Fermion signs and higher Tc

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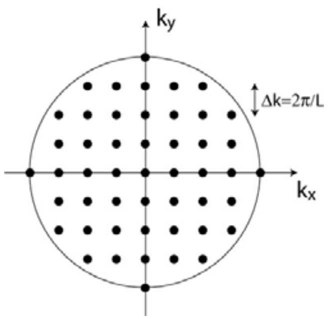
The nodal hypersurface

Antisymmetry of the wave function

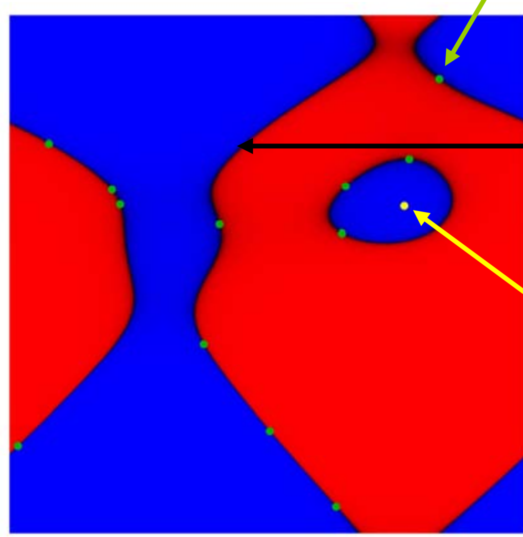
$$\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_j,\ldots,\mathbf{r}_N) = -\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_j,\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_N)$$

Free Fermions

$$\Psi_0(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\mathbf{r}_j}\right)_{ij}$$



d=2



Pauli hypersurface

$$P = \bigcup_{i\neq j} P_{ij}$$

$$P_{ij} = \{\mathbf{R}\in\mathbb{R}^{Nd} | \mathbf{r}_i = \mathbf{r}_j\}$$

$$\dim P = Nd - d$$

Nodal hypersurface

$$\Omega = \{\mathbf{R}\in\mathbb{R}^{Nd} | \Psi(\mathbf{R}) = 0\}$$

$$\dim \Omega = Nd - 1$$

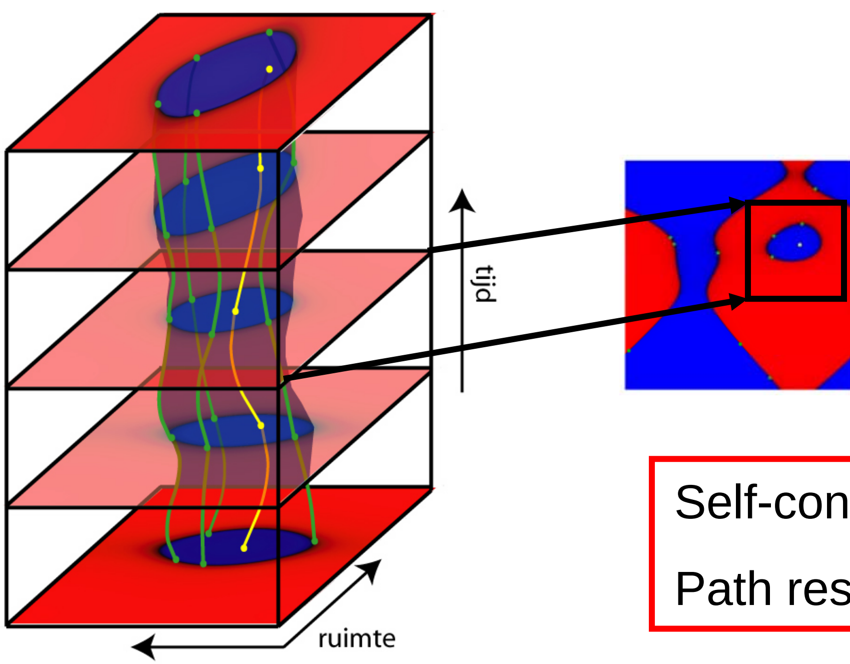
Test particle

Constrained path integrals

Formally we can solve the sign problem!!

$$\rho_F(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P},\mathrm{even}} \int_{\gamma:\mathbf{R}\to\mathcal{P}\mathbf{R}}^{\gamma\in\Gamma(\mathbf{R},\mathcal{P}\mathbf{R})} \mathcal{D}\mathbf{R}(\tau) \exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T} \mathrm{d}\tau \left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau) + V(\mathbf{R}(\tau))\right)\right\}$$

$$\Gamma(\mathbf{R},\mathbf{R}') = \{\gamma : \mathbf{R} \to \mathbf{R}' | \rho_F(\mathbf{R},\mathbf{R}(\tau);\tau) \neq 0\}$$



Ceperley, J. Stat. Phys. (1991)

Self-consistency problem:

Path restrictions depend on ρ_F !

Reading the worldline picture

Fermi-energy: confinement energy imposed by local geometry

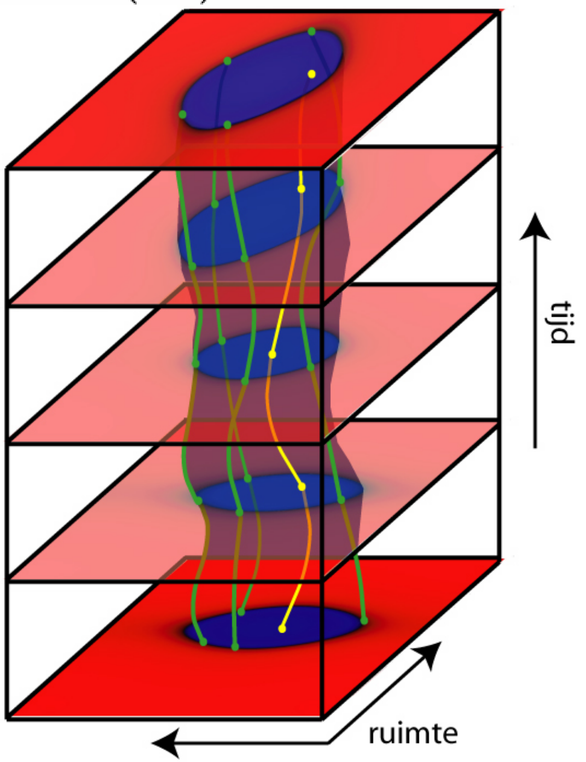
$$l^2(\tau) = \langle(\mathbf{r}_i(\tau) - \mathbf{r}_i(0))^2\rangle = 2d\mathcal{D}\tau = 2d\frac{\hbar}{2m}\tau$$

$$l^2(\tau_c) \simeq r_s^2 \rightarrow \tau_c \simeq \frac{1}{2d}\frac{2m}{\hbar}n^{-2/d}$$

$$\hbar\omega_c = \frac{\hbar}{\tau_c} \simeq d\frac{\hbar^2}{2m}n^{2/d} \simeq E_F$$

Average node to node spacing

$$\sim r_s = \left(\frac{V}{N}\right)^{1/d} = n^{-1/d}$$



Fermi surface encoded globally: $\rho_F = Det\left(e^{ik_i r_j}\right) = 0$

Change in coordinate of one particle changes the nodes everywhere

Finite T: $\rho_F = (4\pi\lambda\beta)^{-dN/2} Det\left[\exp\left(-\frac{(r_i - r_{j0})^2}{4\lambda\tau}\right)\right]$

$\lambda = \hbar^2/(2M)$

Non-locality length: $\lambda_{nl} = v_F \tau_{inel} = v_F\left(\frac{E_F}{k_B T}\right)\left(\frac{\hbar}{k_B T}\right)$

Vacuum structure

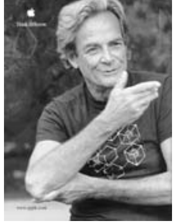
Long time, zero temperature:

$$\rho_F\left(R, R(\tau); \tau \rightarrow \infty\right) = \Psi^*\left(R\right)\Psi\left(R(\infty)\right)$$

IR fermionic information encoded in the ground state wavefunction.

Need a wave function ansatz!

Turning on the backflow



Nodal surface has to become fractal !!!

↓

Try backflow wave functions

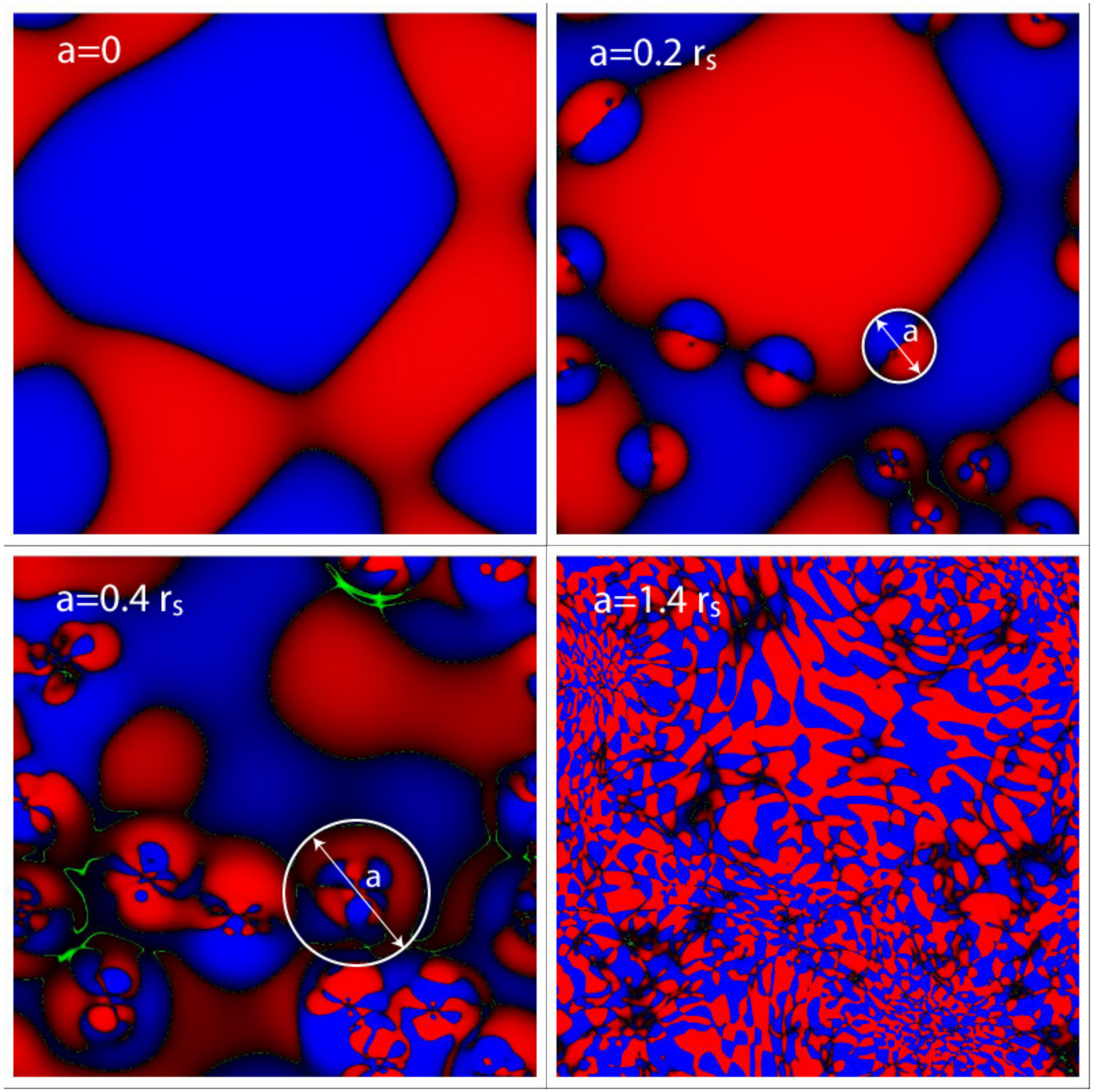
$$\psi_{bf}(\mathbf{R}) \sim \text{Det}\left(e^{i\mathbf{k}_i\tilde{\mathbf{r}}_j}\right)_{ij}$$

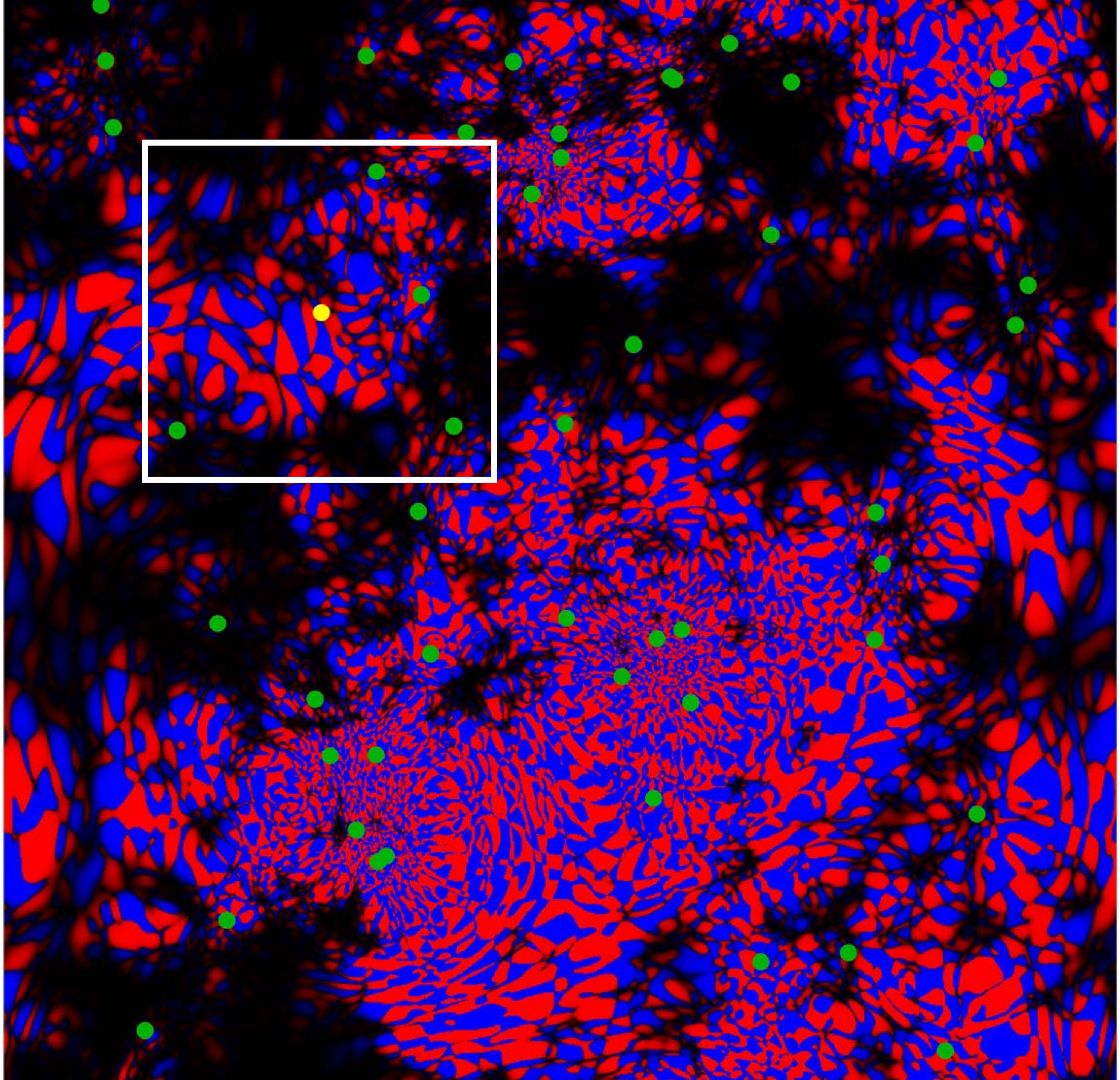
$$\tilde{\mathbf{r}}_j = \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

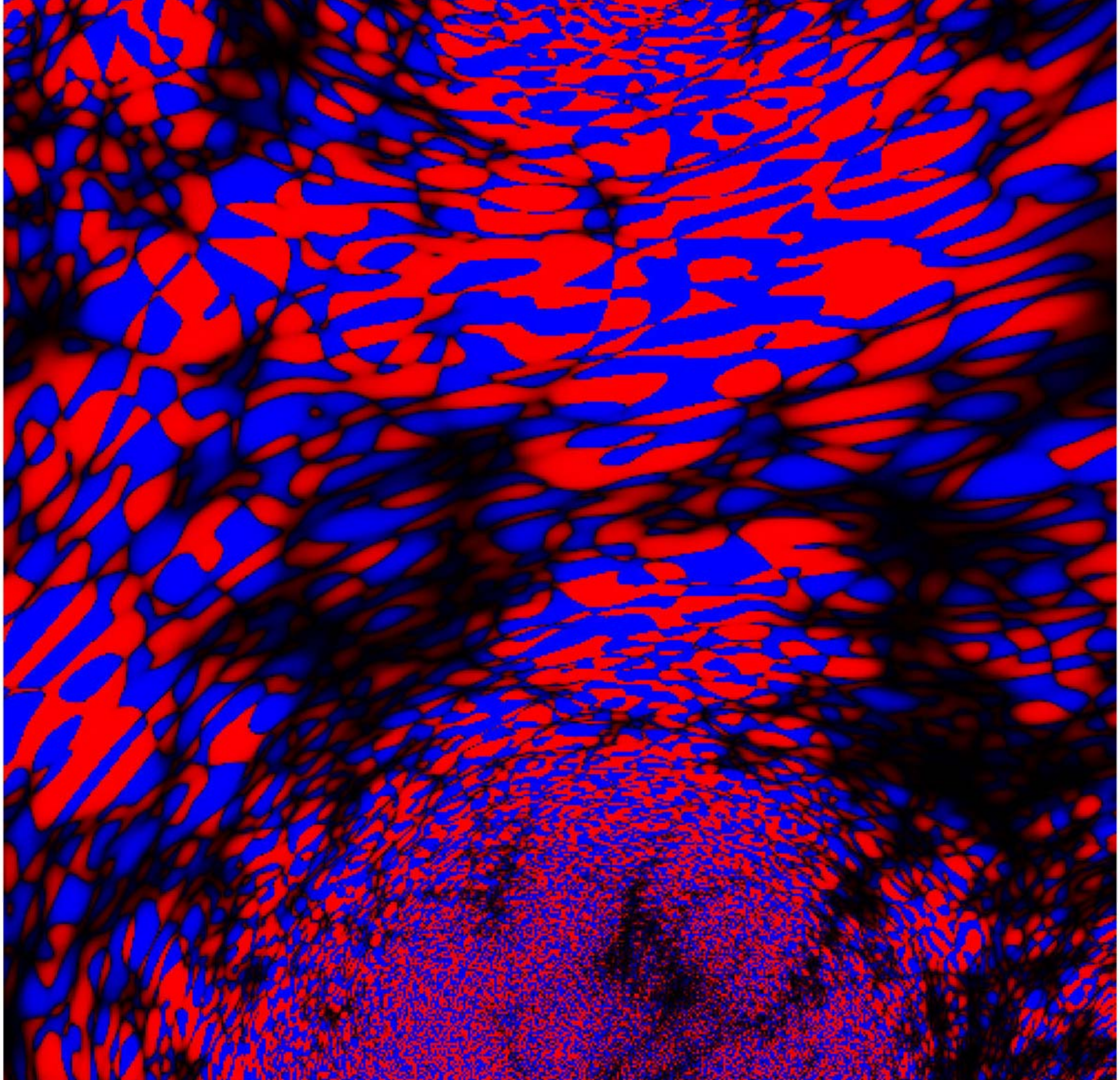
$$\eta(r) = \frac{a^3}{r^3 + r_0^3}$$

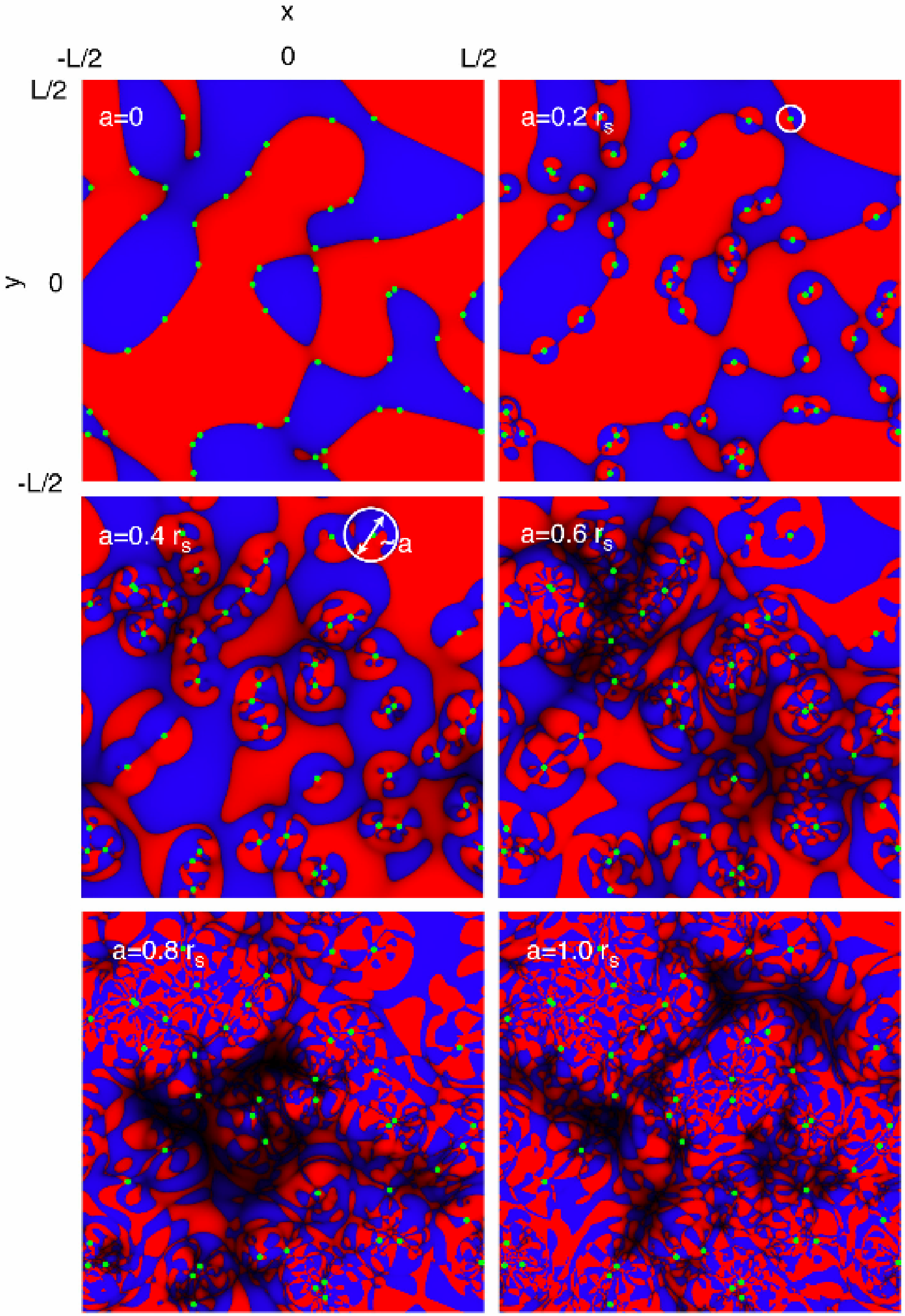
Collective (hydrodynamic) regime:

$$a \gg r_s$$



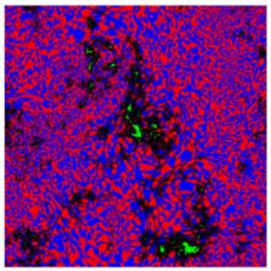






x
-L/2
0
L/2
L/2
y
0
-L/2
a=0
a=0.2 r$_s$
a=0.4 r$_s$
a
a=0.6 r$_s$
a=0.8 r$_s$
a=1.0 r$_s$

Extracting the fractal dimension



The Hausdorff dimension. The Hausdorff dimension of a metric space X, $\dim_H(X)$, is the infimimum of the numbers α with the following property: For any $\epsilon > 0$ there is a $\delta > 0$ and a cover $\mathfrak{U}$ of X such that the sets $B \in \mathfrak{U}$ all have diameter $|B|$ smaller than δ and

$$\sum_{B \in \mathfrak{U}} (|B|)^\alpha < \epsilon.$$

The correlation integral:

$$\begin{aligned} C(r) &= \lim_{n\to\infty} \frac{1}{n^2} \sum_{i,j=1}^{n} \Theta(r - |\mathbf{r}_i - \mathbf{r}_j|) \\ &= \int_0^r \mathrm{d}^D r' c(\mathbf{r}') \end{aligned}$$

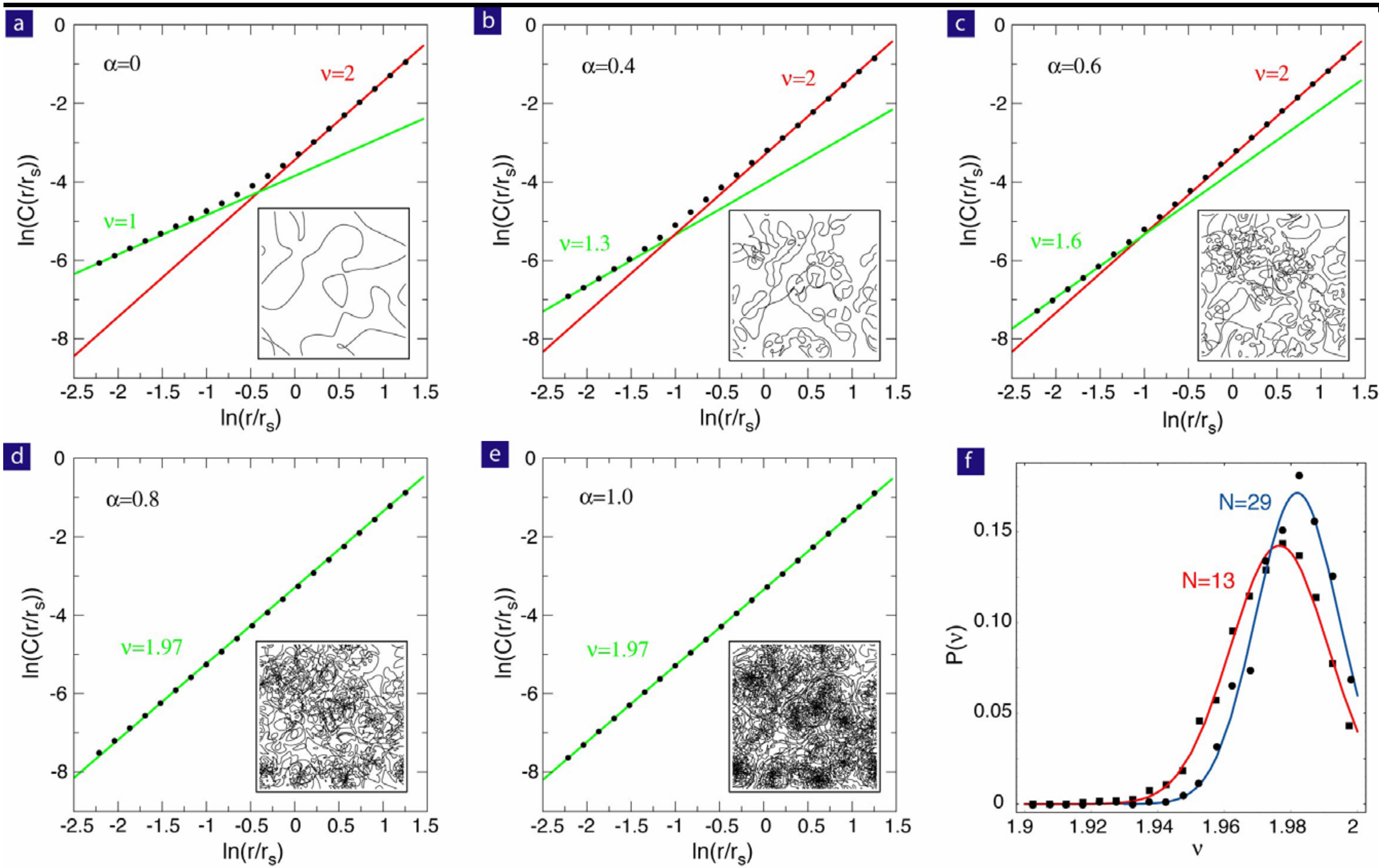
For fractals:

$$C(r) \sim r^\nu, \quad \nu \leq \dim_H$$

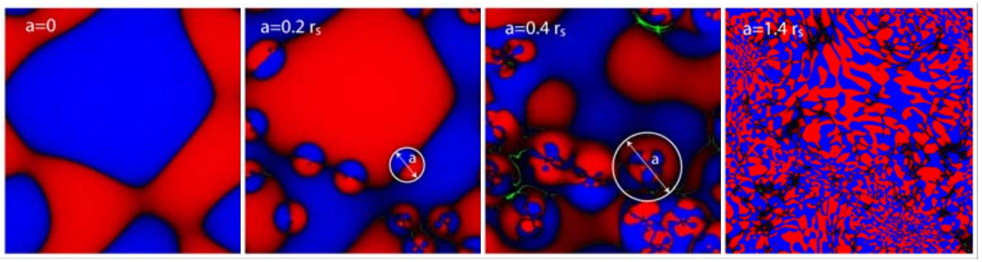
Inequality very tight, relative error below 1%

Grassberger & Procaccia, PRL (1983)

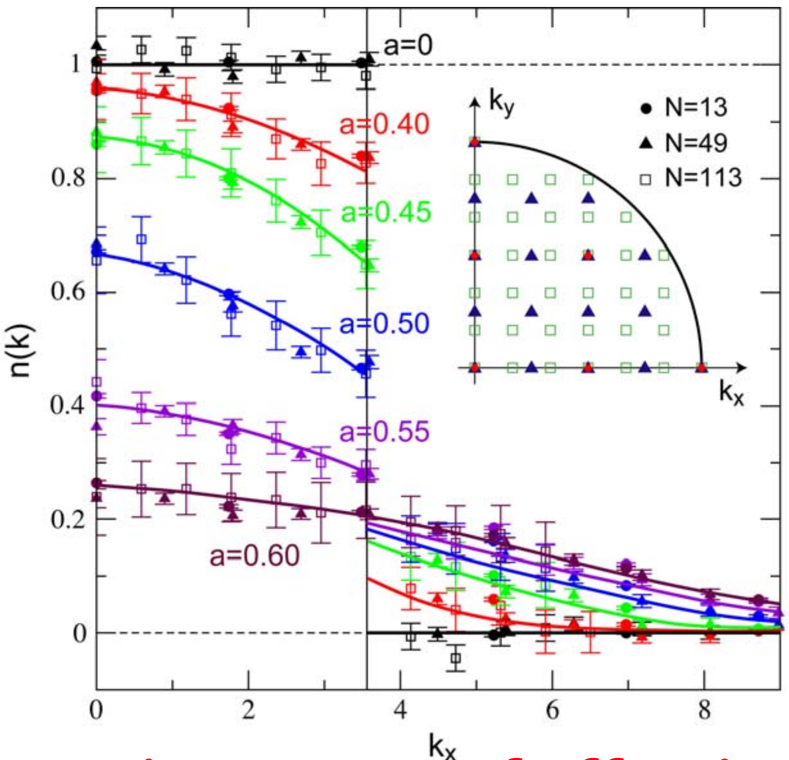
Geometrical correlation length

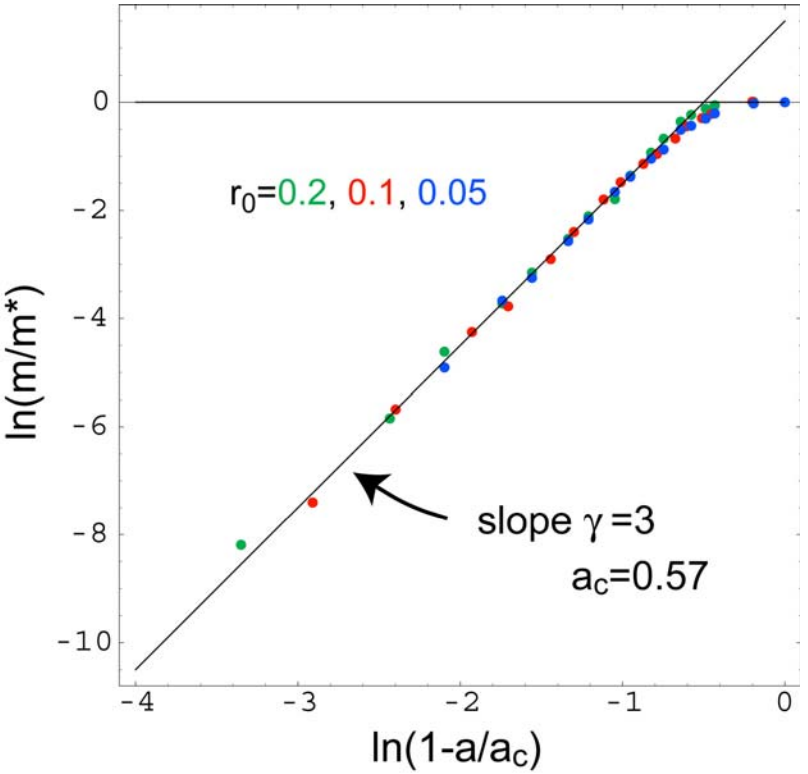


MC calculation of n(k)



$$\frac{m}{m^*} \propto \left(1 - \frac{a}{a_c}\right)^3$$





Divergence of effective mass as $a \rightarrow a_c$

The fixed point Hamiltonian

$$\psi_{bf}(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\tilde{\mathbf{r}}_j}\right)_{ij}$$

$$\tilde{\mathbf{r}}_j = \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

$$\Rightarrow \quad \left|k_1,\ldots,k_N\right\rangle_{bf} = \int_{q_1,\ldots,q_N} \Gamma_{q_1,\ldots,q_N} \left|k_1+q_1,\ldots,k_N+q_N\right\rangle_{bare}$$

turns singular at the QPT.

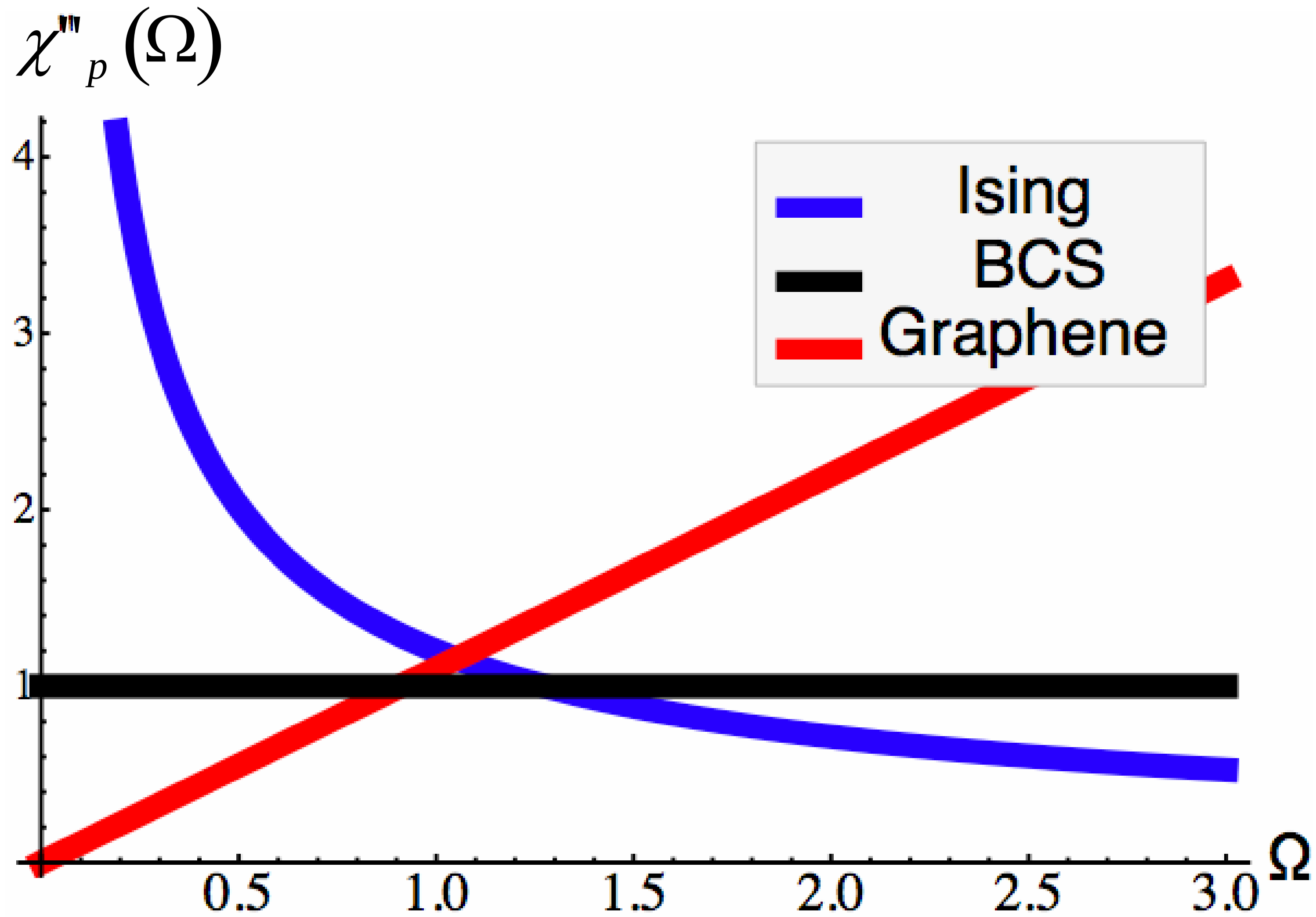
It is the ground state of a Fermi-gas of backflow particles: $H = \sum_k \varepsilon_k \widehat{c}_k^+ \widehat{c}_k$

Expressed in bare particles: $H \propto \sum_k \varepsilon_k c_k^+ c_k + \sum_{N=2}^{\infty} \left(\frac{a}{r_s}\right)^N \sum_{\{kq\}} f(\{k,q\}) \left(c_{\ldots}^+ c_{\ldots}\right)^N$

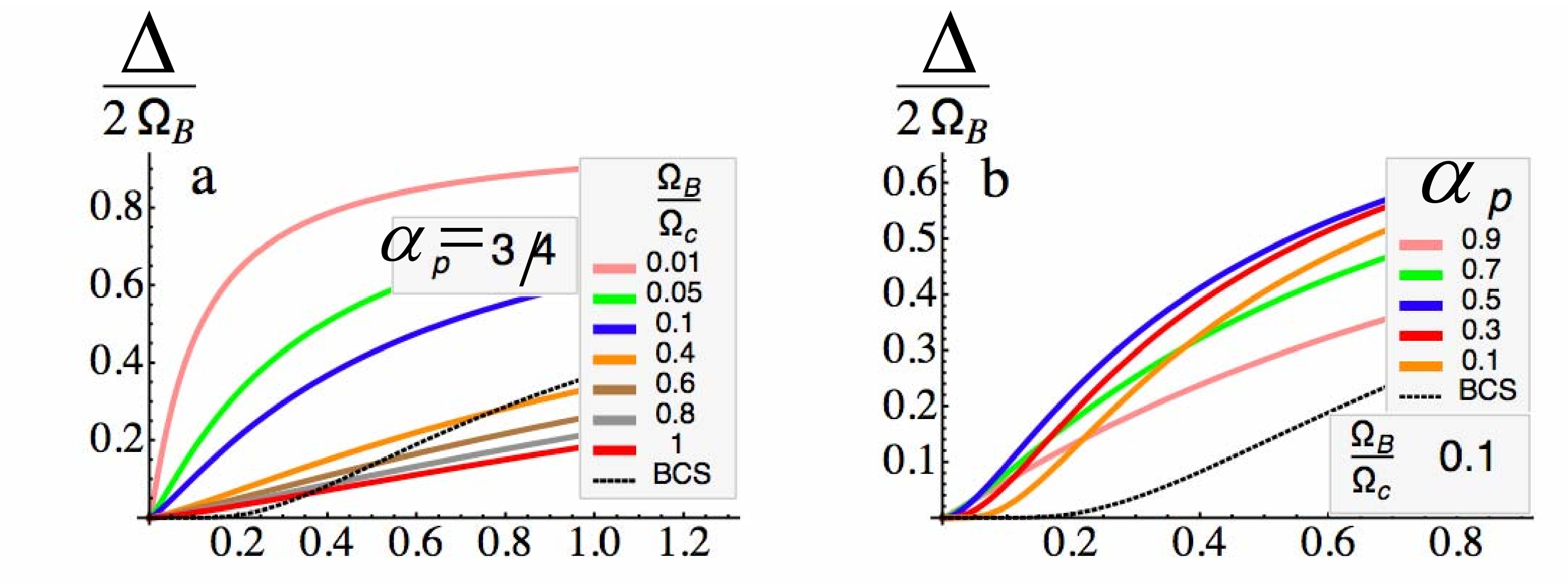
- At the critical point $a \to r_s$ the fixed point Hamiltonian reveals a divergence in N where N refers to N-body interaction!

- No symmetry change, criticality is entirely of 'statistical' nature (information in nodal surface)!

Pair susceptibilities



Huang's equation at work



$$\Delta_0 = 2\hbar\omega_B e^{-1/\lambda}$$

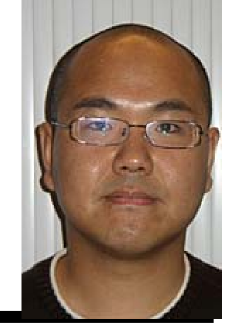
versus

$$\Delta_0 = 2\hbar\omega_B\left(\frac{\hat{\lambda}}{\hat{\lambda}+\left(2\omega_B/\omega_c\right)^{(2-\eta_{pp})/z}}\right)^{\frac{z}{2-\eta_{pp}}}$$

$$\lambda = \frac{V}{E_F}$$

$$\hat{\lambda} = 2\left(\frac{1-\alpha_p}{\alpha_p}\right)\left(\frac{V}{\omega_c}\right)$$

Huang's equation versus high Tc



J.-H. She

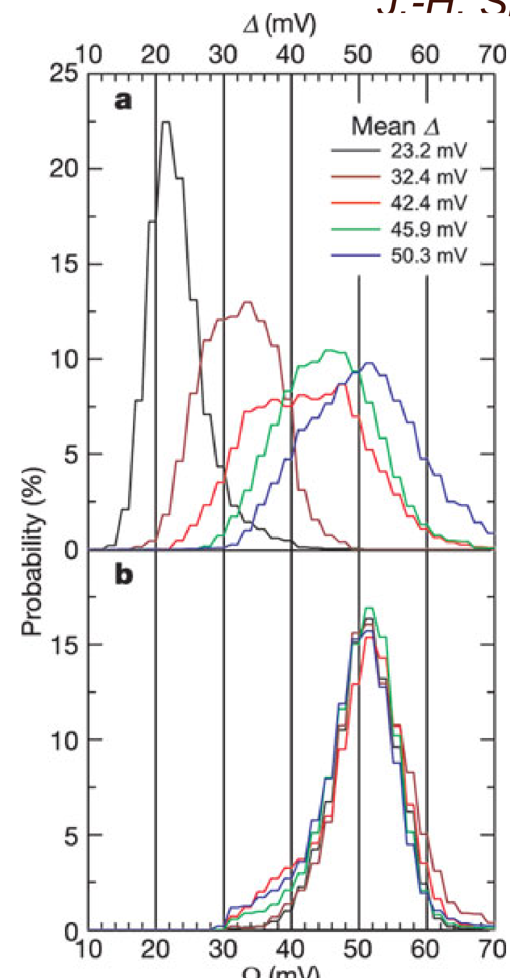
E.g. 1+1D Ising: $\eta_{pp} = 1/4, \quad z = 1$

Typical phonon-, cut-off energy: $\frac{\omega_B}{\omega_c} = \frac{50\ meV}{500\ meV}$

Typical gap: $\Delta_0 = 40\, meV$

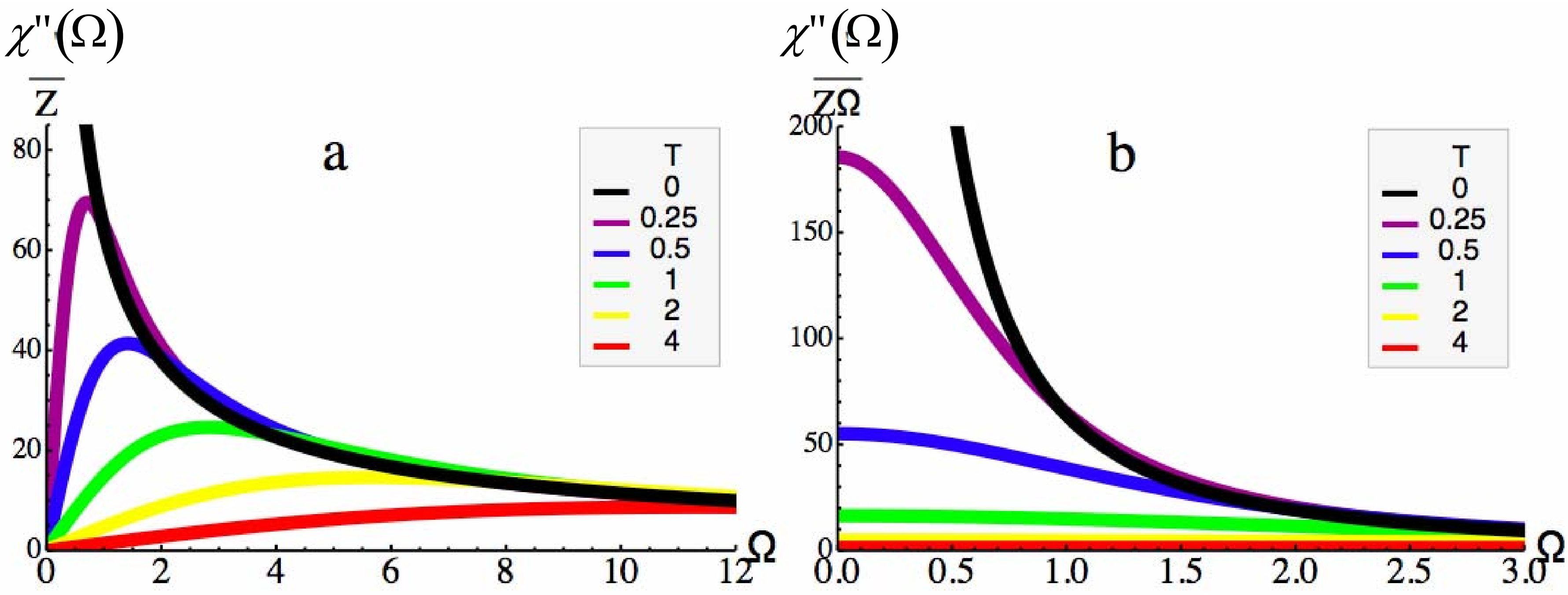
Fermi-liquid: $\lambda \approx 1.1$

Critical case: $\lambda \approx 0.4$!!!

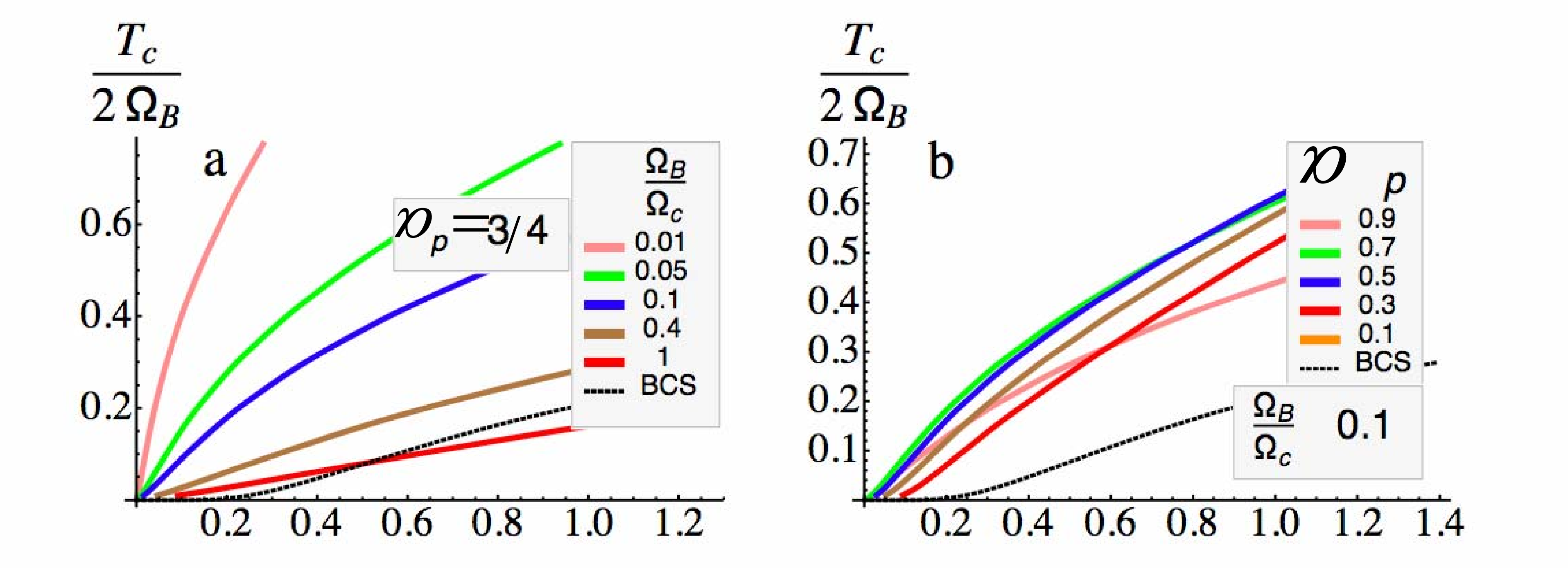


Davis, Balatsky

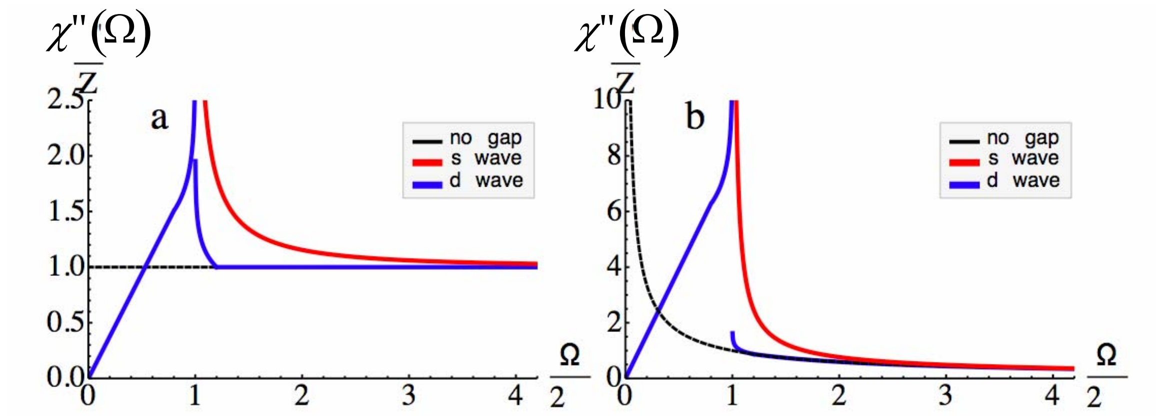
1+1D conformal finite temperature susceptibility

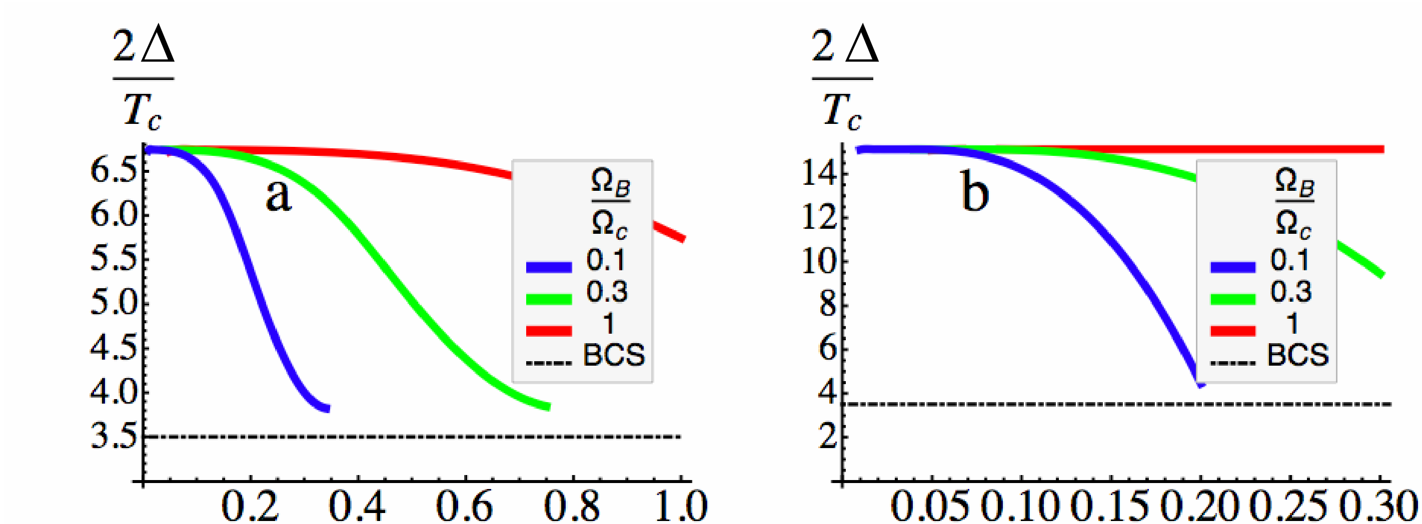


Tc using 1+1D conformal fields

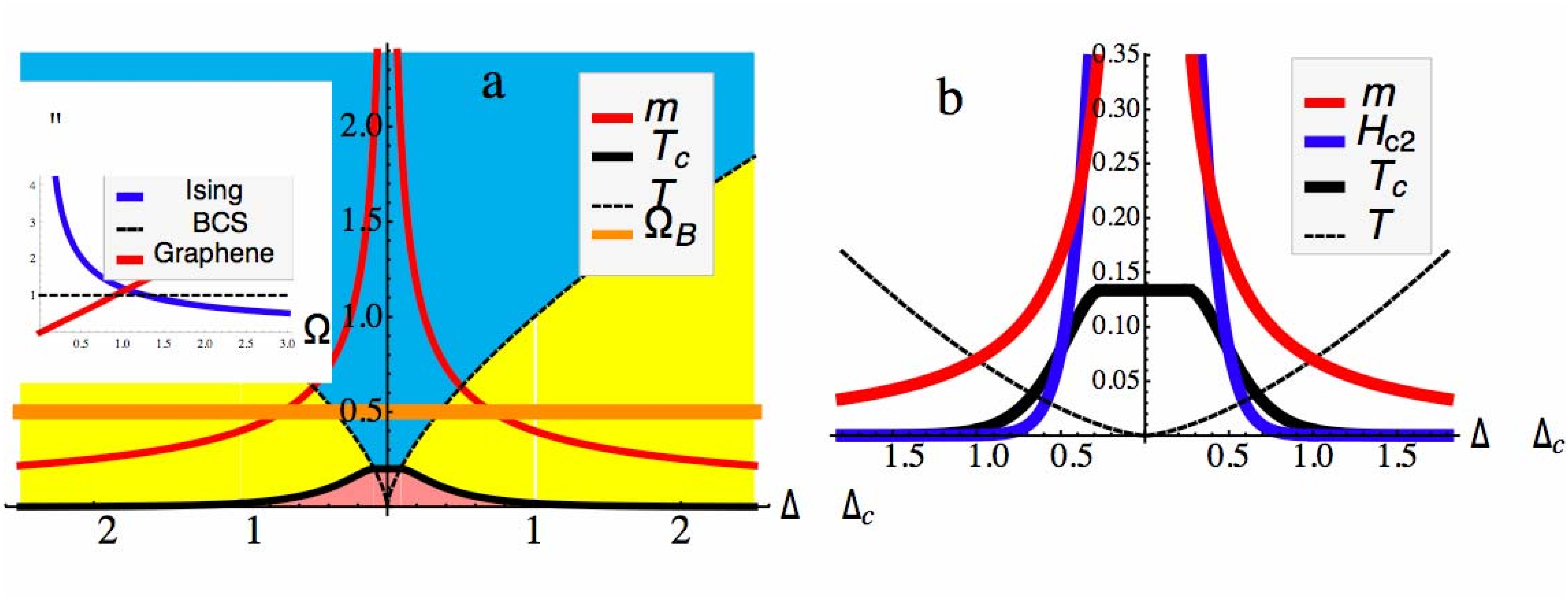


d/s wave and the gap to Tc ratio





SC domes and Hc2



Empty