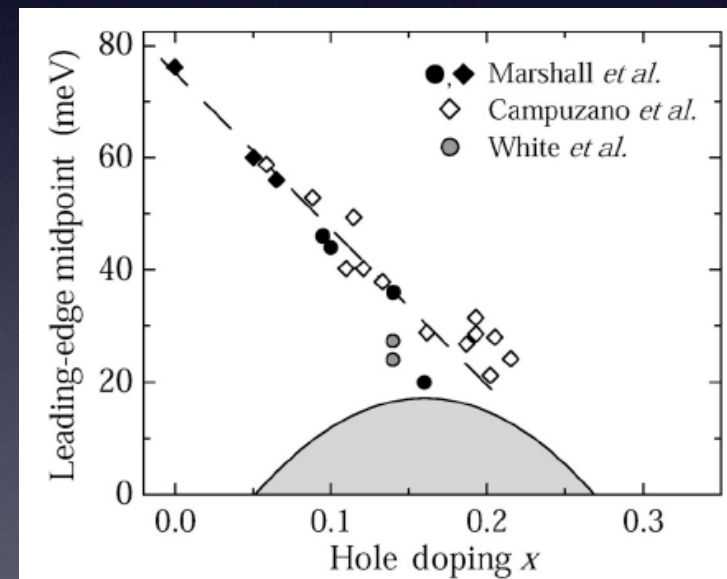
Superconducting state emerges from doping a Mott insulator

# The pseudogap phase



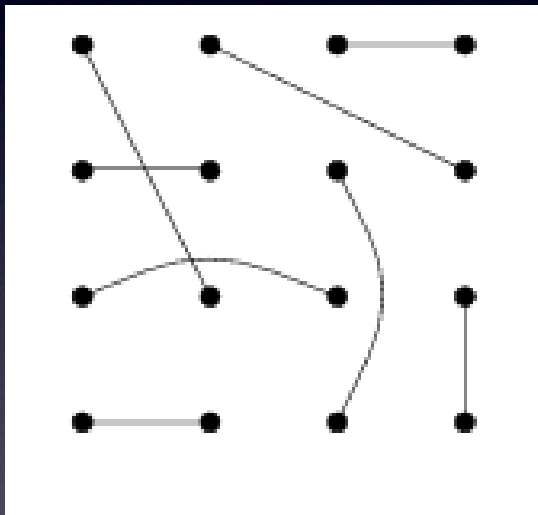
( $\pi, 0$ ) vs.  $T$   
 Fedorov 1999  
 Bi2212  
 $T_c = 91$  K

Existence of a pseudo-gap but  
 no superconductivity  
 and no quasiparticles



# Resonating Valence Bond state

- \* Mott physics: no double occupancies
- \* Antiferromagnetic term important



Non-magnetic ground state:  
good for low spin,  
low dimensionality

$$\bullet - \bullet = \frac{1}{\sqrt{2}} (\uparrow_i \downarrow_j - \downarrow_i \uparrow_j)$$

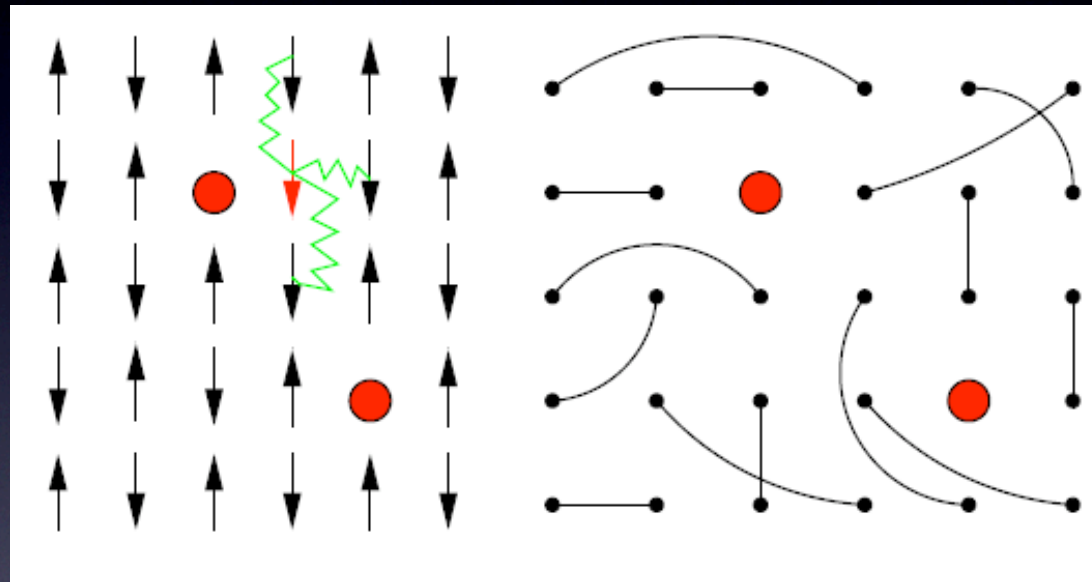
[Anderson, Science 1987]

RVB: liquid of singlets of spins which resonate



# (Simple minded) RVB scenario

Holes frustrate antiferromagnetism

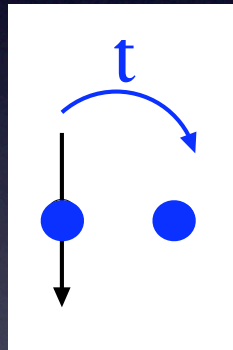


The RVB state regains the lost AF exchange  
by the resonance between many different configurations

The RVB state naturally becomes a superconductor  
since the pairing already exists

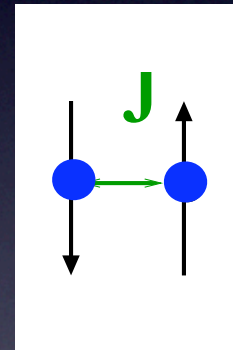
# The t-J model

$$H_{tJ} = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + h.c. + J \sum_{\langle ij \rangle} S_i \cdot S_j$$

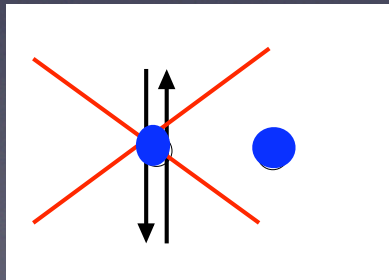


KINETIC  
TERM

+



AF  
EXCHANGE  
TERM



CONSTRAINT of  
NO DOUBLE OCCUPANCIES

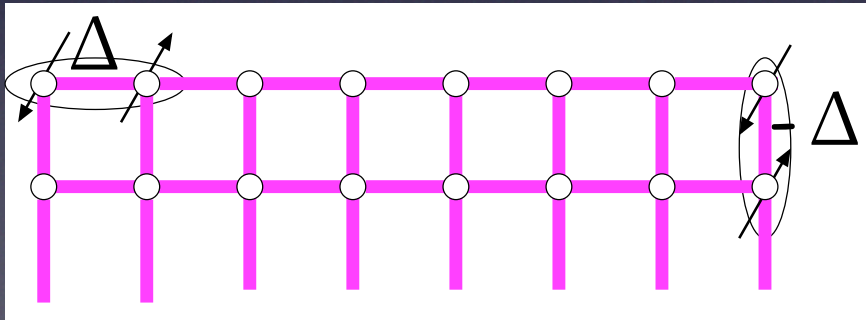
# RVB variational state

$$H_{BCS} = H_{kin} + \sum_{ij} \Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger + \mu \sum_i n_i + h.c.$$

Uncorrelated state  $|D\rangle$

$$|\Psi_{RVB}\rangle = \prod_i (1 - n_{i\uparrow} n_{i\downarrow}) |D\rangle$$

Strongly  
correlated  
wavefunction

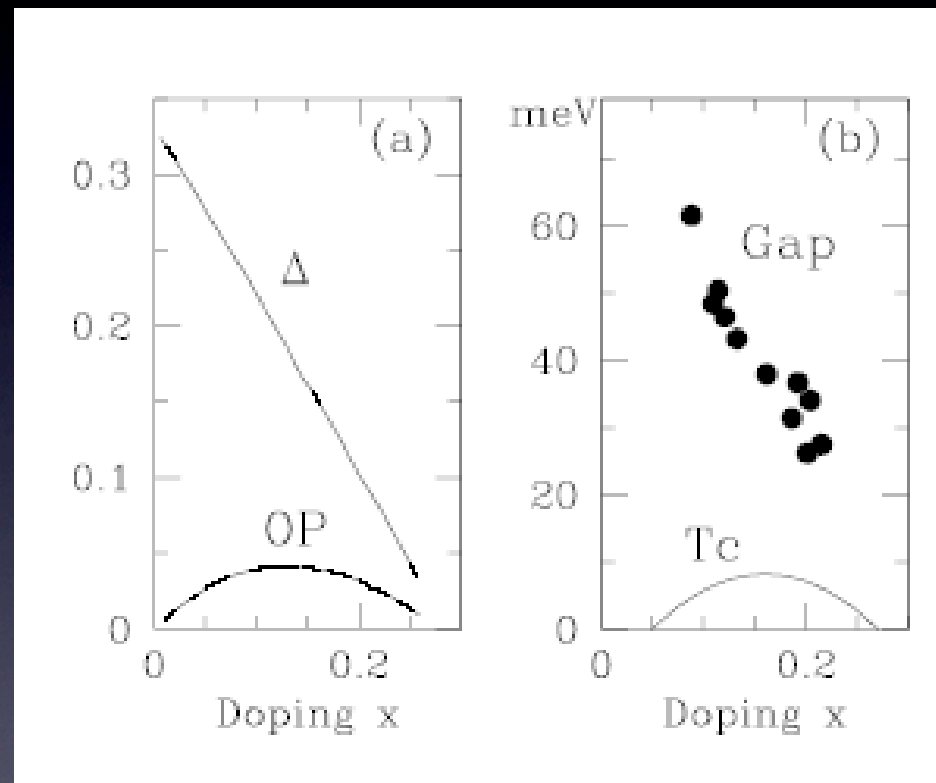


$\Delta, \mu$  are variational  
parameters

All  $\Delta_{ij}$  uniform with d-wave symmetry

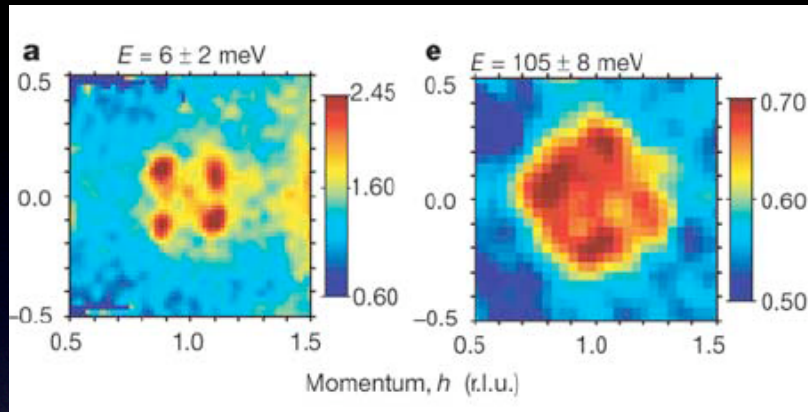


RVB => Correct behavior of  
pseudo-gap & SC order parameter



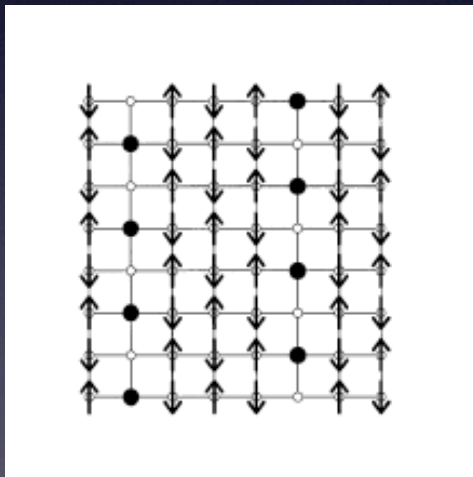
Anderson et al. J.Phys. C 2004

# Neutron scattering: AF Stripes



$\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$   
at doping  $x=1/8$

[Tranquada et al. Nature 1995]

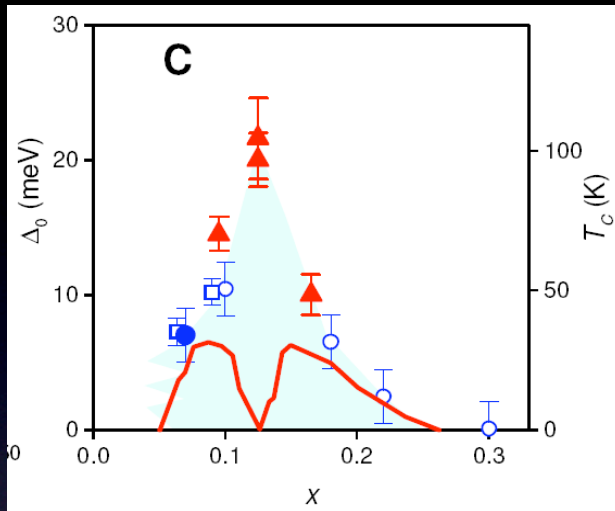


## ANTIFERROMAGNETIC STRIPE SCENARIO

Spatially ordered state with holes concentrated *unidirectionally* between AF domains

Could the SC state become also stripy ?

# Stripes are compatible with pairing !

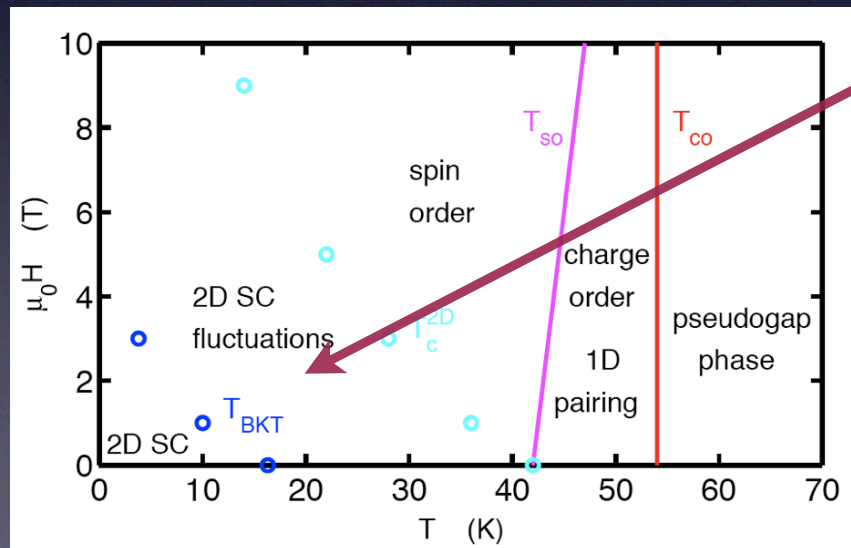


LaBaCuO:  $T_c \sim 0$  at doping  $x = 1/8$   
but (ARPES + STM) d-wave gap  
still there!

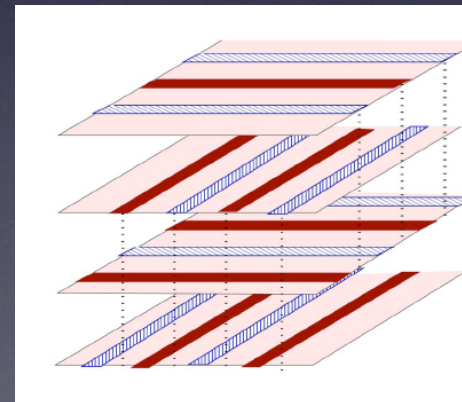
[Valla et al., Science 2006]

Planes are (Josephson) decoupled  
but pairing exists!

[Berg et al., PRL 2007]

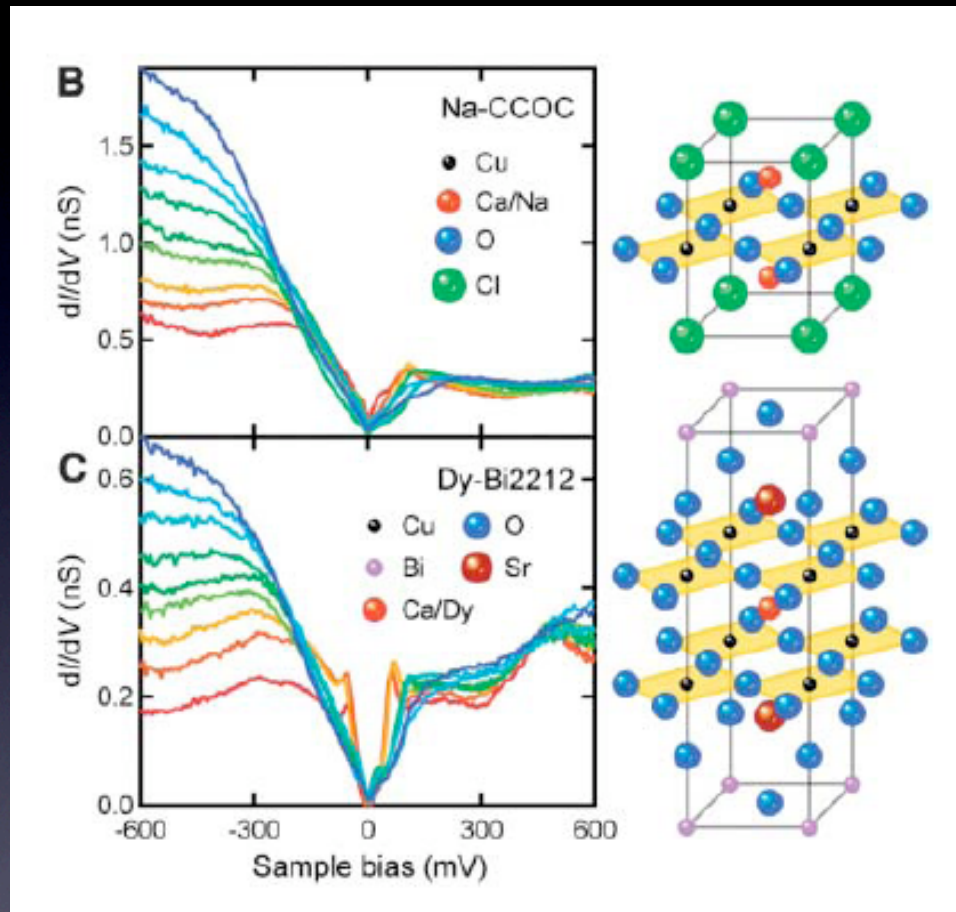


[Li et al., PRL 2007]





# STM Experiments:



DIFFERENTIAL CURRENT

$$dI/dV(r,V) = f(r,z) N(r,E=eV)$$

\* $N(r,E)$  LOCAL DOS

\* $f(r,z)$  tunnelling matrix element  
(unknown)

J.C. Davis' group

EXTRACT  $e^-$

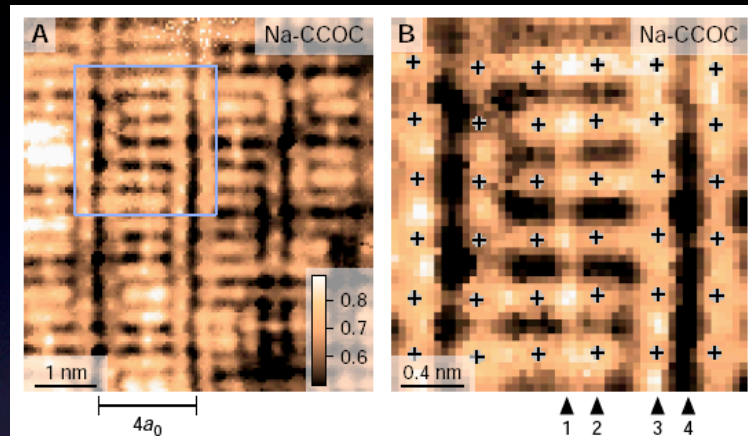
INSERT  $e^-$

Ground state properties ?

# STM-experiments: R-maps

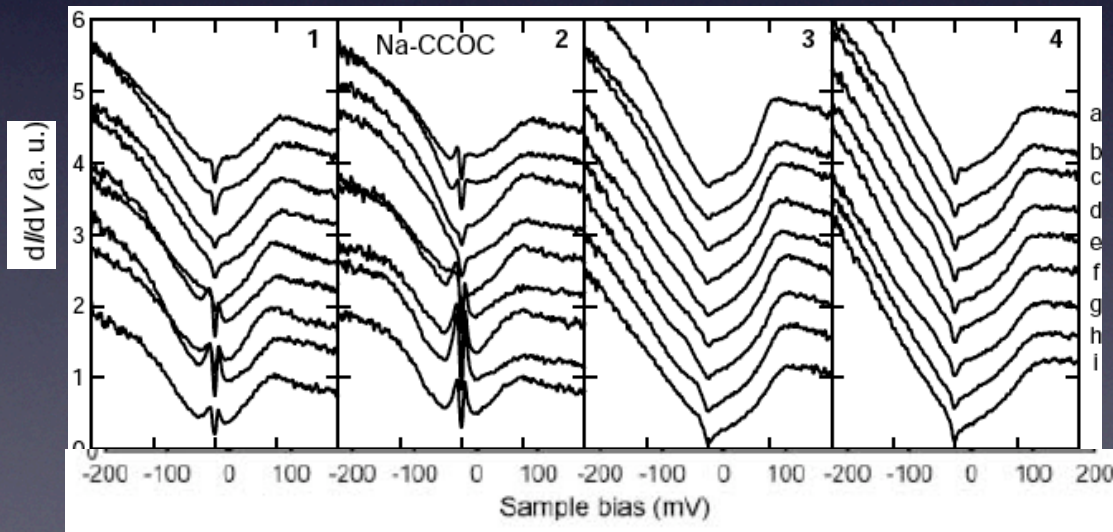
$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$  and  $\text{Dy-Bi2212}$  (at  $T < T_c$ )

[Kohsaka et al. Science 2007]



$$R(r, z, V) = \frac{I(r, z, +V)}{I(r, z, -V)} \sim \frac{x(r)}{1 - x(r)}$$

→ Extract hole density  $x(r)$



HOLE RICH

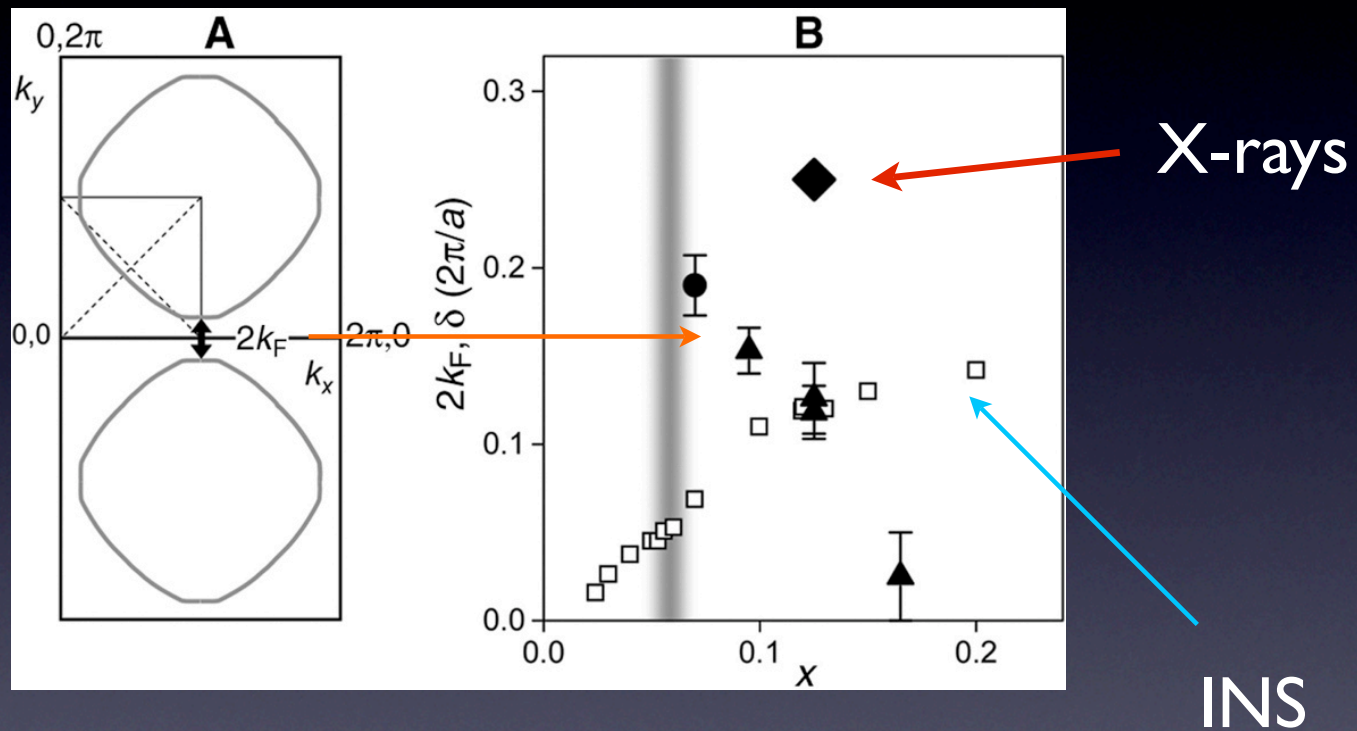
HOLE POOR

Bond-centered  
unidirectional  
patterns

Different low-energy  
properties



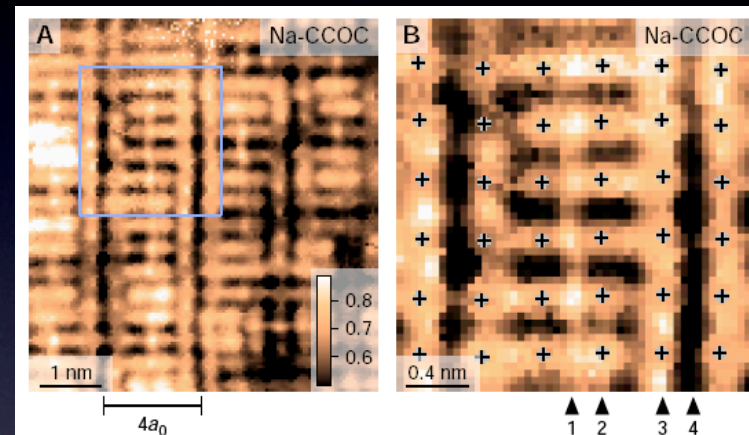
# Not a Fermi surface nesting mechanism !



[Valla et al., Science 2006]



# SPATIAL ORDER + SUPERCONDUCTIVITY



GOAL: describe **superconducting hole-stripes**  
within RVB framework ?

# RVB variational state revisited

$$H_{BCS} = H_{kin} + \sum_{ij} \Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger + \mu \sum_i n_i + h.c.$$

$t_{ij}$  and  $\Delta_{ij}$  become bond dependent !!

Uncorrelated state  $|D\rangle$

$$|\Psi_{RVB}\rangle = \prod_i (1 - n_{i\uparrow} n_{i\downarrow}) |D\rangle$$

Strongly  
correlated  
wavefunction

A convenient guide:  
Gutzwiller approximation  
+  
Mean-field

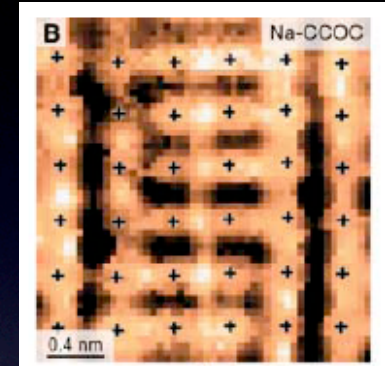
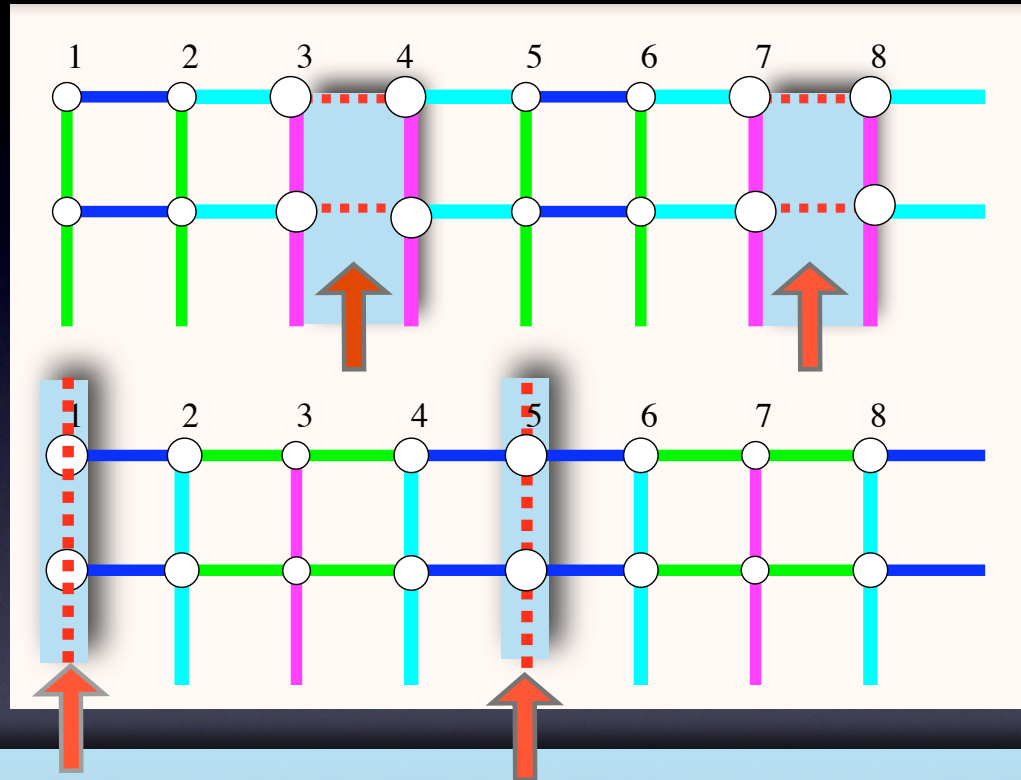
VMC : optimization scheme  
developped in S. Sorella's group,  
up to 16x16 clusters with PBC

# Superconducting stripes

We allow for inhomogeneous  $\Delta_{ij}$

Bond  
centered

Site  
centered

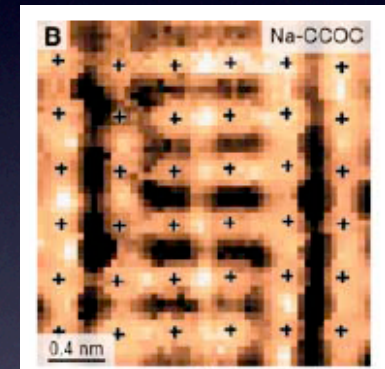
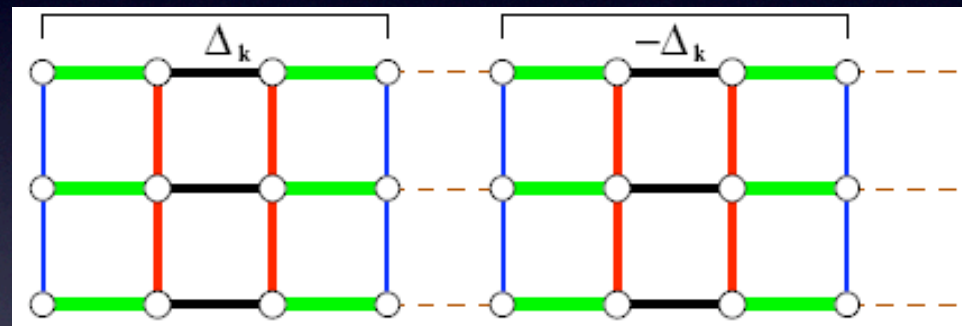


Create line-defects in the RVB state  
Impose  $\Delta_{ij}=0$  along one direction,  
with periodicity  $1/2x$



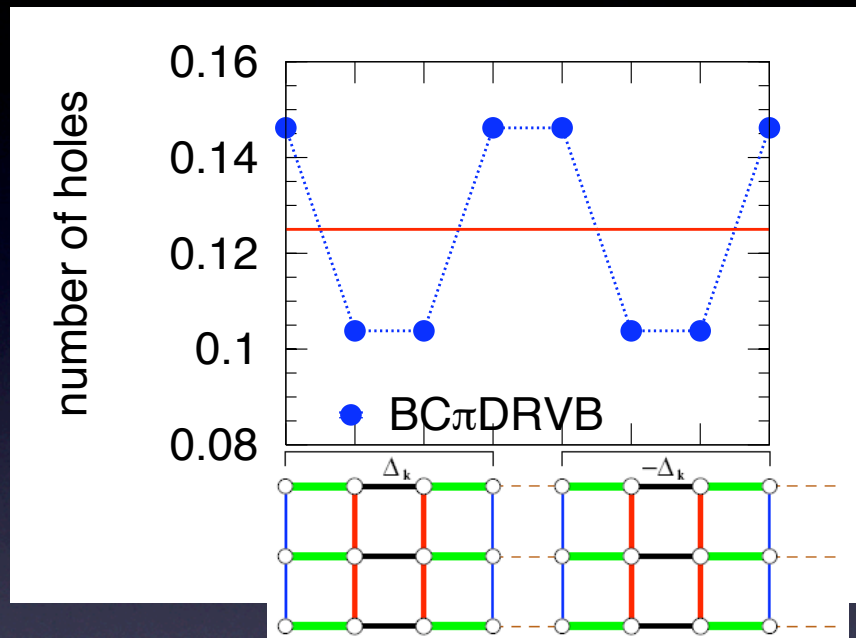
# Superconducting stripes (II): pi-domain RVB stripes

Bond-centered symmetry



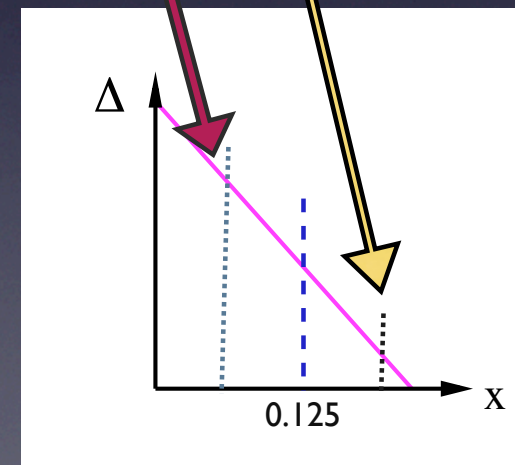
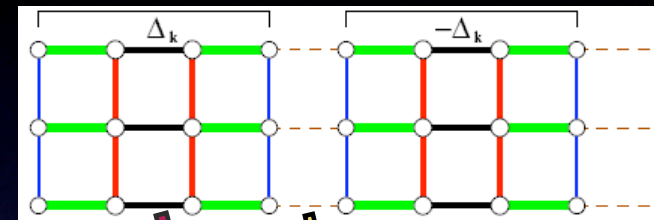
The pi-phase shift in  $\Delta_k$  implies regions with domain walls in the pairing, with  $\Delta_{ij}=0$

# Charge modulation is there!

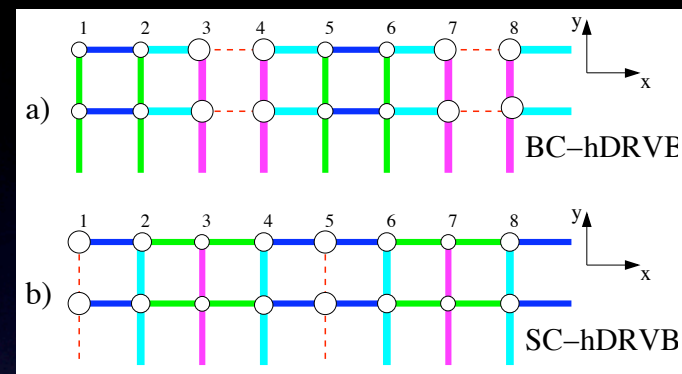
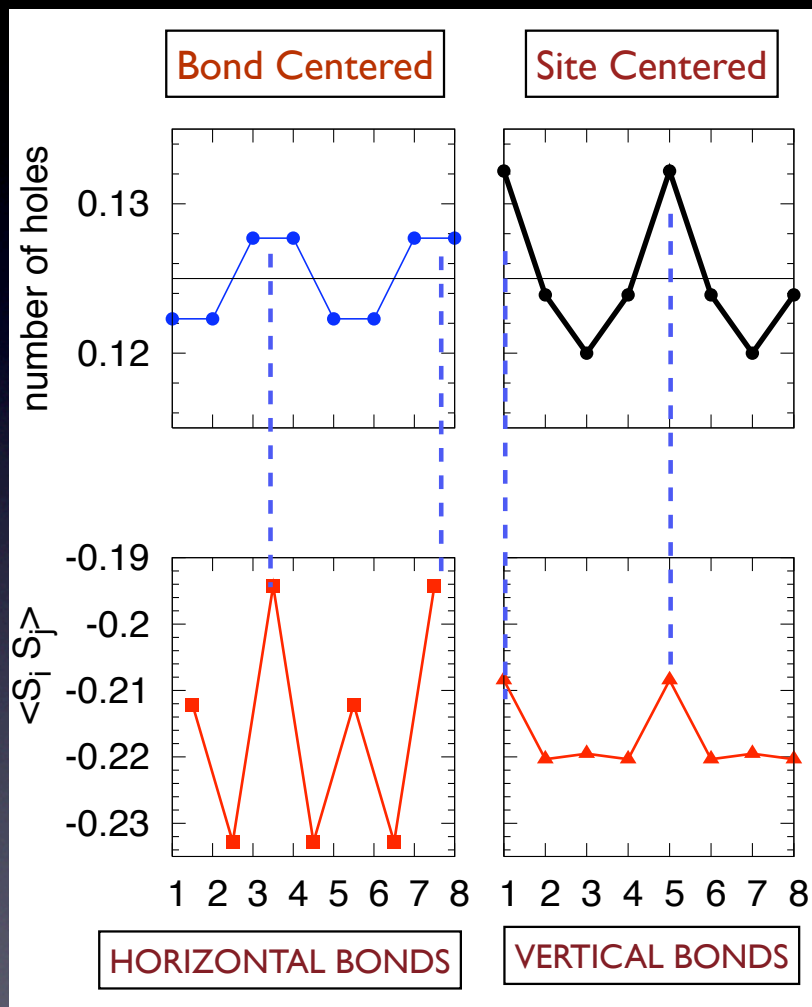


## 2 NON-EQUIVALENT SITES

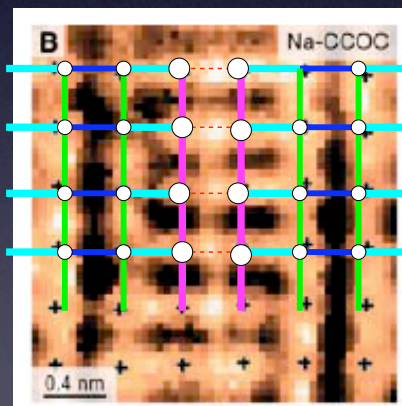
Holes concentrate where  
spin-pairing is smaller  
(around the domain walls)



# In-phase domains



Hole stripes emerge

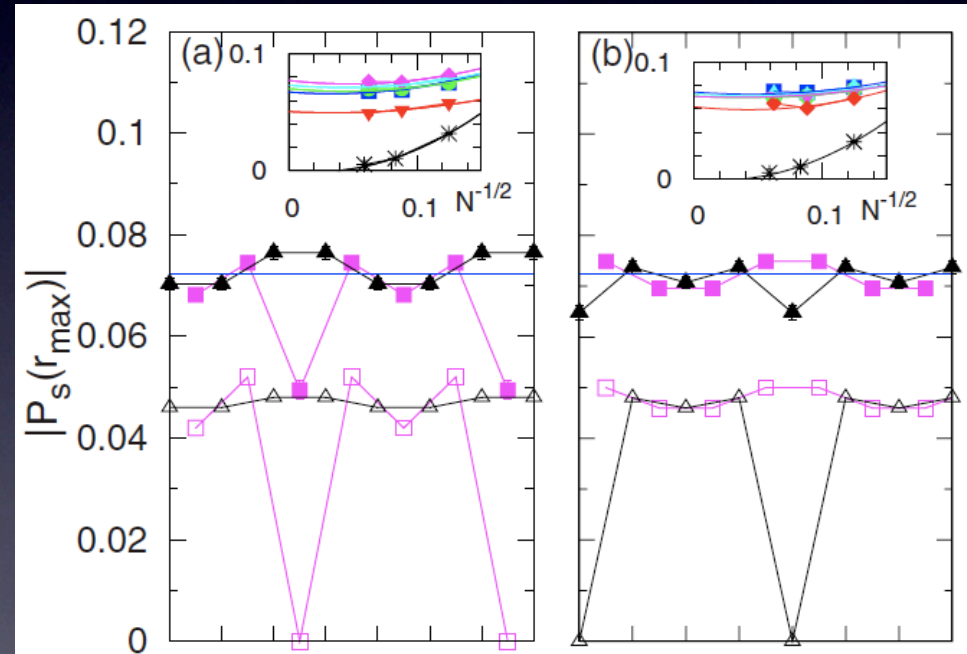
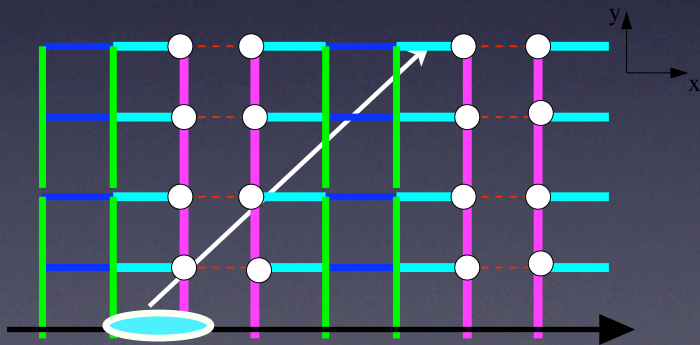




# Superconductivity is modulated !

$$P_s^2(r) = \langle \tilde{\Delta}_{s+r}^\dagger \tilde{\Delta}_s \rangle$$

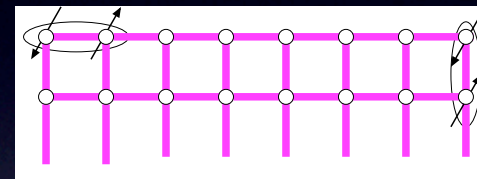
$$\tilde{\Delta}_s^\dagger = c_{s\uparrow}^\dagger c_{s+a\downarrow}^\dagger - c_{s\downarrow}^\dagger c_{s+a\uparrow}^\dagger$$



# Energies are really close ( $\sim 10^{-4} t$ )

$t/J=3$ ,  
doping  $1/8$   
up to  
 $16 \times 16$  clusters

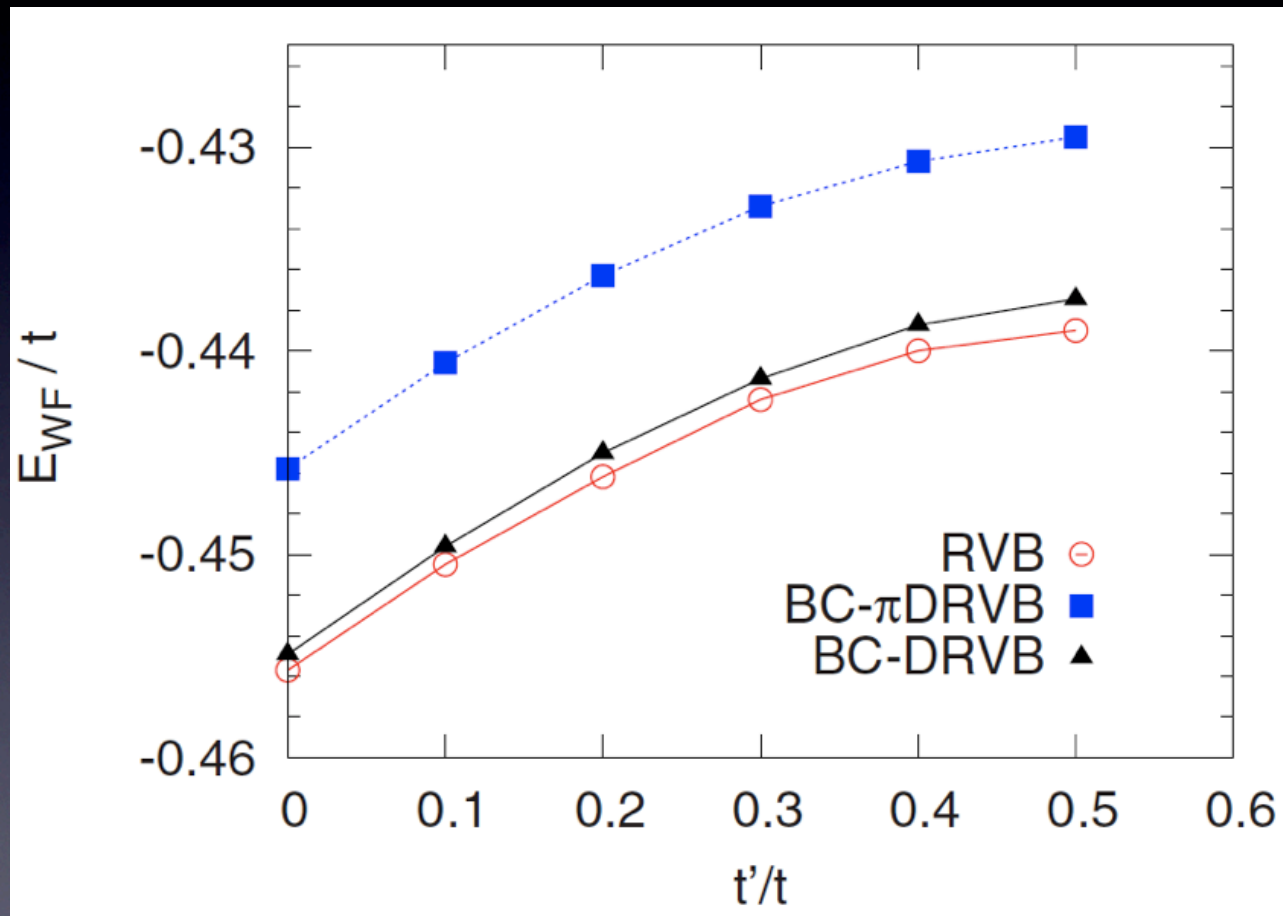
WF	$E_{\text{VMC}} [t]$
RVB	-0.45564(3)
SFP	-0.44630(3)
$\pi$ -DRVb	-0.44529(3)
BC-hDRVb	-0.45490(3)
SC-hDRVb	-0.45530(3)



Anti-phase

In-phase

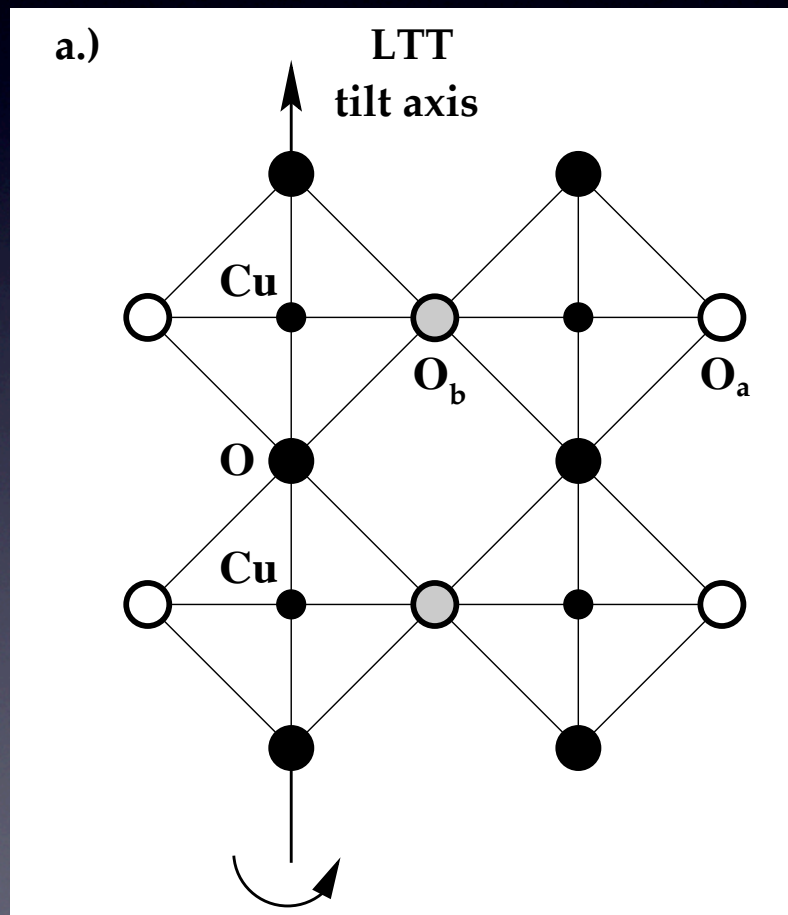
# Role of $t'$





# Lattice distortion

$$H_{tJ}^{\alpha} = -t \sum_{\langle ij \rangle, \sigma} \alpha_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. + J \sum_{\langle ij \rangle} \alpha_{ij}^2 S_i \cdot S_j$$

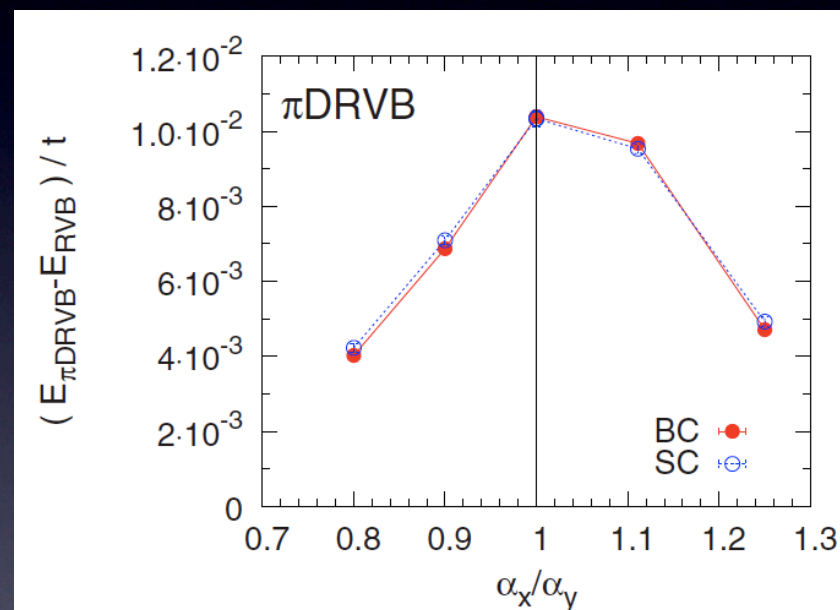
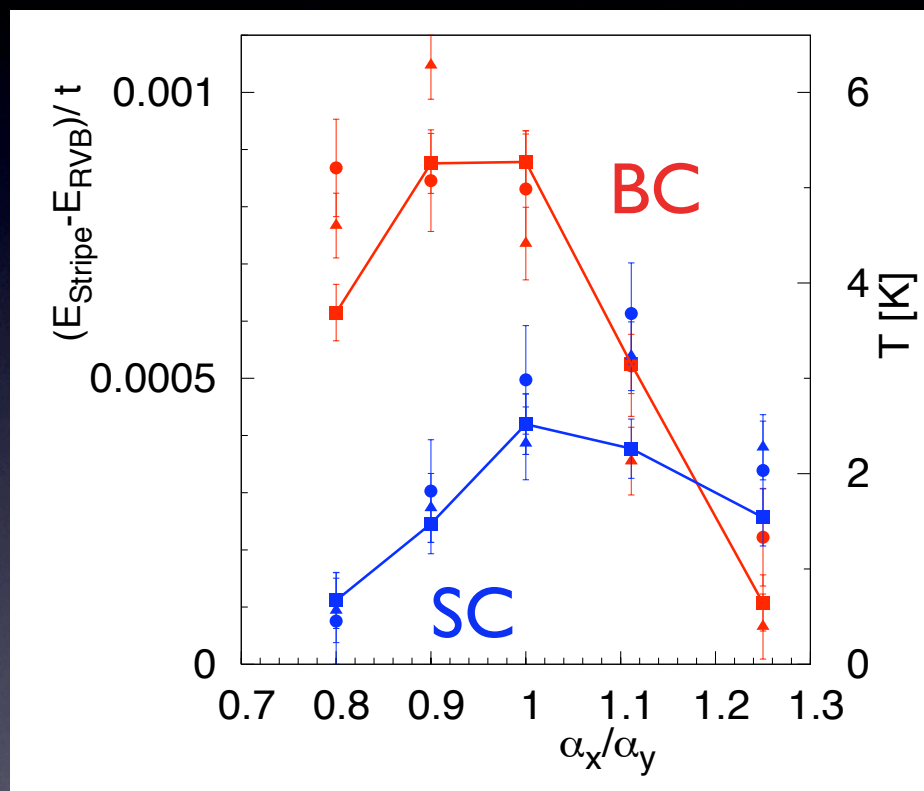


The tilt in the oxygen octahedra induces a different  $t$  and  $J$  along  $x$  and  $y$ :

tilt axis along  $y$ :  
 $\alpha_x < 1$  and  $\alpha_y = 1$

# Lattice LTT distortion further stabilizes the superconducting stripes

In-phase



Anti-phase

Tilt axis along y

Tilt axis along x

# Other related work

- \* Himeda, Kato & Ogata, PRL 2002  
[ simple cosine modulation of SC ]
- \* Berg, Fradkin, Kim, Kivelson, Oganessian, Tranquada & Zhang, PRL (2007)  
[Dynamical layer decoupling scheme]
- \* Yang, Chen, Rice, Sigrist and Zhang, arXiv:0807.3789  
[Mean-field RVB including spin ordering]

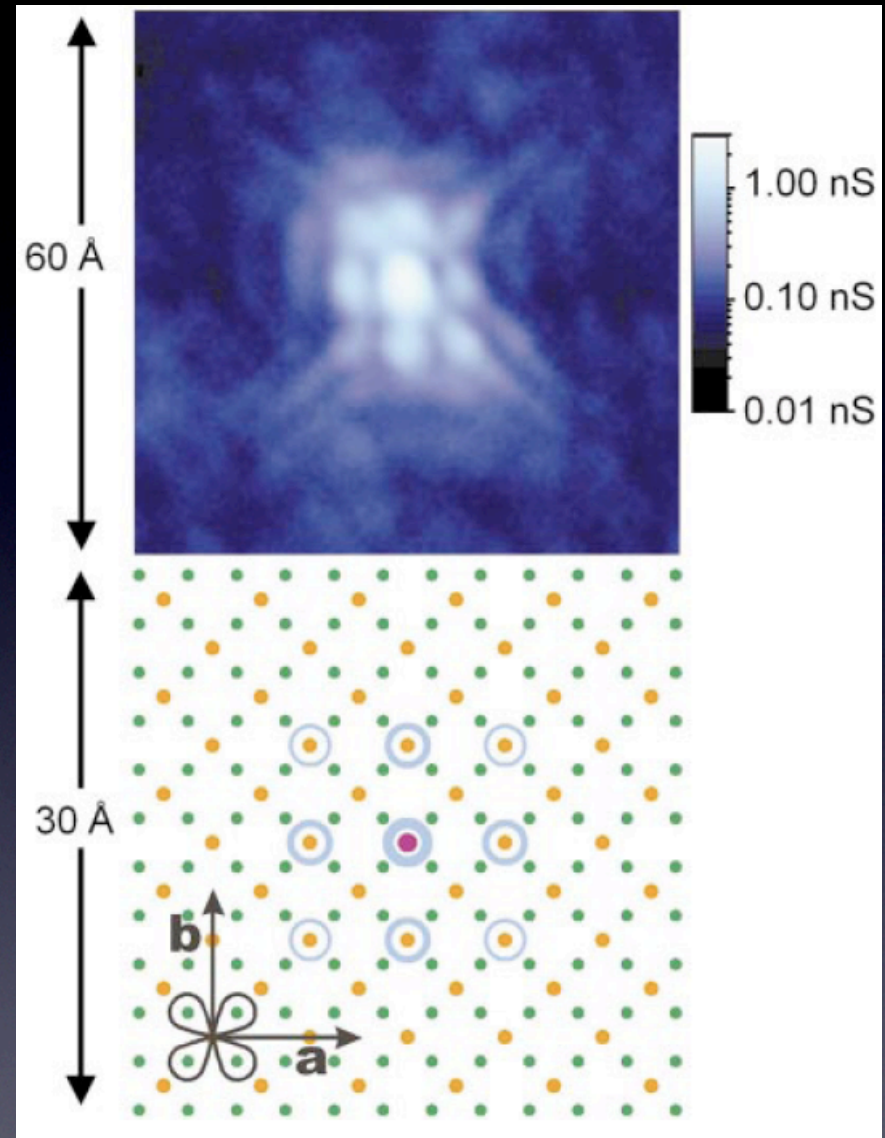


# Role of impurities ?

Low-energy DOS around  
Zinc impurity in SC state

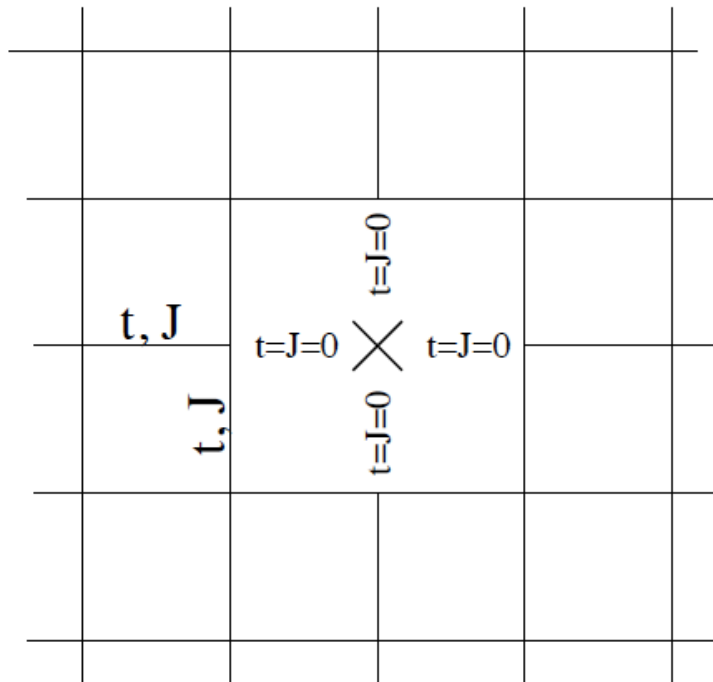
- Large DOS along nodal directions
- Suppression of SC within  $\sim 15\text{\AA}$  from Zn

Pan et al., Nature 403, 746 (2000)



$\text{Bi}_2\text{Sr}_2\text{Ca}(\text{Cu}_{1-x}\text{Zn}_x)_2\text{O}_{8+\delta}$  single crystals

Controlled impurity doping offers a stringent test  
for correlated models and RVB wf's !

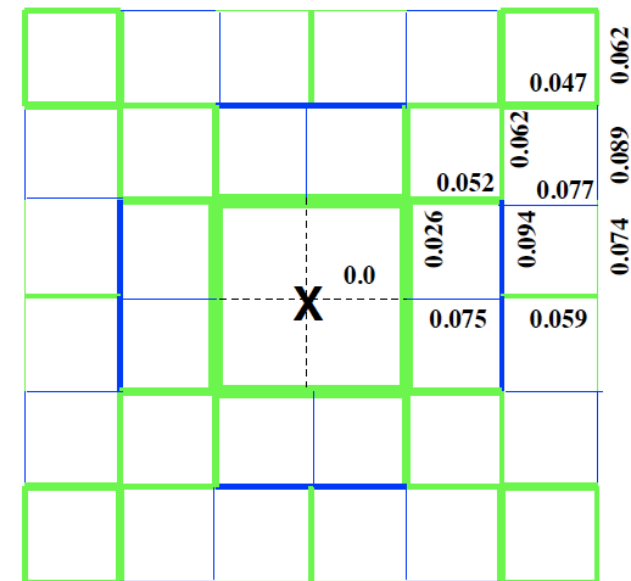


+

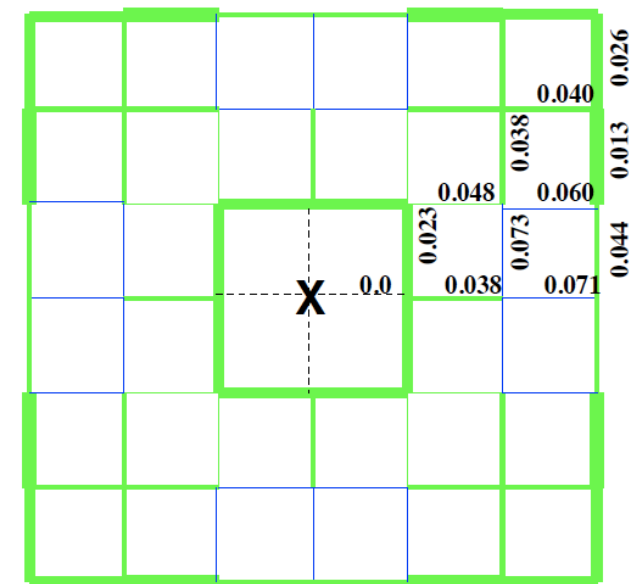
Variational Gutzwiller-  
projected RVB wavefunction  
(16x16 clusters)

Suppression of  
pairing correlations  
over large distances

$x=12\%$



$x=7\%$



16x16 cluster  
(only central region shown)



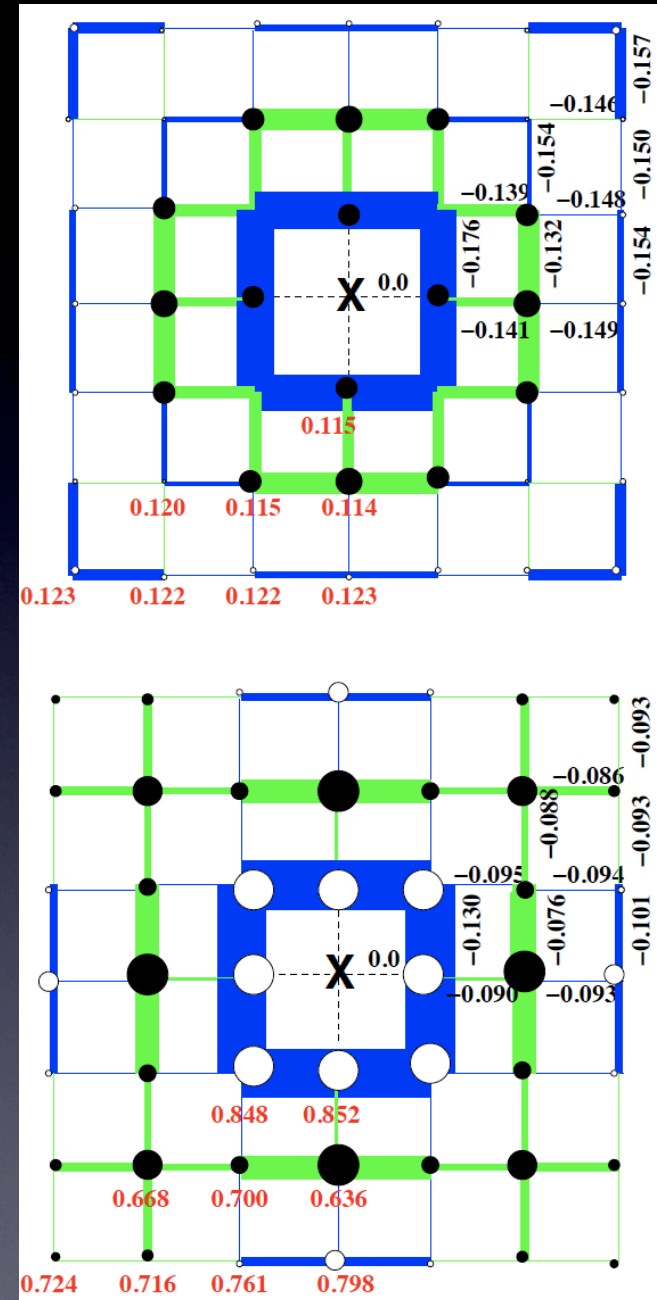
Strong modulation of  
local hole density

$x=12\%$

But needs STM R-maps to  
compare to experiments !!

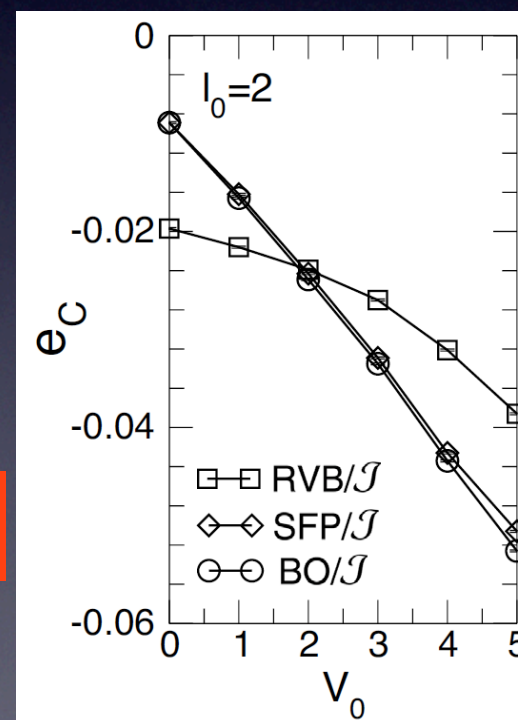
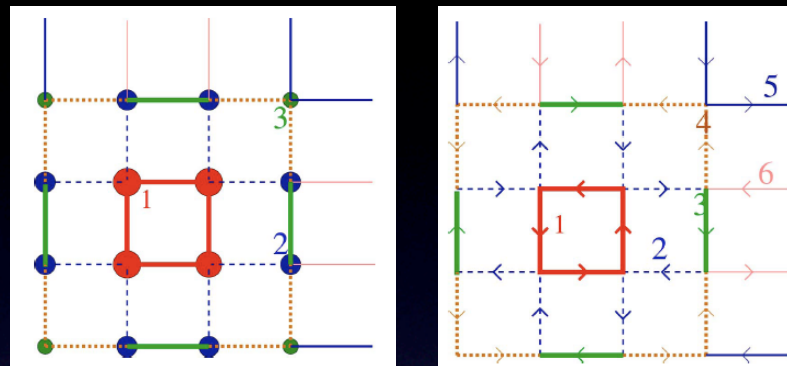
$x=7\%$

Open issue: magnetism around  
Zn ? To compare to NMR ...



# Competition with orbital-current phases ?

- Motivation from early numerics:
  - scalar-chiral spin correlations (DP, Riera, Dagotto, 91)
  - current-current (Leung, 2000) correlations
- DMRG (White-Scalapino) shows  $t'$  destabilizes SDW stripes
- 4x4 “checkerboard” ?



DP, PRB 2005  
C. Weber et al., PRB 2006

➡ Needs more VMC / DMRG simulations !

# Conclusions

- Striped superconducting states are competitive w.r.t. the uniform RVB but have higher energies within our extended t-J model ...
- Out-of-phase SC domains cost more than in-phase: in agreement with DMRG (S.White et al.)
- Impurities like Zinc induce large domains of modulated SC regions: might have a role in the STM patterns seen
- 4x4 checkerboard charge ordered state seems energetically very competitive: relevant competing non-SC state with TRS breaking ?

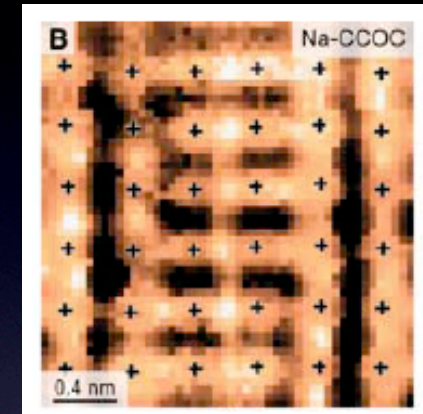
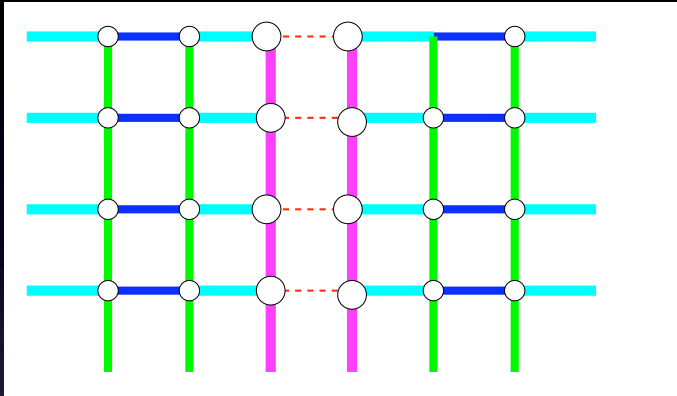
## References:

- (I) Raczkowski et al., PRB (RC) **76**, 140505 (2007)
- (II) Capello et al., PRB **77**, 224502 (2008)
- (III) Capello and DP, PRB **79**, 224507 (2009)

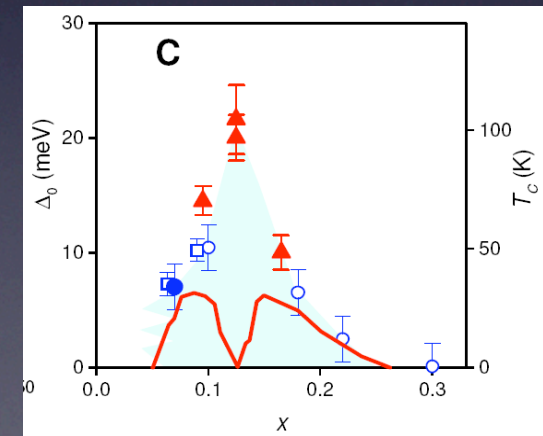
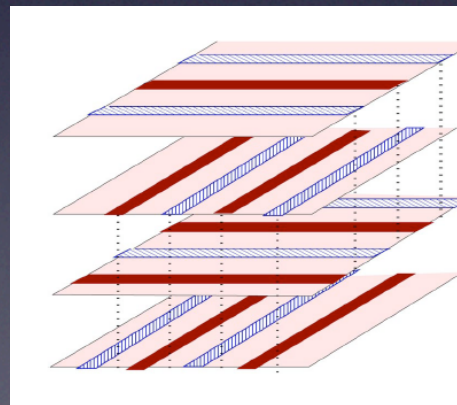
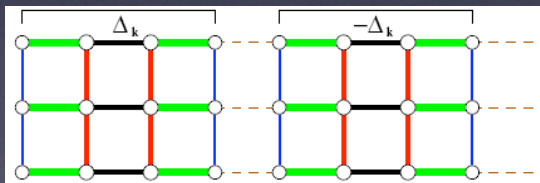


# Partial summary

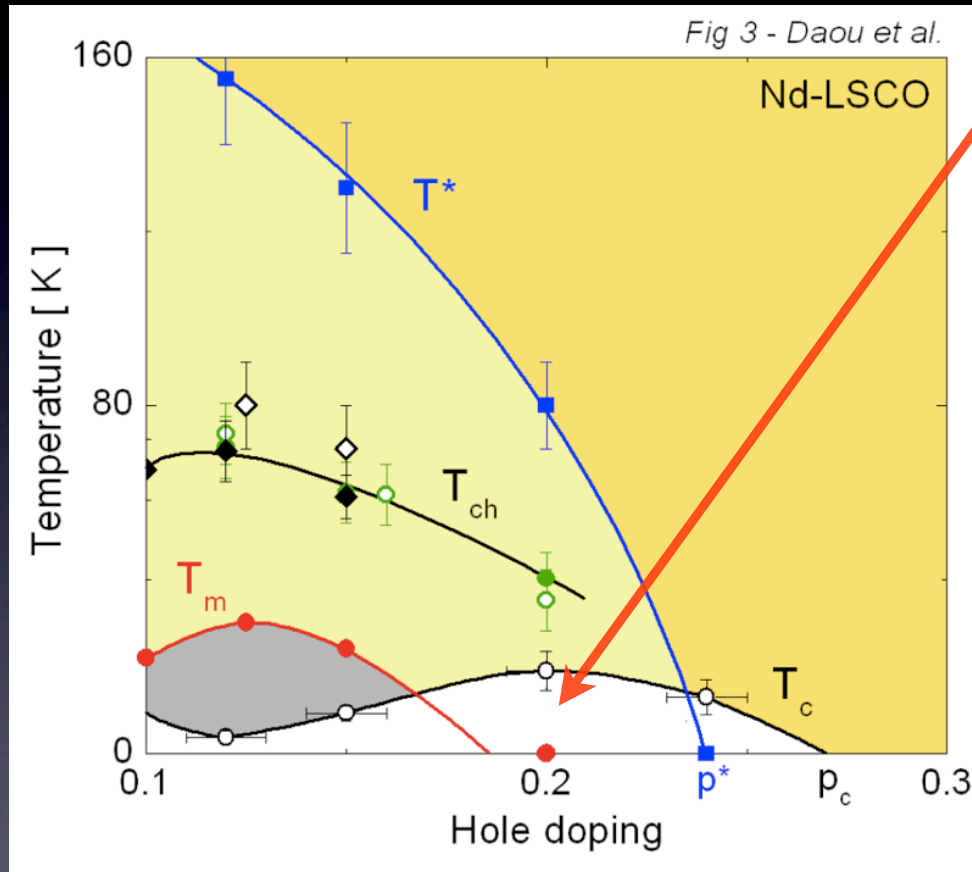
\* superconducting RVB hole stripes



\* **pi-shift** RVB hole stripes

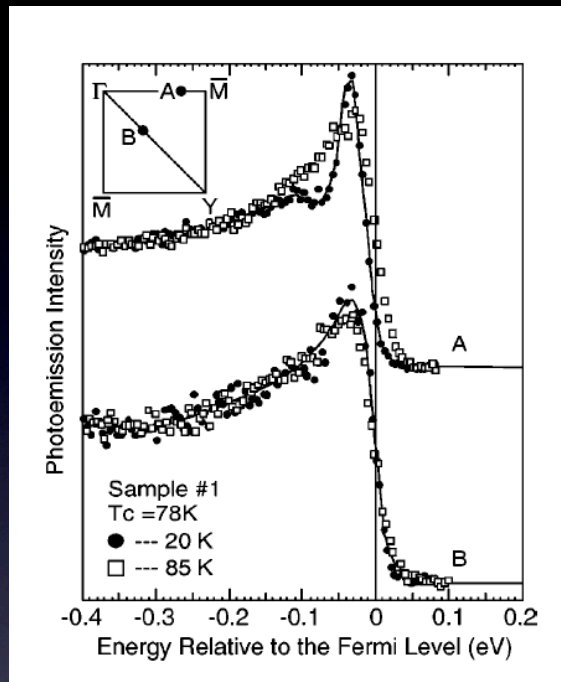


Charge and Superconductivity might  
coexist even without spin order !

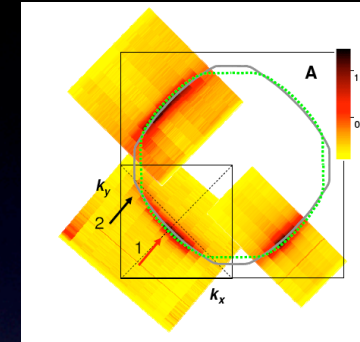


L. Taillefer's group  
[arXiv:0806.2881]

# ARPES: the d-wave gap

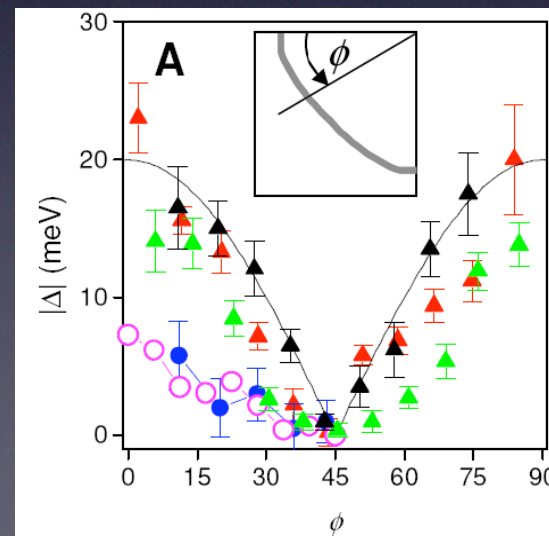


( $\pi, 0$ ) vs ( $\pi/2, \pi/2$ )  
Shen 1993  
Bi2212  
T<sub>c</sub> = 88 K



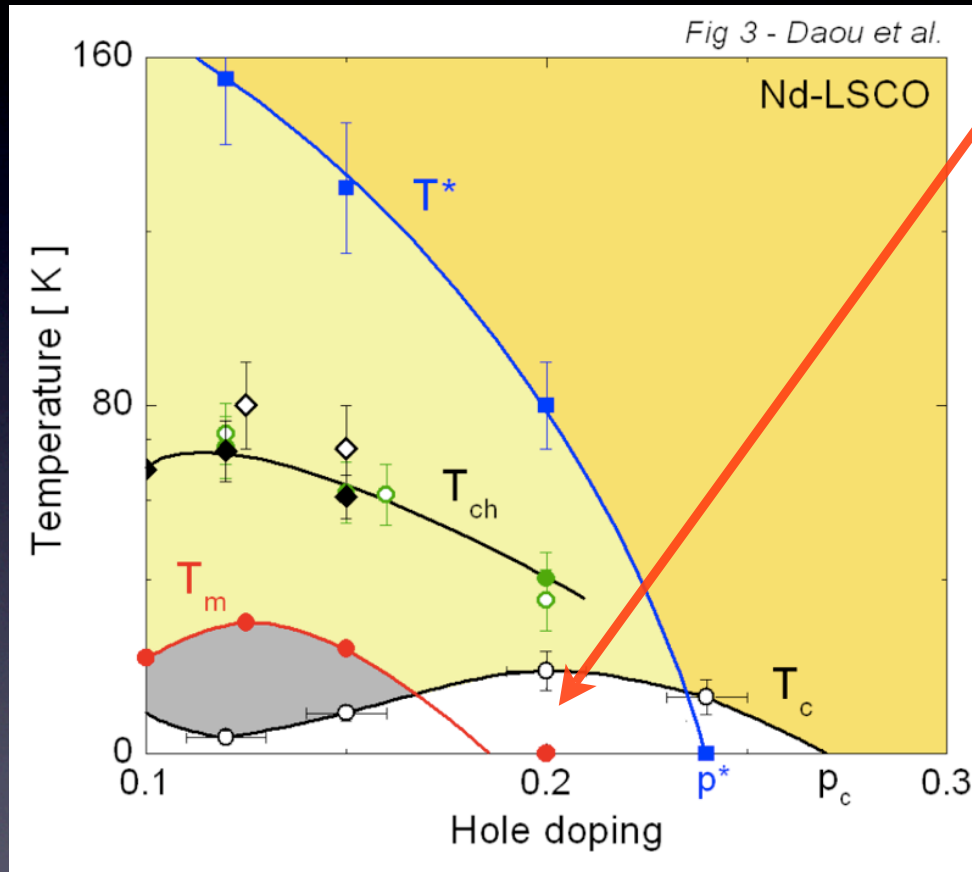
Nodal quasiparticles  
at  $\pi/2, \pi/2$

The gap closes  
at  $\pi/2, \pi/2$





Charge and Superconductivity might  
coexist even without spin order !



L. Taillefer's group  
[arXiv:0806.2881]

# RVB theory: mathematical framework

Correlated wavefunctions  
Gutzwiller projected HF d-wave BCS:

$$P|\Phi\rangle = P \prod_{\vec{k}} \left( u_{\vec{k}} + v_{\vec{k}} c_{\vec{k}\uparrow}^{\dagger} c_{-\vec{k}\downarrow}^{\dagger} \right) |0\rangle$$

$$P = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})$$

→ Variational Monte Carlo

→ Mean field theory

F.C. Zhang et al., Supercond. Sci. Technol. **1**, 36 (1988).

Gutzwiller approximation

$$\langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle = g_t \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle_0$$

$$\langle S_i \cdot S_j \rangle = g_S \langle S_i \cdot S_j \rangle_0$$

↓

$$H_{eff} = g_t T + g_S J \sum S_i \cdot S_j$$

Competing phases:  
d-wave RVB ↔ staggered flux  
Affleck-Marston 1988

# Meanfield Fermionic theory

## Extend RVB picture & formalism to inhomogeneous case

$$\begin{aligned} H_{\text{MF}} = & -t \sum_{\langle ij \rangle \sigma} g_{ij}^t (c_{i,\sigma}^\dagger c_{j,\sigma} + h.c.) - \mu \sum_{i\sigma} n_{i,\sigma} \\ & - \frac{3}{4} J \sum_{\langle ij \rangle \sigma} g_{i,j}^J (\chi_{ji} c_{i,\sigma}^\dagger c_{j,\sigma} + h.c. - |\chi_{ij}|^2) \\ & - \frac{3}{4} J \sum_{\langle ij \rangle \sigma} g_{i,j}^J (\Delta_{ji} c_{i,\sigma}^\dagger c_{j,-\sigma}^\dagger + h.c. - |\Delta_{ij}|^2), \end{aligned}$$

- + usual MF self-consistent equations
- Site dependent g's, bond amplitudes and site densities

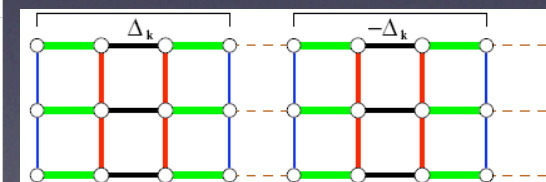
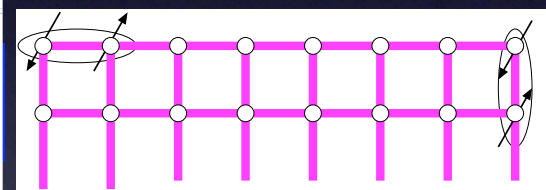


# Energetics for the t-J model

$$H_{tJ} = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + h.c. + J \sum_{\langle ij \rangle} S_i \cdot S_j$$

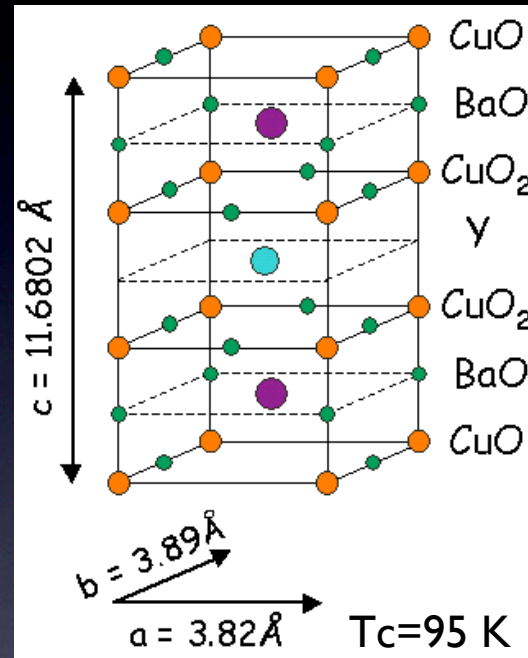
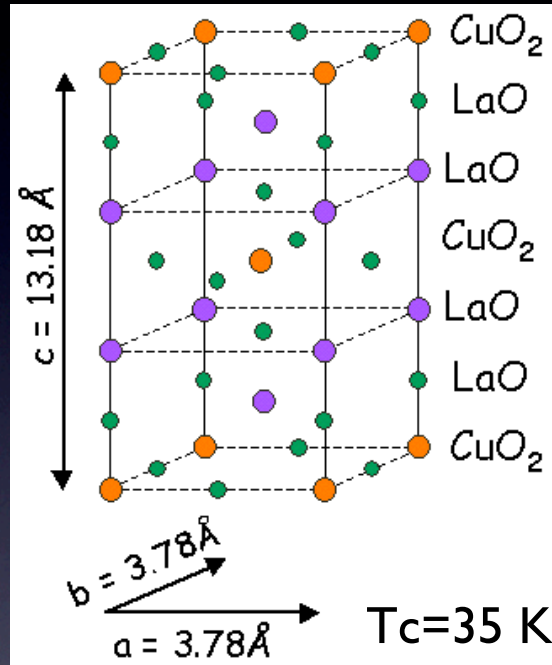
$t/J=3$ ,  
doping 1/8  
up to  
16x16 clusters

WF	$E_{\text{RMFT}} [t]$	$E_{\text{VMC}} [t]$
RVB	-0.4549	-0.45564
SFP	-0.4284	-0.44630
pi-DRVb	-0.4412	-0.44529

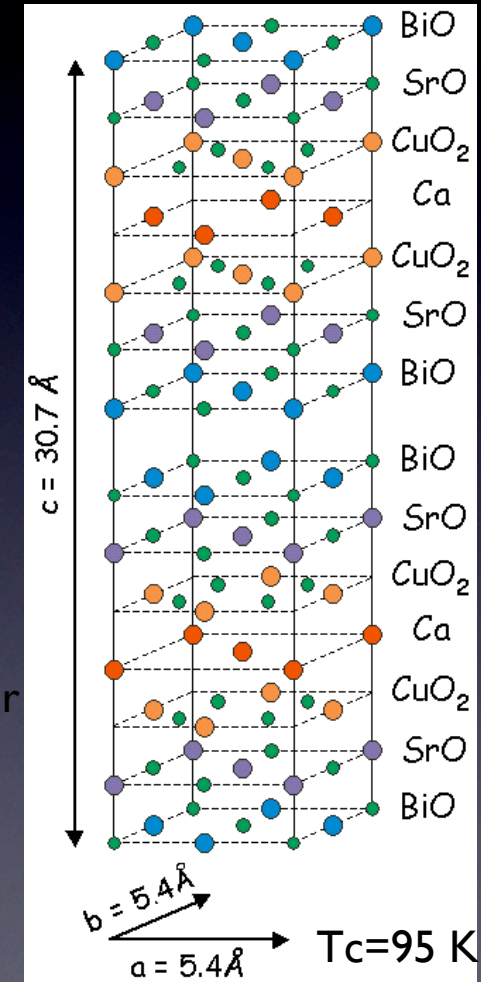


Very close energies but pi-shift in  $\Delta_k$  has a cost

# Cuprates Structure

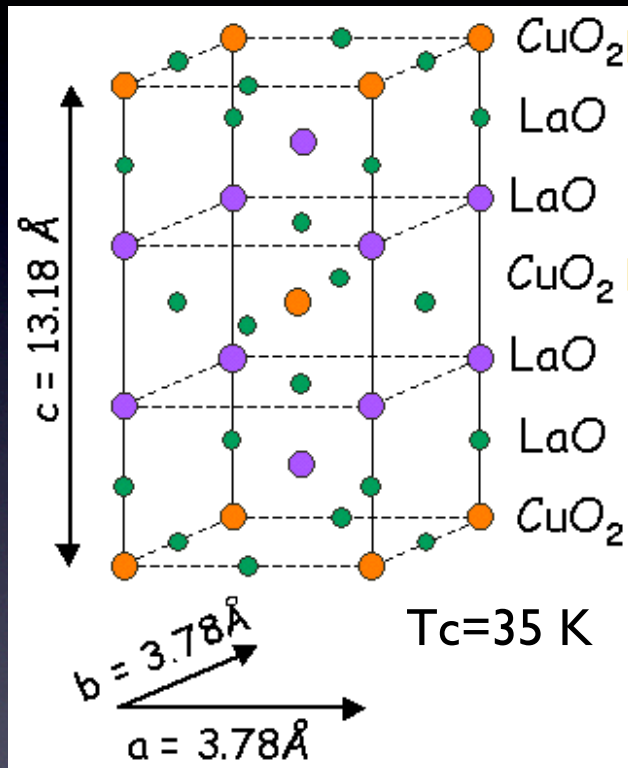


chains charge reservoir

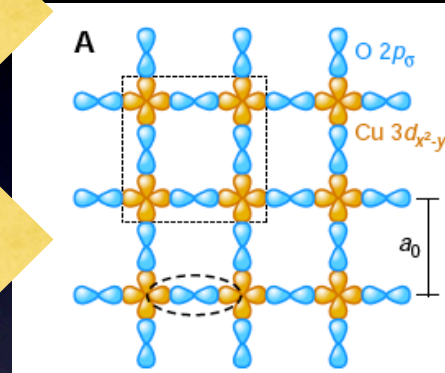


Layered structure with  $\text{CuO}_2$  planes  
 +  
 charge reservoirs (La, Y, Ba, Ca, O)

# The Cuprates



Layered structure with  $\text{CuO}_2$  planes



Cu d-orbitals:  
small overlap  
strong correlation

2D square lattice

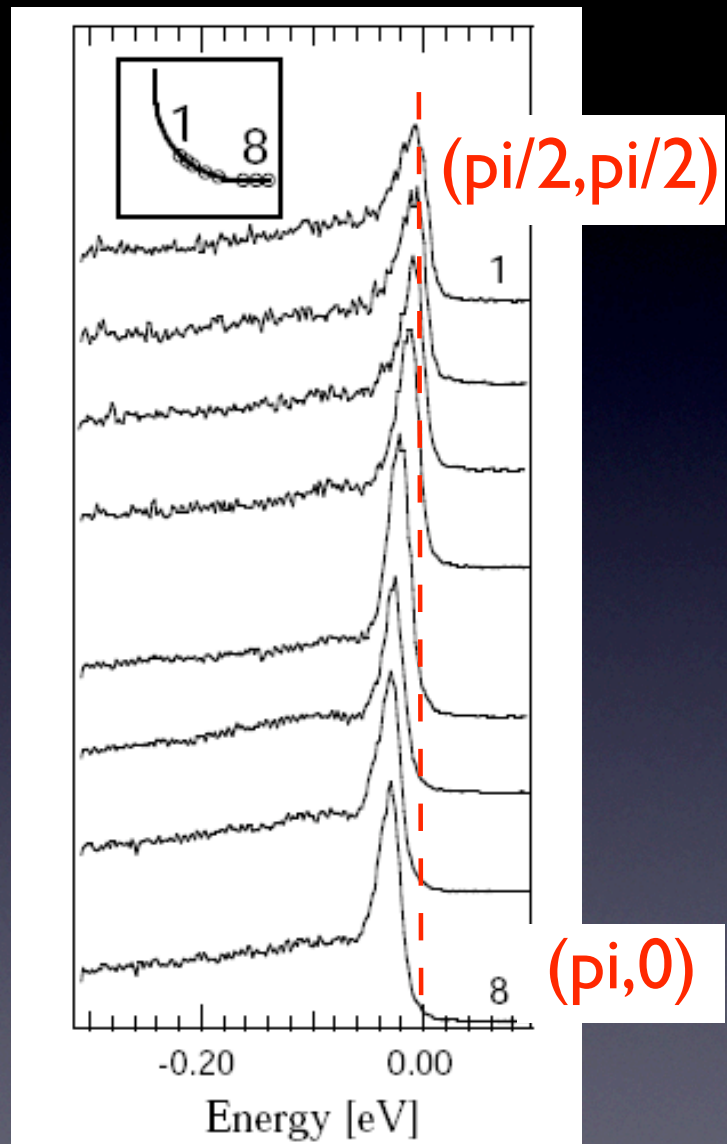
Interesting Physics upon doping

\*Undoped:  $\text{La}_2\text{CuO}_4$ : 1 electron per site

\*Doped:  $\text{La}^{3+}$  substituted with  $(\text{Ba}, \text{Sr})^{2+}$   
introduction of extra carriers (holes)  
in the planes

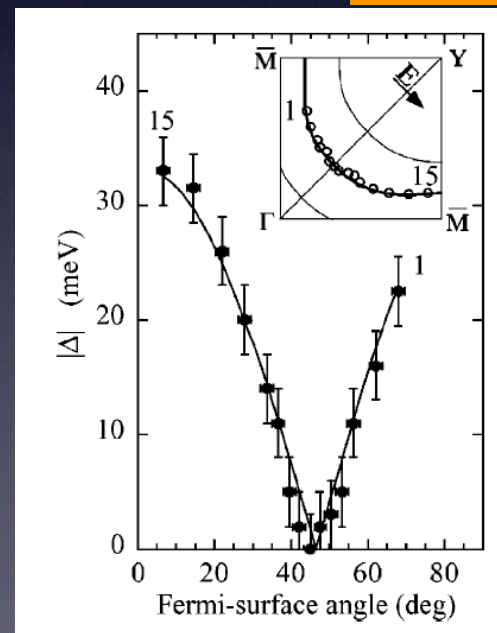


# The d-wave superconducting gap



Defined quasiparticles  
in the superconducting state

d-wave gap:  
the gap closes  
at  $(\pi/2, \pi/2)$



[Bi2212, Kaminski PRL 2001]

[Bi2212, Ding, Norman 1996]