

Beyond $U = \infty$: Hidden Charge $2e$ boson

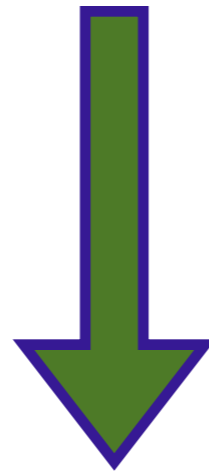
Thanks to: T.-P. Choy, R. G. Leigh, S. Chakraborty
PRL, 99, 46404 (2007);
PRB, 77, 14512 (2008); *ibid*, 77, 104524 (2008));
ibid, 79, 245120 (2009);...., DMR/NSF-ACIF

Mott gap

dynamical spectral
weight transfer

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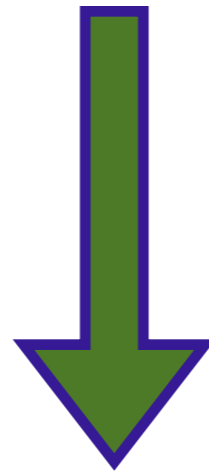


mediated by

composite or bound states not in UV theory

Mott gap

dynamical spectral
weight transfer



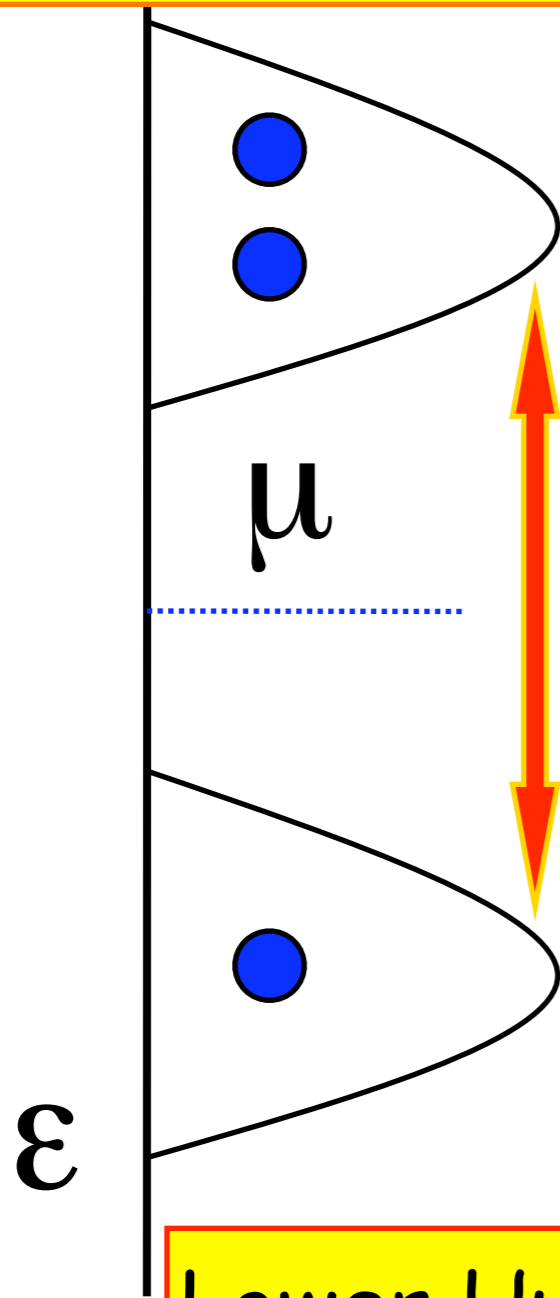
mediated by

composite or bound states not in UV theory

Strong Coupling

First Problem: Mott gap

Upper Hubbard band



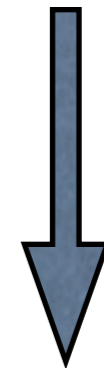
Lower Hubbard band

$$c_{i\sigma} n_{i-\sigma}$$

U

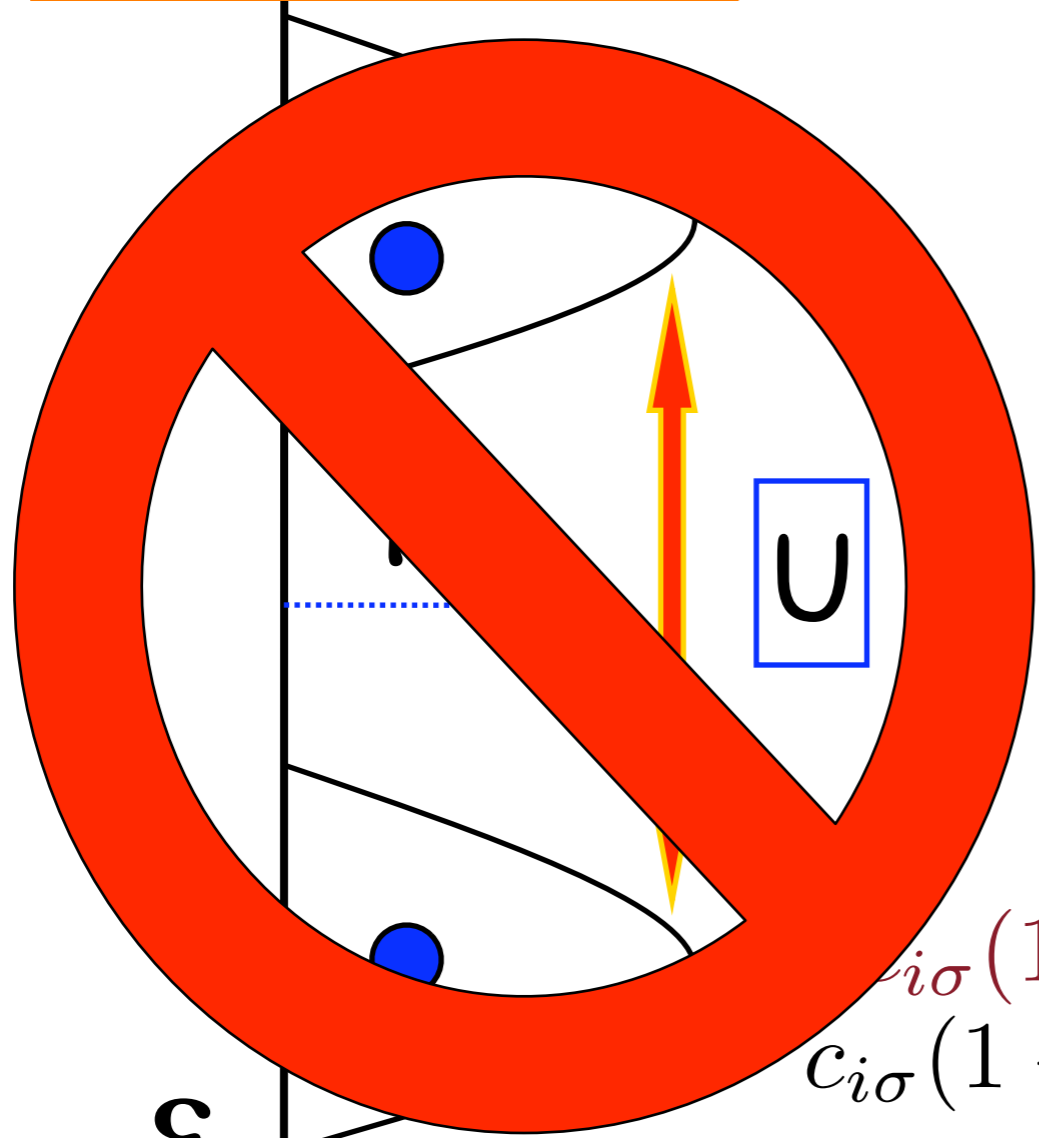
$$c_{i\sigma} (1 - n_{i-\sigma})$$
$$c_{i\sigma} (1 - n_{i\sigma} n_{i-\sigma})$$

do not propagate independently



not the origin of the gap

Upper Hubbard band

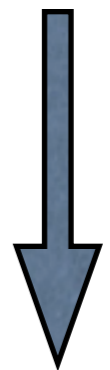


Lower Hubbard band

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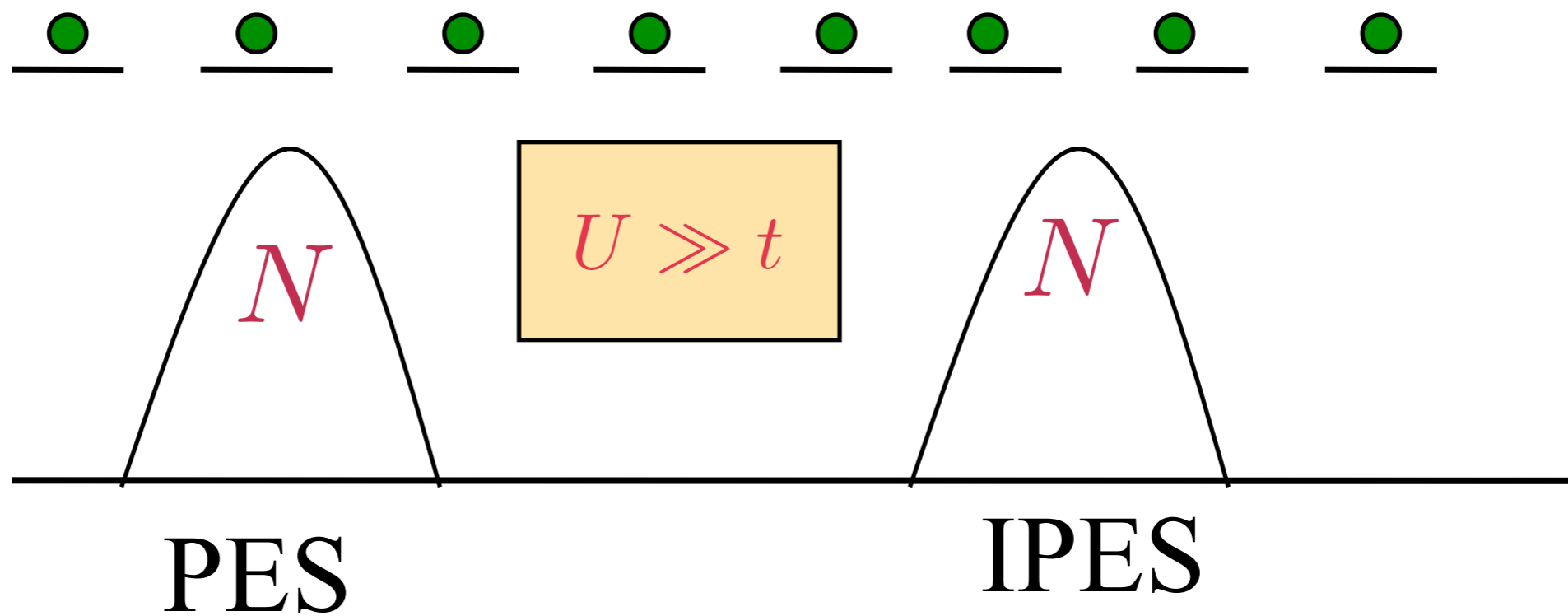
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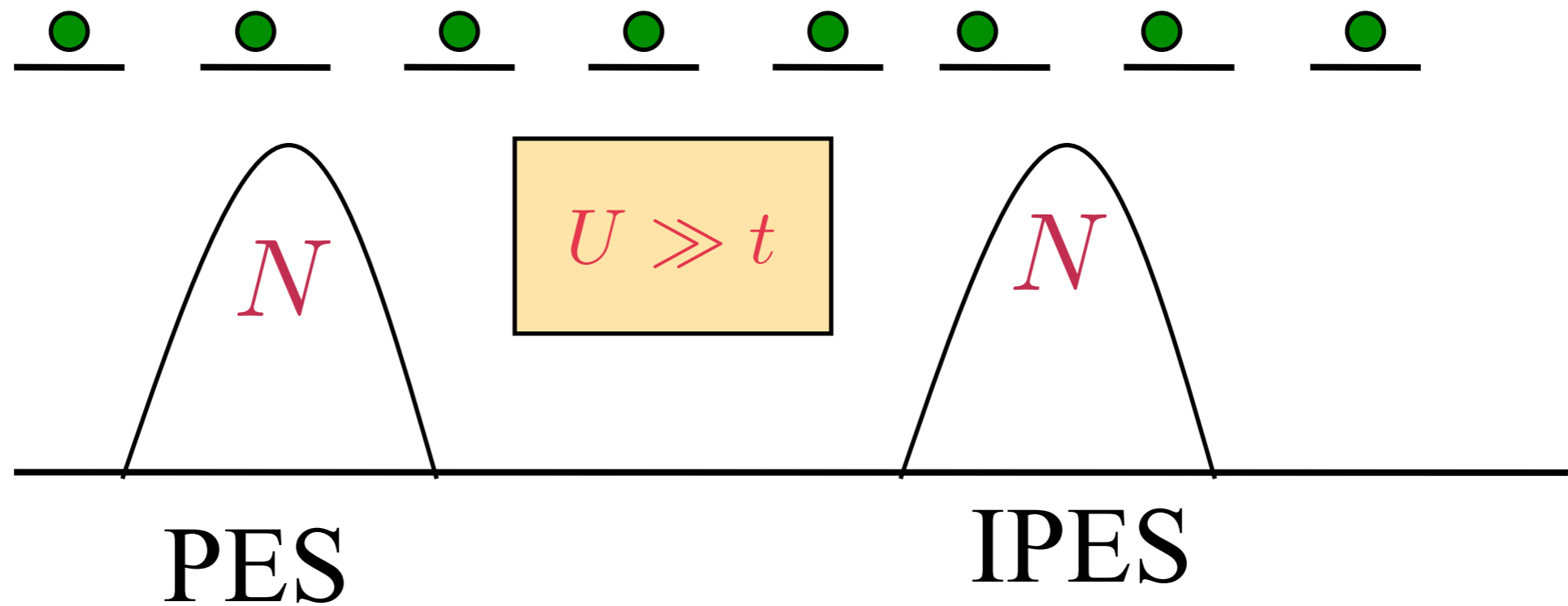


not the origin of the gap

Mott Problem

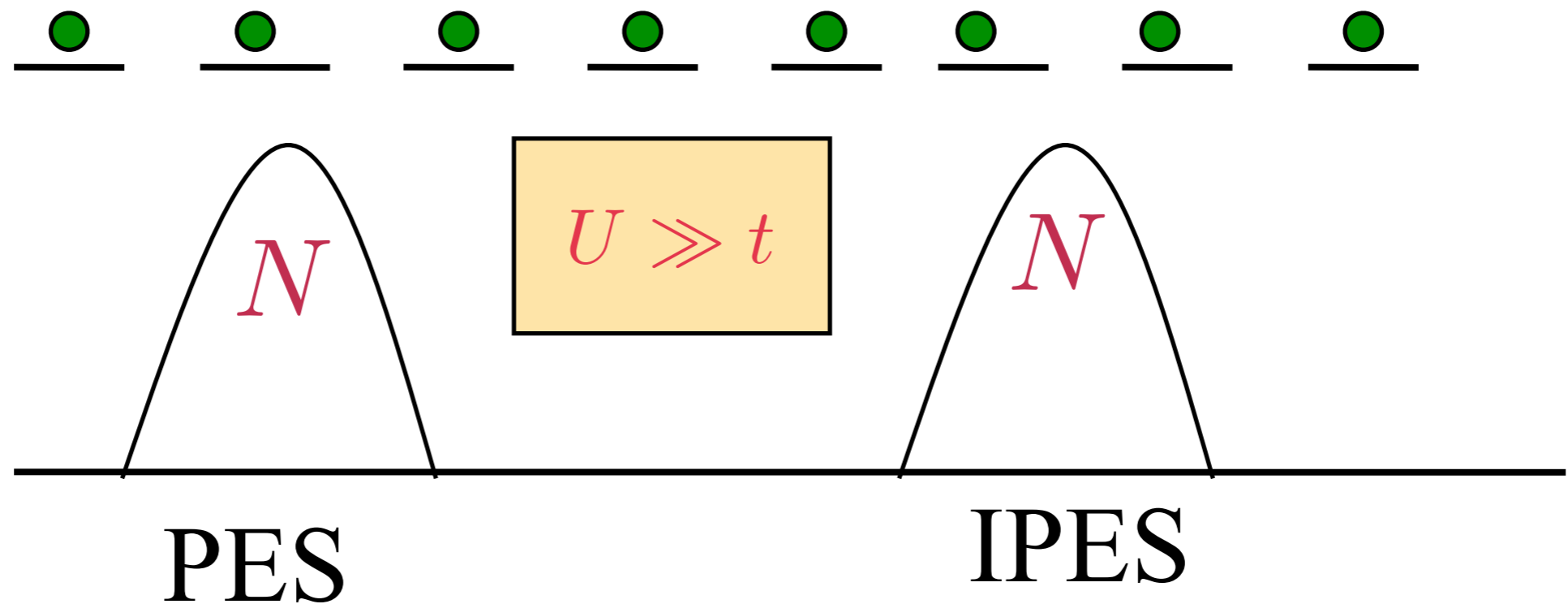


Mott Problem



What are the propagating degrees of freedom that produce the gapped spectrum?

Mott Problem



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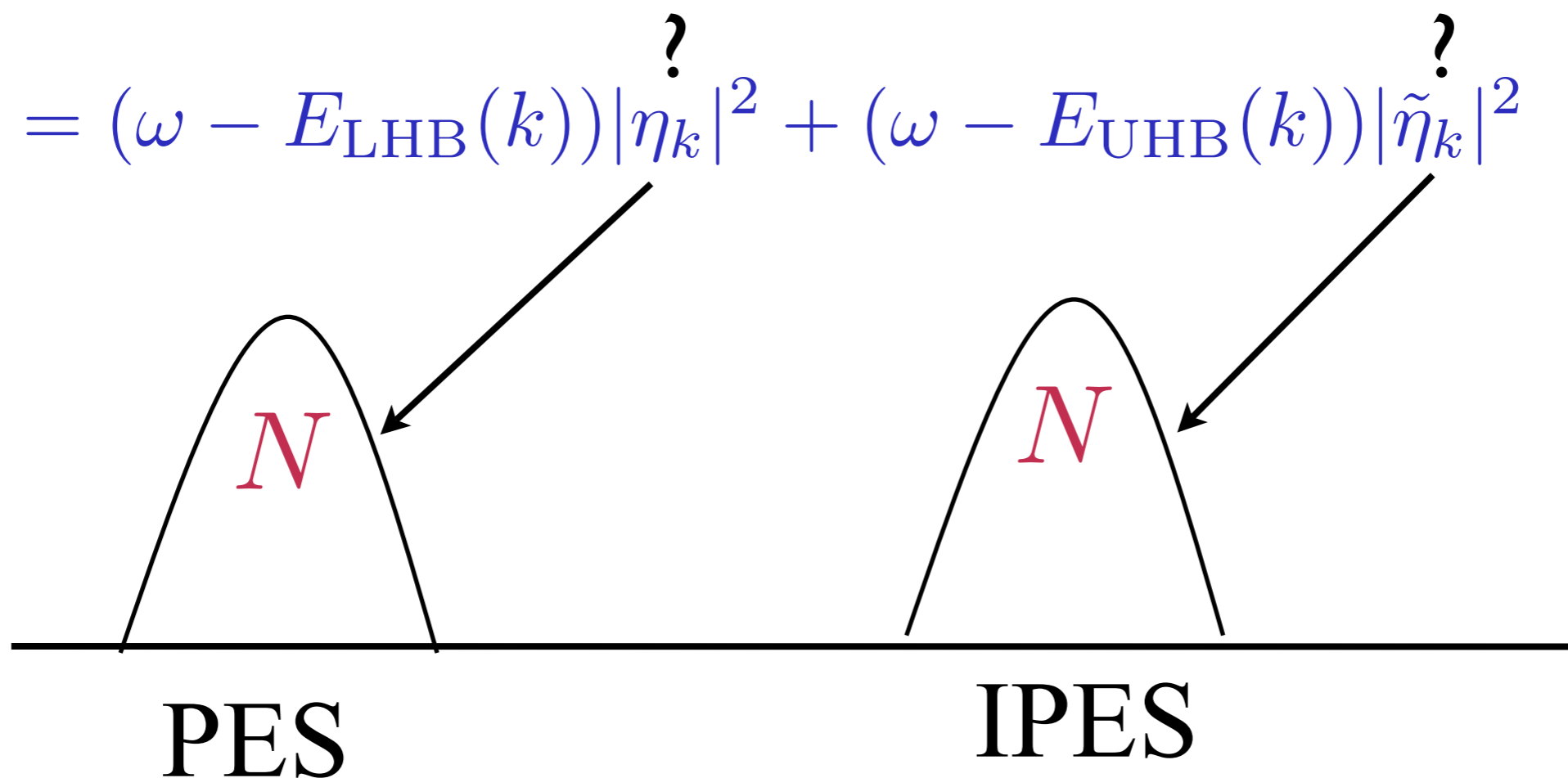
not electrons: $G(\omega = 0, p = p_L) = 0$

Fermi-liquid analogy

$$L_{\text{FL}} \propto (\omega - \epsilon_k) |\psi_k|^2$$

Mott Problem?

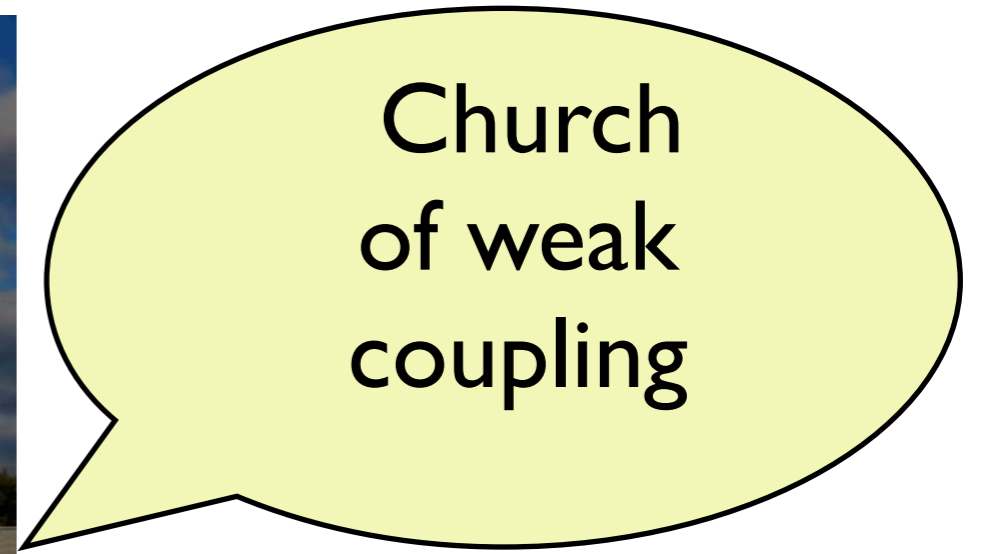
$$L_{\text{MI}} = (\omega - E_{\text{LHB}}(k)) |\eta_k|^2 + (\omega - E_{\text{UHB}}(k)) |\tilde{\eta}_k|^2$$



A Critique of Two Metals

R. B. Laughlin

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.



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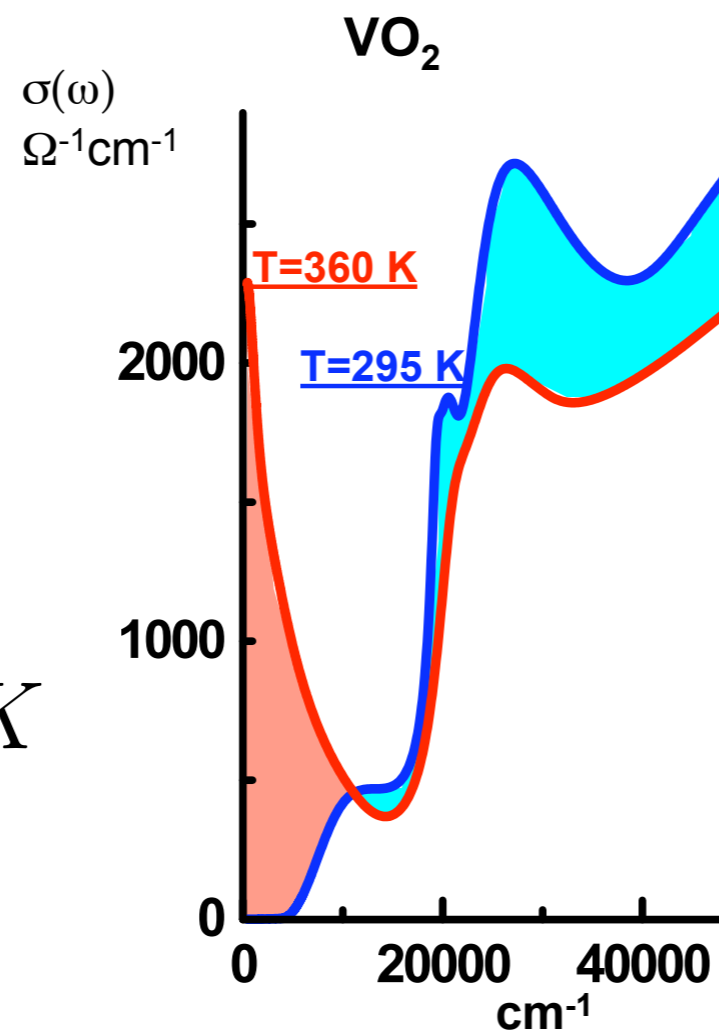
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Church
of weak
coupling

Beliefs:
Mott gap is heresy?
HF is the way!
No UHB and LHB!

$$\Delta = 0.6eV > \Delta_{\text{dimerization}} \quad (\text{Mott, 1976})$$

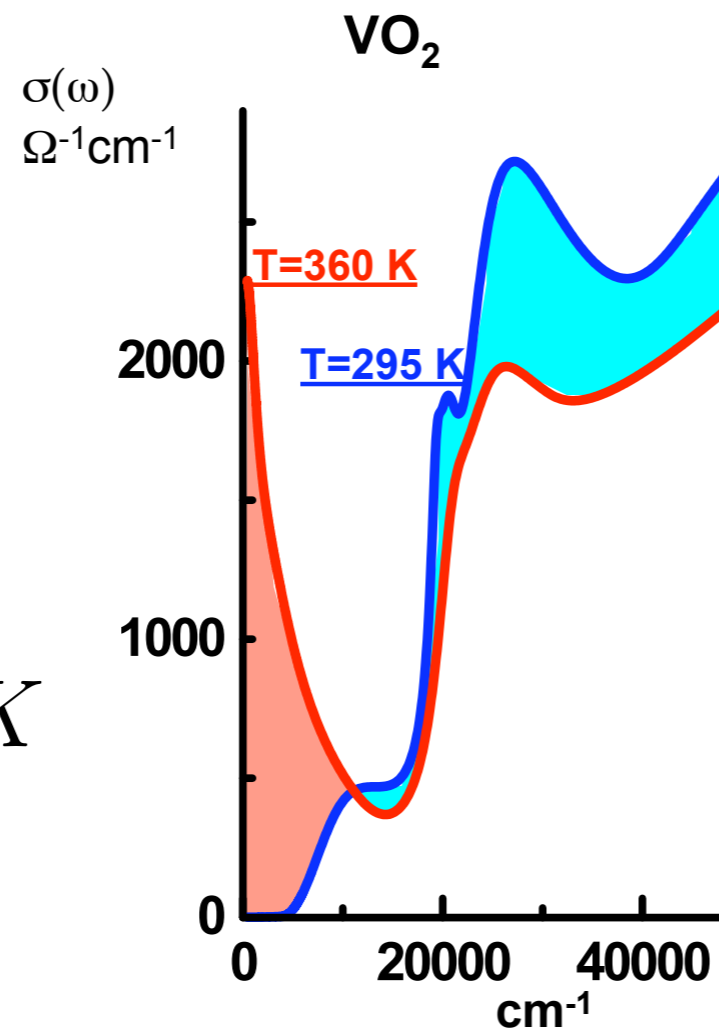
Recall,
 $eV = 10^4 K$



transfer
of spectral
weight to
high energies
beyond any ordering
scale

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Recall,
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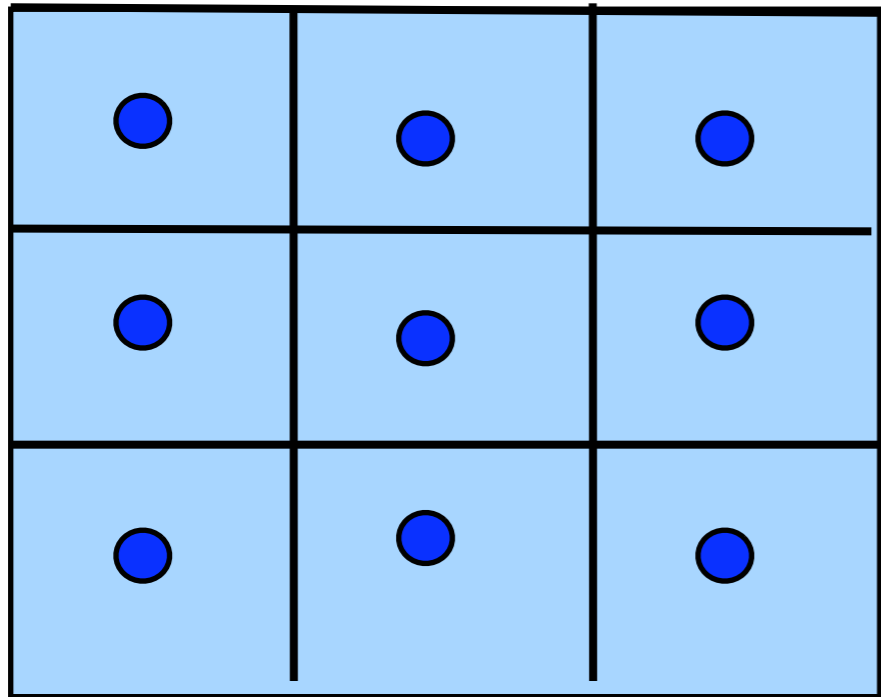


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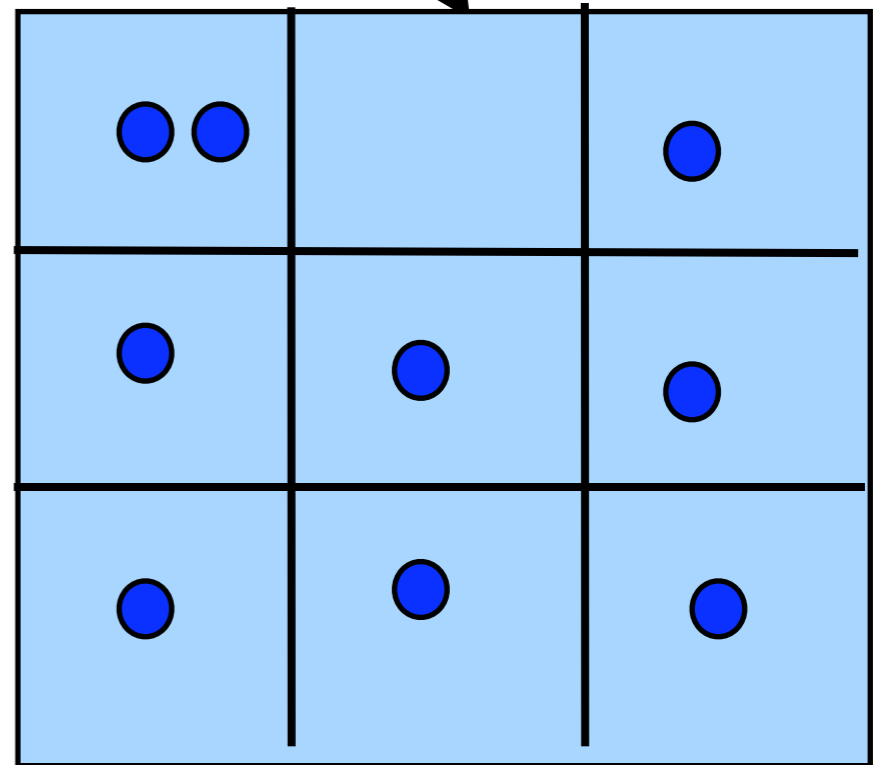
*M. M. Qazilbash, K. S. Burch, D. Whisler,
D. Shrekenhamer, B. G. Chae, H. T. Kim,
and D. N. Basov PRB 74, 205118 (2006)*

Collective Phenomena

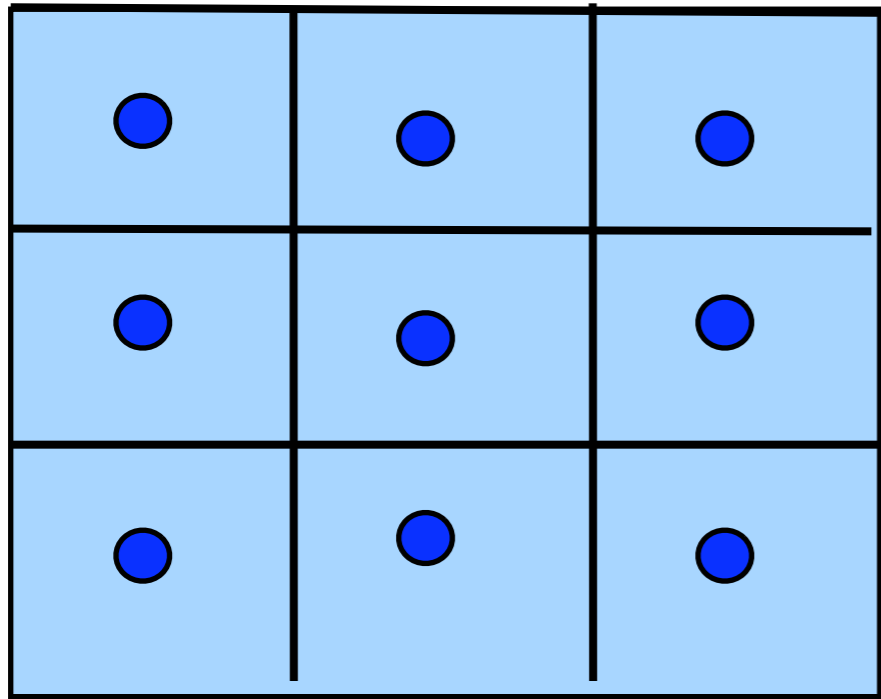
is the empty site
mobile??



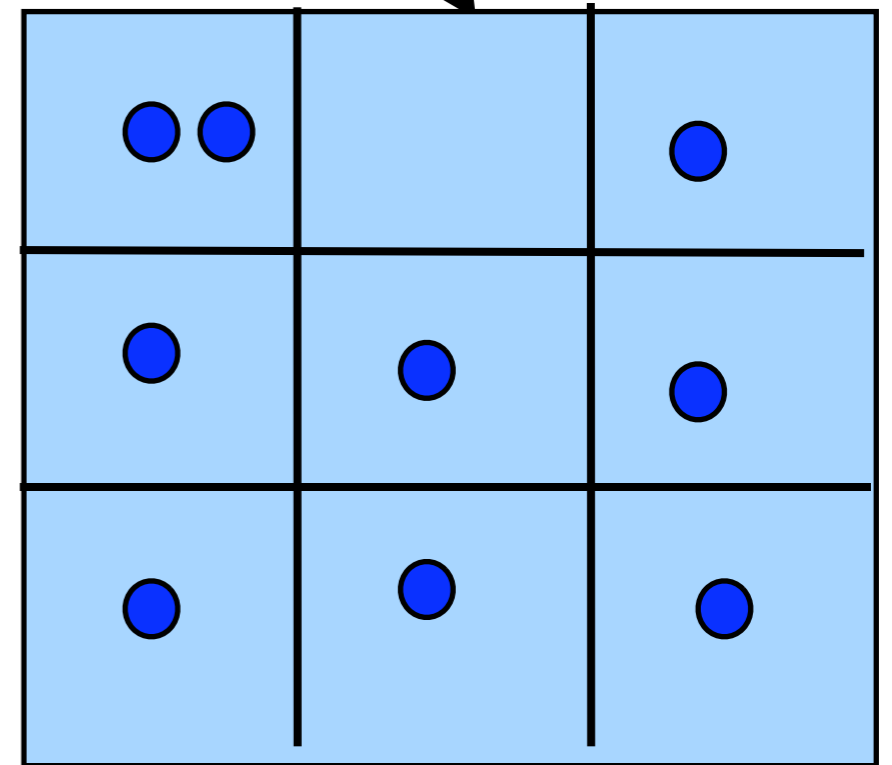
+ t/U



is the empty site
mobile??

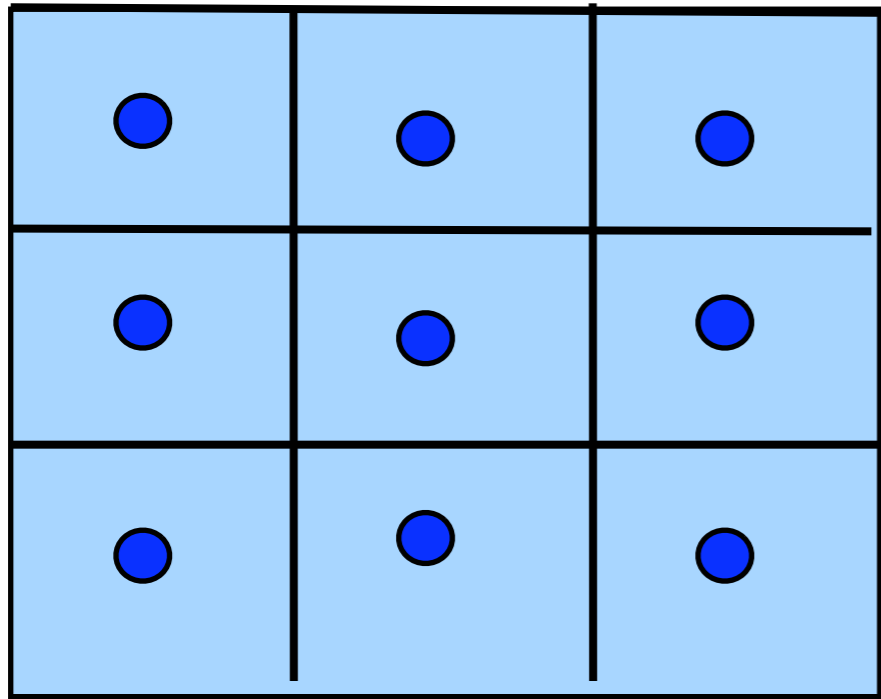


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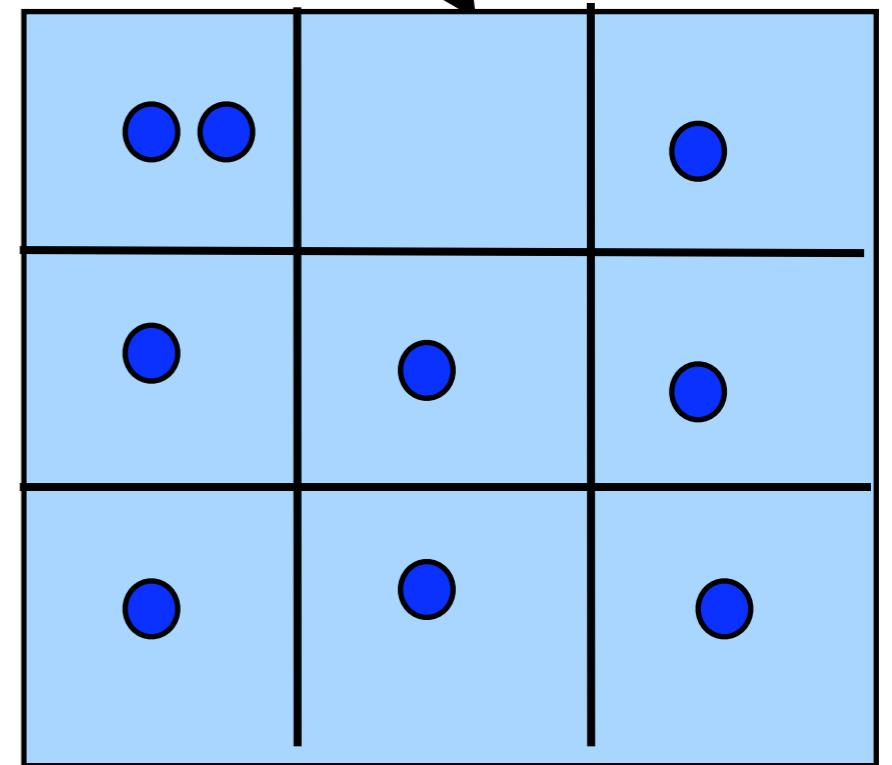


if yes, then 1.)
Mott insulator is a metal,
2.) no magnetic order

is the empty site
mobile??

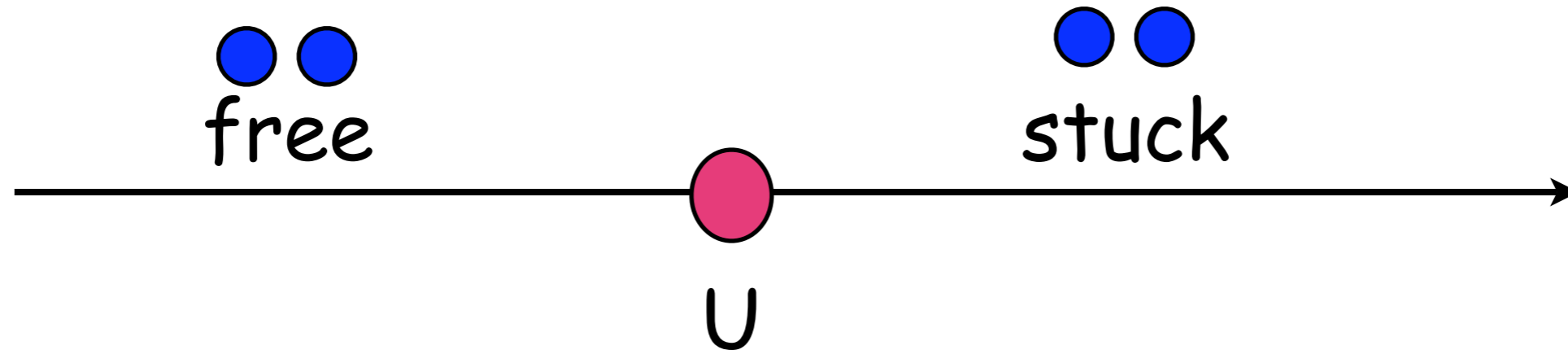


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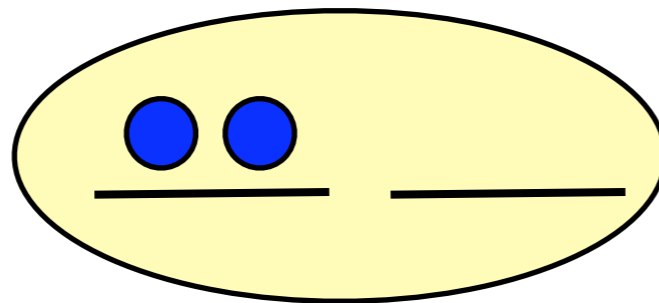
localisation criterion



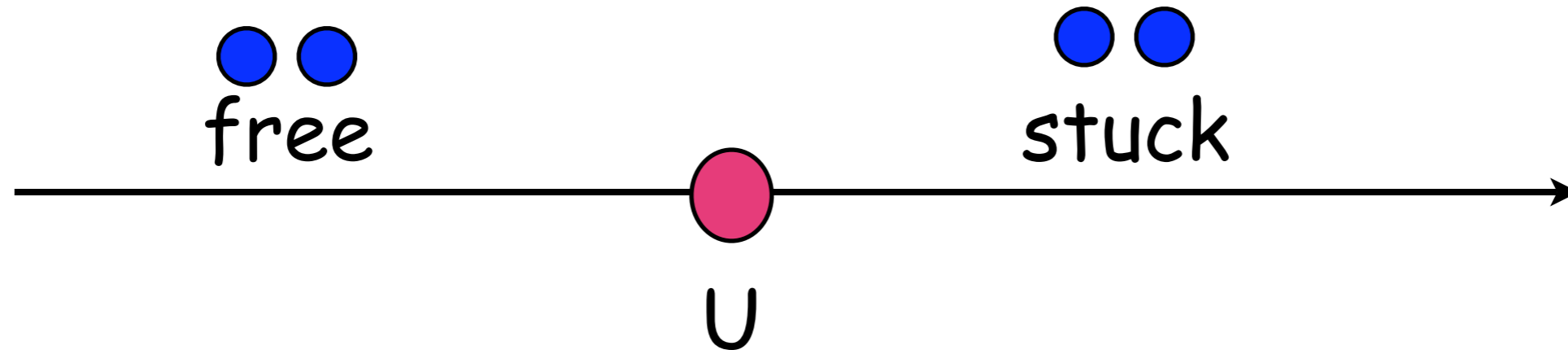
Mott Problem: what is the dynamical degree of freedom that makes this happen?

new bound states

Kohn, Mott,
Castellani,
others



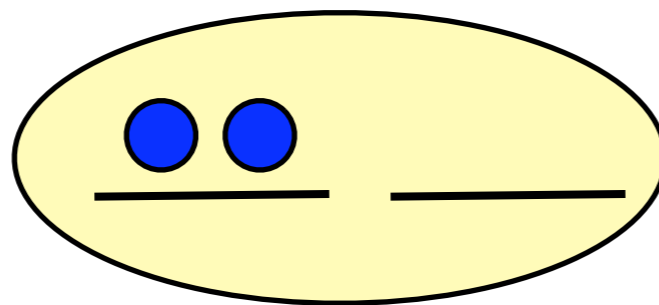
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Mott Problem: what is the dynamical degree of freedom that makes this happen?

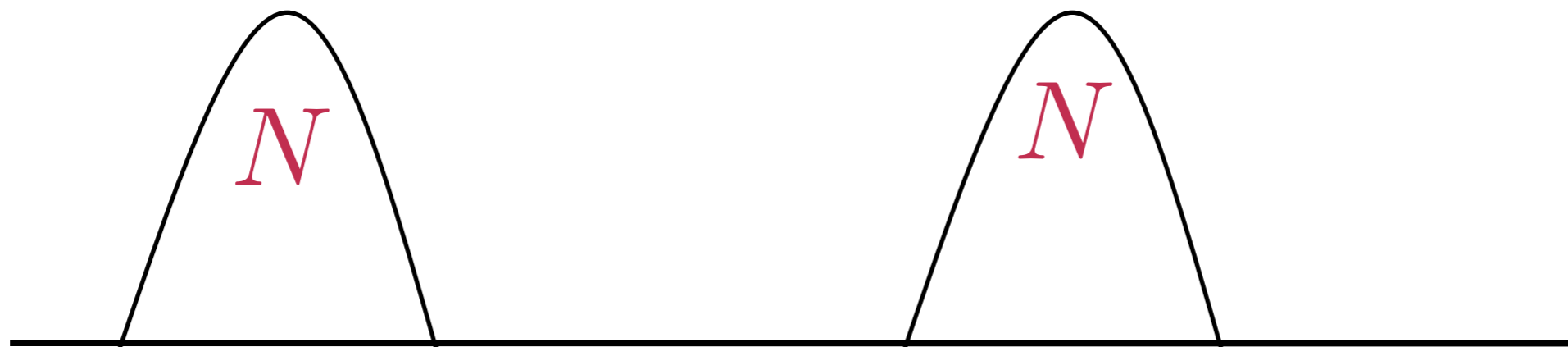
new bound states

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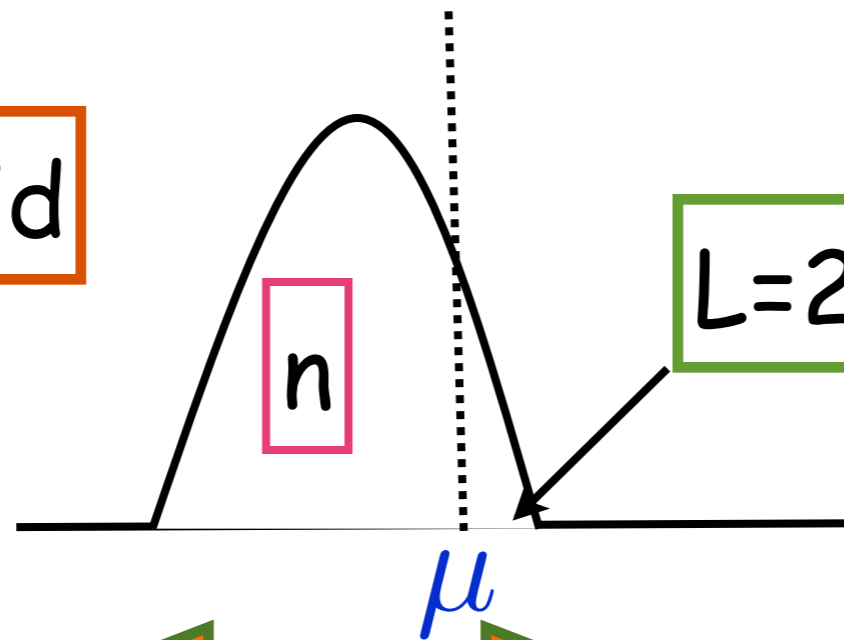
No proof exists?
Mottness is ill-defined

Second Problem: spectral weight transfer



the weight of each band depends on the filling!

Fermi liquid



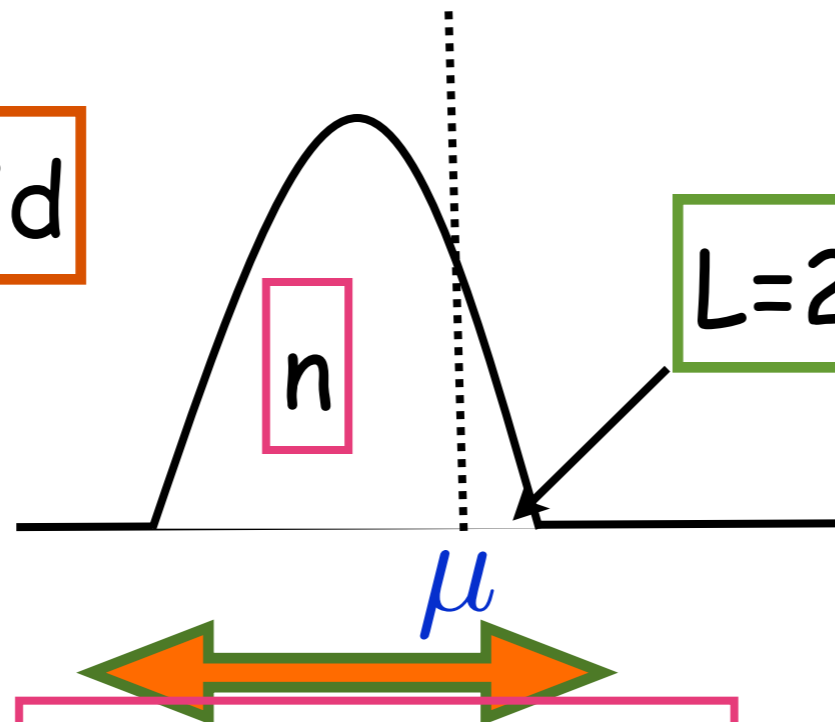
$$L=2-n$$

= Area/# of sites

$$\int_{\mu}^{\Lambda} N(\omega) d\omega$$

total weight=2

Fermi liquid



= Area/# of sites

$$\int_{\mu}^{\Lambda} N(\omega) d\omega$$

L

2

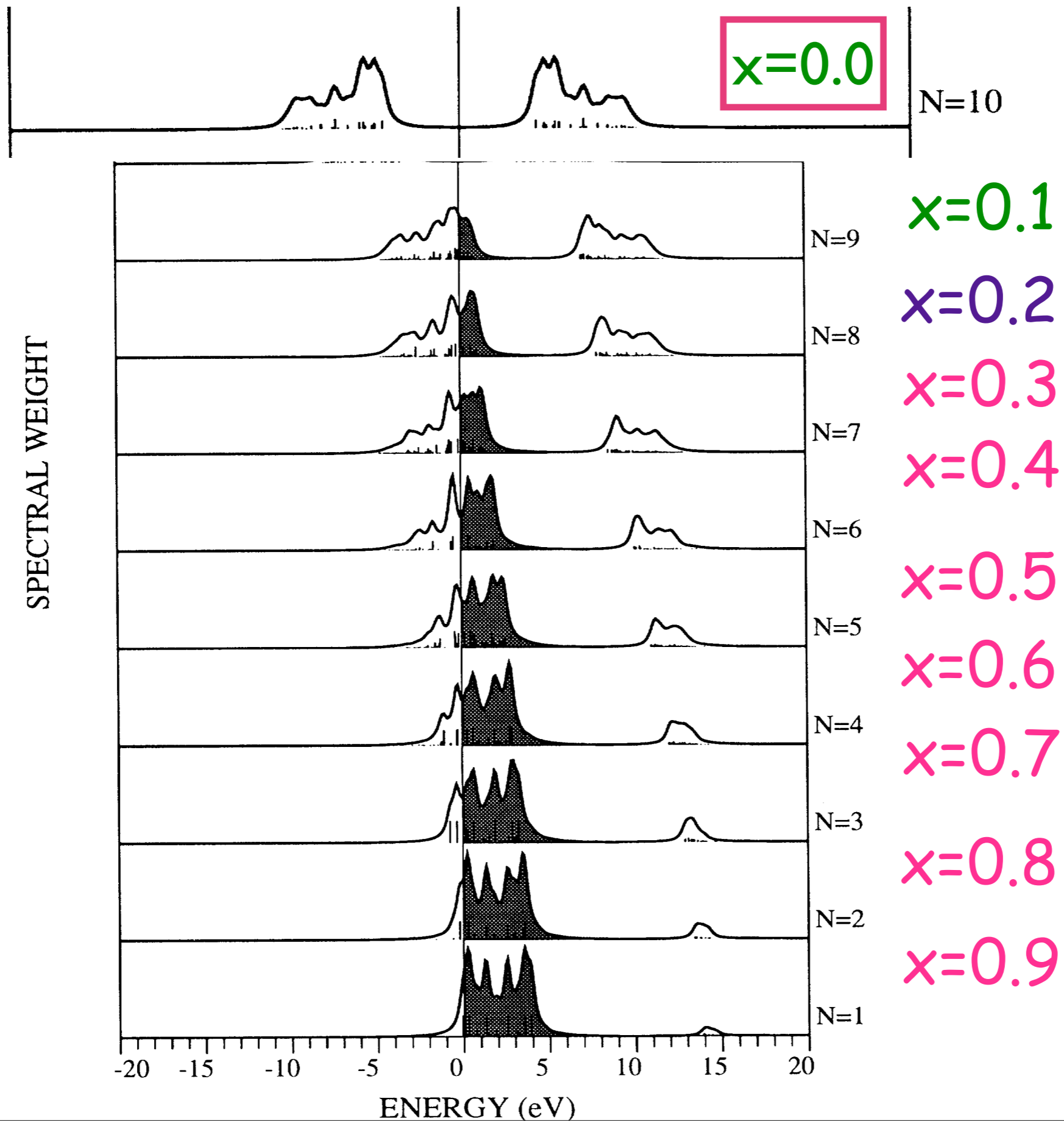
0

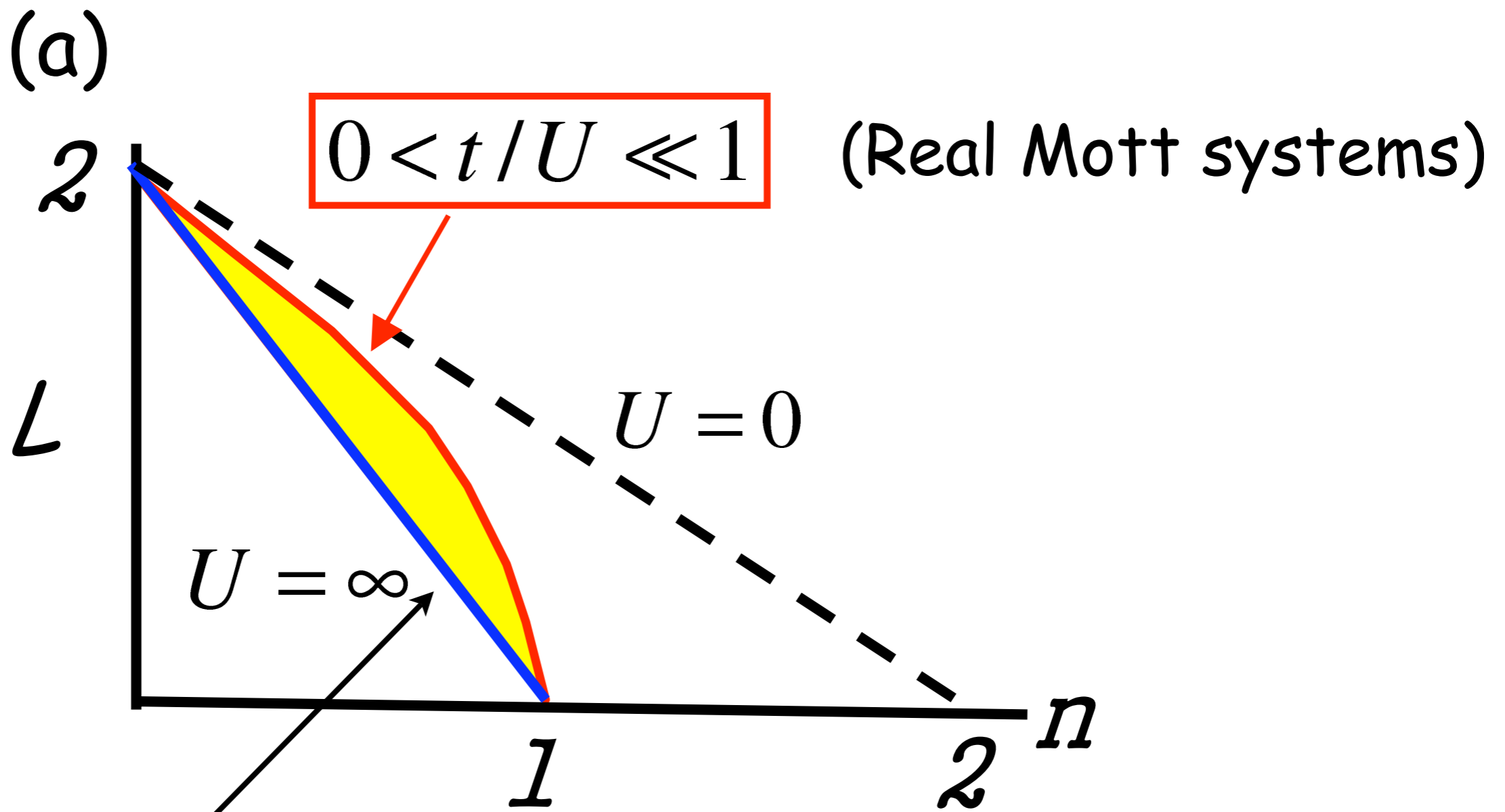
filling (n)

$$n_h = 2 - n \rightarrow L/n_h = 1$$

of ways
electrons can be
added

Mott gap

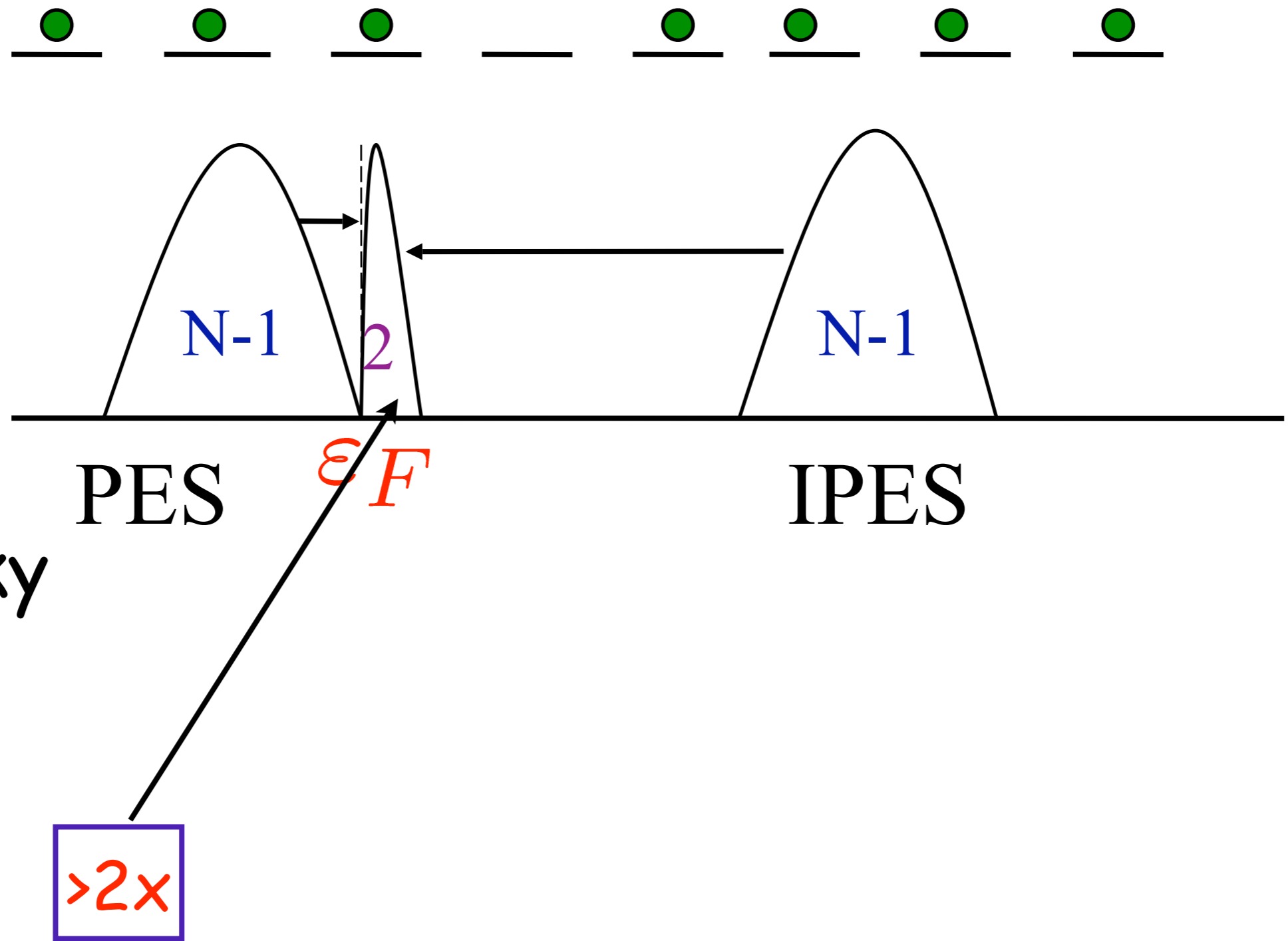


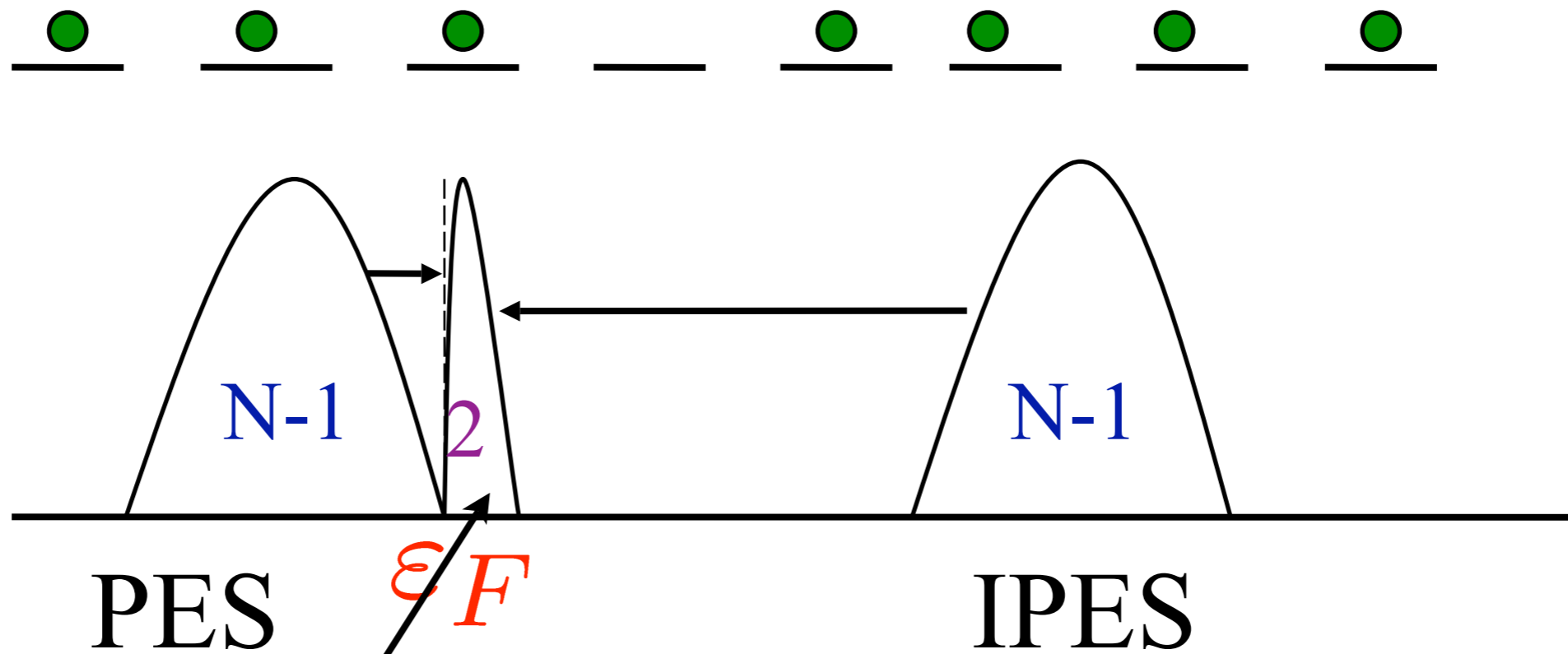


(t - J with hard projection)

Why?

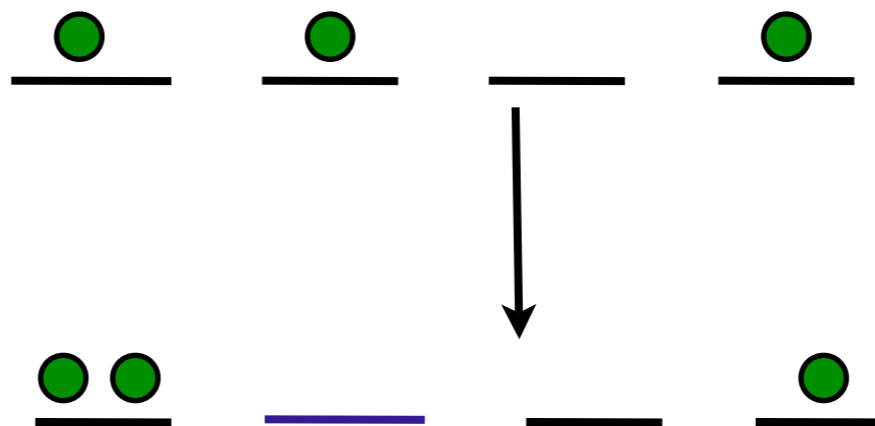
Sawatzky

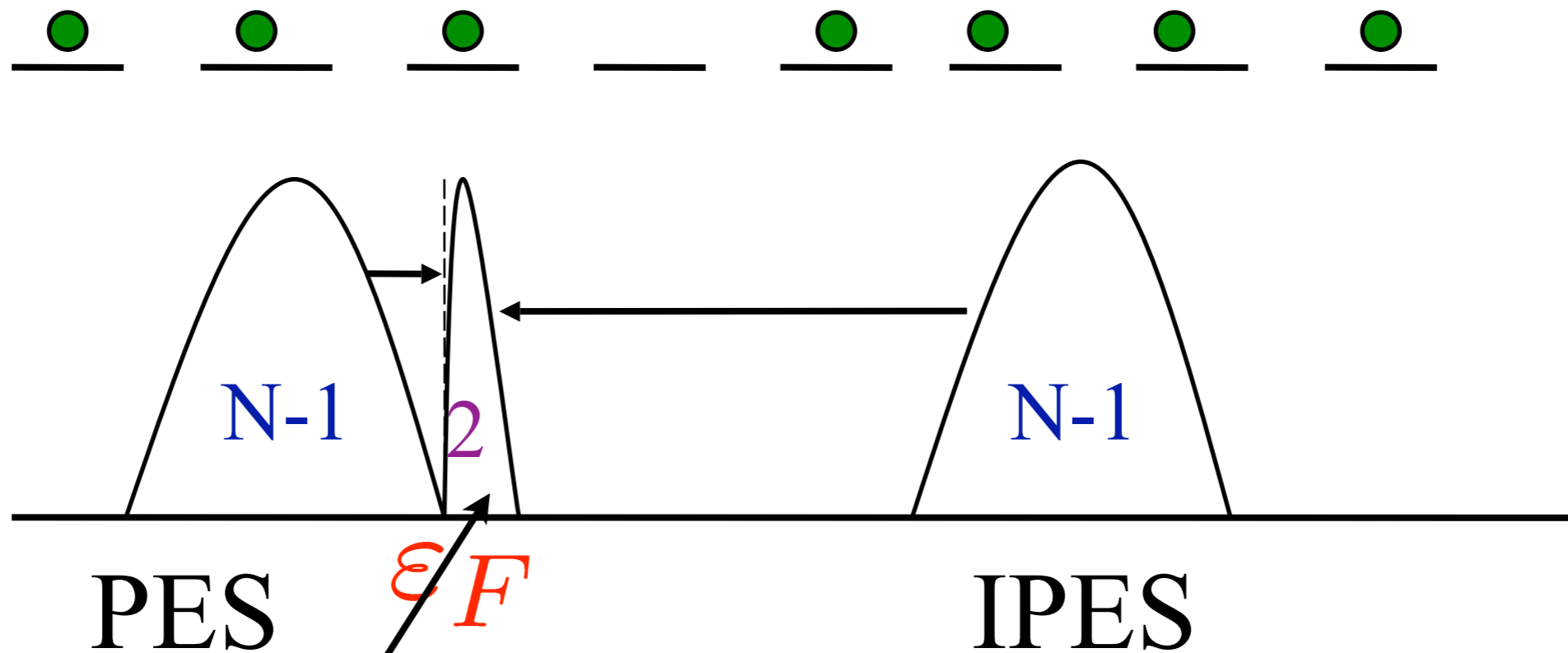




Sawatzky

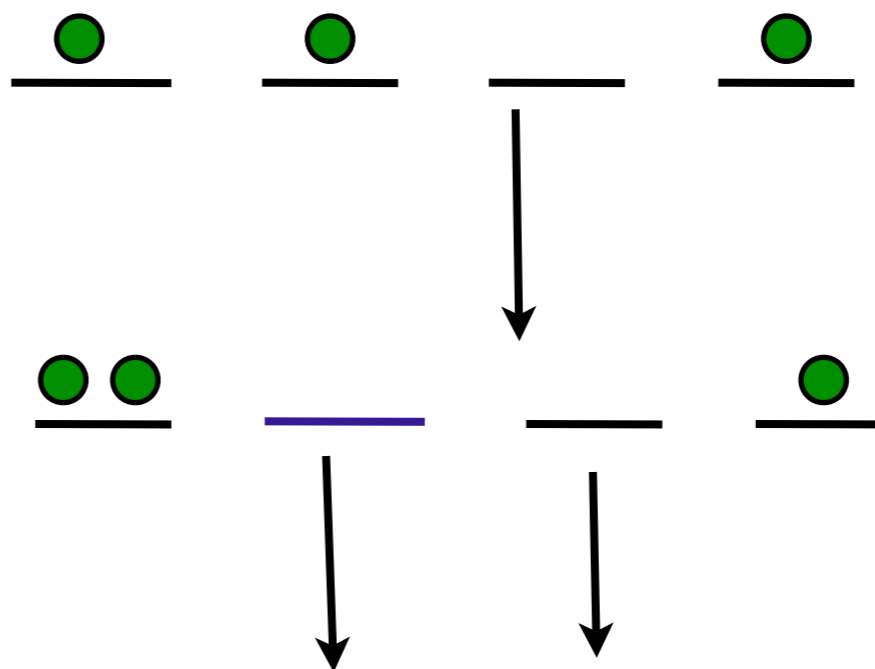
$>2x$





Sawatzky

$>2x$



$$L = 2(t/U)f(x) + 2x > 2x$$

Number of low energy states per
electron per spin > 1

Mottness

L-2x of the states
are orthogonal to
electron addition

new degree of freedom
is relevant

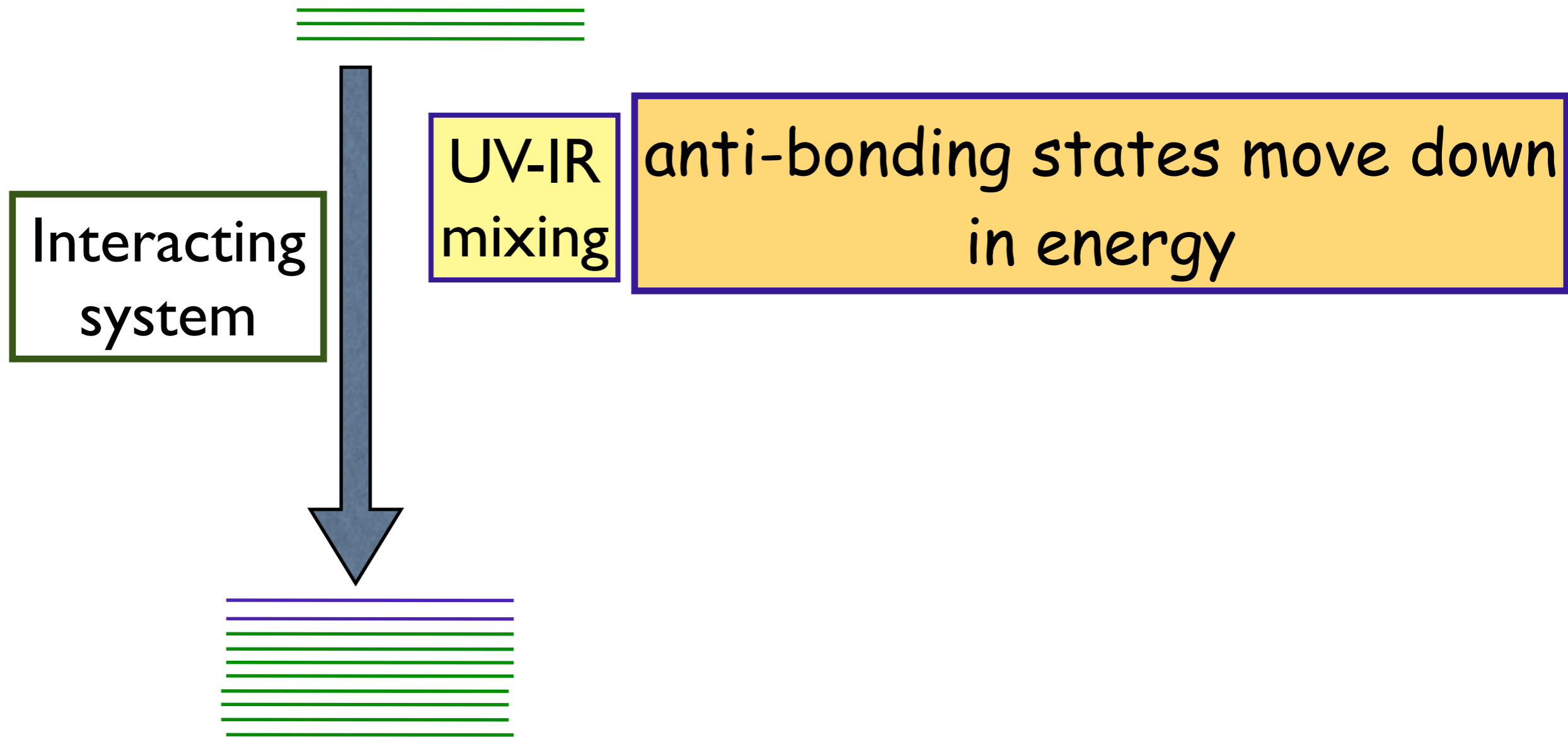
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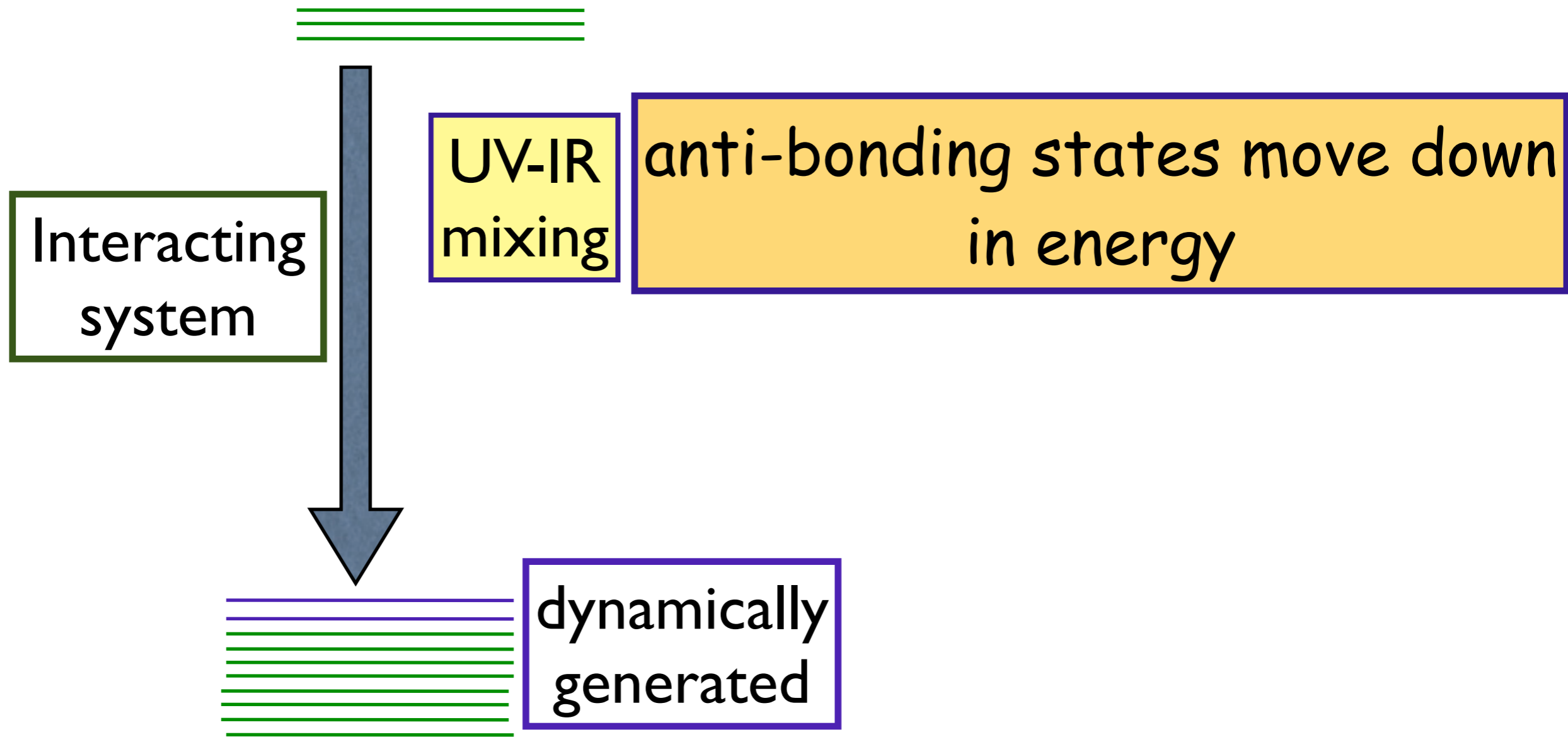
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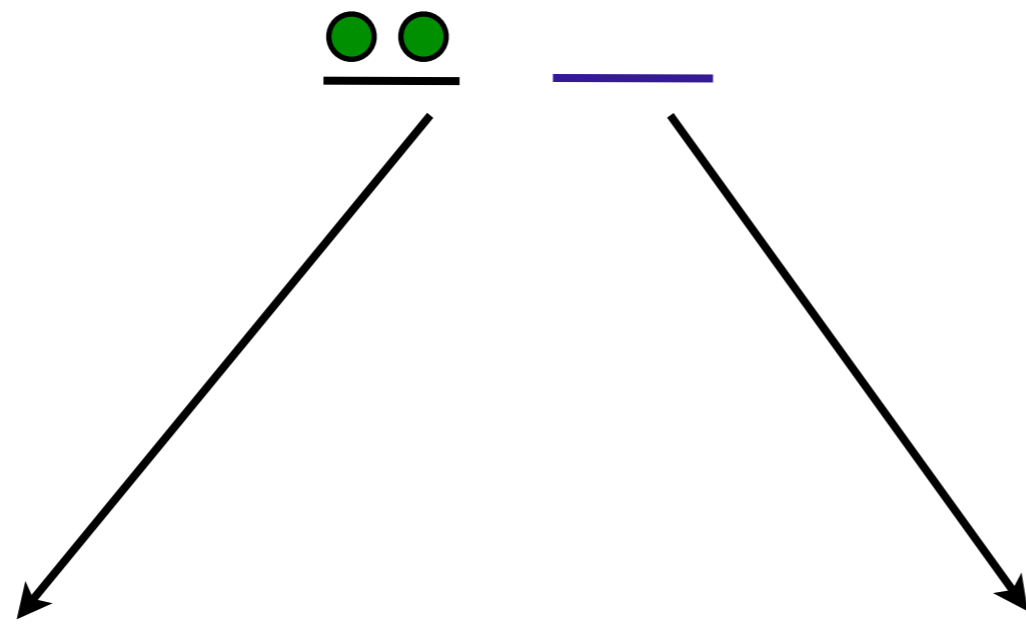
pseudogap

new degree of freedom
is relevant





composite excitation: bound state



half-filling:
Mott gap

doping:
SWT, pseudogap?

charge $2e$ boson

identifying the propagating degrees of freedom

Effective Theories:

$S(\phi)$ at half-filling

Integrate
Out high
Energy fields

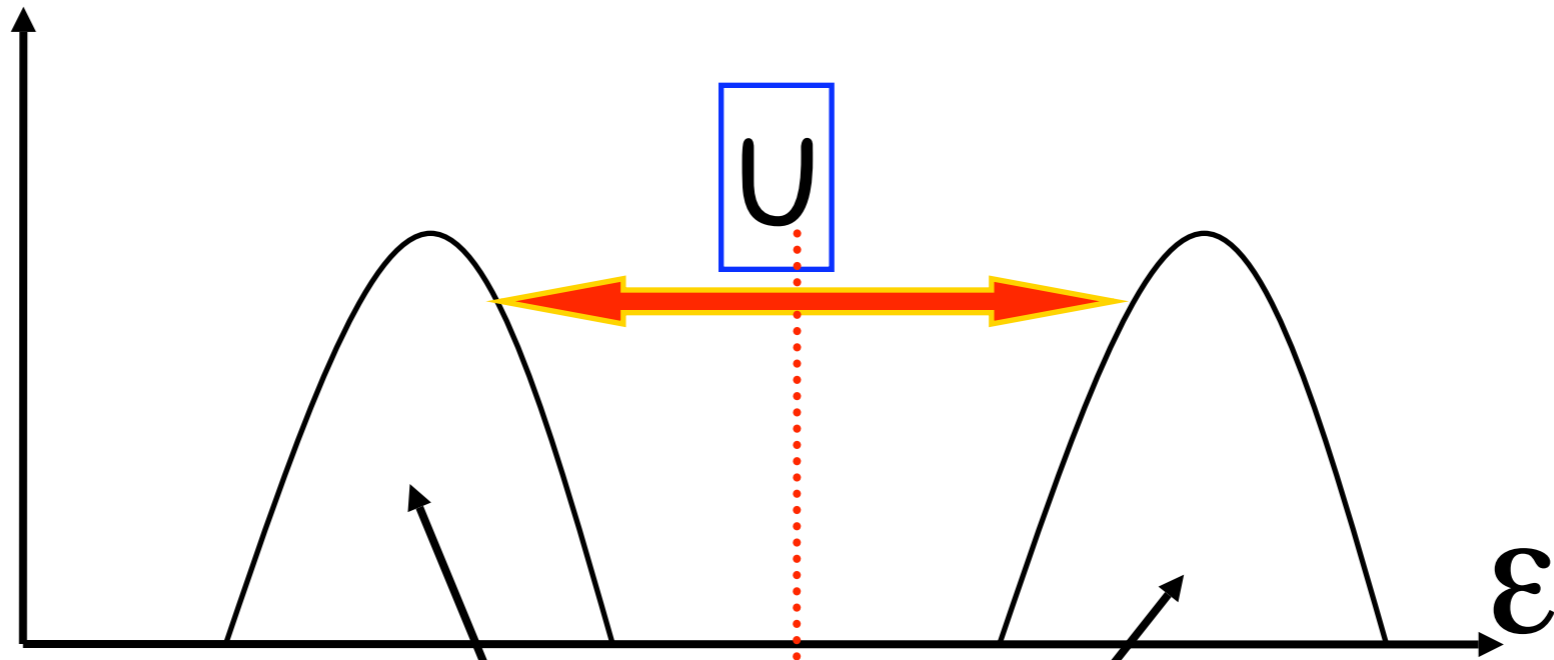
$$\phi = \phi_L + \phi_H$$

$$e^{-S_{\text{eff}}[\phi_L]} = \int d\phi_H \exp -S(\phi_L, \phi_H)$$

Low-energy theory of M I

Half-filling

$N(\omega)$

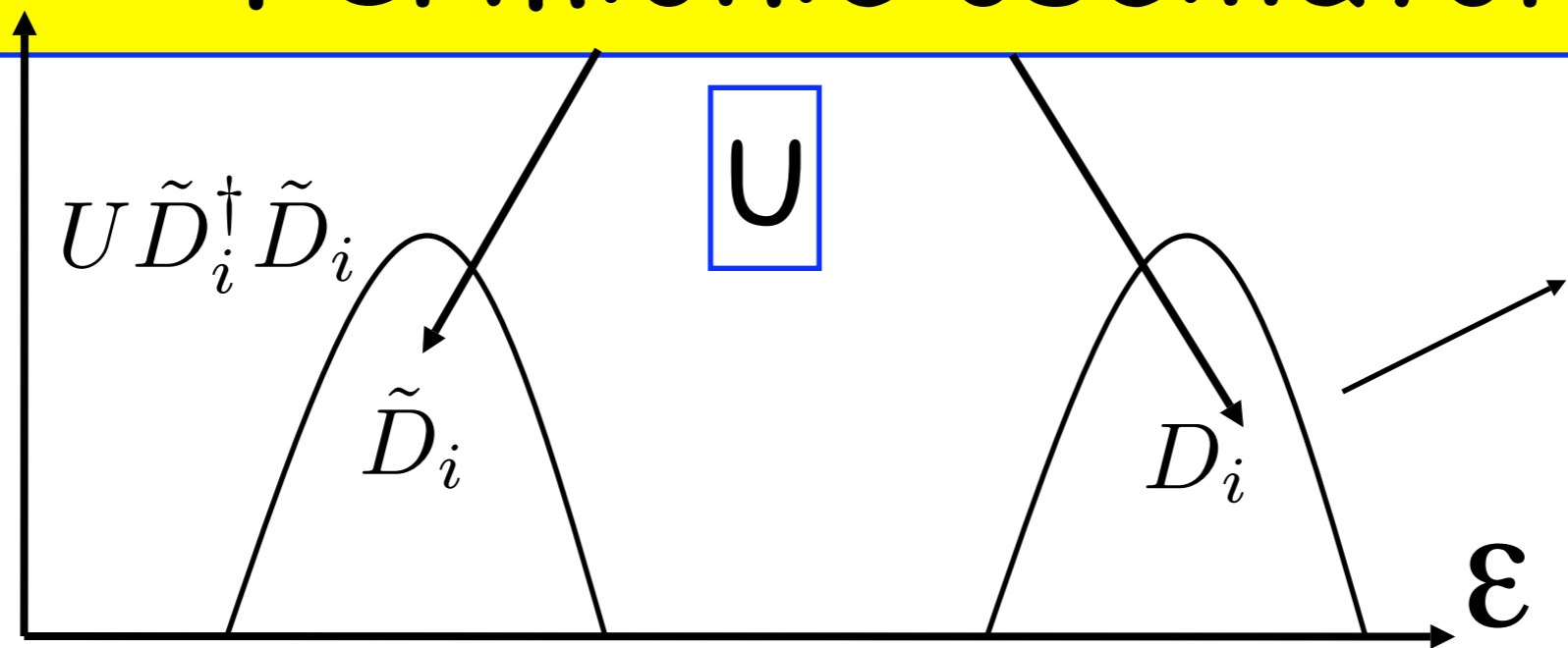


Integrate out both

Key idea: similar to Bohm/Pines

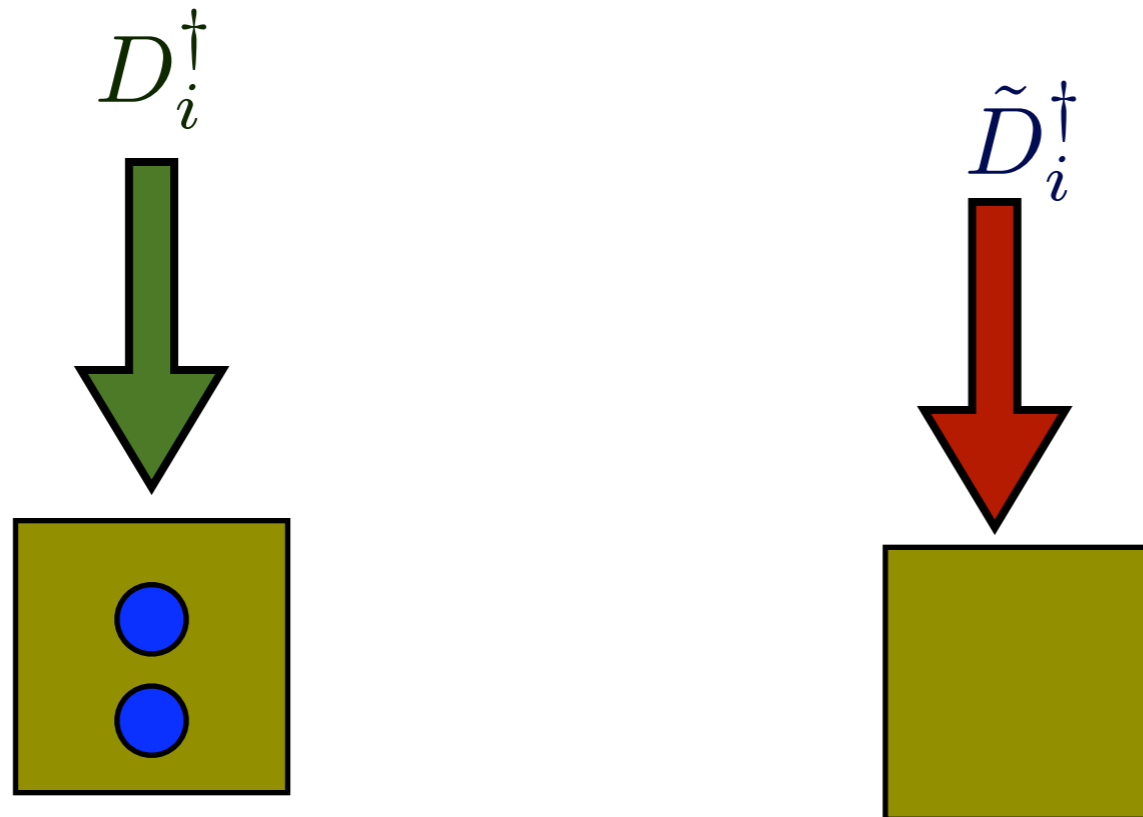
Extend the Hilbert space:
Associate with U-scale new
Fermionic oscillators

$N(\omega)$



$$U D_i^\dagger D_i$$

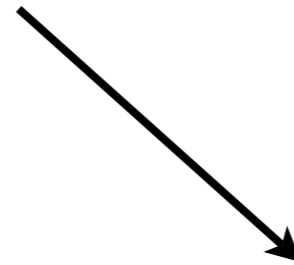
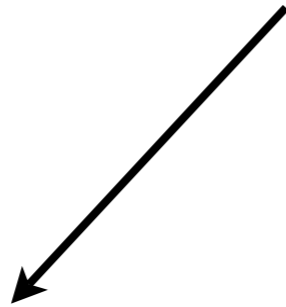
Impose Constraint:



How is this possible with
Fermions?

$$D_i^\dagger$$

Fermionic

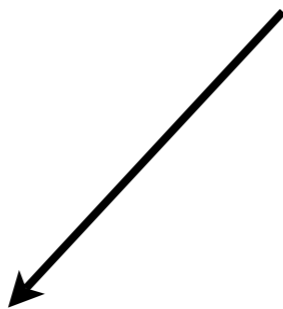
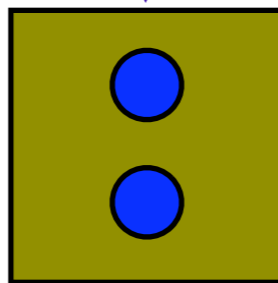


one per site
(fermionic)

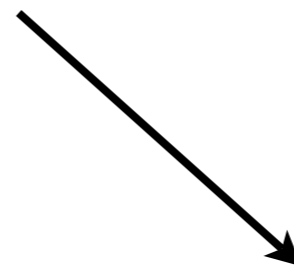
transforms as a boson

$$D_i^\dagger$$

Fermionic



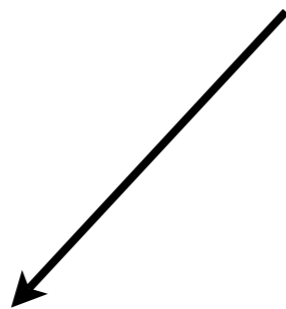
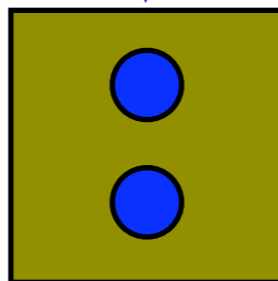
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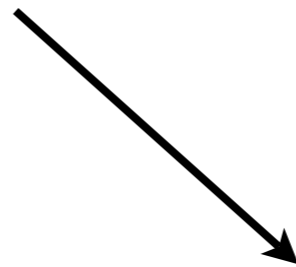
transforms as a boson

D_i^\dagger

Fermionic



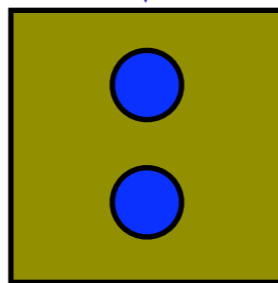
one per site
(fermionic)



transforms as a boson

'supersymmetry'

D_i^\dagger Fermionic



one per site
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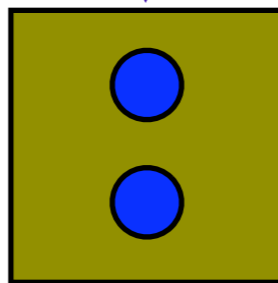
transforms as a boson

'supersymmetry'

Grassmann

$$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$$

D_i^\dagger Fermionic



one per site
(fermionic)

transforms as a boson

'supersymmetry'

Grassmann

$$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$$

super field:

$$X^\mu(\sigma, \theta) = X^\mu(\sigma) + \theta \gamma^\mu(\sigma) \quad (\text{Ramond})$$

$$\begin{aligned}
L_{\text{UV}}^{\text{hf}} = & \int d^2\theta \left[iD^\dagger \dot{D} - i\dot{\tilde{D}}^\dagger \tilde{D} - \frac{U}{2} (D^\dagger D - \tilde{D}\tilde{D}^\dagger) \right. \\
& + \frac{t}{2} D^\dagger \theta b + \frac{t}{2} \bar{\theta} b \tilde{D} + h.c. + s\bar{\theta} \varphi^\dagger (D - \theta c_\uparrow c_\downarrow) \\
& \left. + \tilde{s}\bar{\theta} \tilde{\varphi}^\dagger (\tilde{D} - \theta c_\uparrow^\dagger c_\downarrow^\dagger) + h.c. \right], \tag{1}
\end{aligned}$$

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\end{aligned}$$

$$b_i = \sum_j g_{ij} (c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow})$$

$$|b_i|^2 \propto S_i \cdot S_j$$

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charge $|2e|$ boson

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\end{aligned}$$

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$$|b_i|^2 \propto S_i \cdot S_j$$

charge $|2e|$ boson

solve constraint

$$\int d^2\theta \bar{\theta} \theta L_{\text{Hubb}} = \sum_{i,\sigma} c_{i,\sigma}^\dagger \dot{c}_{i,\sigma} + H_{\text{Hubb}},$$

Exact low-energy Lagrangian

$$L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons})$$

$$+ f(\omega)L_{\text{int}}(c, \varphi) + \tilde{f}(\omega)L_{\text{int}}(c, \tilde{\varphi})$$

$$f(\omega) = 0$$

dispersion
of propagating
modes

composite excitations

$$H_{IR}(n = 1) \neq H_{\text{spinmodel}}$$

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Boson breaks local
SU(2) symmetry
Of Heisenberg model!!

$$H_{IR}(n = 1) \neq H_{\text{spinmodel}}$$

Boson breaks local
SU(2) symmetry
Of Heisenberg model!!

$$\Psi = \begin{pmatrix} c_{\uparrow} & c_{\downarrow} \\ c_{\downarrow}^{\dagger} & -c_{\uparrow}^{\dagger} \end{pmatrix}$$

$$S = \Psi^{\dagger} \Psi \quad \text{local SU(2)}$$

$$\Psi = h \Psi \quad h = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$S \rightarrow \Psi^{\dagger} h^{\dagger} h \Psi = S$$

Exact IR Lagrangian

bare fields have no dynamics

$$L_{\text{IR}}^{\text{hf}} \rightarrow 2 \frac{|s|^2}{U} |\varphi_\omega|^2 + 2 \frac{|\tilde{s}|^2}{U} |\tilde{\varphi}_{-\omega}|^2 + \frac{t^2}{U} |b_\omega|^2$$

$$\left\{ \begin{aligned} &+ s \gamma_{\vec{p}}^{(\vec{k})}(\omega) \varphi_{\omega, \vec{k}}^\dagger c_{\vec{k}/2 + \vec{p}, \omega/2 + \omega', \uparrow} c_{\vec{k}/2 - \vec{p}, \omega/2 - \omega', \downarrow} \\ &+ \tilde{s}^* \tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) \tilde{\varphi}_{-\omega, \vec{k}} c_{\vec{k}/2 + \vec{p}, \omega/2 + \omega', \uparrow} c_{\vec{k}/2 - \vec{p}, \omega/2 - \omega', \downarrow} + h.c. \end{aligned} \right.$$

bosons
and fermions
are strongly coupled

$$\gamma_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U - t \varepsilon_{\vec{p}}^{(\vec{k})} - 2\omega}{U} \sqrt{1 + 2\omega/U}$$


$$\tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U + t \varepsilon_{\vec{p}}^{(\vec{k})} + 2\omega}{U} \sqrt{1 - 2\omega/U}.$$


$$\varepsilon_{\vec{p}}^{(\vec{k})} = 4 \sum_{\mu} \cos(k_{\mu} a/2) \cos(p_{\mu} a)$$

turn-on of spectral
weight governed
by composite
excitations (CEXONS)

$$L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons})$$

$$+ f(\omega)L_{\text{int}}(c, \varphi) + \tilde{f}(\omega)L_{\text{int}}(c, \tilde{\varphi})$$


$$\Psi^\dagger \Psi$$

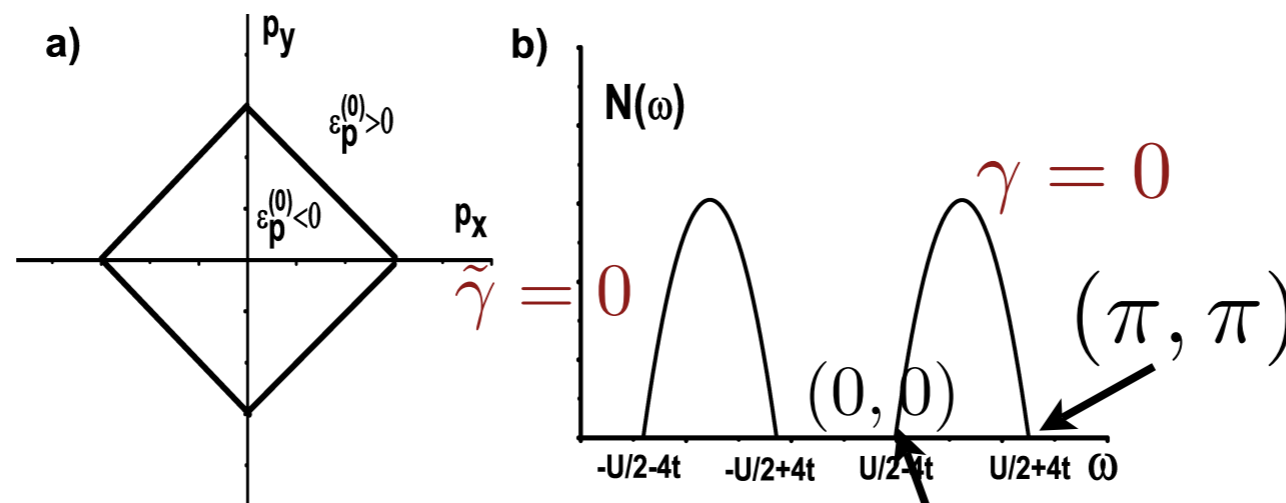

$$\tilde{\Psi}^\dagger \tilde{\Psi}$$

**quadratic form:
composite or bound
excitations of
 $\varphi^\dagger c_{i\sigma}$**

composite excitations determine spectral density

$$\gamma_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U - t\varepsilon_{\vec{p}}^{(\vec{k})} - 2\omega}{U} \sqrt{1 + 2\omega/U}$$

$$\tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U + t\varepsilon_{\vec{p}}^{(\vec{k})} + 2\omega}{U} \sqrt{1 - 2\omega/U}.$$

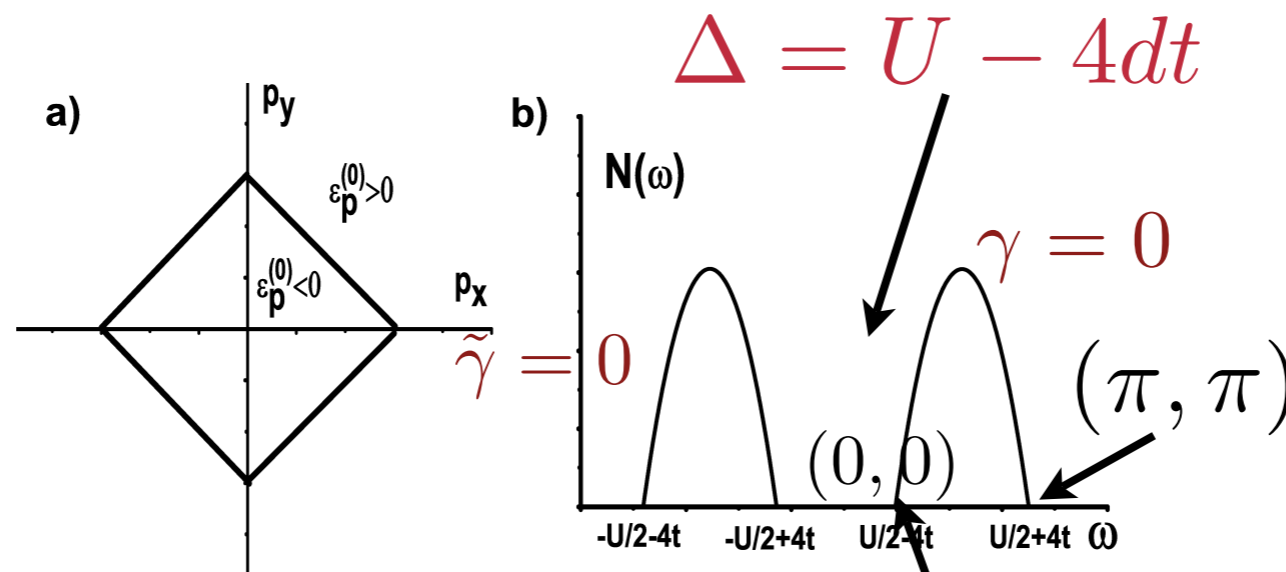


each momentum has SD at two distinct
energies

composite excitations determine spectral density

$$\gamma_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U - t\varepsilon_{\vec{p}}^{(\vec{k})} - 2\omega}{U} \sqrt{1 + 2\omega/U}$$

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each momentum has SD at two distinct energies

electron transform at low energies

$$L_{\text{UV}}^{\text{hf}} \rightarrow L_{\text{UV}}^{\text{hf}} + \int d^2\theta J_{i,\sigma} \left[V_\sigma D_i^\dagger c_{i,-\sigma} \theta + V_\sigma \bar{\theta} c_{i,-\sigma} \tilde{D}_i \right] + h.c.$$

UV

IR

$c_{i\sigma}^\dagger$

$$c_{i,\sigma}^\dagger \rightarrow \tilde{c}_{i,\sigma}^\dagger \equiv -V_\sigma \frac{t}{U} \left(c_{i,-\sigma} b_i^\dagger + b_i^\dagger c_{i,-\sigma} \right) + V_\sigma \frac{2}{U} \left(s\varphi_i^\dagger + \tilde{s}\tilde{\varphi}_i \right) c_{i,-\sigma}$$

spin-fluctuations

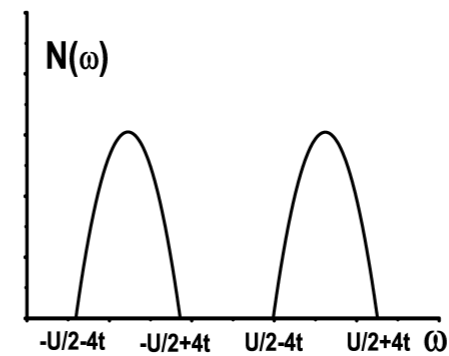
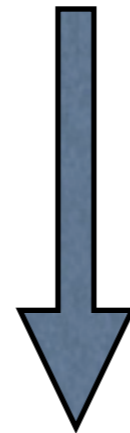
CEXON

UV-IR mixing: spectral weight transfer

electron overlap with composite excitations?

$$O = |\langle c^\dagger | \tilde{c}^\dagger \rangle \langle \tilde{c}^\dagger | \Psi \rangle|^2 P_\Psi$$

destructive interference at low energy

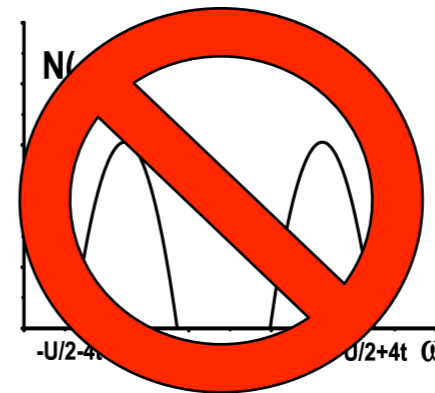
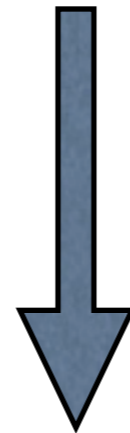


$$\Delta_{\text{electron}} > \Delta_{\text{composites}}$$

electron overlap with composite excitations?

$$O = |\langle c^\dagger | \tilde{c}^\dagger \rangle \langle \tilde{c}^\dagger | \Psi \rangle|^2 P_\Psi$$

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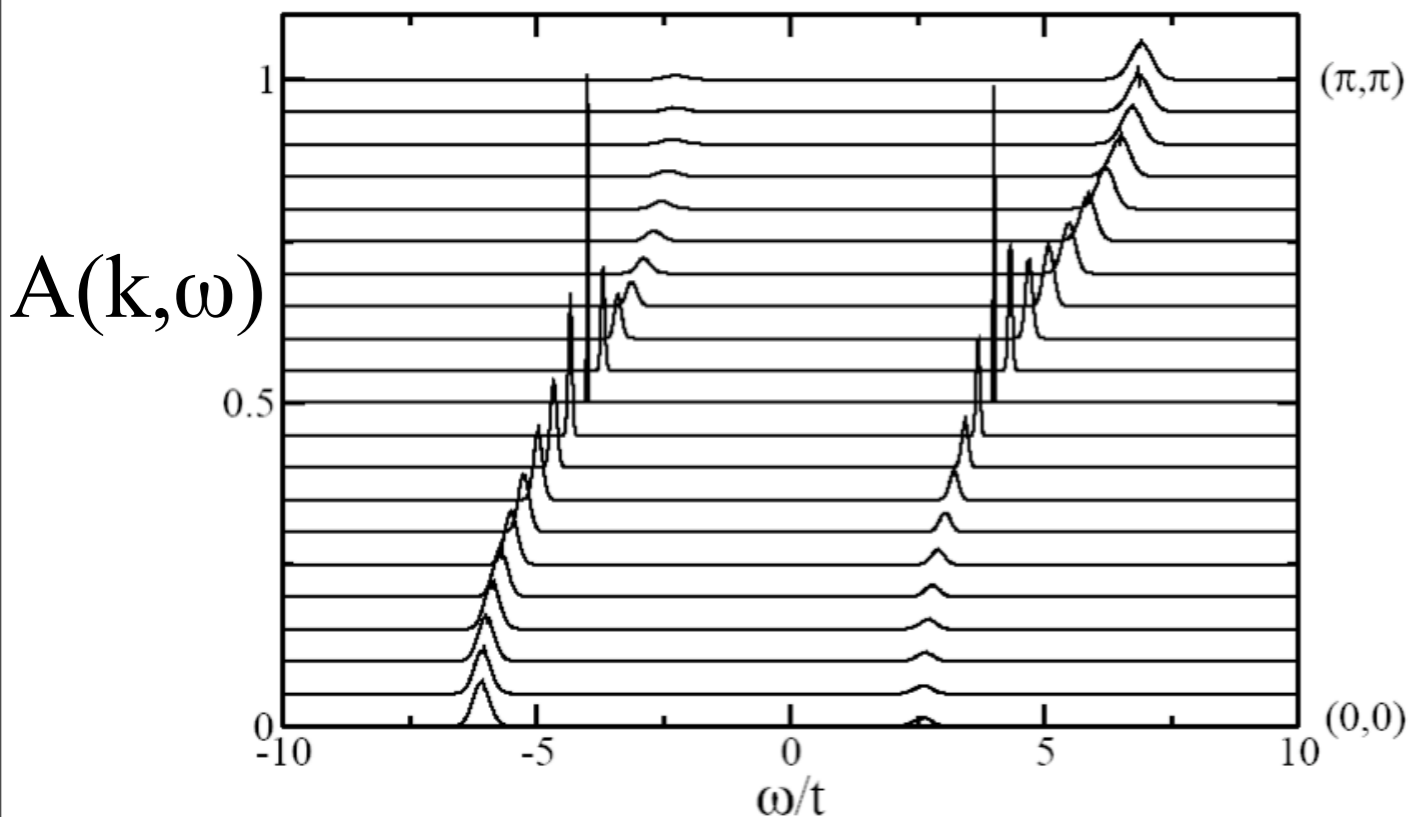


$$\Delta_{\text{electron}} > \Delta_{\text{composites}}$$

$$H(\varphi, \tilde{\varphi}, c, c^\dagger)$$

- 1.) no derivative couplings with respect to bosonic field
- 2.) spatially uniform
- 3.) spin-spin sub-dominant (non-dispersive)

consistent
with exact argument



bosonic field
contains all
the Mott physics
(numerically verified
that spatial variation of
bosonic field does nothing)
spin part irrelevant

Strong-coupling antiferromagnet

$$B_{ij} = \langle g_{ij} \varphi_i^\dagger c_{i,\uparrow} c_{j,\downarrow} \rangle.$$

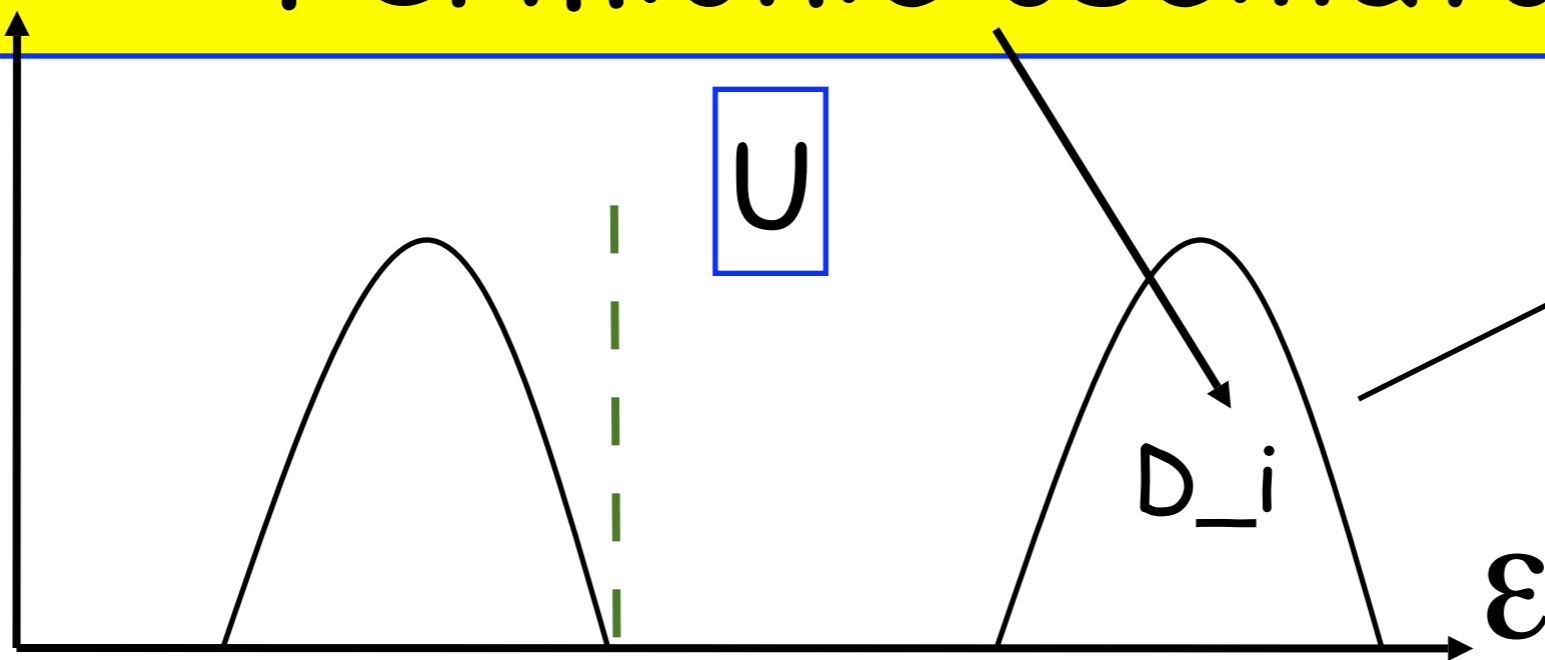
No adiabatic connection
with weak-coupling AF

bound states of the boson
mediate the Mott gap:
mechanism of localization of double occupancy

hole-doping

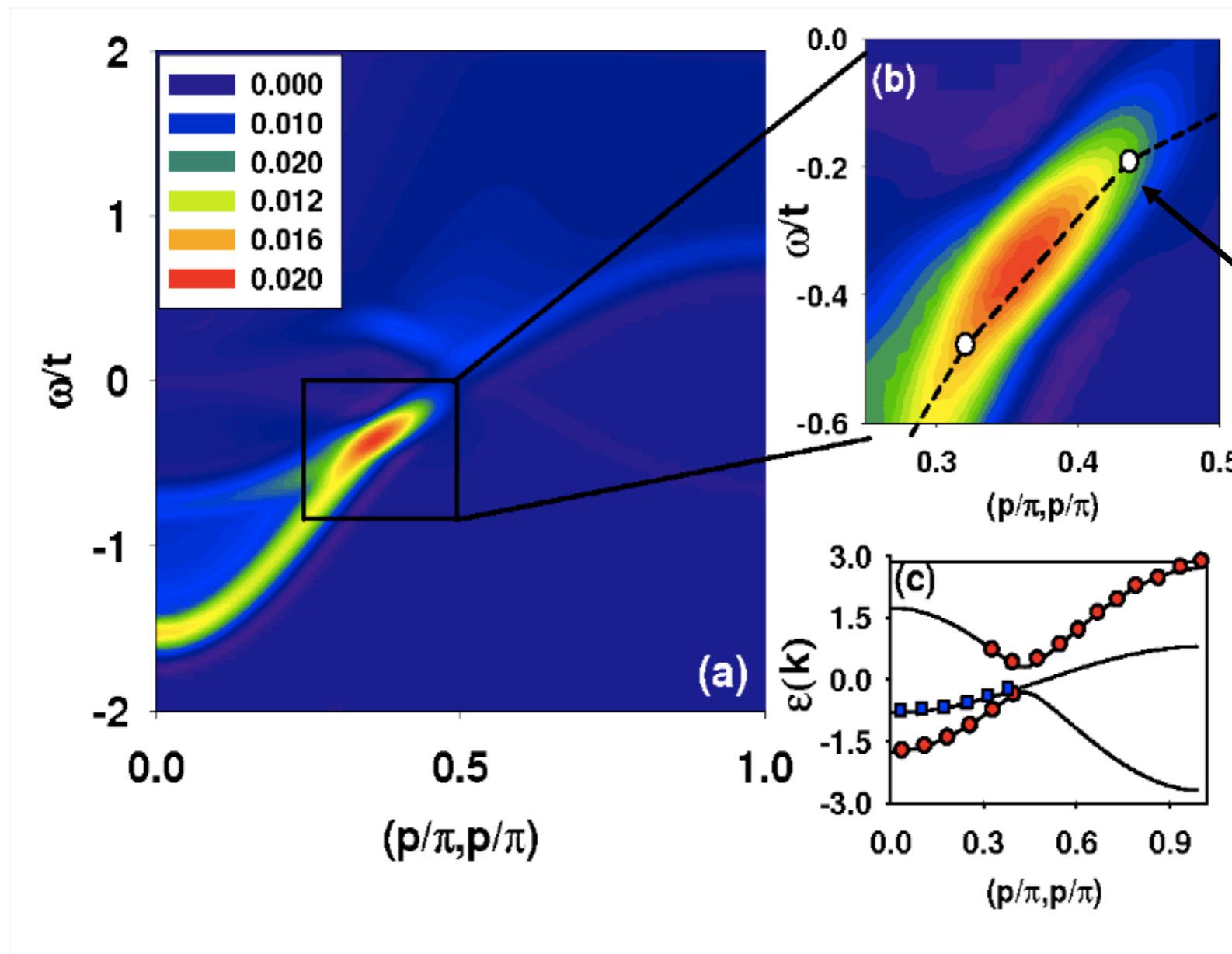
Extend the Hilbert space:
Associate with U-scale a new
Fermionic oscillator

$N(\omega)$



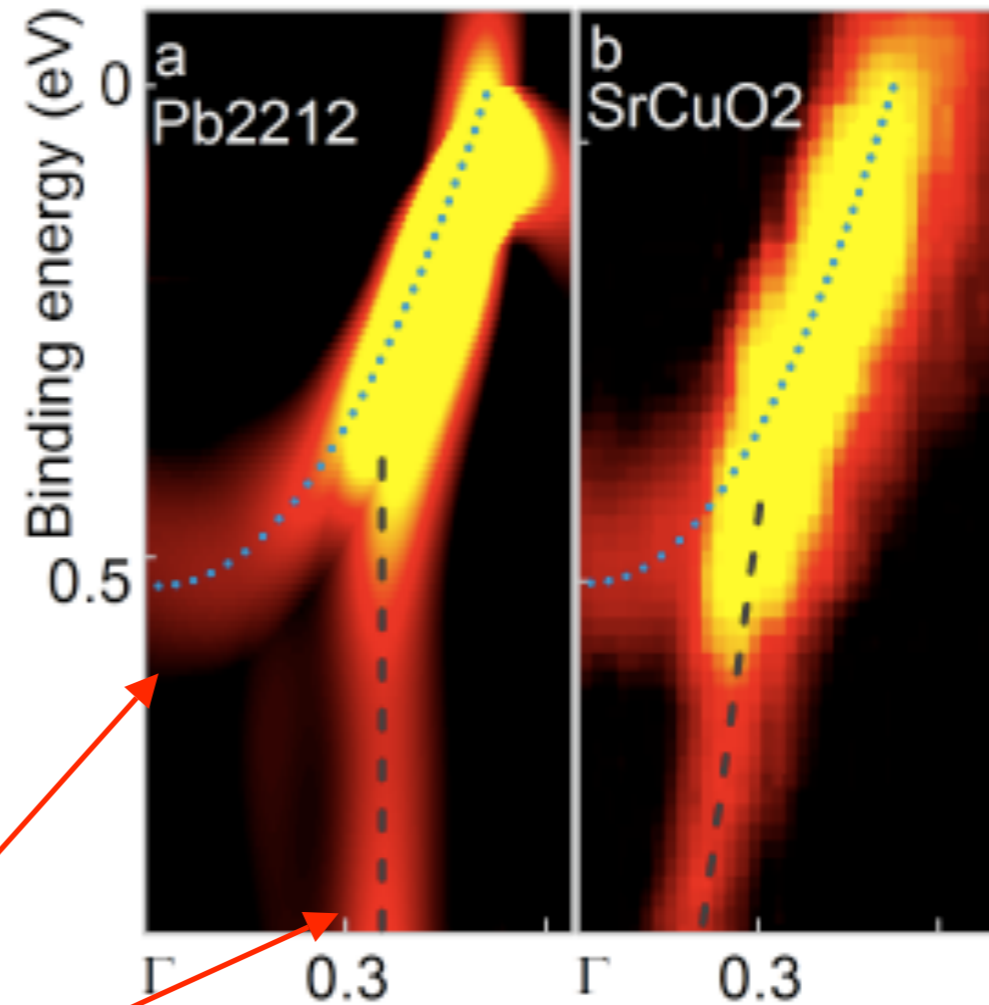
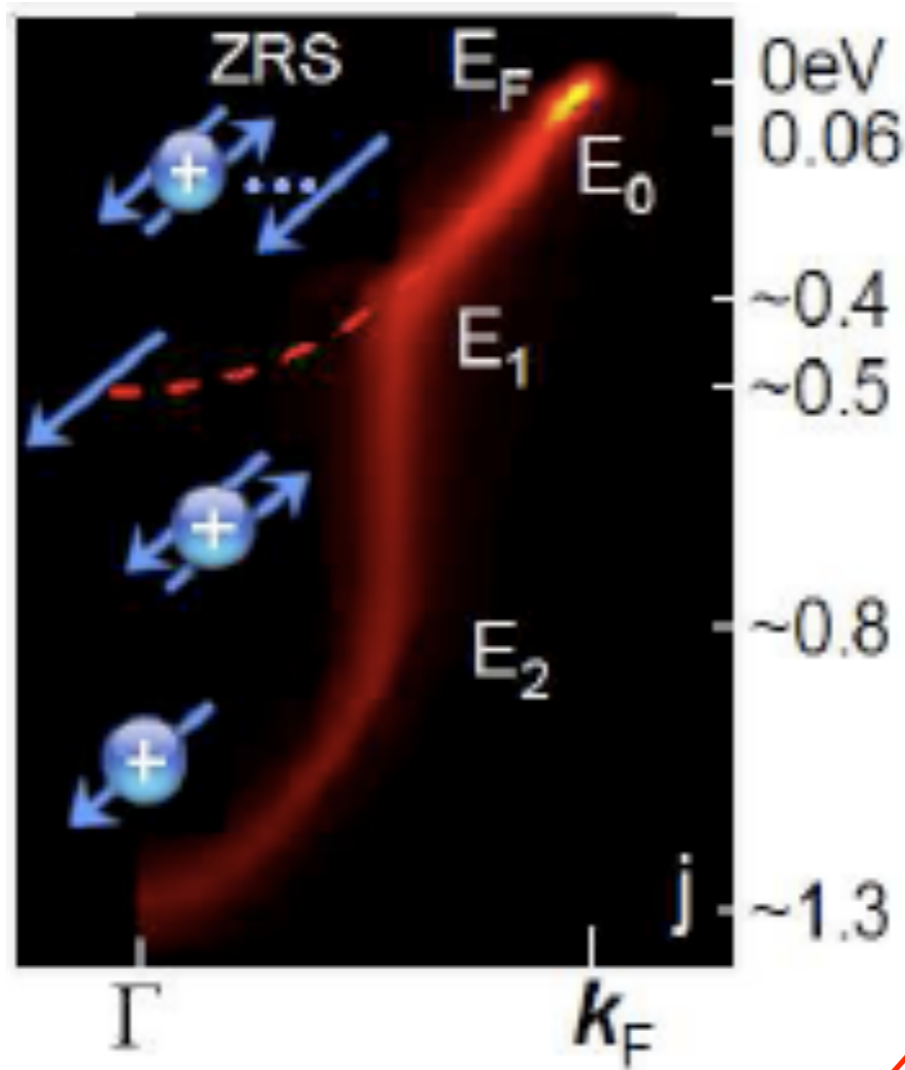
$$U D_i^\dagger D_i$$

Electron spectral function



$t^2/U \sim 60 \text{meV}$

Graf, et al. PRL vol. 98, 67004 (2007).




Two bands!!

Spin-charge separation?

Graf, et al. PRL vol. 98, 67004 (2007).

Origin of two bands

Two charge e excitations


$$c_{i\sigma}(1 - n_{i-\sigma})$$

$$\varphi_i^\dagger c_{i\bar{\sigma}}$$

Graf, et al. PRL vol. 98, 67004 (2007).

Origin of two bands

Two charge e excitations

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New bound state

Graf, et al. PRL vol. 98, 67004 (2007).

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φ_i is confined (no kinetic energy)

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Graf, et al. PRL vol. 98, 67004 (2007).

Origin of two bands

Two charge e excitations

$$c_{i\sigma}(1 - n_{i-\sigma})$$

φ_i is confined (no kinetic energy)

$$\varphi_i^\dagger c_{i\bar{\sigma}}$$

New bound state

Pseudogap

two types of charges

'free'

bound

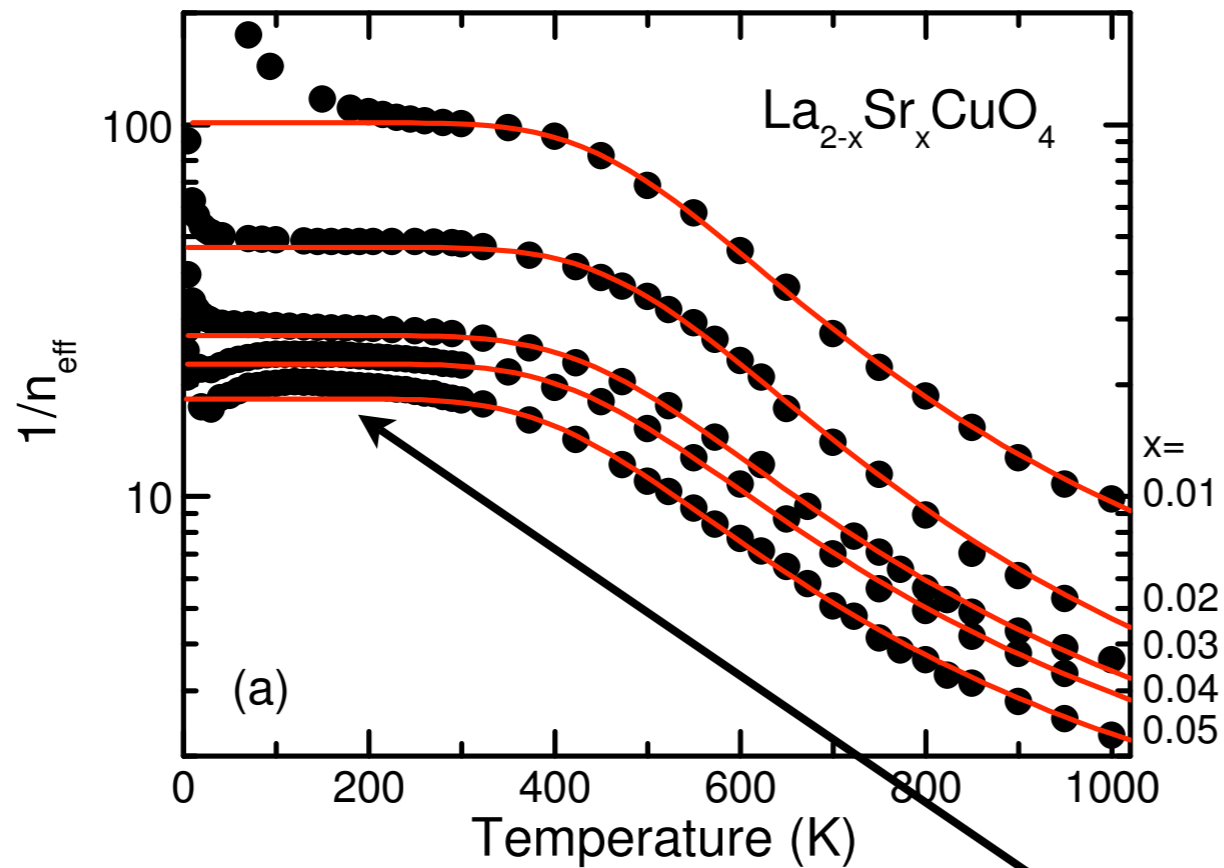
direct evidence

direct evidence

charge carrier density:

direct evidence

charge carrier density:

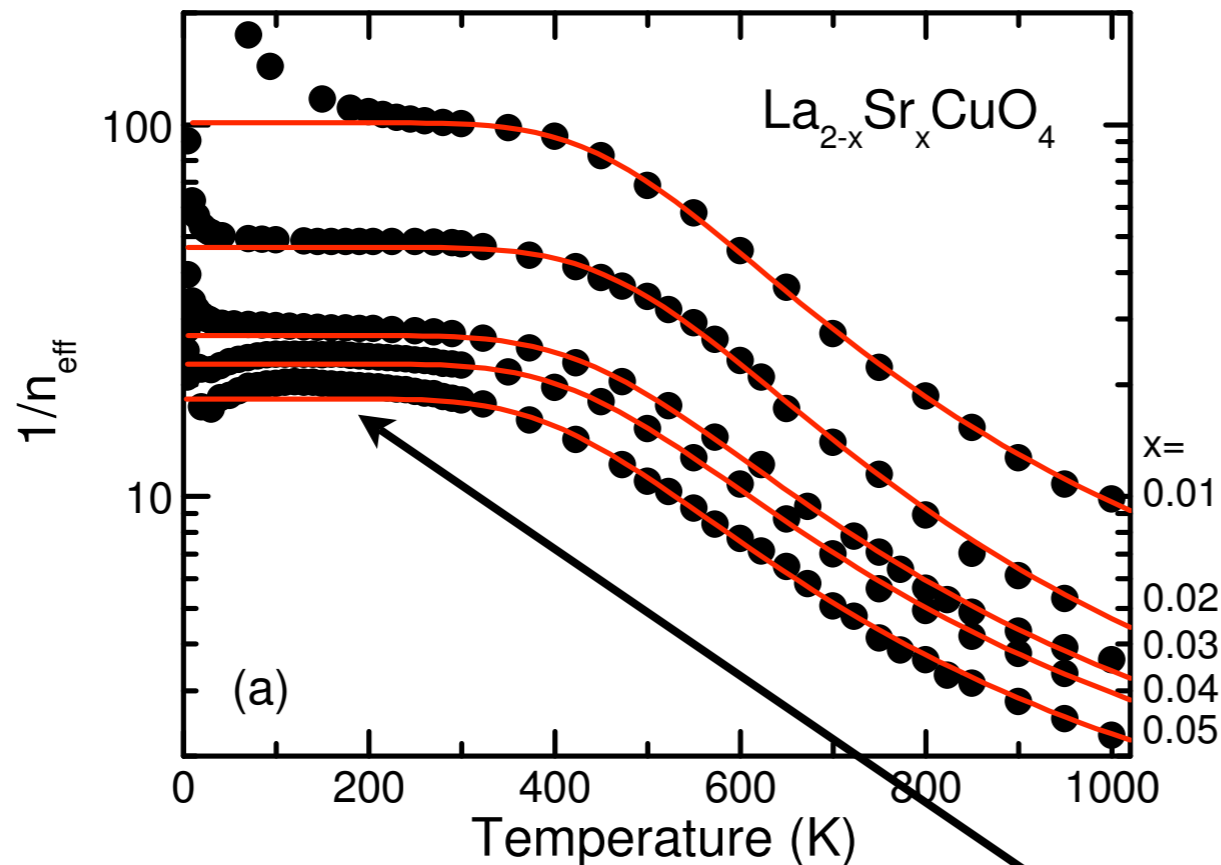


Ono, et al., Phys. Rev. B 75, 024515
(2007)

$$n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp(-\Delta(x)/T), \quad \text{PRL, vol. 97, 247003 (2006).}$$

direct evidence

charge carrier density:

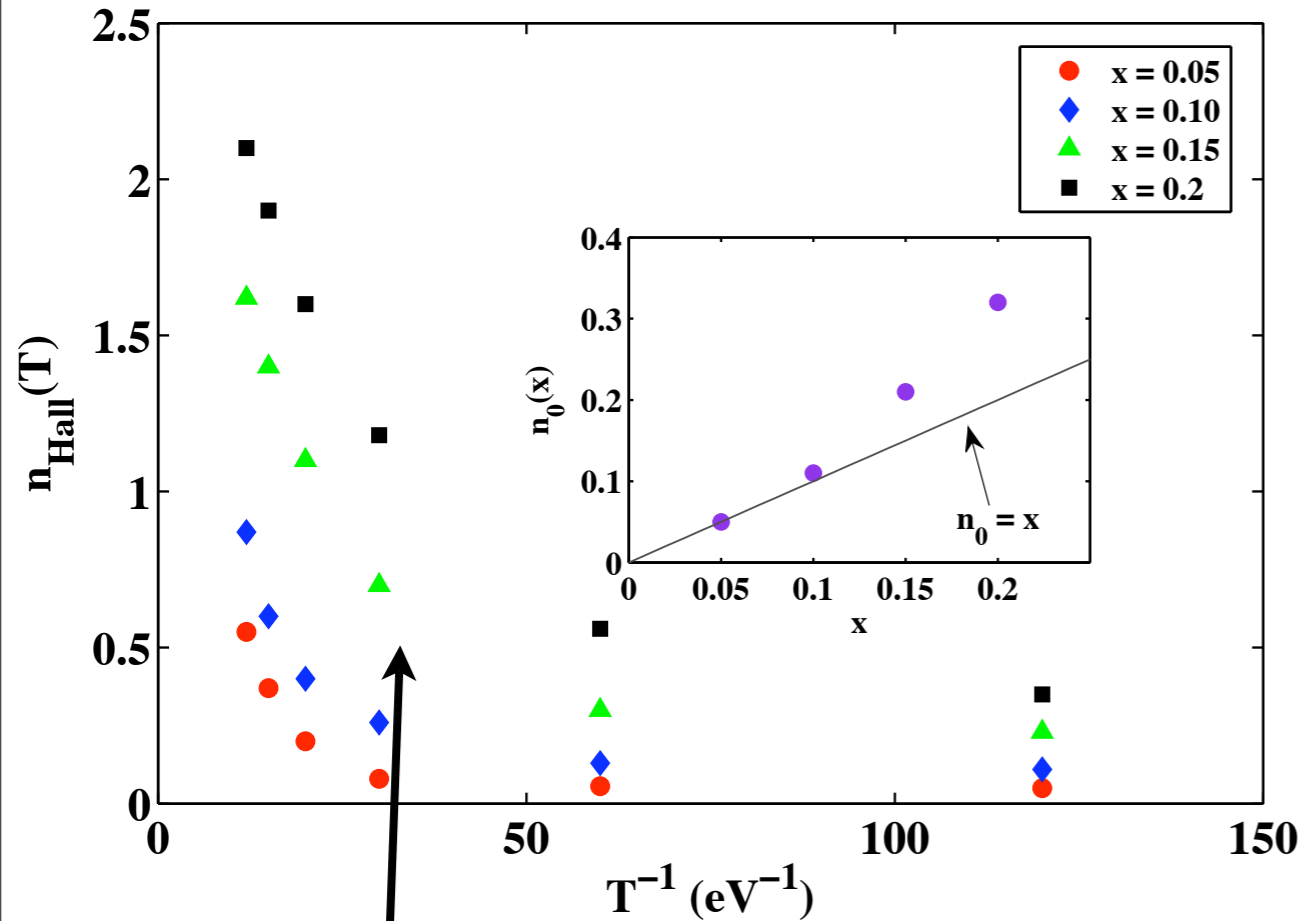


Ono, et al., Phys. Rev. B 75, 024515
(2007)

$$n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp(-\Delta(x)/T), \quad \text{PRL, vol. 97, 247003 (2006).}$$

exponentially suppressed: confinement

Our Theory

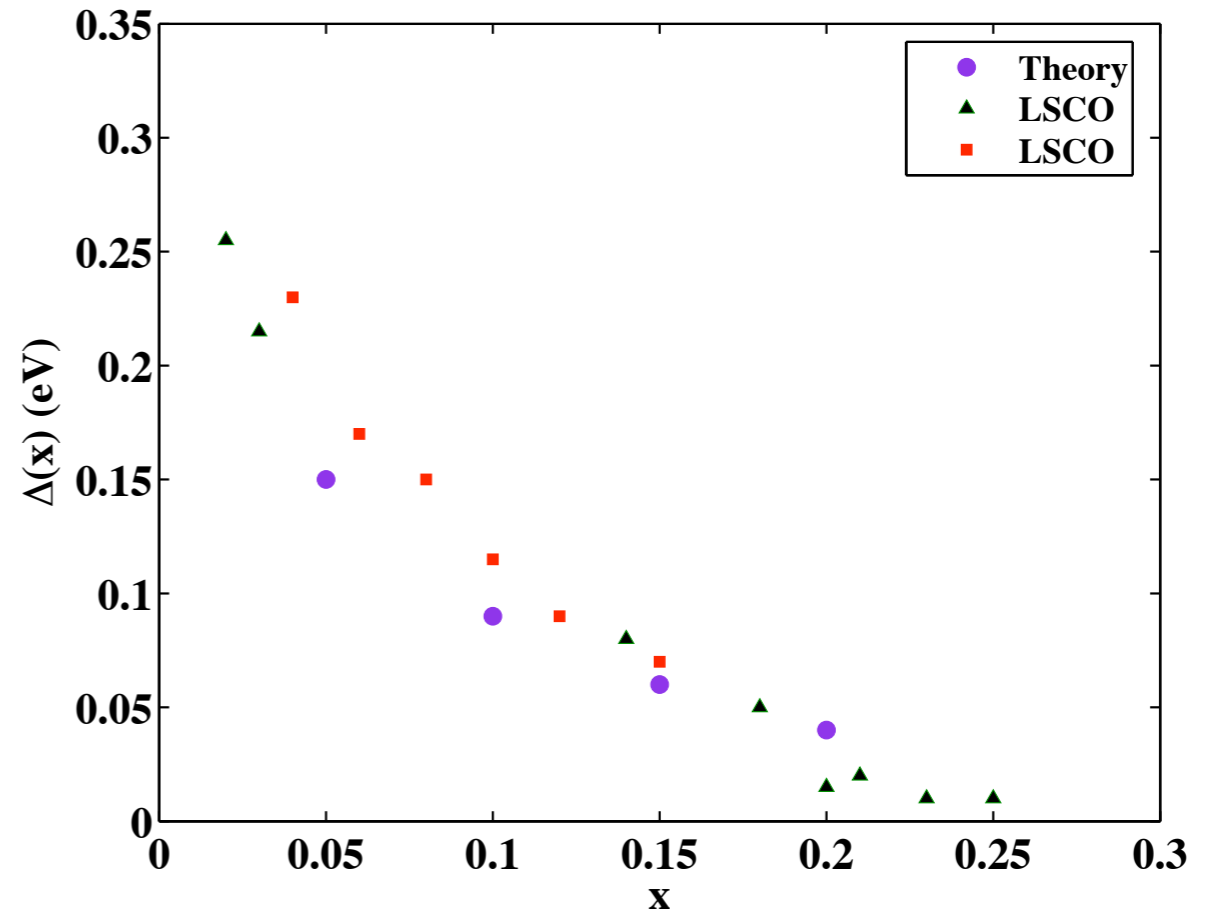
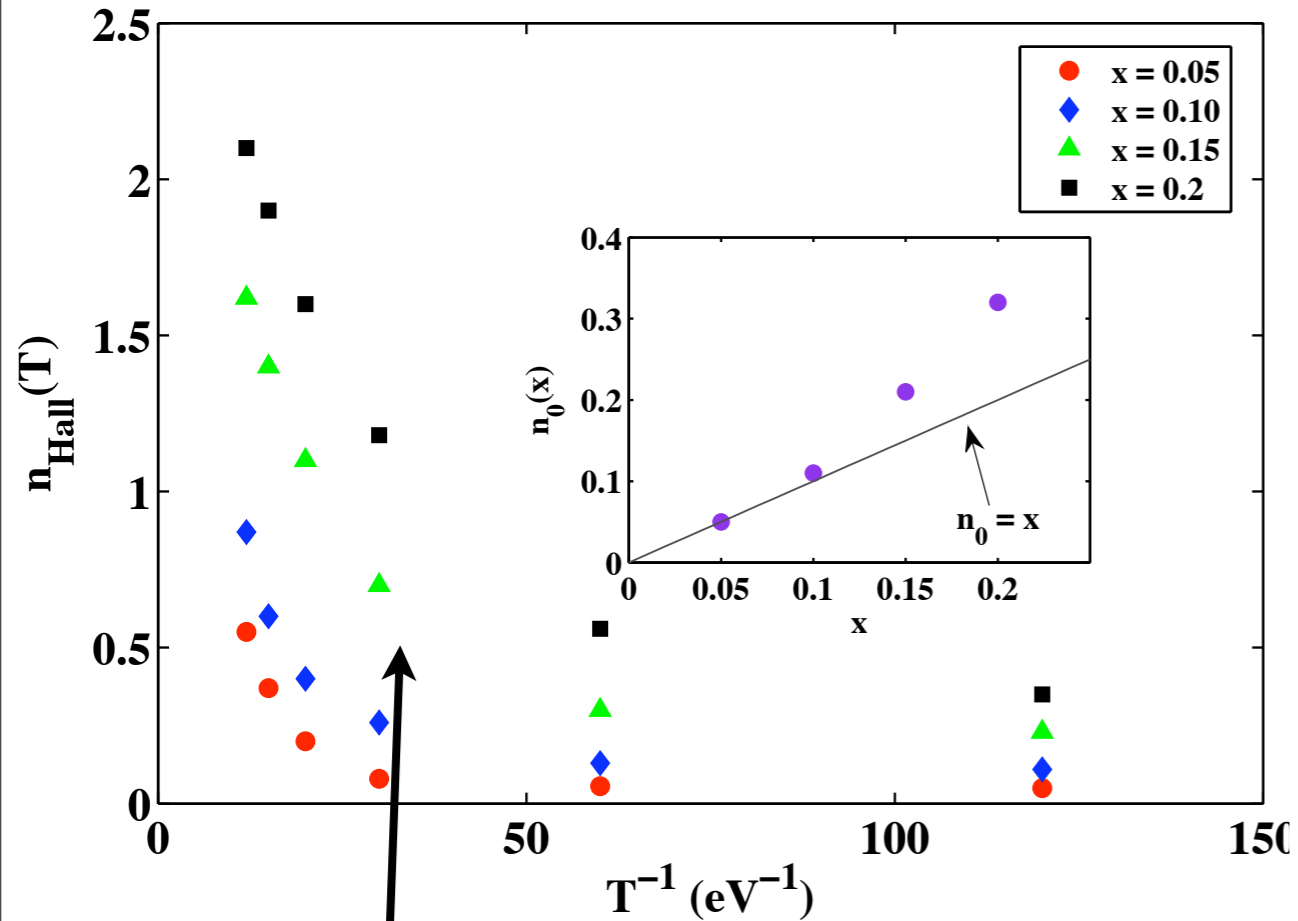


exponential
T-dependence

no model-dependent
free parameters: just
 t/U

Our Theory

gap

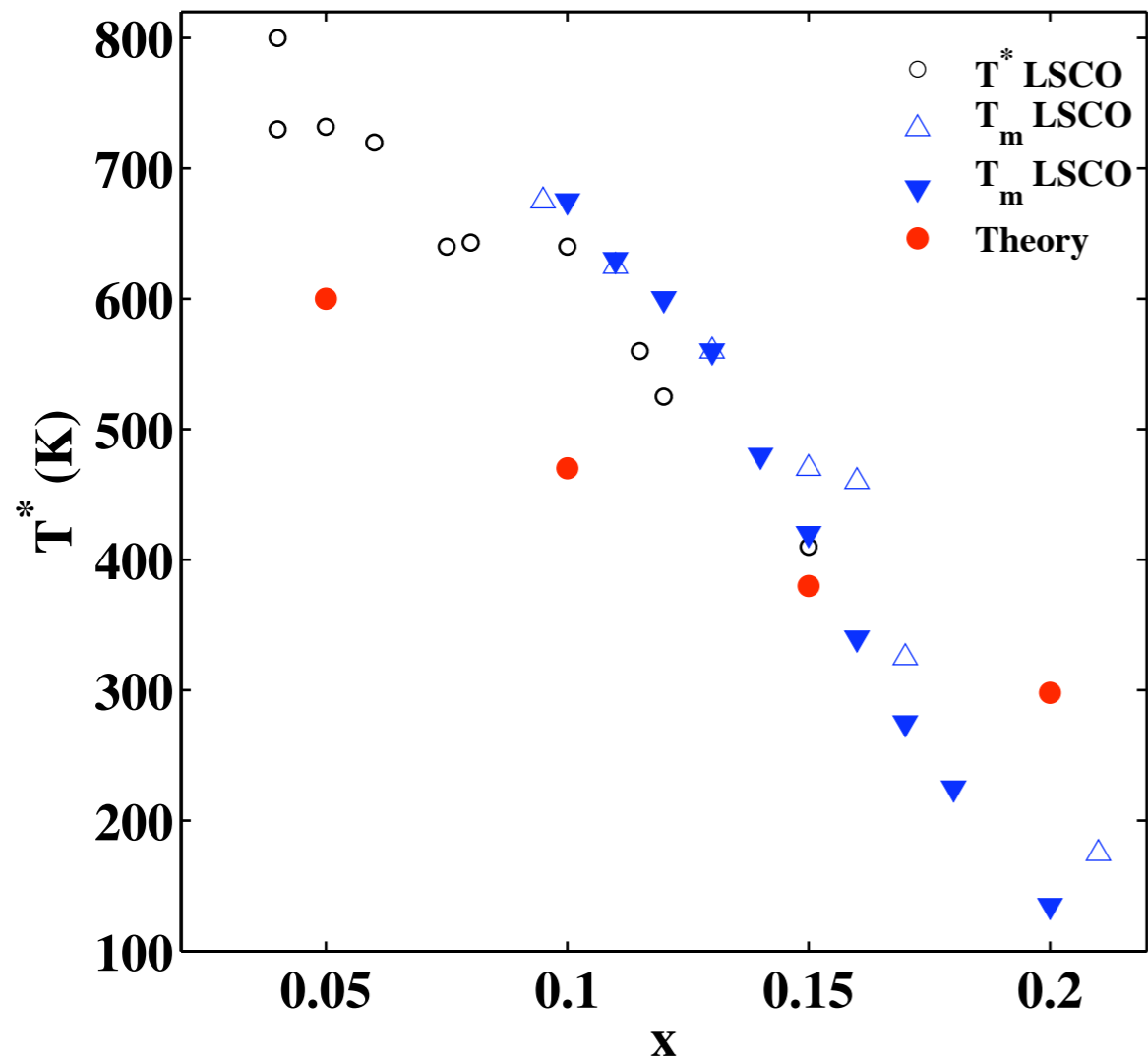


exponential
T-dependence

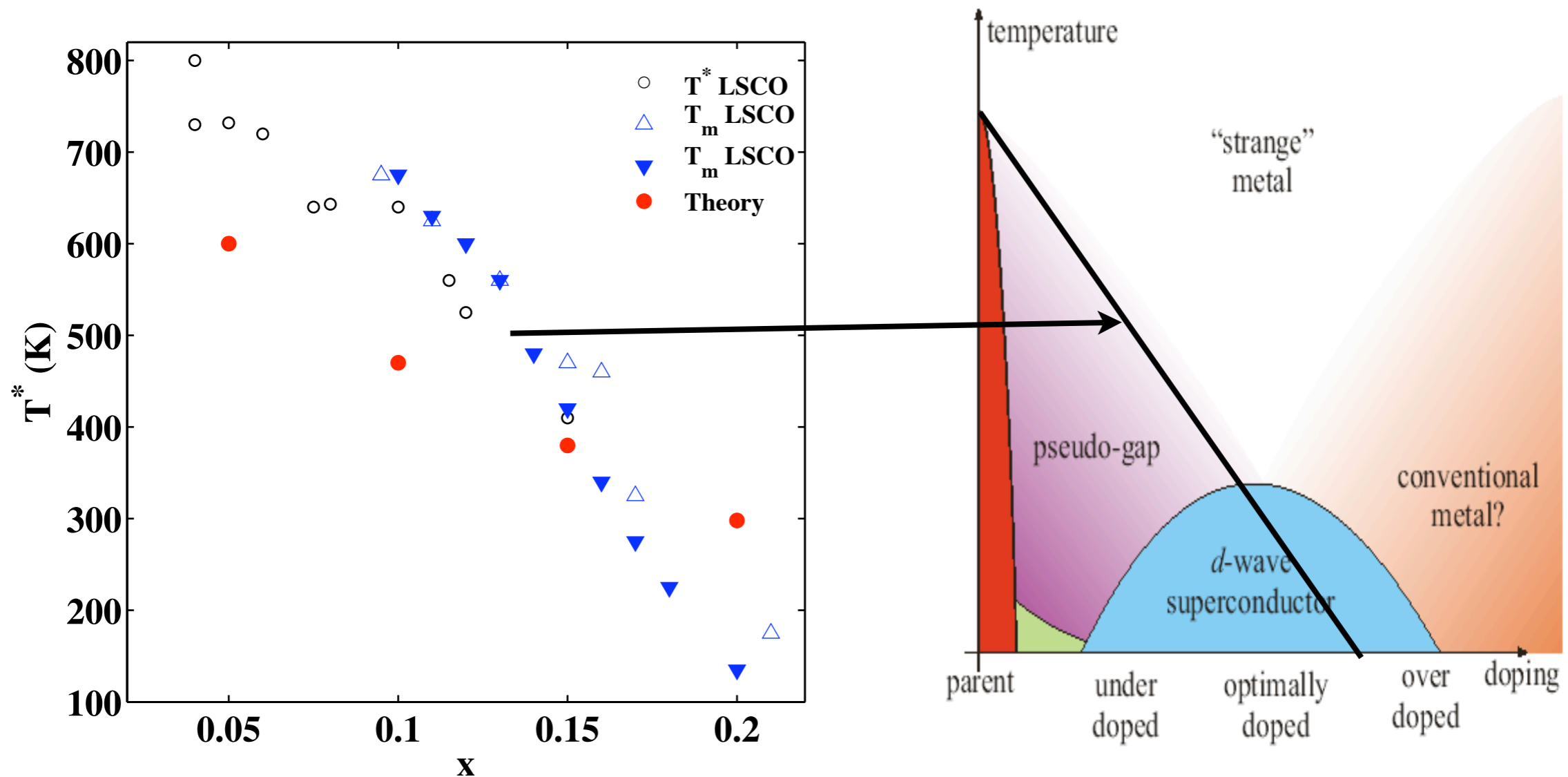
no model-dependent
free parameters: just
 t/U

Like Mott gap,
Pseudogap is a bound-state
problem with new IR modes

strange metal: breakup ('deconfinement') of bound states



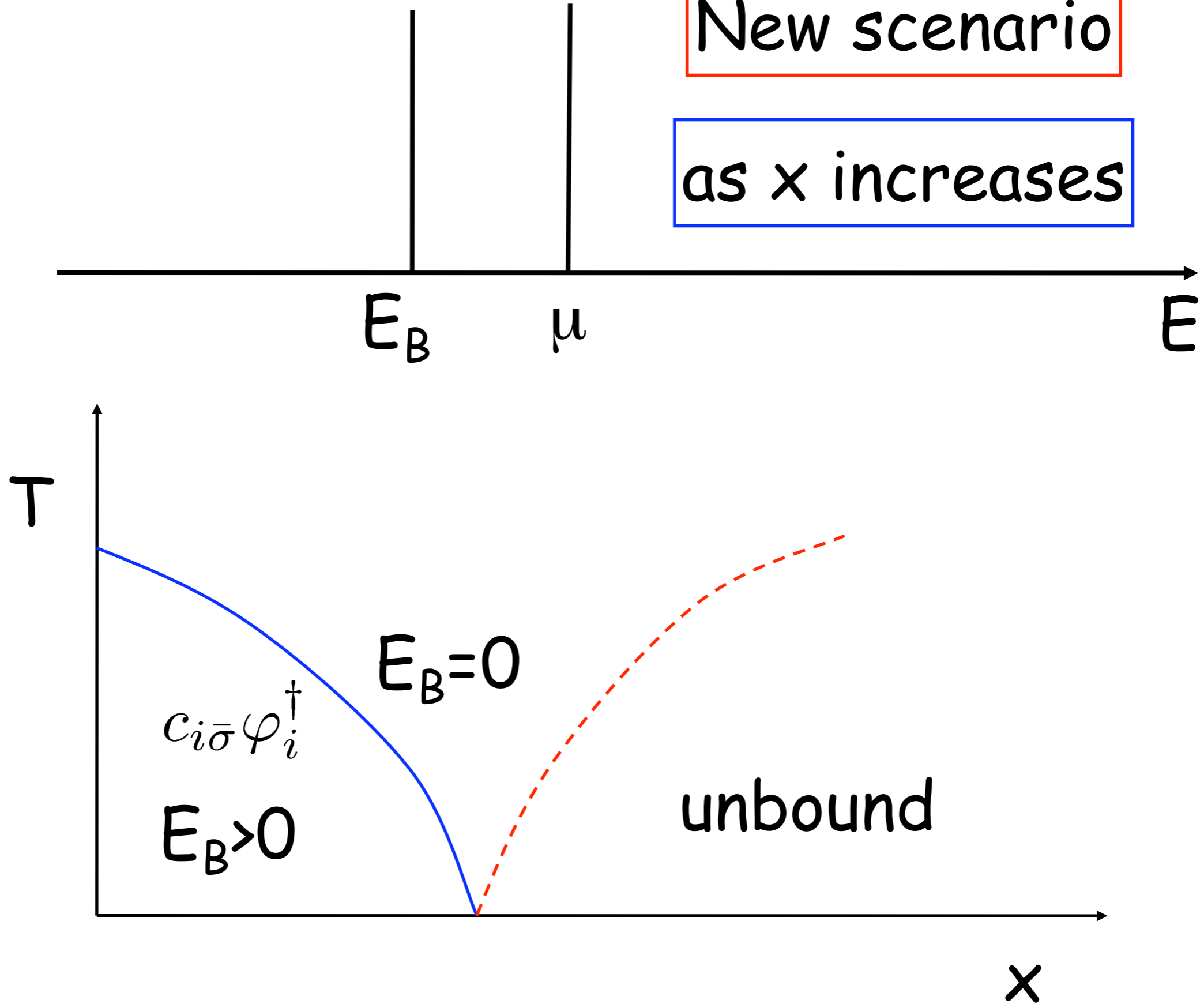
strange metal: breakup ('deconfinement') of bound states



T-linear resistivity

New scenario

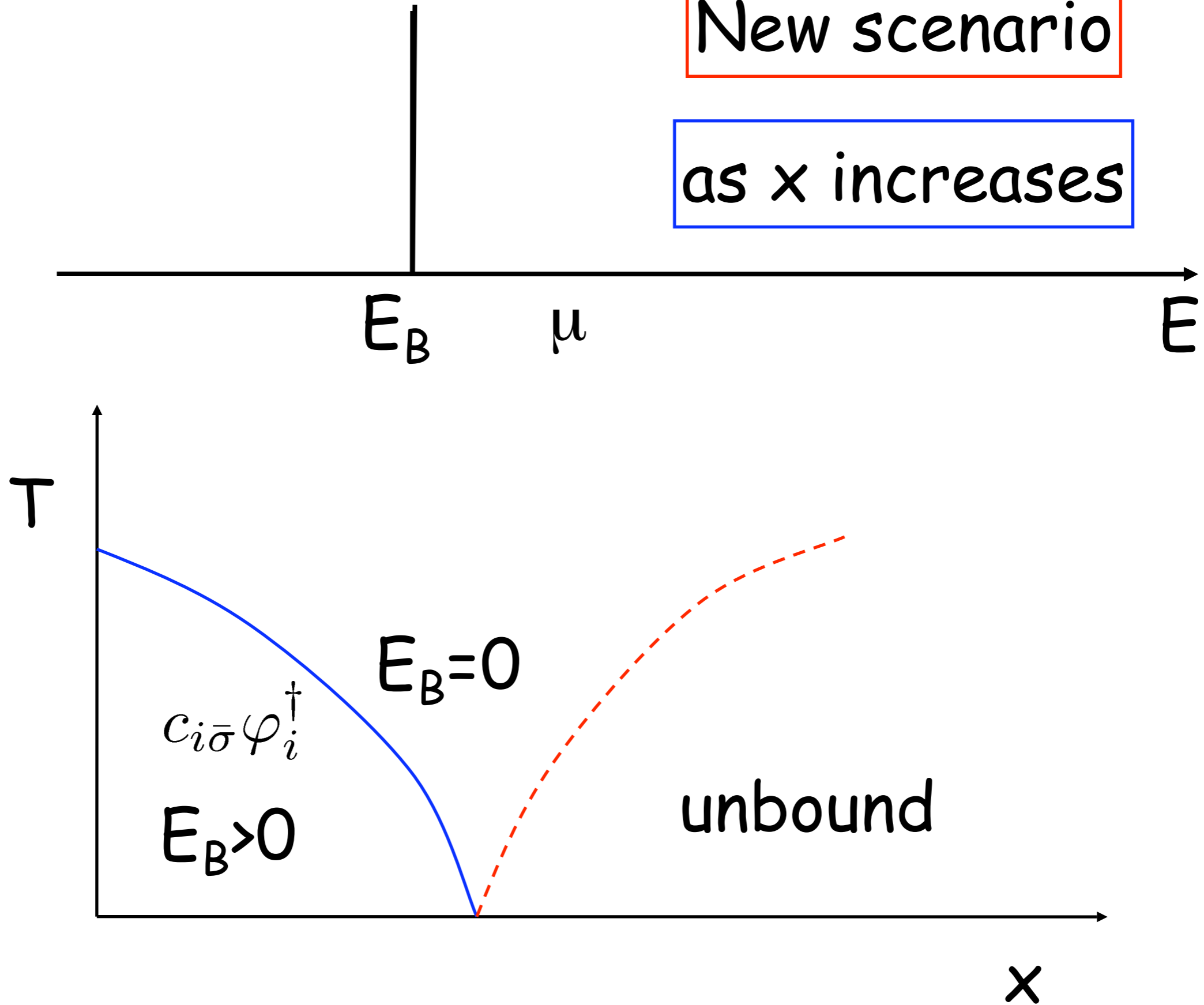
as x increases



T-linear resistivity

New scenario

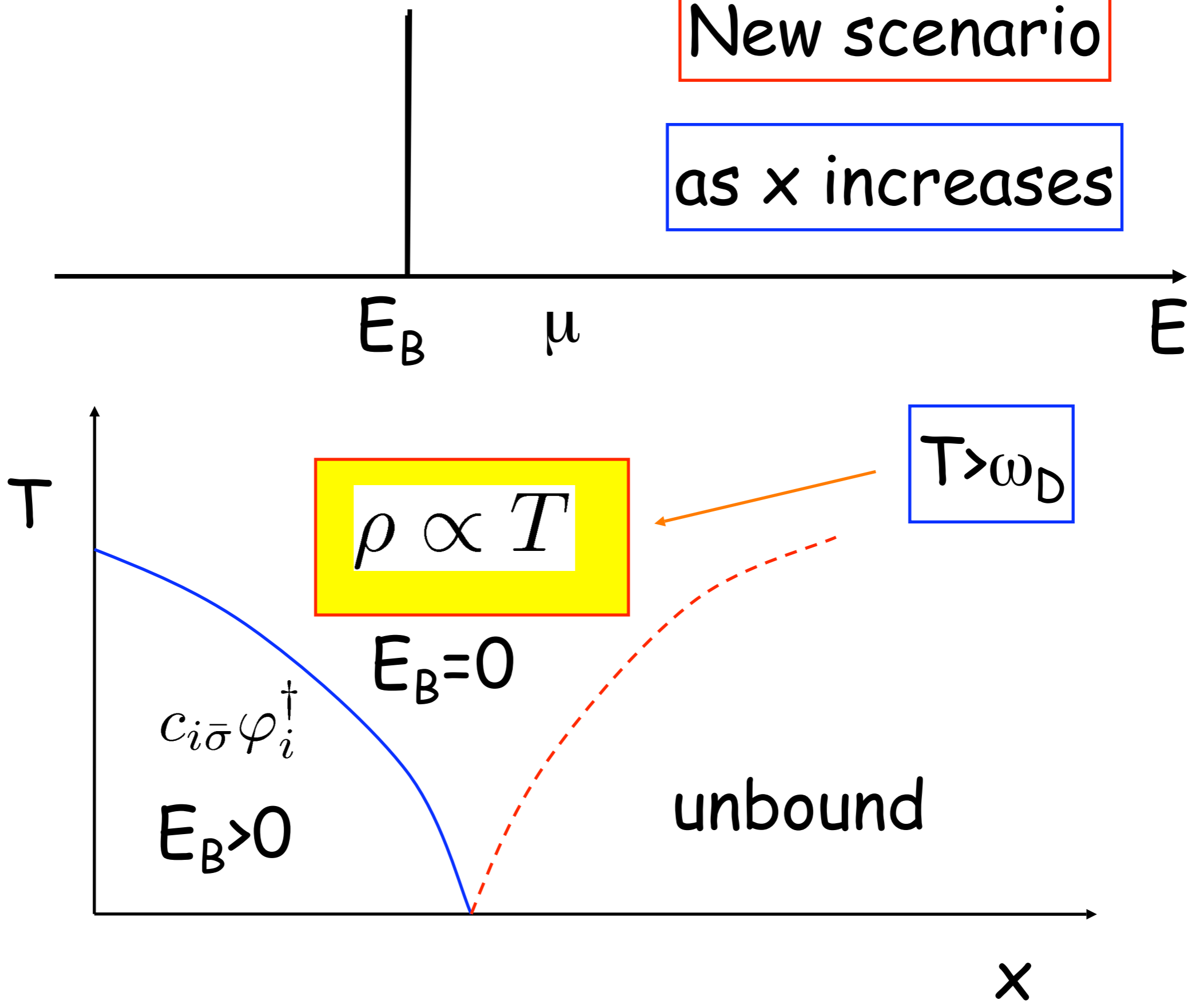
as x increases



T-linear resistivity

New scenario

as x increases



Motttness: Strong Coupling



low-energy reduction

Mottness: Strong Coupling



low-energy reduction

bare particles

Mottness: Strong Coupling

low-energy reduction

~~bare particles~~

Mottness: Strong Coupling


low-energy reduction

~~bare particles~~

Pseudogap = 'confinement'

Mottness: Strong Coupling

low-energy reduction

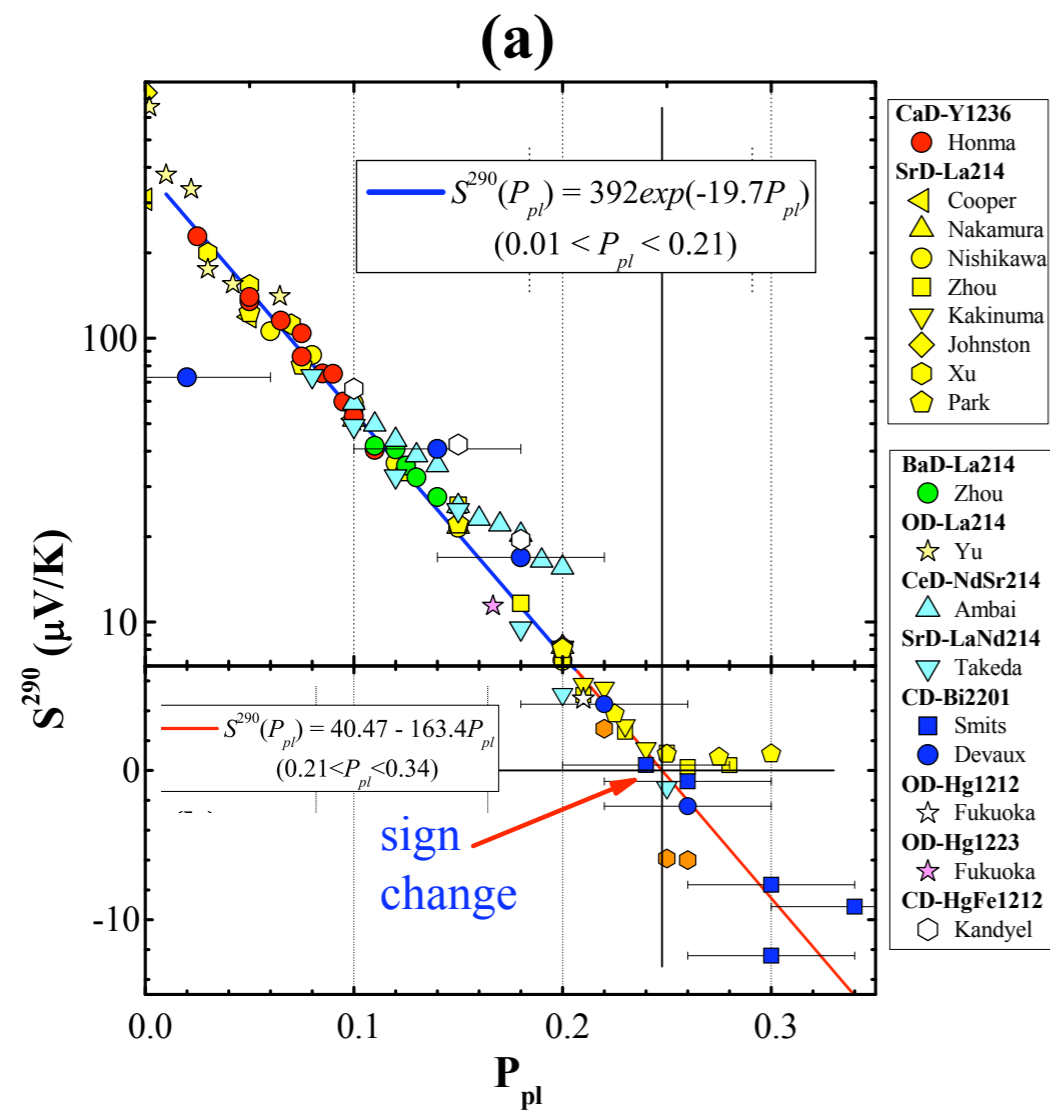
bare particles

composite or bound states not in UV theory

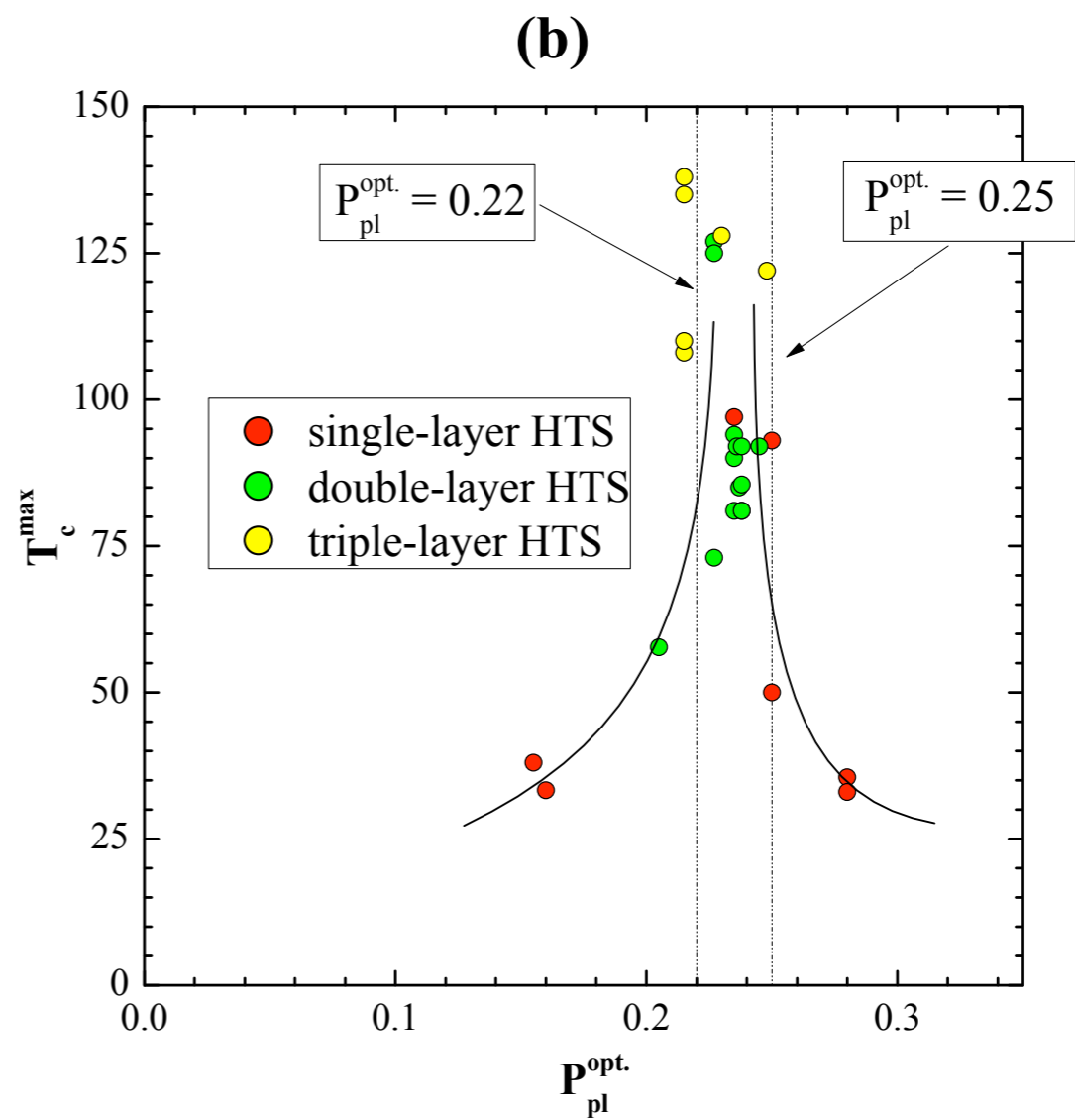
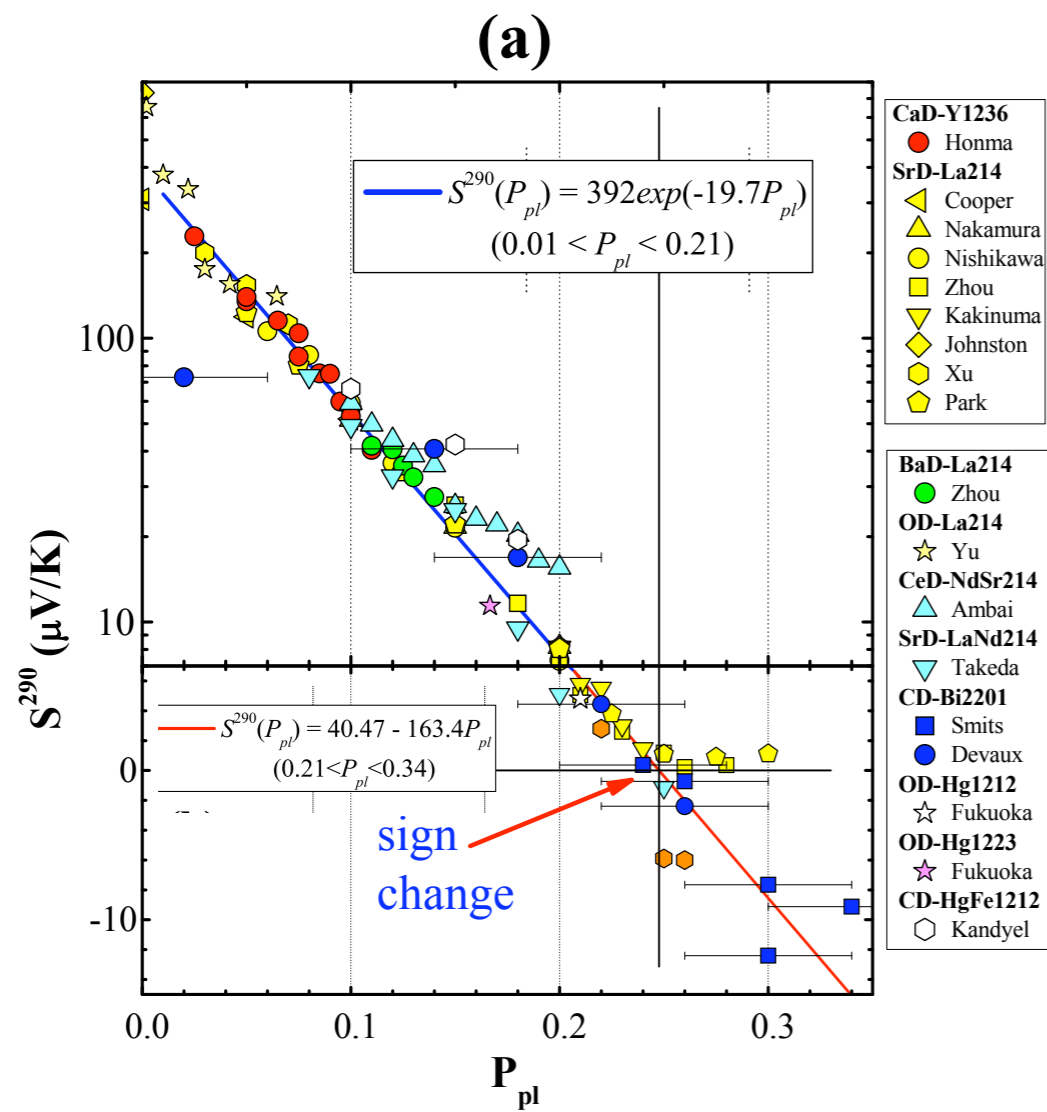
Pseudogap= 'confinement'

Third Problem:
Correlate of superconductivity?

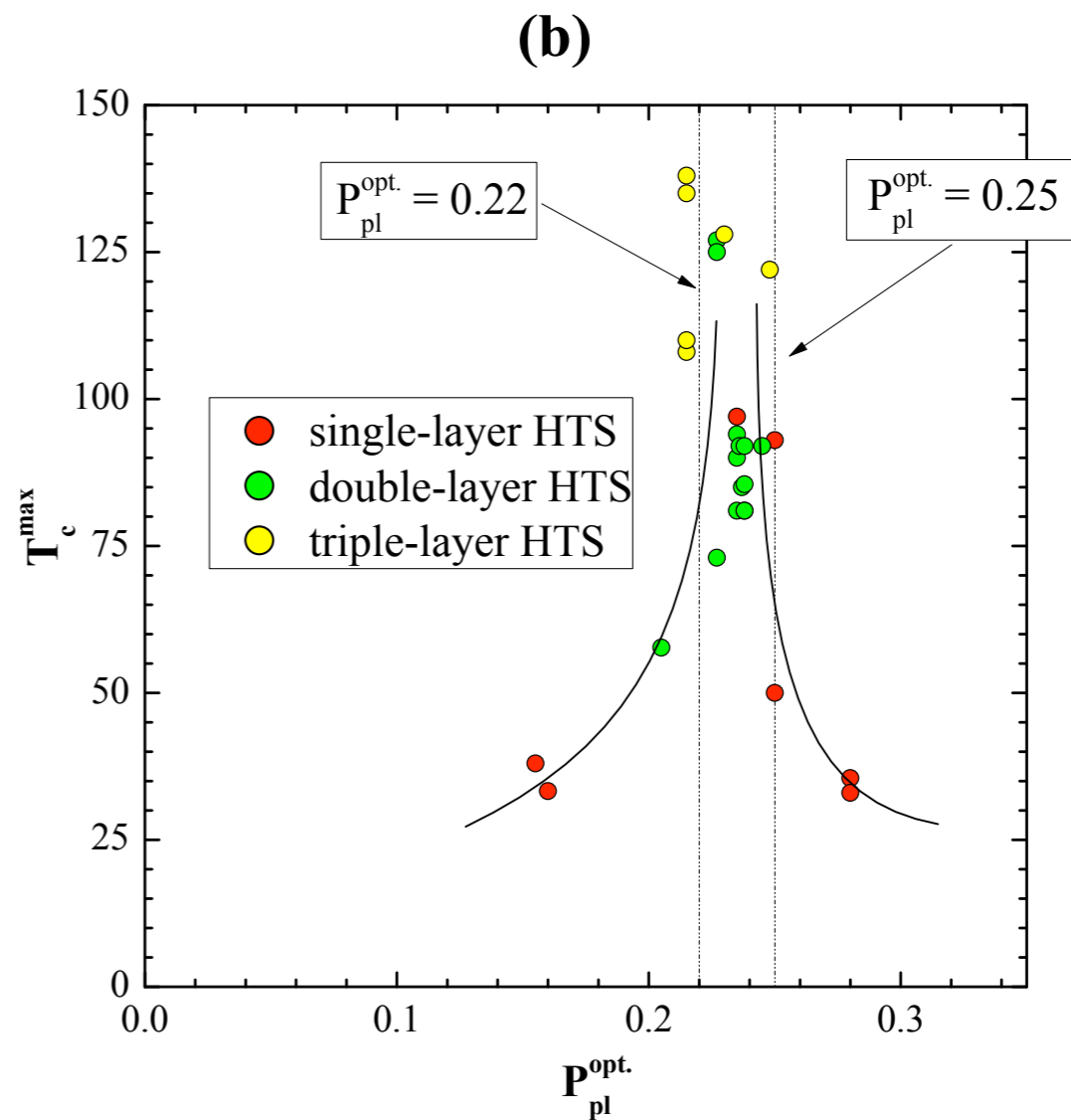
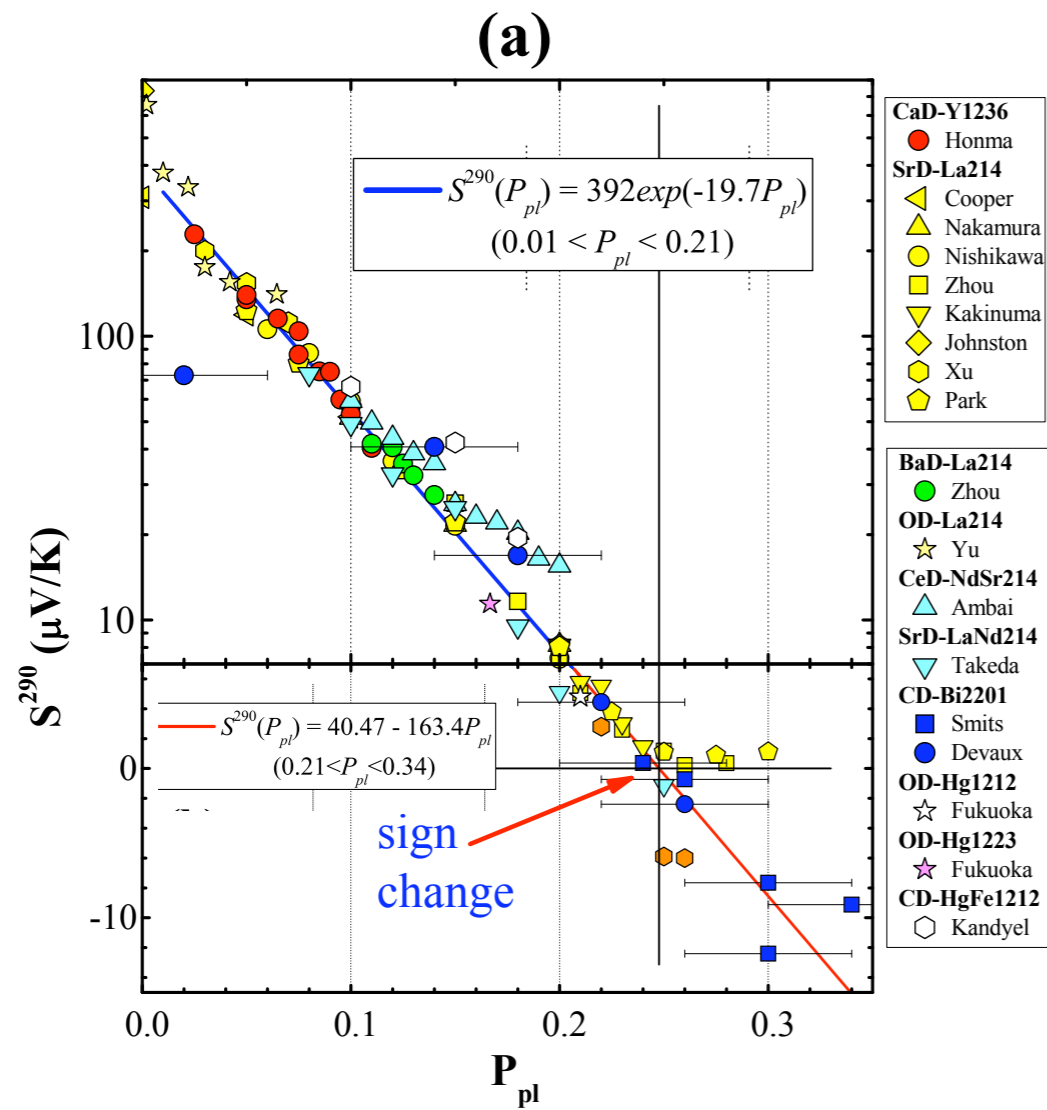
Universal sign change of thermopower



Universal sign change of thermopower

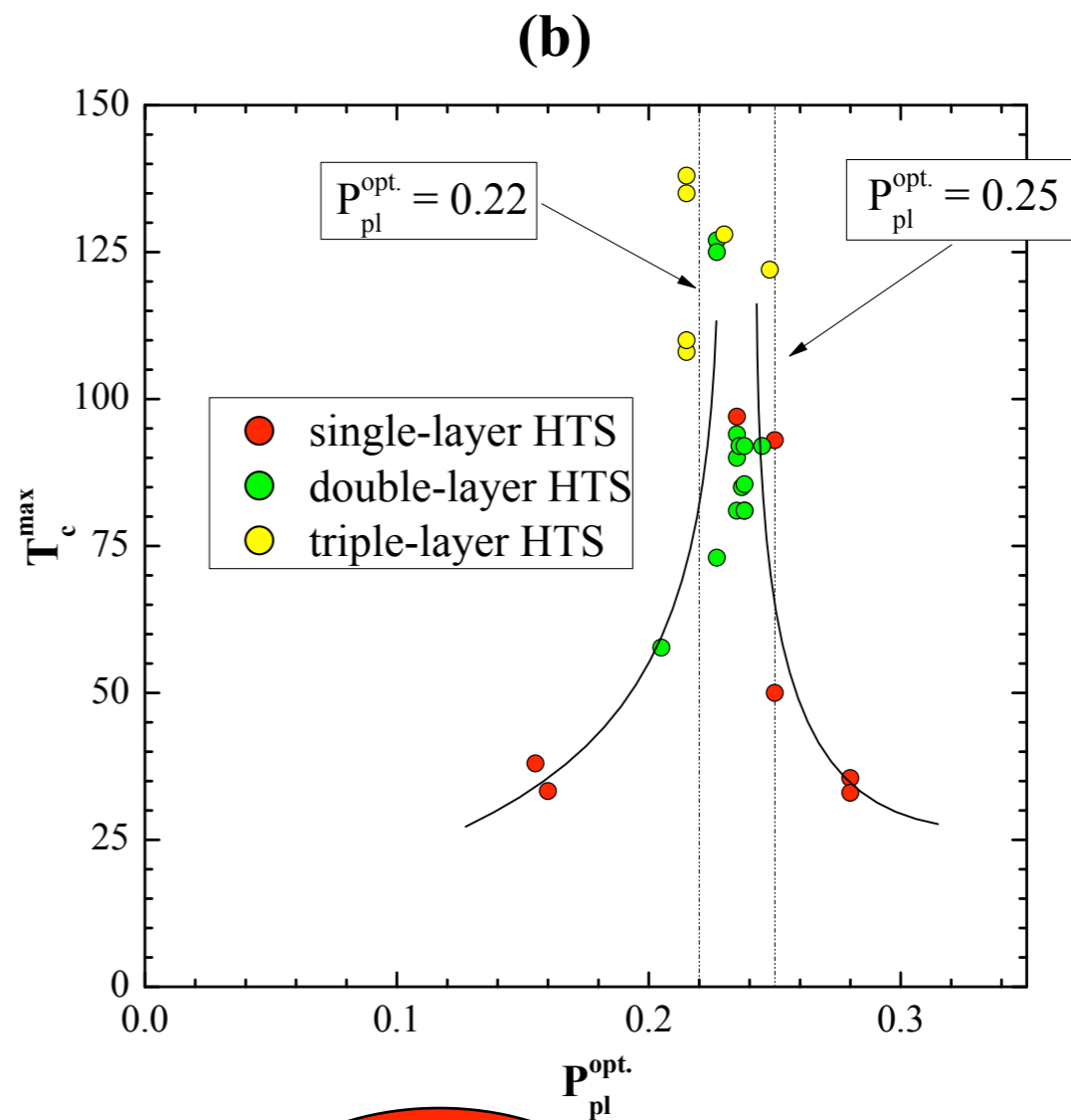
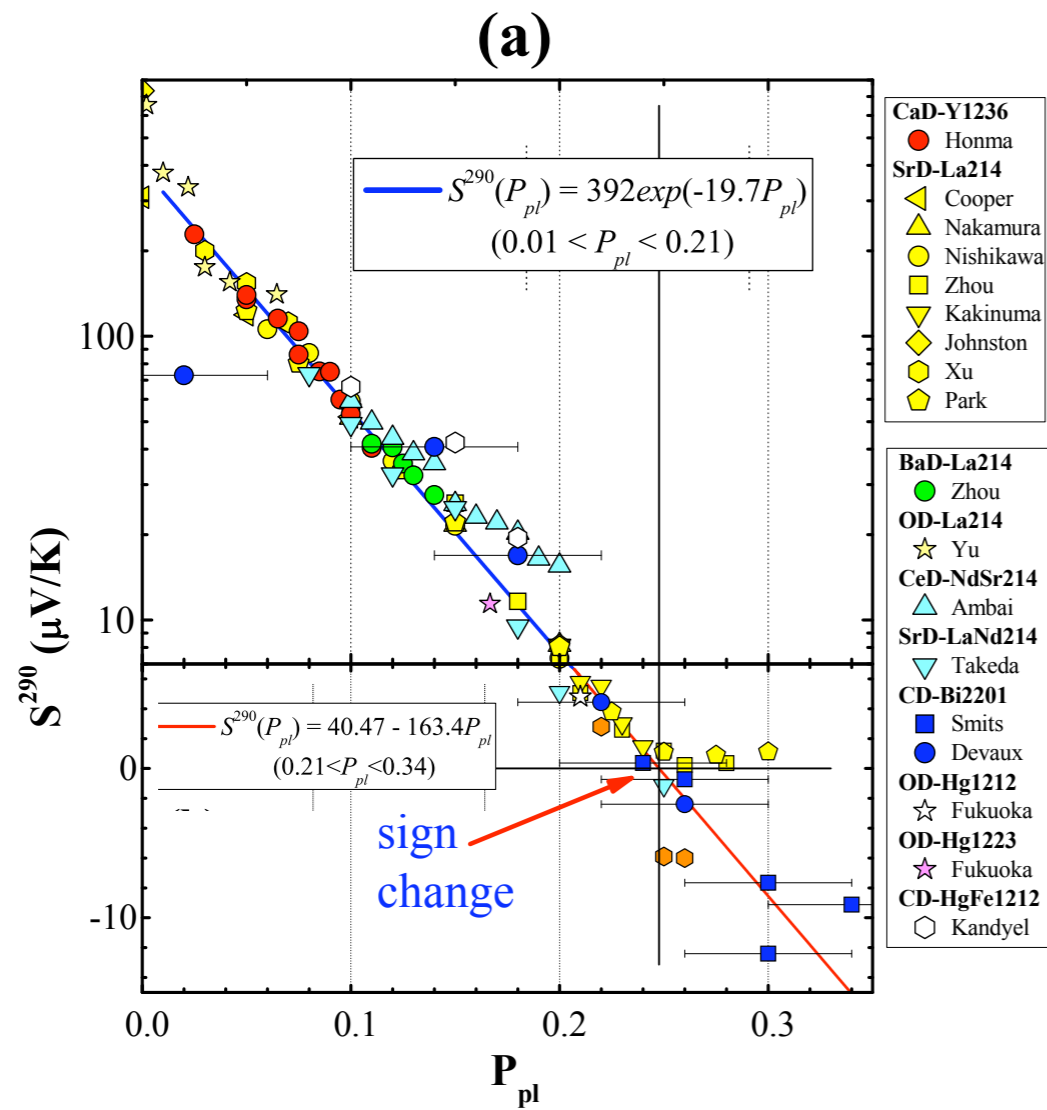


Universal sign change of thermopower



$$1 - \frac{T_c}{T_c^{\max}} = 82.6(x - 0.16)^2.$$

Universal sign change of thermopower



~~$1 - \frac{T_c}{T_{c\max}} = 2.6(x - 16)^2$~~

Why?

Mottness

G. Beni, Phys. Rev. B vol. 10, 2186
(1973).



Exact calculation
of S for atomic ($t=0$)
limit of Hubbard model

$$S = -\frac{k_B}{e} \ln \frac{2x}{1-x}$$

G. Beni, Phys. Rev. B vol. 10, 2186
(1973).

Exact calculation
of S for atomic ($t=0$)
limit of Hubbard model



band insulator
(free electrons)

$$S = -\frac{k_B}{e} \ln \frac{2x}{1-x}$$

$$\ln \frac{x}{2-x}$$

vanishes at
half-filling

G. Beni, Phys. Rev. B vol. 10, 2186
(1973).

Exact calculation
of S for atomic ($t=0$)
limit of Hubbard model



band insulator
(free electrons)

$$\ln \frac{x}{2-x}$$

vanishes at
half-filling

$$S = -\frac{k_B}{e} \ln \frac{2x}{1-x}$$



$S=0$ when $x=1/3$; WHY?

What is so
special about
 $2x$ and $1-x$?

Thermopower Primer

$$S = -\frac{k_B}{e} \beta \frac{L_{12}}{L_{11}}$$

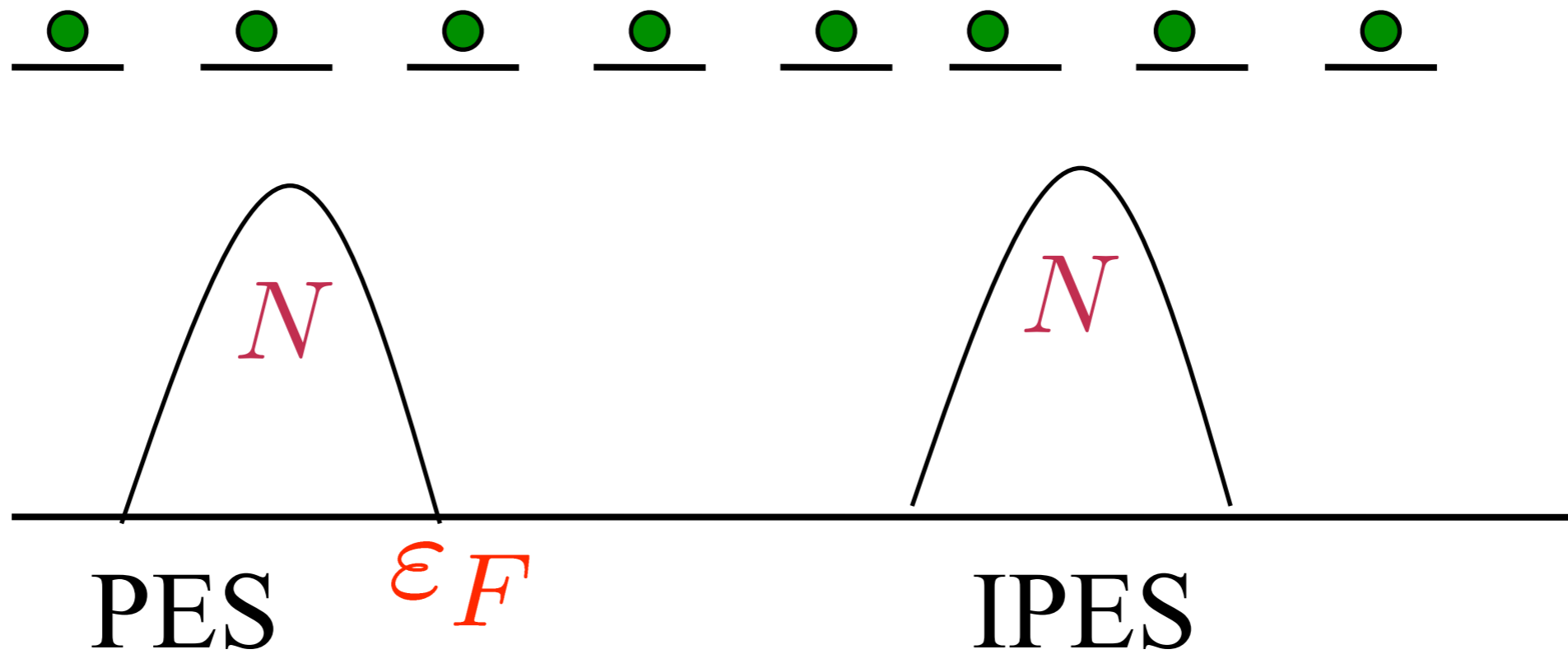
$$L_{ij} = \int_{-\infty}^{\infty} d\omega \left(-\frac{\partial f(\omega)}{\partial \omega} \right) \tau^i(\omega) \omega^{j-1}$$

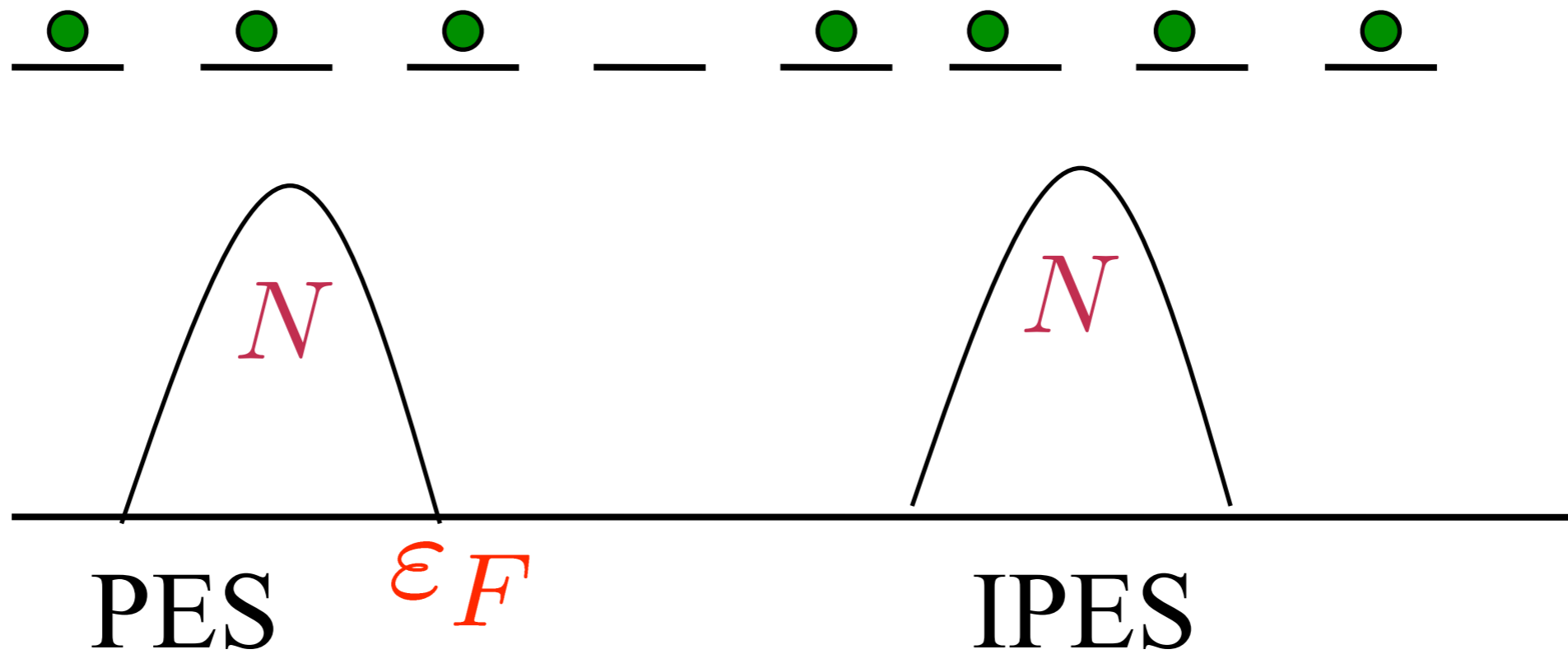
$$\tau(\omega) = \frac{1}{N} \sum_{\mathbf{k}, \sigma} \left(\frac{\partial \epsilon_{\mathbf{k}}}{\partial k_x} \right)^2 A^2(\mathbf{k}, \omega)$$

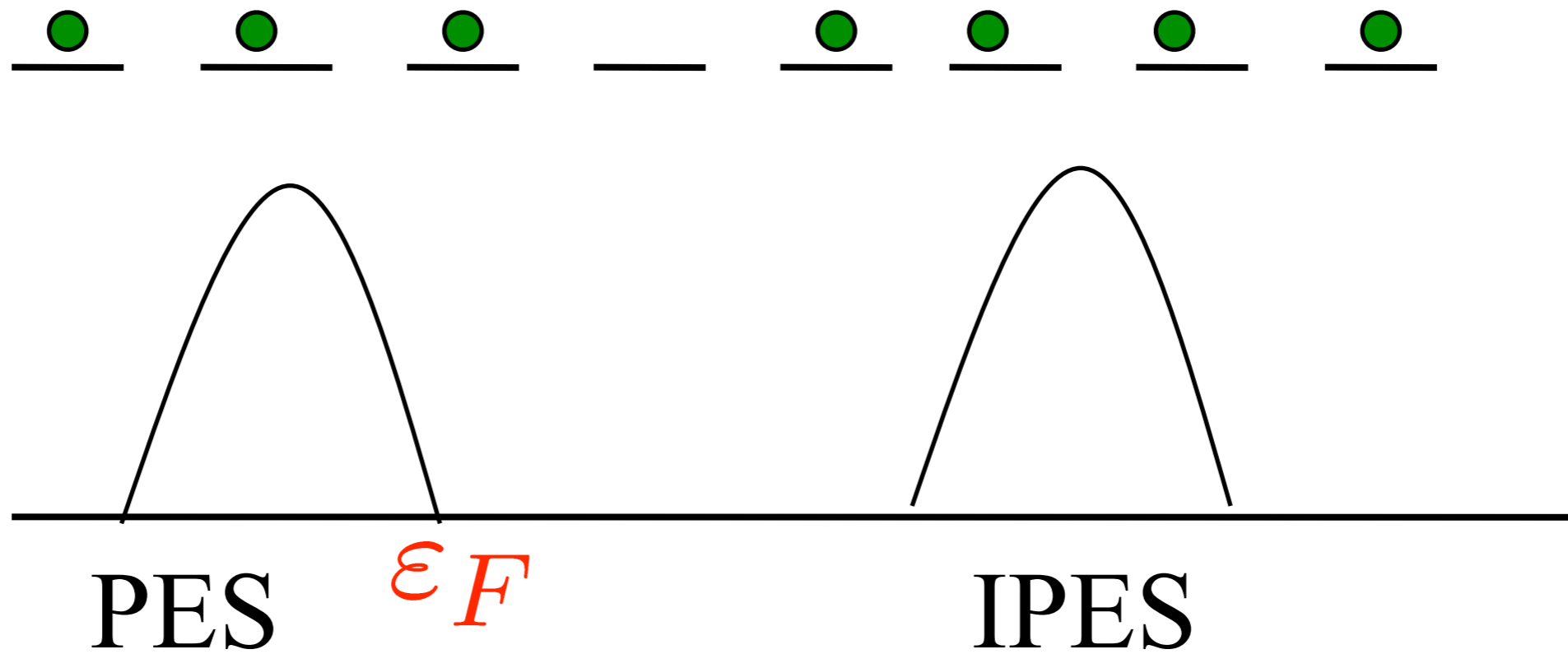
spectral
function

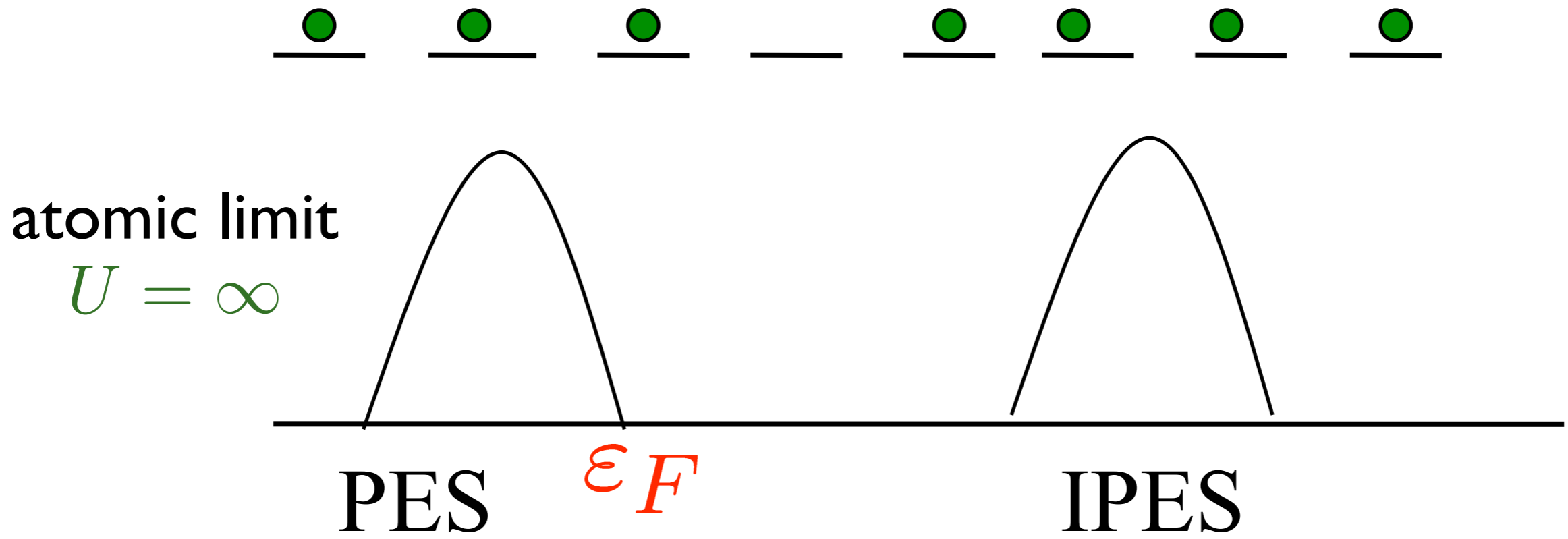
1.) \mathcal{T} must be symmetric about the chemical potential for $S=0$

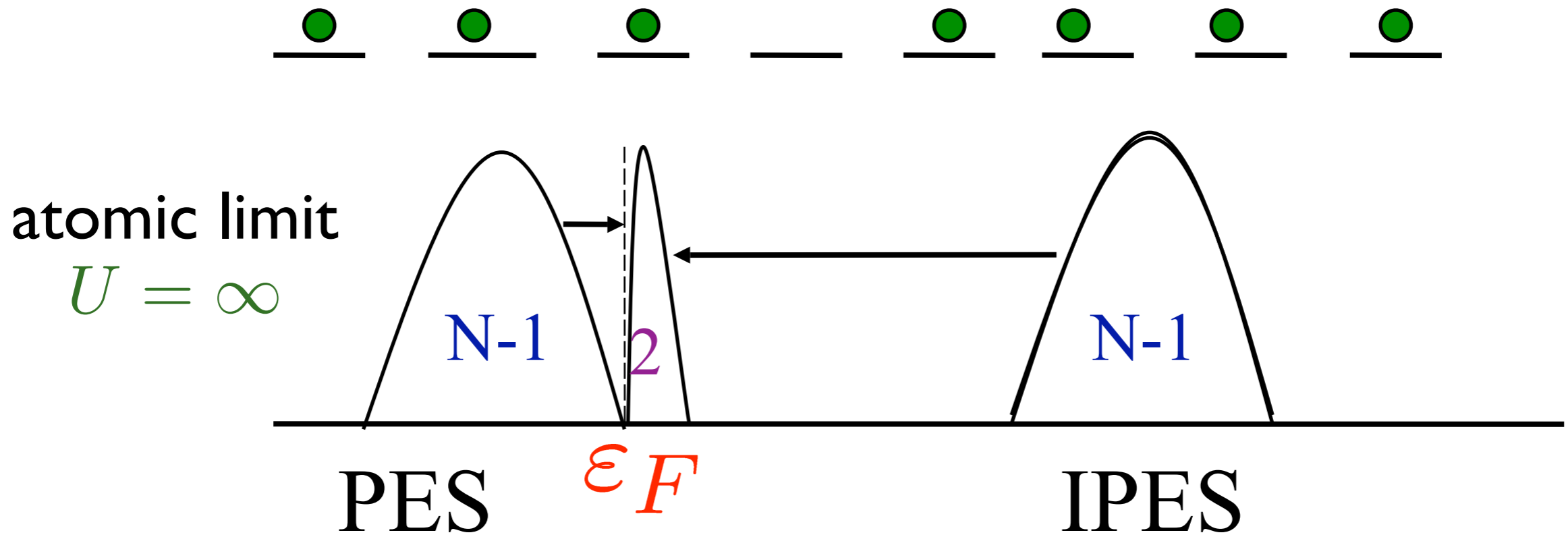
2.) but if A is momentum-independent, $S=0$ by particle-hole symmetry

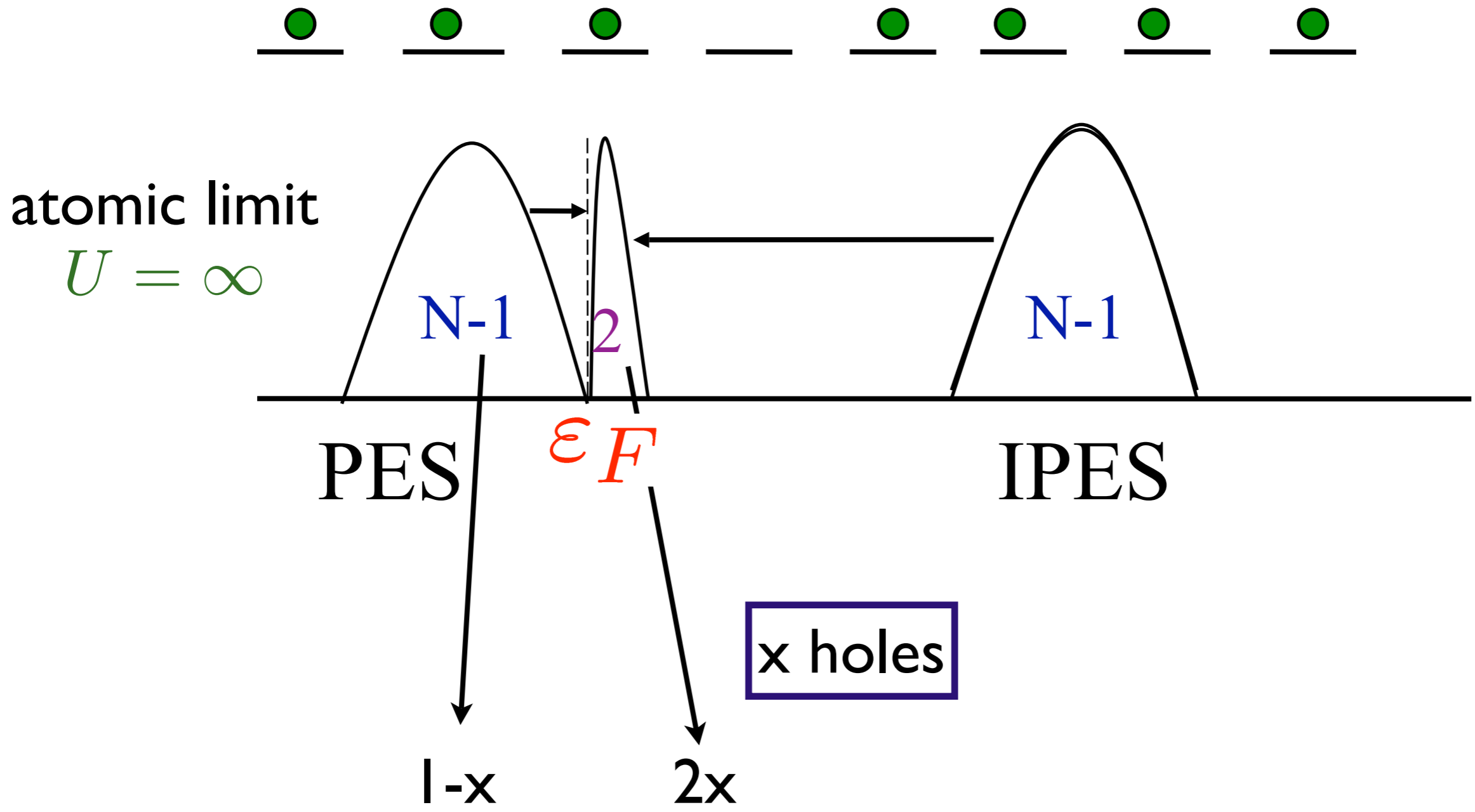


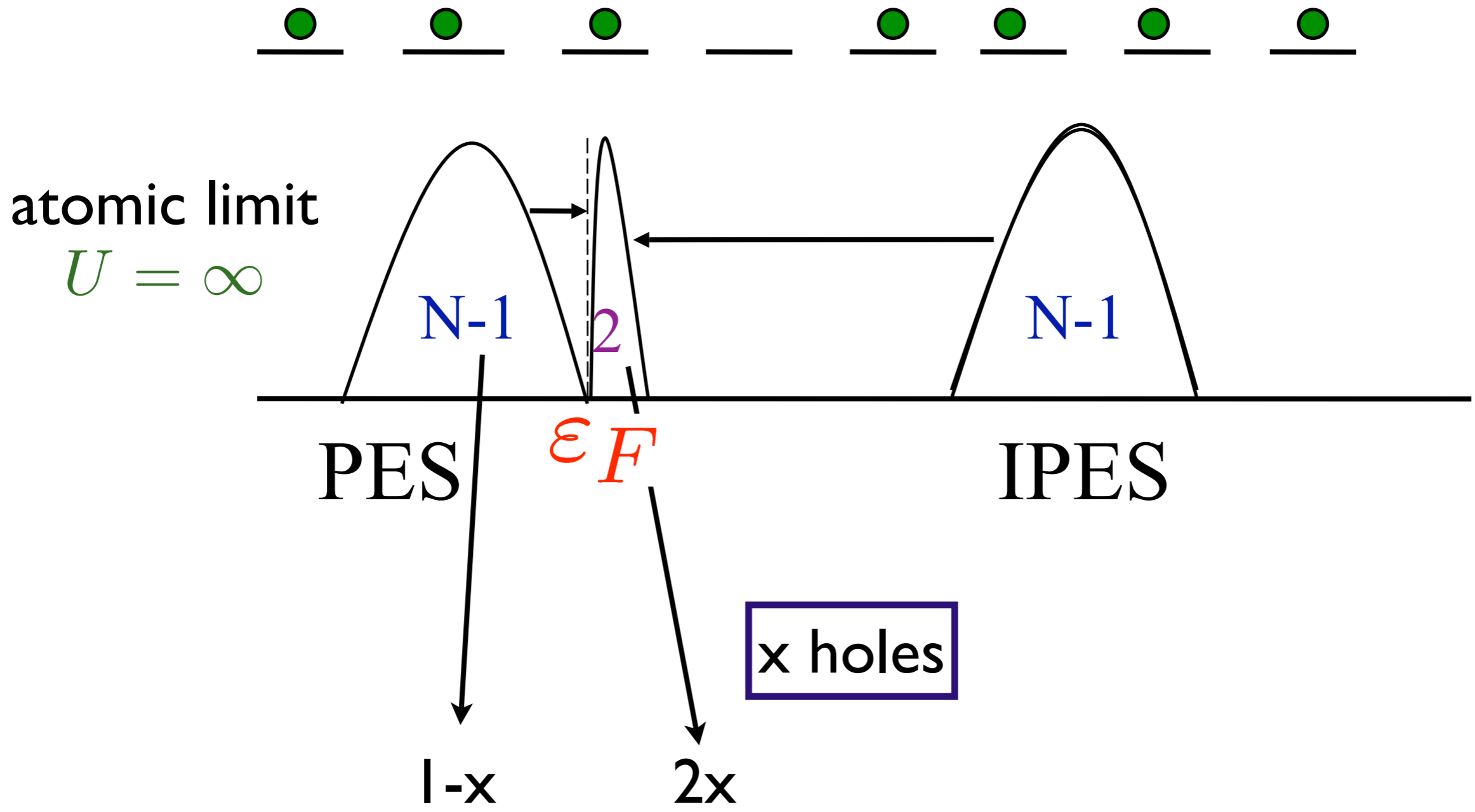






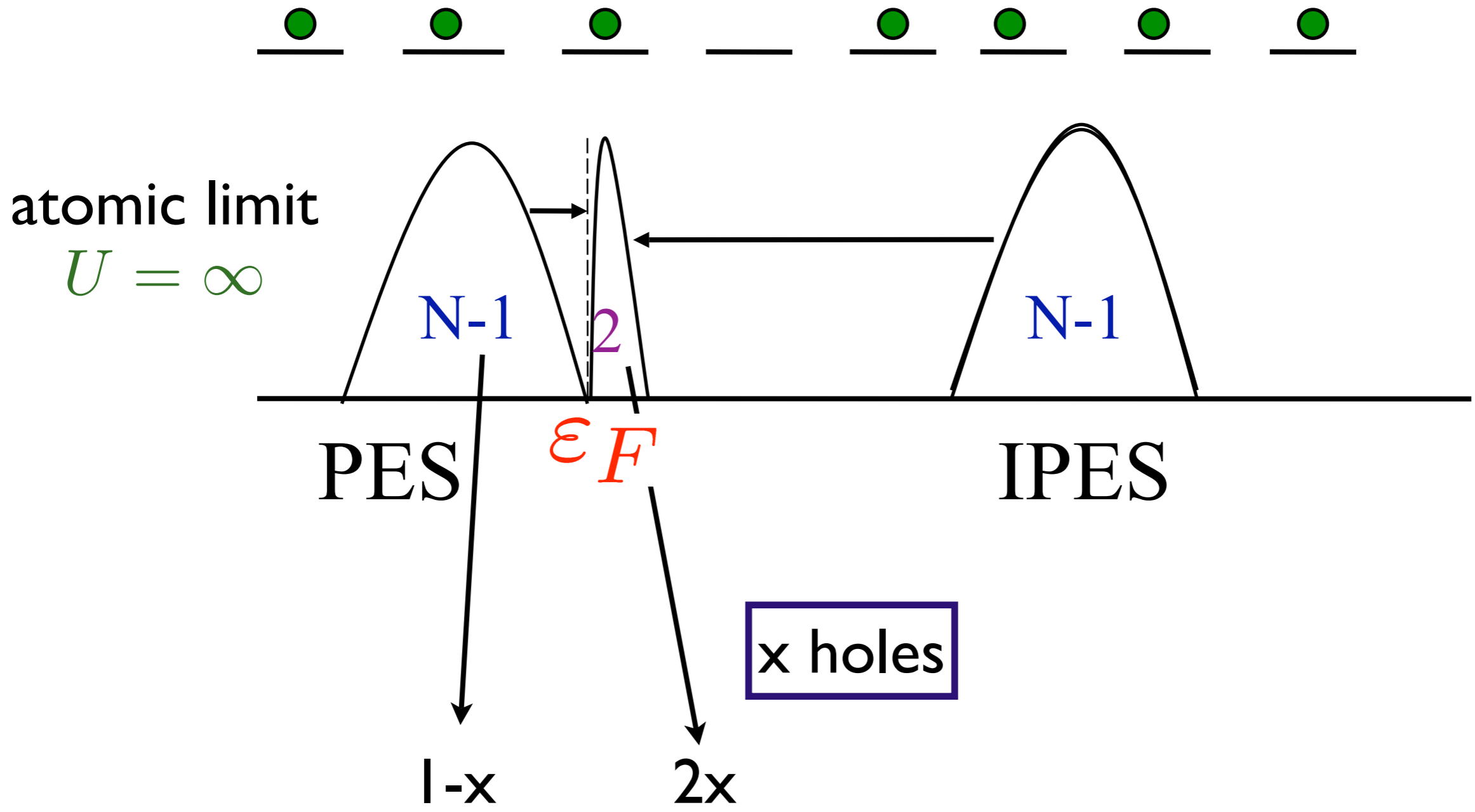






criterion for vanishing of thermopower

LESW above ϵ_F = LESW below ϵ_F



critierion for vanishing of thermopower

LESW above ϵ_F = LESW below ϵ_F

$2x = 1 - x \rightarrow x = 1/3!$

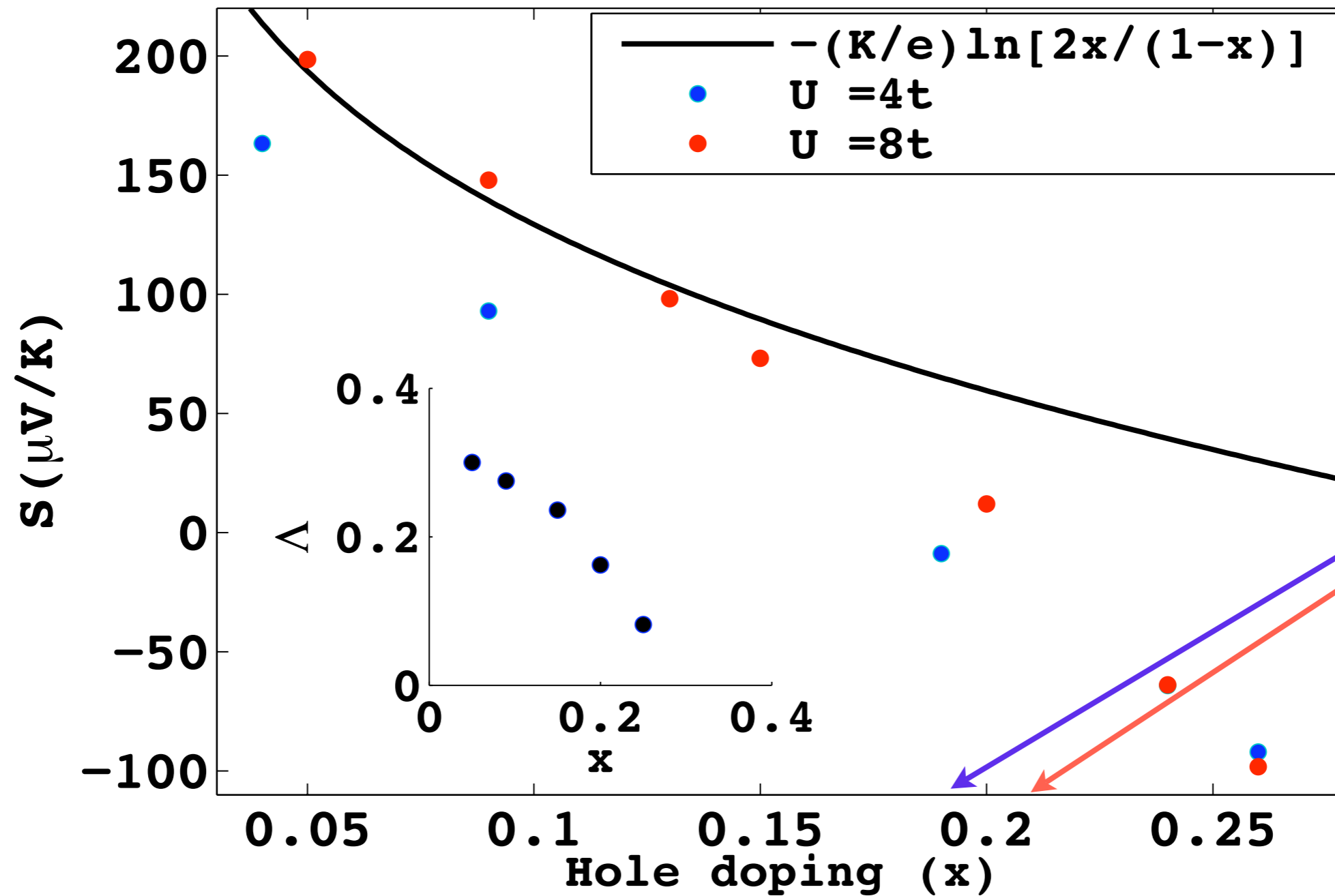
$$S \approx -\frac{k_B}{e} \ln \frac{\mathcal{L}}{1-x}$$

S must change sign before $x=1/3$
(atomic limit)

experiments: $x_c=.24$

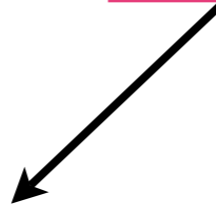
Hubbard Model

$$T = 0.1t, \quad t = 0.5 \text{ eV}$$



sign
change

$$L > 2x$$

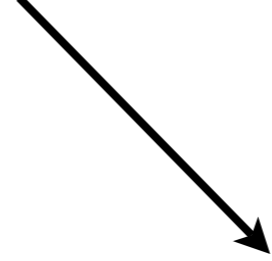
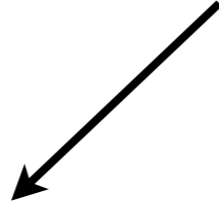


Sign change of S



T_c^{\max}

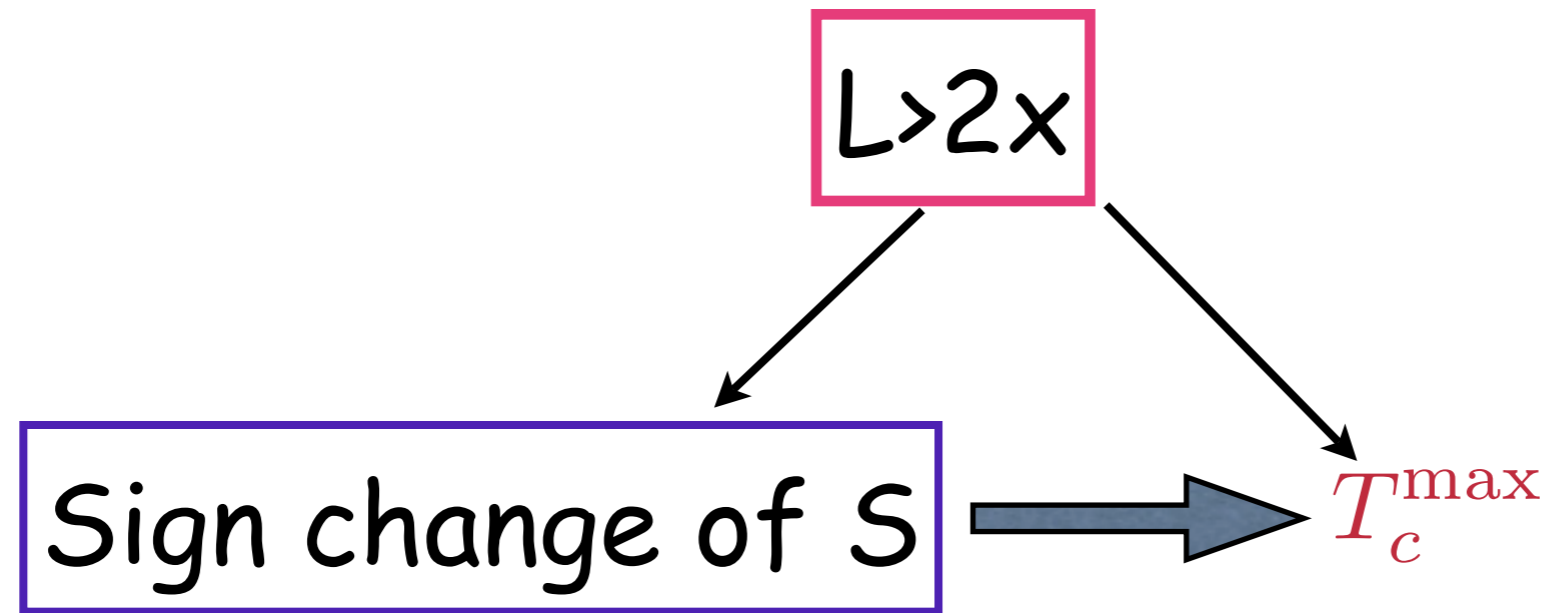
$$L > 2x$$



Sign change of S



T_c^{\max}



Is $L > 2x$ important qualitatively?

summary

low-energy theory: non-electron
Quantum numbers emerge---
SC boson-fermion model

Thanks to R. G. Leigh and Ting-
Pong Choy, Shiladitya
Chakraborty
and DMR-NSF/ACIF

summary

Thanks to R. G. Leigh and Ting-
Pong Choy, Shiladitya
Chakraborty
and DMR-NSF/ACIF

summary

Boson=normal state properties of cuprates

Thanks to R. G. Leigh and Ting-Pong Choy, Shiladitya Chakraborty and DMR-NSF/ACIF