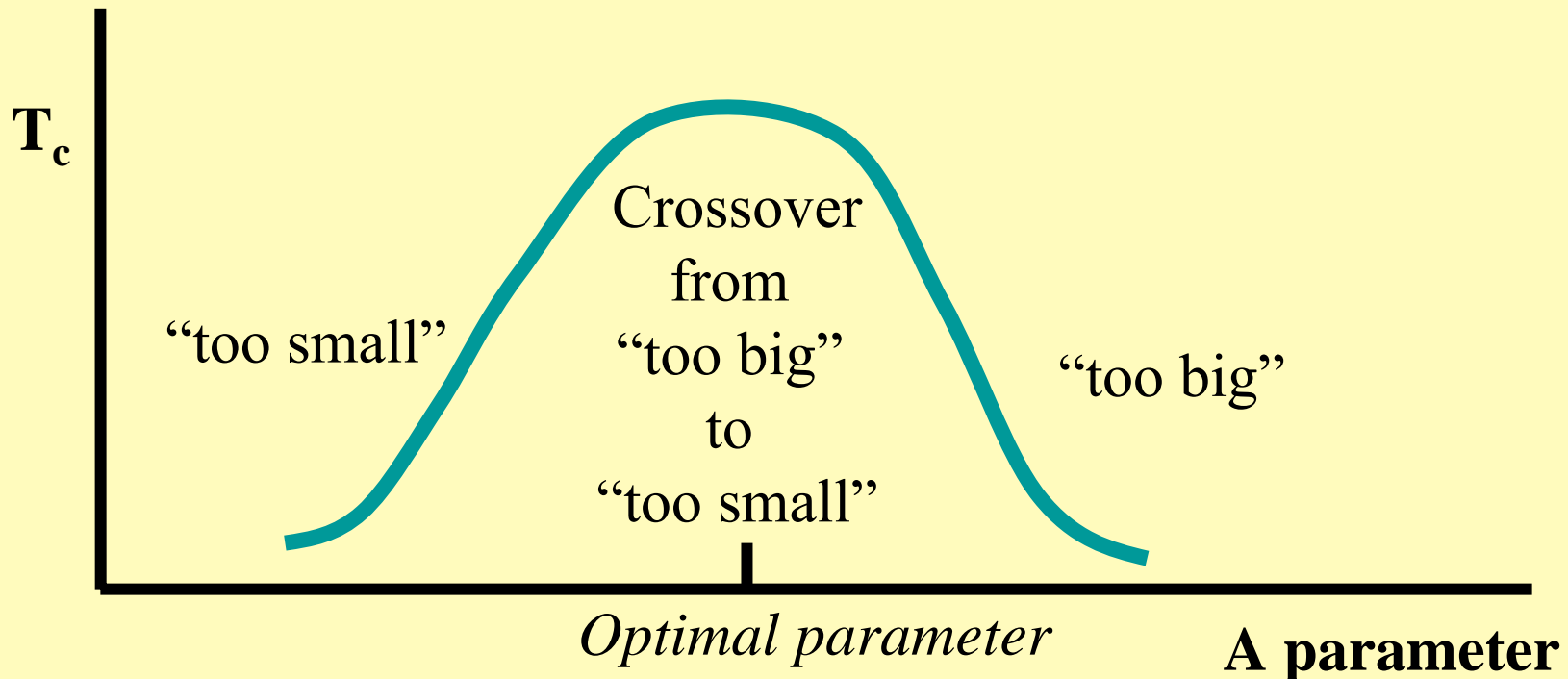


# High Temperature Superconductivity

is a

## Crossover Phenomenon

Steven Kivelson, Stanford



# “The Goldilocks Principle”

V.J. Emery and E.Fradkin,

H.Yao, W-F. Tsai, E. Berg,

S. White, G. Karakonstantakis

A.Kapitulnik, S. Chakravarty, S.Raghu

J.Tranquada, E-A.Kim, E.Carlson, D.Orgad

D.J.Scalapino, S.White, A.Lauchli

E.Arrigoni, I.Martin, D. Poldolsky, O Zachar

S.A.K. and E.Fradkin, “How optimal inhomogeneity produces high temperature superconductivity,” in "Treatise of High Temperature Superconductivity" edited by J. R. Schrieffer and J. Brooks

1) Unless the superconducting phase is cut off by a (first order) transition to a competing phase, optimal  $T_c$  occurs at a crossover from a pairing dominated regime -  $T_c \sim \Delta_0$  to a condensation regime -  $T_c \sim T_\theta$

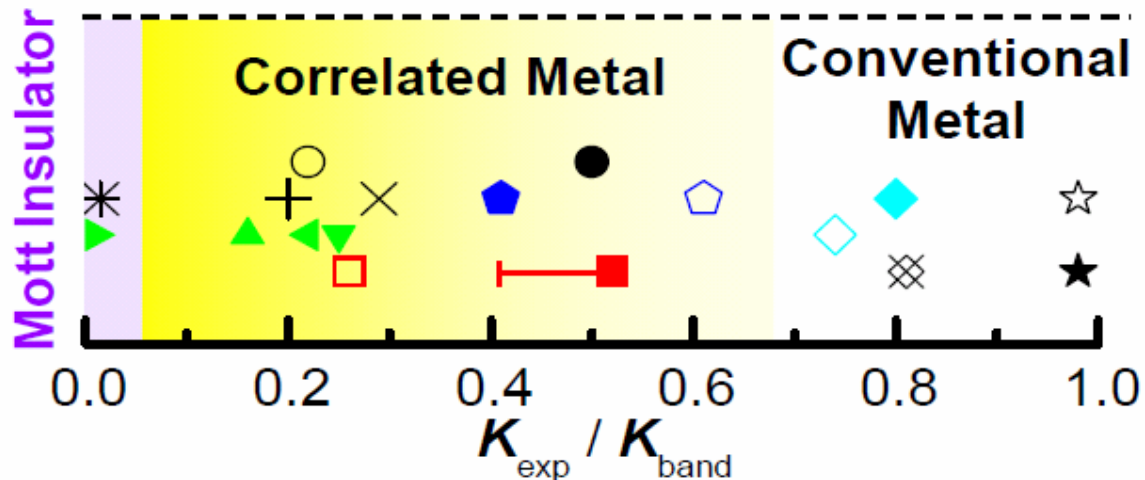
2) High temperature superconductivity occurs at intermediate coupling, where neither weak nor strong coupling approaches are well justified:  
 $U \sim$  Band-width.

3) Certain forms of nano-scale structuring of materials produce enhanced pairing (enhancement of  $\Delta_0$ ) at the expense of a reduction of  $T_\theta$ .  
There exists an “*optimal inhomogeneity*” for  $T_c$ .

# Electronic correlations in the iron pnictides

*Qazibash, Hamlin, Baurbach Zhang, Singh, Maple, and Basov*

- |   |                                    |
|---|------------------------------------|
| ■ LaFePO  | ● VO <sub>2</sub>                  |
| □ BaFe <sub>2</sub> As <sub>2</sub>                           | ○ V <sub>2</sub> O <sub>3</sub>    |
| ▶ La <sub>2</sub> CuO <sub>4</sub>                            | ◆ Sr <sub>2</sub> RuO <sub>4</sub> |
| ▲ La <sub>2-x</sub> Sr <sub>x</sub> CuO <sub>4</sub> (x=0.1)  | ◇ SrRuO <sub>3</sub>               |
| ◀ La <sub>2-x</sub> Sr <sub>x</sub> CuO <sub>4</sub> (x=0.15) | ◇ CrO <sub>2</sub>                 |
| ▼ La <sub>2-x</sub> Sr <sub>x</sub> CuO <sub>4</sub> (x=0.2)  | ◆ Cr                               |
| * Nd <sub>2</sub> CuO <sub>4</sub>                            | ⊗ MgB <sub>2</sub>                 |
| + Nd <sub>2-x</sub> Ce <sub>x</sub> CuO <sub>4</sub> (x=0.1)  | ★ Ag                               |
| × Nd <sub>2-x</sub> Ce <sub>x</sub> CuO <sub>4</sub> (x=0.15) | ☆ Cu                               |



Why is it that high temperatures superconductors are in an intermediate coupling regime?

Is it only to make the theorist's life complicated (and to keep us employed)?

Or is it that when we choose to study materials which are in some sense optimal superconductors, we have implicitly fine tuned the interaction strength to intermediate values?

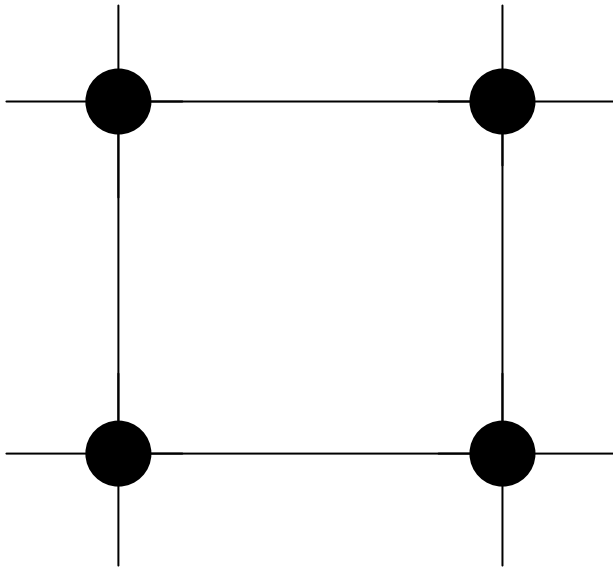
## Some solutions of model problems:

- 1) The negative U Hubbard model

*Crossover between pairing and coherence.*

# The Hubbard model

$$H = - \sum_{ij,\sigma} t_{ij} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_j c_{i,\uparrow}^\dagger c_{j,\downarrow}^\dagger c_{i,\downarrow} c_{j,\uparrow}$$



$U > 0$	Repulsive
$U < 0$	Attractive

$|U| \ll t$       BCS superconductivity

$$T_c \sim t \exp[ - \alpha t/|U| ]$$

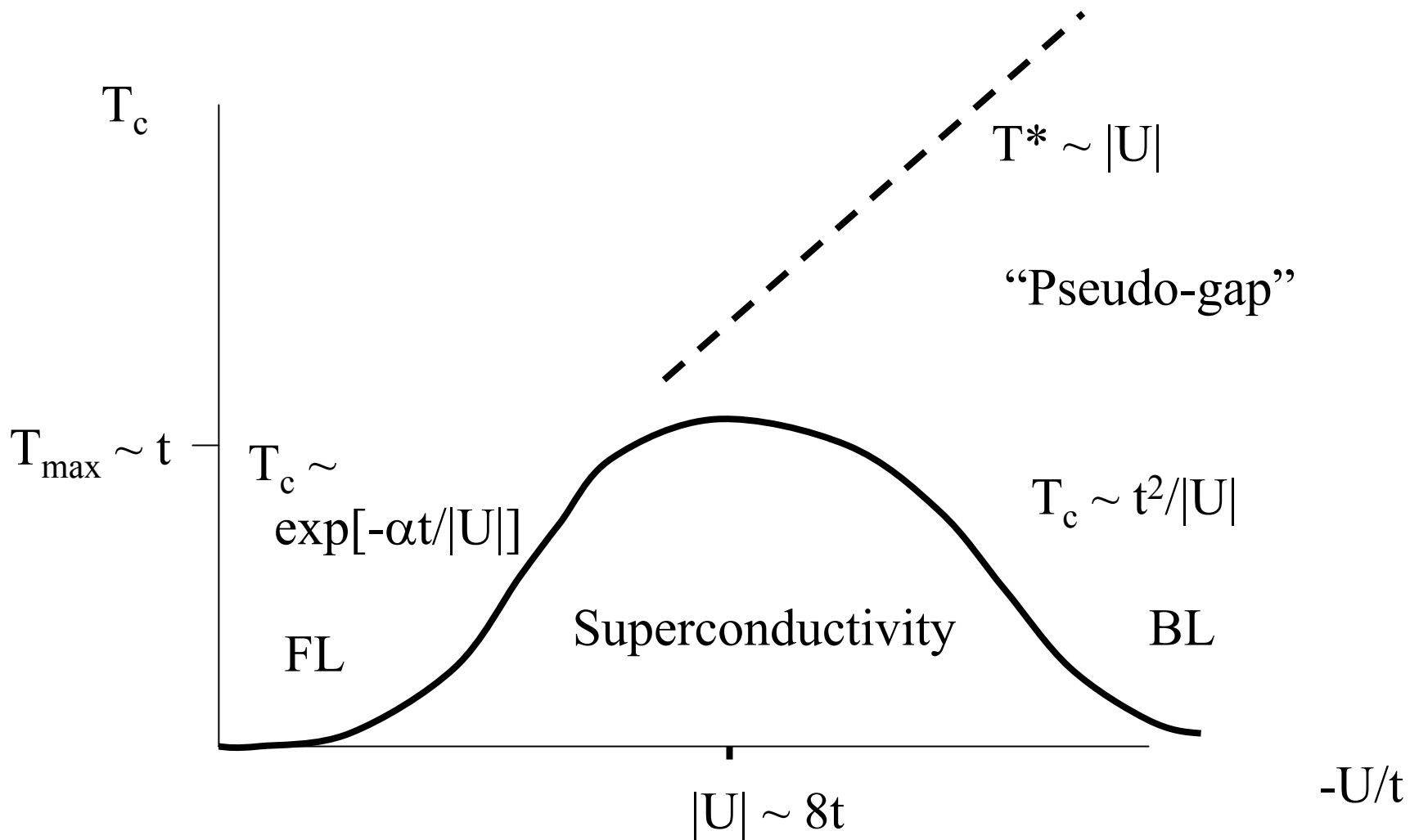
very mean-field like transition.

$|U| \gg t$       BEC superconductivity

pairs form at  $T \sim |U|$  (pseudo-gap)

$$T_c \sim \rho_s \sim t^2 / |U|$$





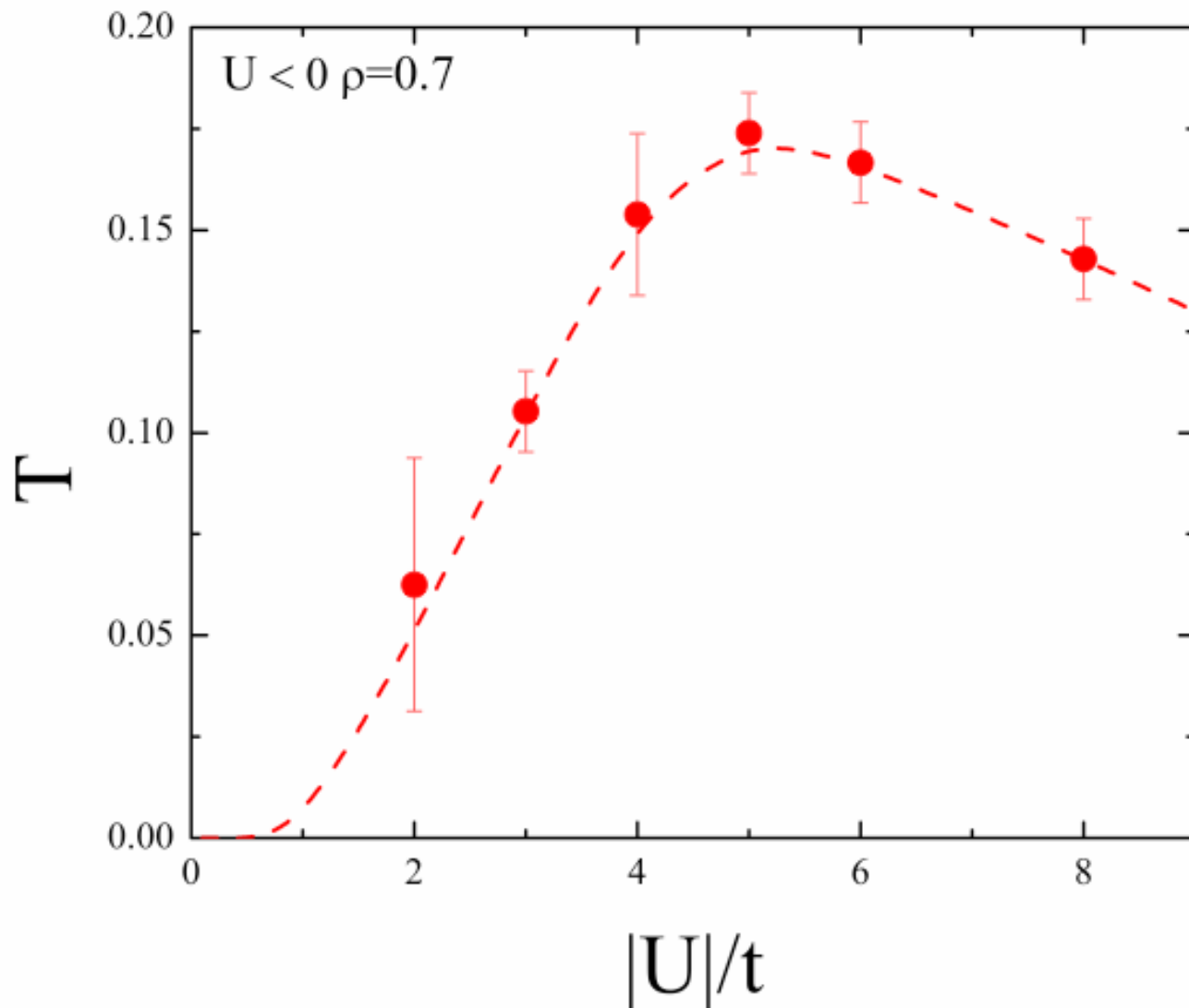
Maximal  $T_c$  occurs at a point of crossover in the physics:  
 crossover from pairing to phase ordering transition  
 crossover from weak coupling to strong coupling

## $T_c$ for 2D negative $U$ Hubbard model with $x=0.3$

*“Real” Monte Carlo version from Paiva, Scalettar, Randeria, Trivedi*

$$(U/t)_{\text{opt}} = 5$$

$$T_{\text{opt}} \approx t / 6$$



## Some solutions of model problems:

1) The negative U Hubbard model

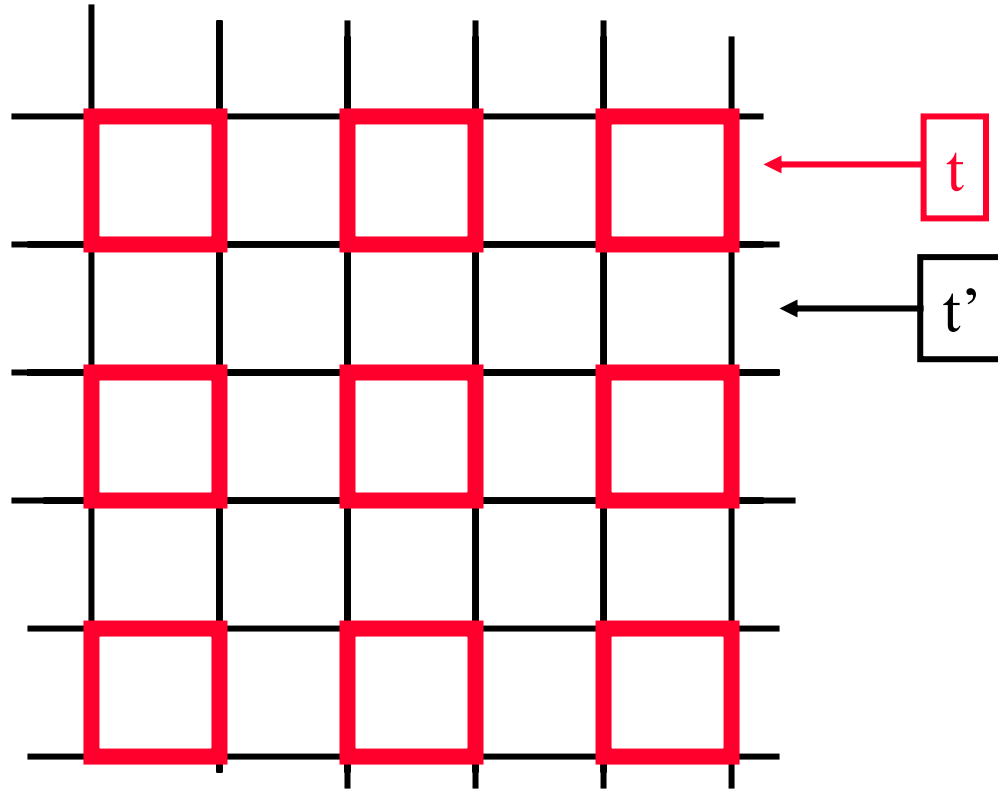
2) The “Checkerboard Hubbard model.”

*Crossover between too homogeneous  
and too inhomogeneous*

*Crossover between strong and weak  
coupling*

# Phase diagram of checkerboard Hubbard model

“Homogeneous” ( $t'/t=1$ ) to “highly inhomogeneous” ( $t'/t \ll 1$ )

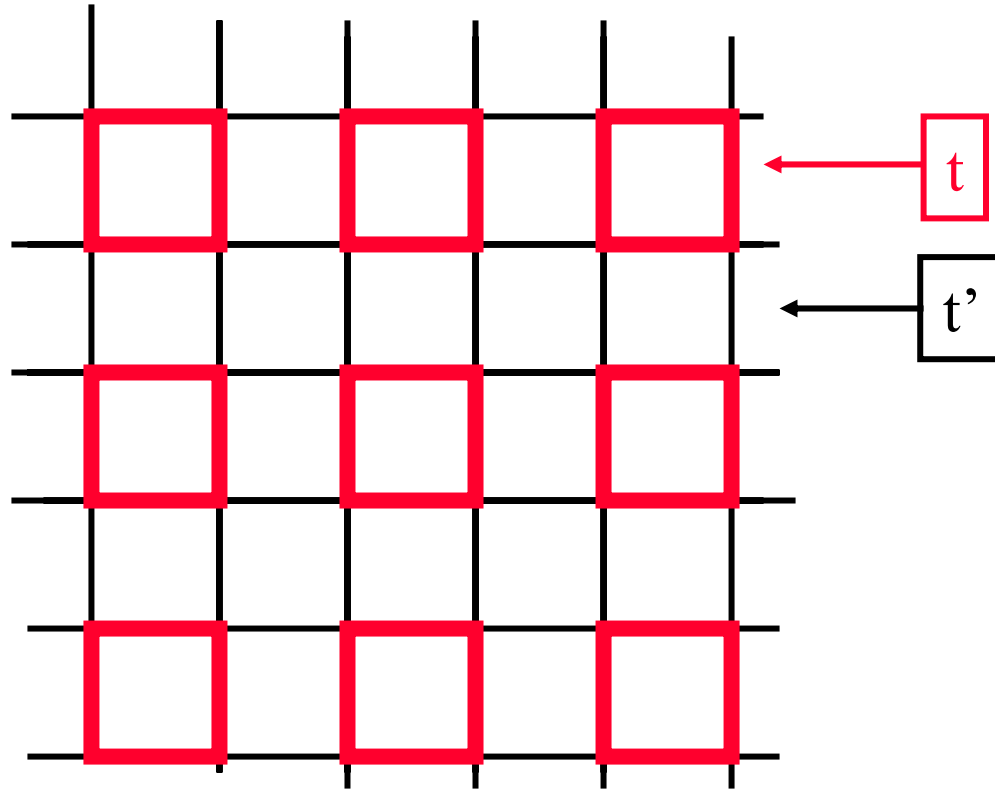


**The** Hubbard model for  $t = t'$

The Checkerboard Hubbard model for  $t > t'$

# Phase diagram of checkerboard Hubbard model

“Homogeneous” ( $t'/t=1$ ) to “highly inhomogeneous” ( $t'/t \ll 1$ )



As a function of  $U/t$  and  $t'/t$ , what are the conditions for the maximal  $T_c$ ?

Can “solve” the checkerboard Hubbard model if:

- 1)  $U/t \ll 1$  (weak coupling physics).  
*(unpublished results)*
- 2)  $N < 20$  by exact diagonalization.  
*(W.F.Tsai et al, PRB 2008)*
- 3) Limit of large inhomogeneity,  $t'/t \ll 1$ .  
*(W.F.Tsai and SAK, PRB 2006;  
H.Yao et al, PRB 2007  
Kocharian et al, 2005, 2006,2008)*
- 4) 1d (“checkerboard ladder”) using DMRG.  
*(E.Berg, G.Karakonstantakis, S.White, SAK)*
- 5) Variational and mean-field (DMFT) approaches,  
CORE, ...  
*(Altman and Auerbach, PRB (2002)  
Jarrell, Maier et al,PRB (2008)*

## Numerical studies of the 16 site Checkerboard Hubbard model:

Pair-binding energy:  $E_{\text{pair}} = 2 E(2n+1) - E(2n) - E(2n+2)$

$E_{\text{pair}} \longrightarrow 2\Delta_{\text{min}}$  as  $N \longrightarrow \infty$  in gapped SC

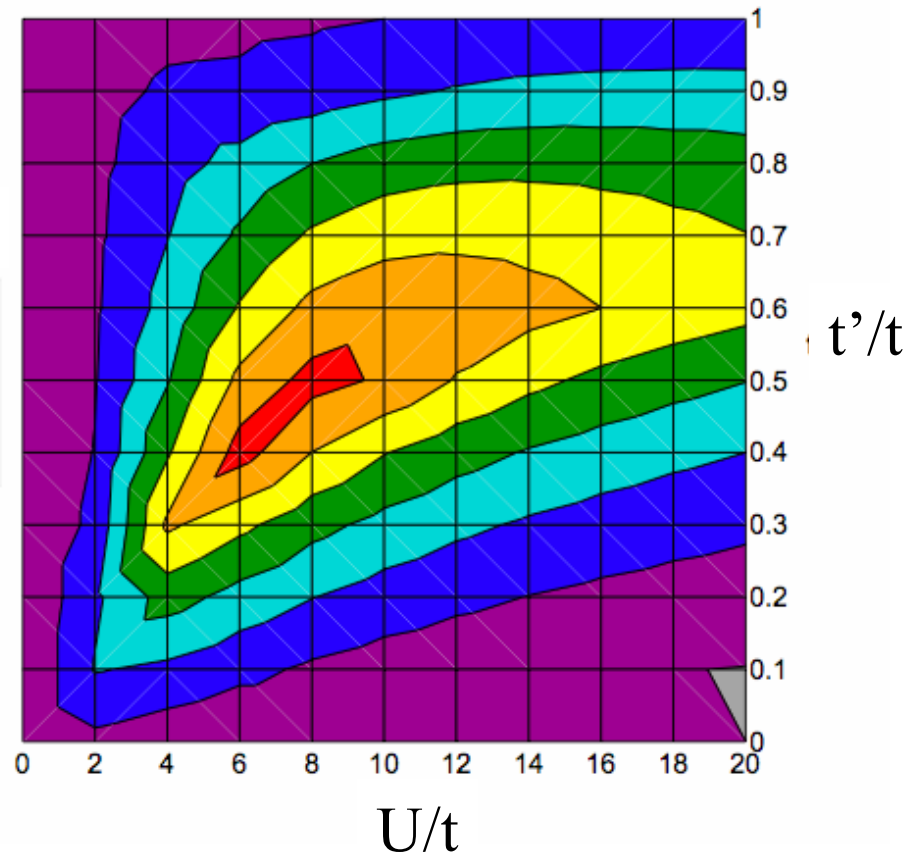
$E_{\text{pair}} \sim 2\Delta_0 N^{-1}$  as  $N \longrightarrow \infty$  in nodal SC

Spin-gap:  $E_{\text{SG}} = E(S=1) - E(S=0)$

Finite size scaling is not possible, but  
sensitivity to boundary conditions is:  
Periodic vs Twisted Periodic B.C.

# Pair-binding energy 16 site checkerboard model with PBC

$E_{\text{pair}}/t$



Max occurs at

$$t' \approx t/2$$

$$U \approx 8t$$

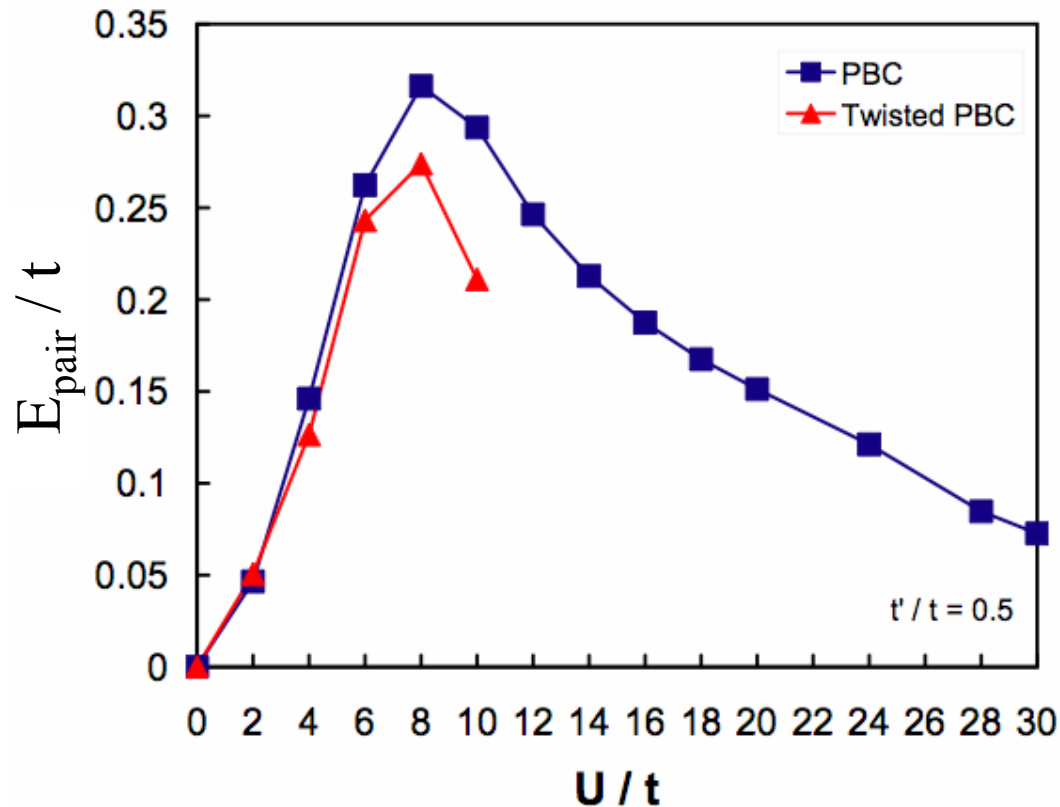
$$E_{\text{pair}} \approx t/3$$

$$(T_{\text{MF}} \sim t/12)$$

$$E_{\text{pair}}(x=1/16) = 2 E(15) - E(16) - E(14)$$



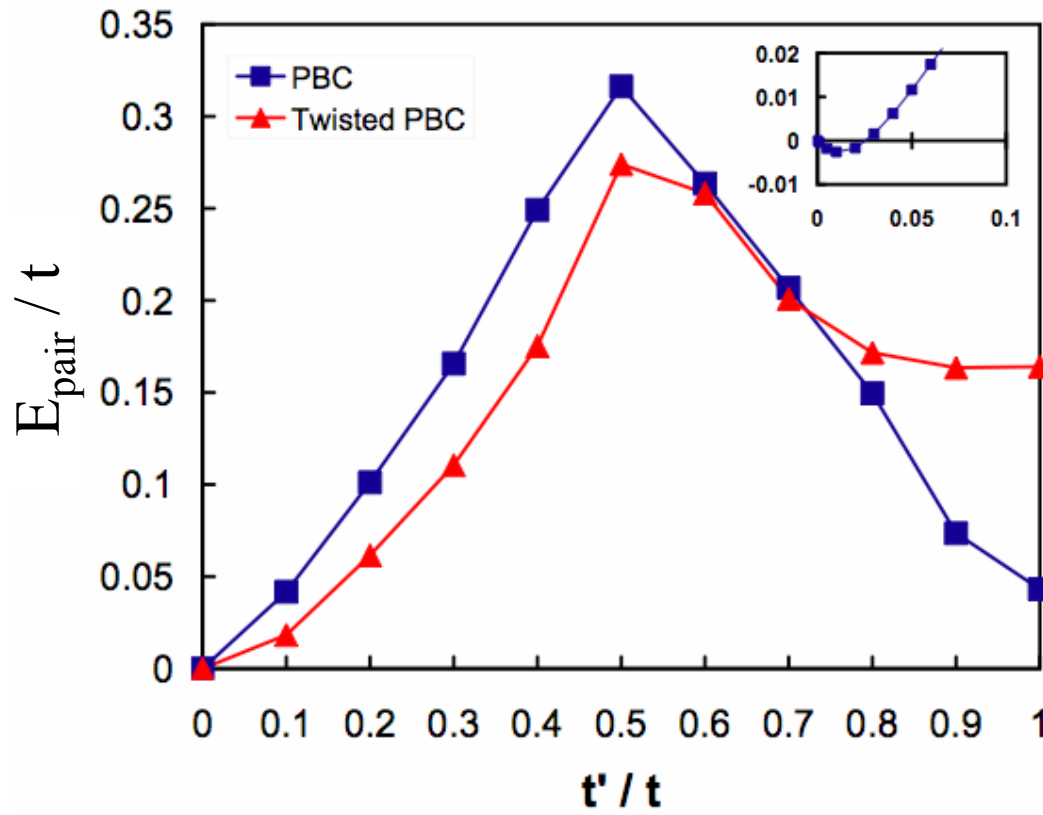
# Pair-binding energy 16 site checkerboard model with $t'=t/2$



$$E_{\text{pair}} = 2 E(15) - E(14) - E(16)$$

High temperature superconductivity occurs  
at a crossover between weak and strong  
coupling regime.

# Pair-binding energy 16 site checkerboard model with $U=8t$



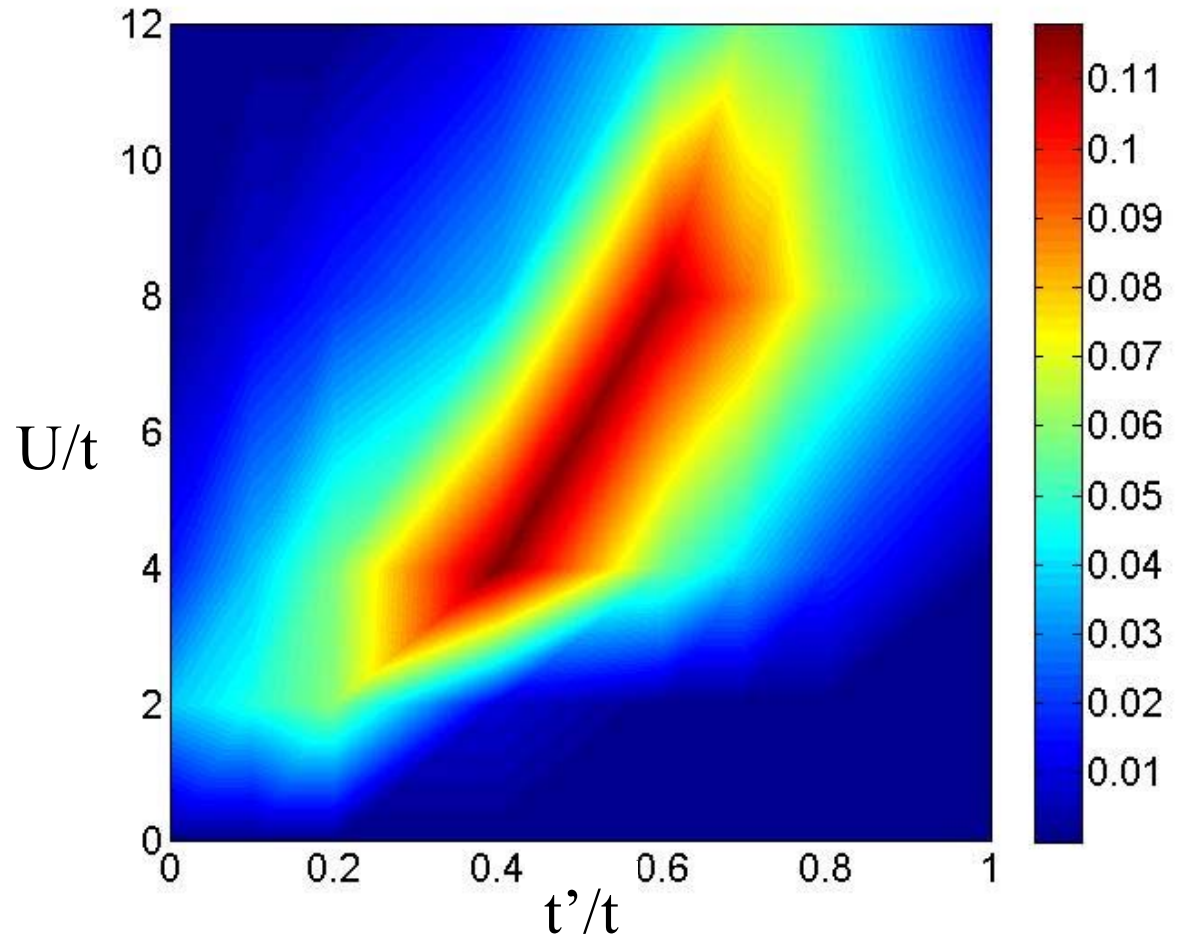
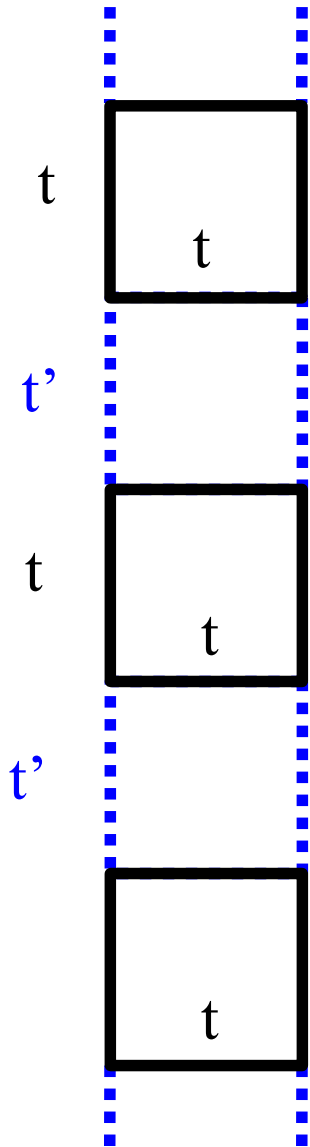
$$E_{\text{pair}} = 2 E(15) - E(14) - E(16)$$

High temperature superconductivity occurs  
at a crossover between too inhomogeneous  
and insufficiently structured.

# Checkerboard Hubbard ladder (DMRG results extrapolated $N$ to $\infty$ )

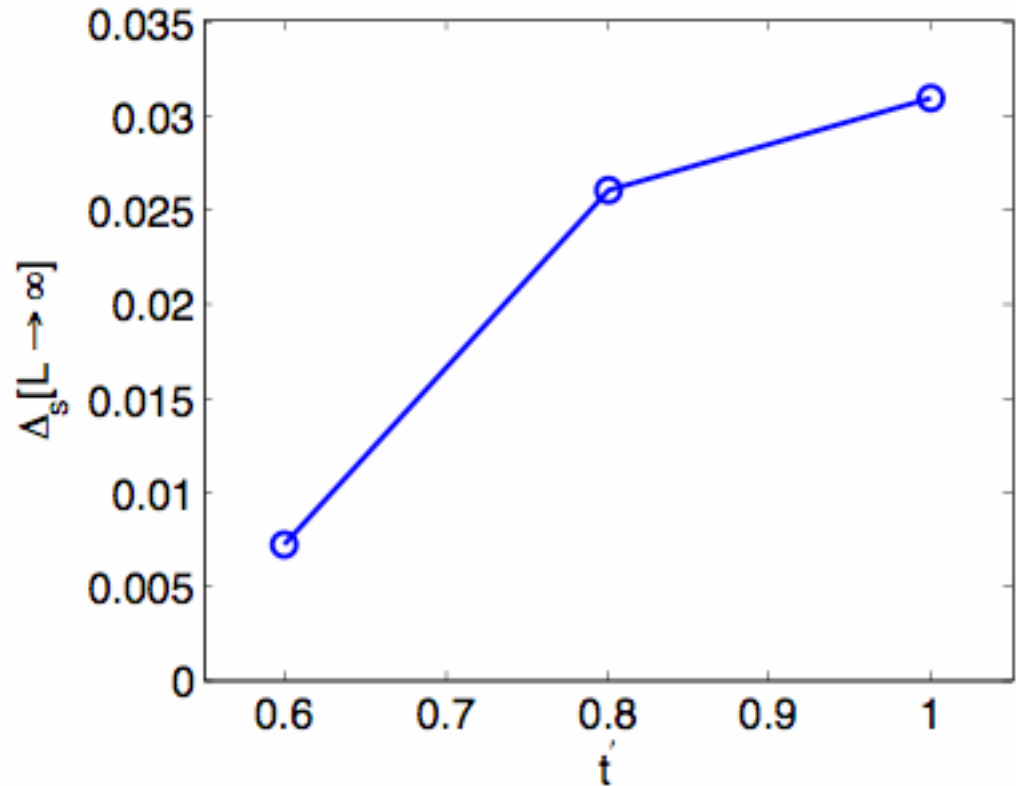
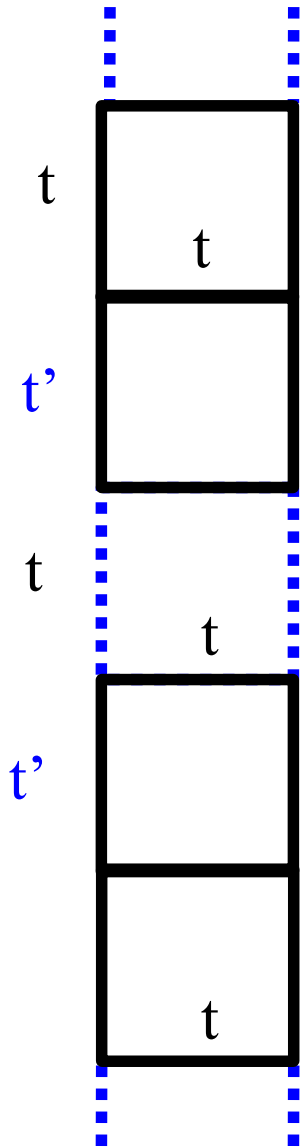
(Results for up to  $2 \times 64$ )

**Spin gap for  $n = 0.875$**



# Other patterned Hubbard ladder (DMRG results extrapolated $N$ to $\infty$ )

Spin gap, optimal for  $t' = t$ .



Checkerboard Hubbard model proves many points of principle.

Can get superconductivity directly from strong repulsion between electrons

Highly non-BCS **mechanism** of SC - no well defined phonon (or any other well defined boson) exchanged, and (typically) no FL “normal” state.

D-wave superconductivity emerges naturally from lattice geometry and strong repulsion.

*(Whether RVB or SFE)*

If there is an “optimal inhomogeneity” for HTC.

Then “stripes” may be essential - a form of self-organized inhomogeneity.

Mesoscale structure of another kind is demonstrably important  $T_c$  rises for  $n=1$  to 2 to 3, then drops for  $n=4$  to 5 ... ( $n$ =number of layers)

Search for ways of making inhomogeneous systems with high pairing regions and highly coherent (itinerant) regions.

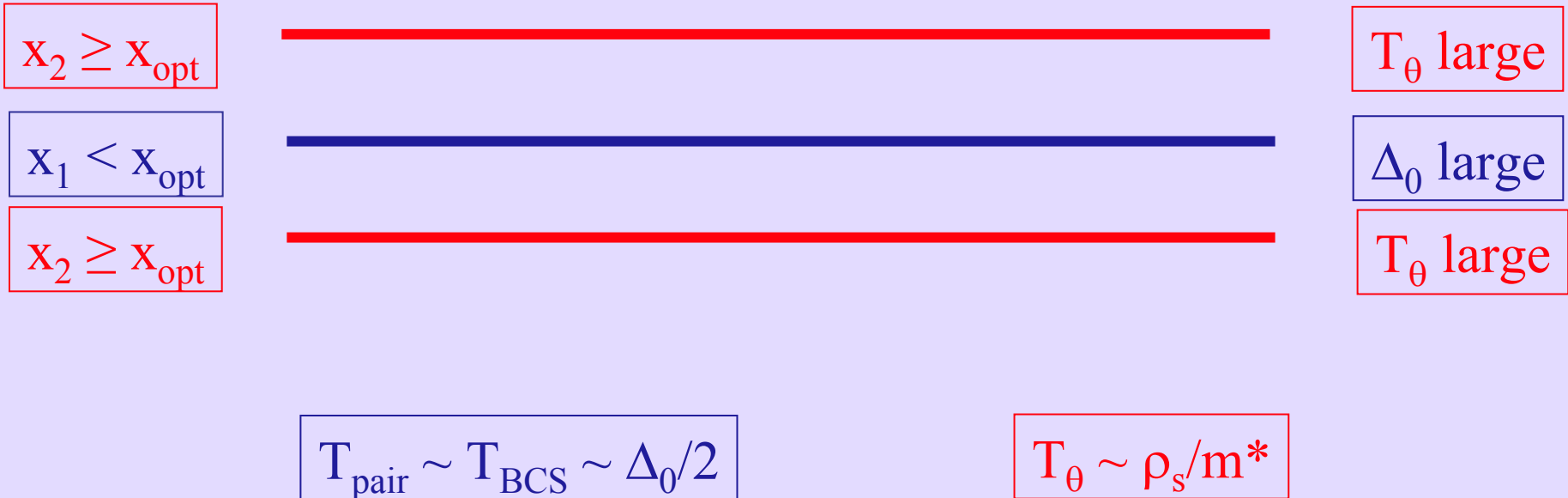


# How to make High $T_c$ higher - a theoretical proposal

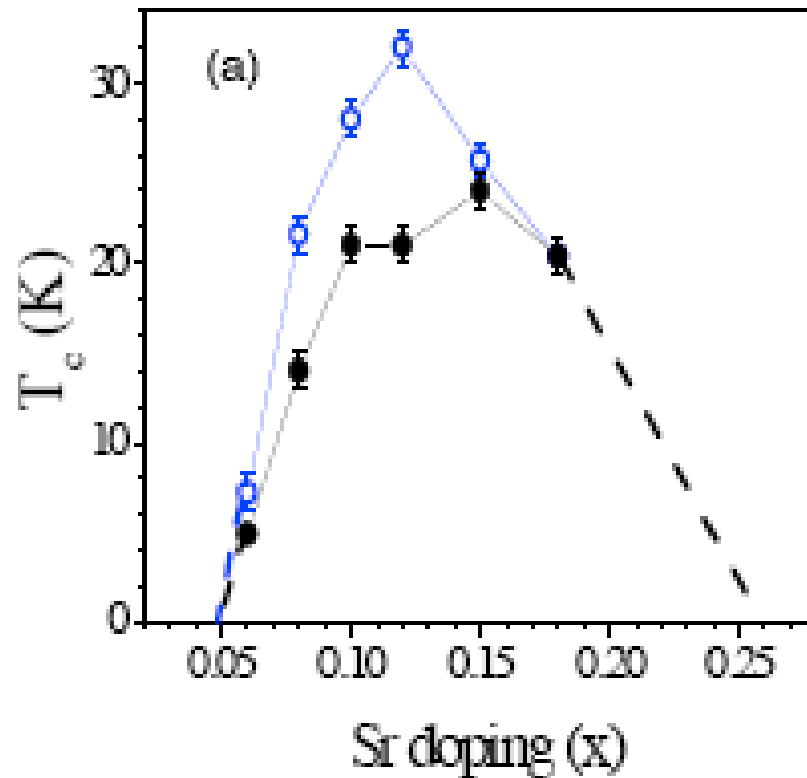
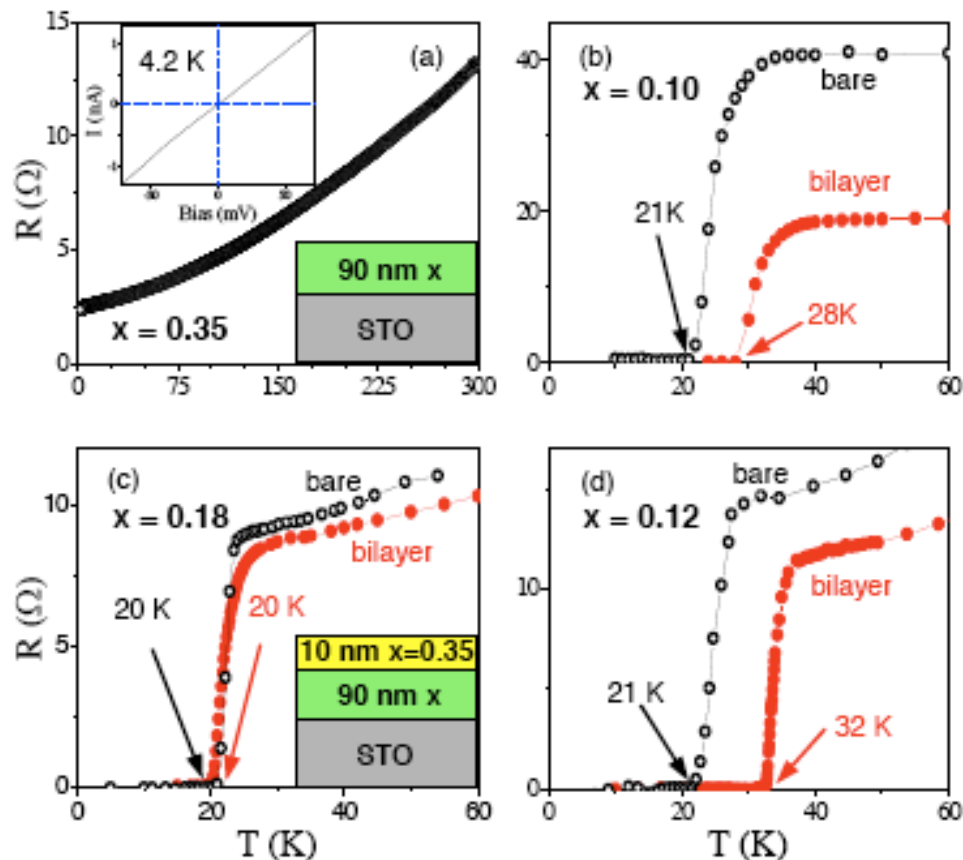
Physica B 318, 61-67 (2002).

Suppose it is true that in underdoped cuprates,  $T_{\text{pair}} \gg T_c$ .

Then, to enhance  $T_c$ , we need to enhance  $T_\theta$ .



# Enhancement of the superconducting transition temperature in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ bilayers



Ofer Yuli, Itay Asulin, Leonid Iomin, Gad Koren, Oded Millo, Dror Orgad arXiv:0805.0405

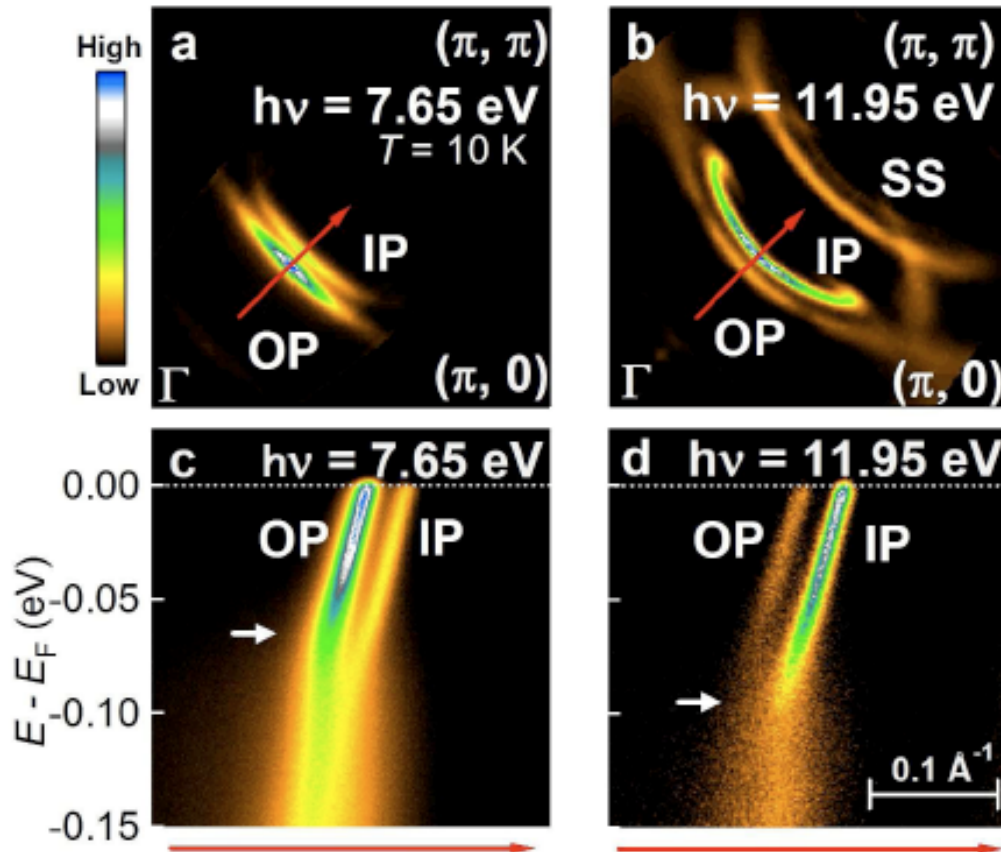
Similar results by Gozar, Logvenov, Kourkoutis, Bollinger, Giannuzzi, Muller, and Bozovic, arXiv:0810.1890  
 However, role of strain complicates the interpretation of both of these results.

Do these considerations have any  
relation to reality?

# Enhanced superconducting gaps in the tri-layer high- $T_c$ cuprate $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$

*Ideta, Takashima, Hashimoto, Yoshida, Fujimori, Anzai, Fujita, Nakashima,  
Ino, Arita, Namatame, Taniguchi, Ono, Kubota, Lu, Shen, Kojima, Uchida*

**ArXiv - 0905.1223**



From the area enclosed by the Fermi surface -

$$x_{\text{IP}} = 6\%$$

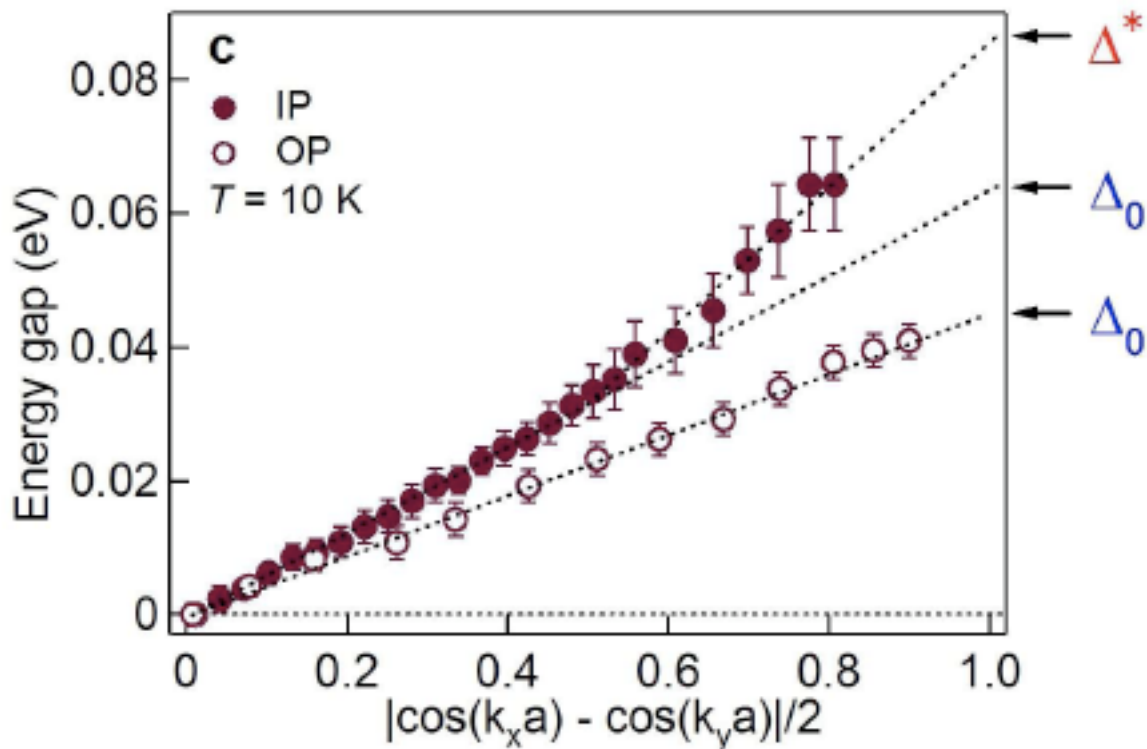
$$x_{\text{OP}} = 26\%$$

$$T_c = T_{\text{opt}} = 110\text{ K}$$

# Enhanced superconducting gaps in the tri-layer high- $T_c$ cuprate $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$

*Ideta, Takashima, Hashimoto, Yoshida, Fujimori, Anzai, Fujita, Nakashima, Ino, Arita, Namatame, Taniguchi, Ono, Kubota, Lu, Shen, Kojima, Uchida*

**ArXiv - 0905.1223**



From the area enclosed by the Fermi surface -

$$x_{\text{IP}} = 6\%$$

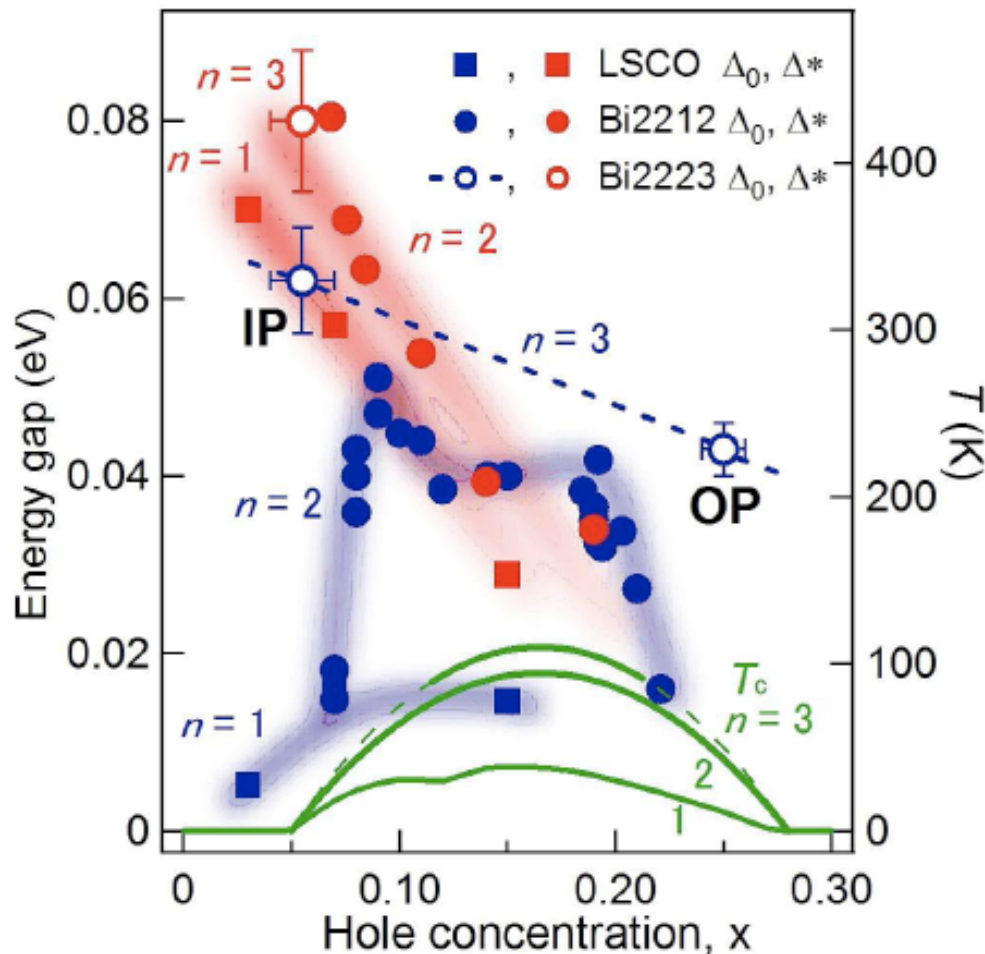
$$x_{\text{OP}} = 26\%$$

$$T_c = T_{\text{opt}} = 110\text{K}$$

# Enhanced superconducting gaps in the tri-layer high- $T_c$ cuprate $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$

*Ideta, Takashima, Hashimoto, Yoshida, Fujimori, Anzai, Fujita, Nakashima, Ino, Arita, Namatame, Taniguchi, Ono, Kubota, Lu, Shen, Kojima, Uchida*

**ArXiv - 0905.1223**



From the area enclosed by the Fermi surface -

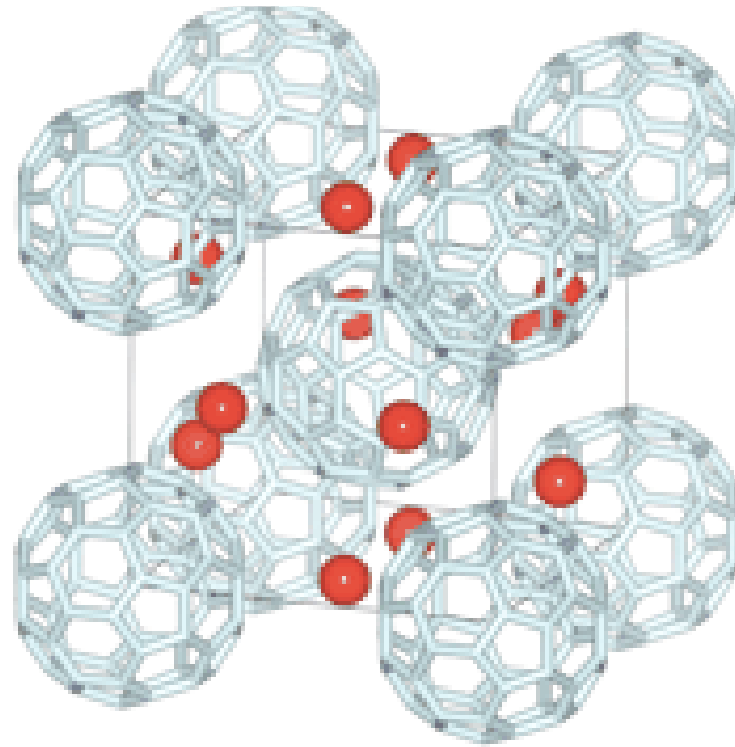
$$x_{\text{IP}} = 6\%$$

$$x_{\text{OP}} = 26\%$$

$$T_c = T_{\text{opt}} = 110\text{K}$$

# The Disorder-Free Non-BCS Superconductor $\text{Cs}_3\text{C}_{60}$ Emerges From An Antiferromagnetic Insulator Parent State

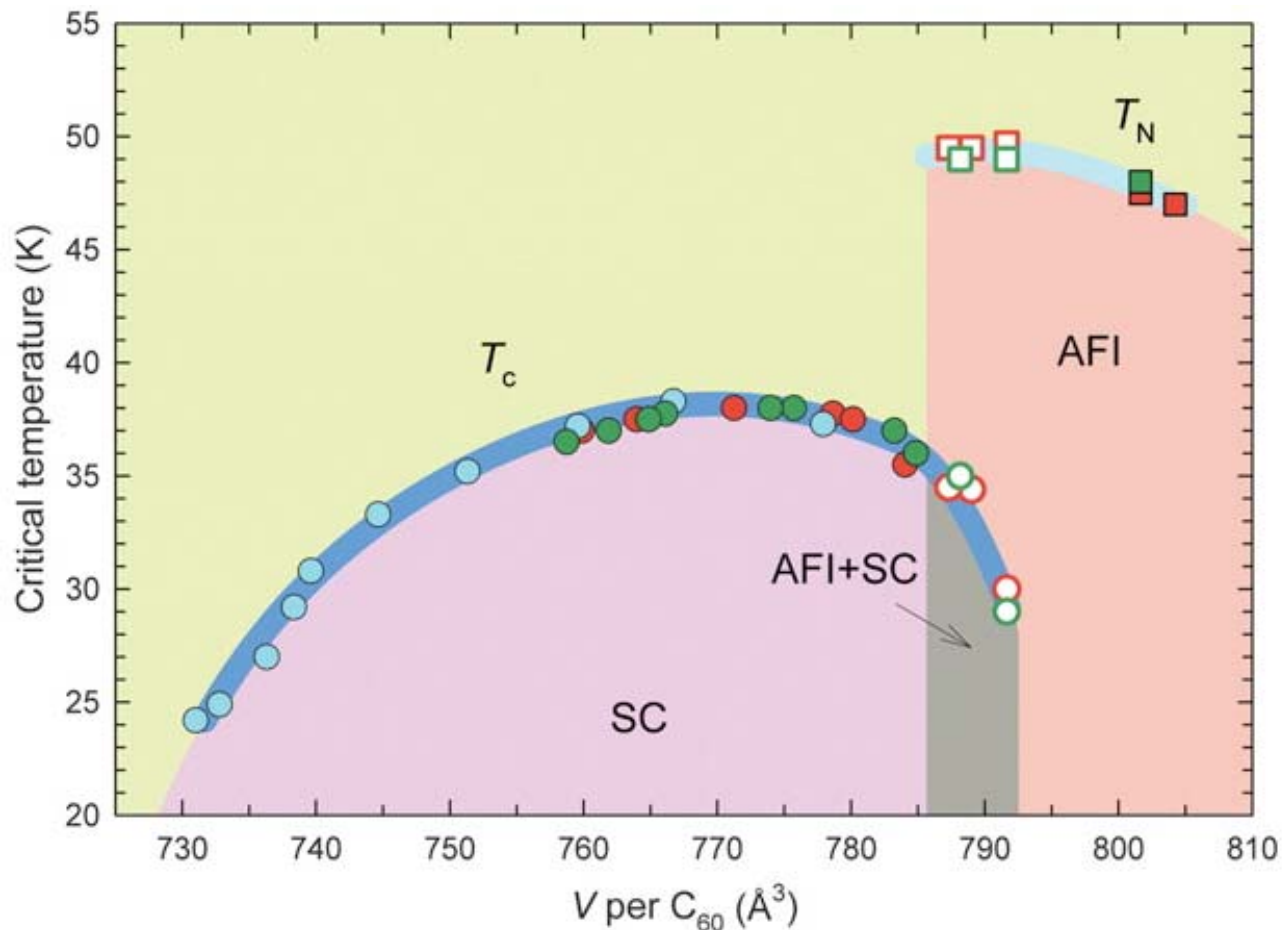
*Takabayashi, Ganin, Jeglic, Arcon, Takano, Iwasa, Ohishi, Takata, Prassides, Rosseinsky, Science 323, 1585 (2009)*



$\text{C}_{60}$  molecule is a mesoscale structure which generates effective pairing correlations.

# The Disorder-Free Non-BCS Superconductor $\text{Cs}_3\text{C}_{60}$ Emerges From An Antiferromagnetic Insulator Parent State

*Takabayashi, Ganin, Jeglic, Arcon, Takano, Iwasa, Ohishi, Takata, Prassides, Rosseinsky, Science 323, 1585 (2009)*





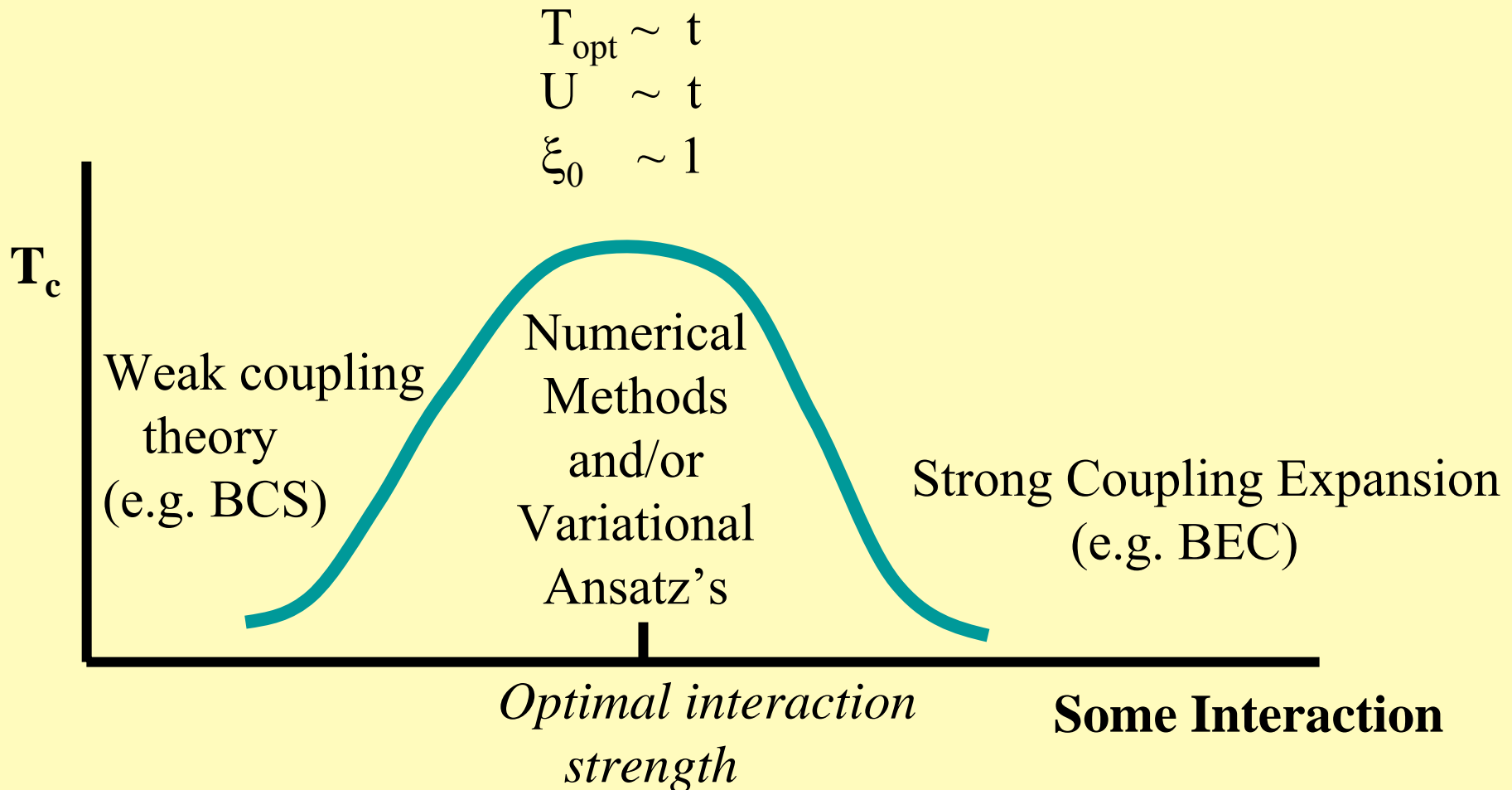
1) Optimal  $T_c$  occurs at a crossover from  
a pairing dominated regime -  $T_c \sim \Delta_0$   
to a condensation regime -  $T_c \sim T_\theta$

2) High temperature superconductors occur at  
a crossover from strong to weak coupling  
regimes.

3) Special forms of mesoscale structure (“optimal  
inhomogeneity”) is favorable for high temperature  
superconductivity.

4) Composite systems consisting of coupled regions of  
strong pairing and large Drude weight  
can produce an optimal combination of large  $\Delta_0$  and  $T_\theta$ .

# HTS is a Crossover Phenomenon: Implications for theory



The end