

Quantum Disentangled Liquids

MPA Fisher

JoeP Fest February 28, 2014

HAPPY BIRTHDAY JOE!

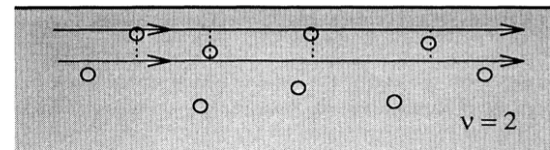


Joe Polchinski

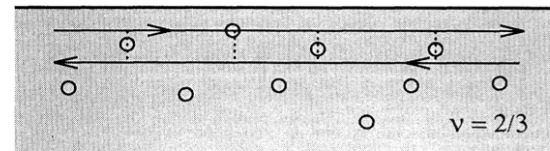
JoeP and CM

- Low Energy Dynamics of the Spinon Gauge Theory: **J. Polchinski** (1994), 118 citations
- Randomness at the Edge: Theory of Quantum Hall Transport at filling $\nu=2/3$, C. Kane, MPAF, **J. Polchinski** (1994), 115 citations

Predicted “upstream” neutral modes
in $\nu=2/3$ fractional quantum Hall states
 (“phenomenology”!!)



(a)



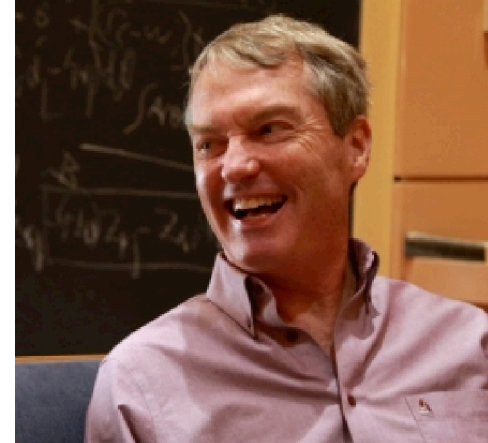
(b)

JoeP and Experiment!

Observation of neutral modes in the fractional quantum Hall regime, M. Heiblum et al, (2010)

But the jury is still out...

Firewalls, Aaaahhhh...



Black Holes event horizon: quantum (or not), very hot or not, thermal or not, entangled but how....

My personal confusion (among many): Equivalence of thermal/entanglement entropy, or not...

Hidden Locality in (some) “Thermal” Eigenstates?

- Thermal/entanglement entropy and Eigen-State Thermalization (ETH)
- Breakdown of ETH?

Yes: Many-Body-Localization, frozen disorder (classical dof)



Tarun Grover

KEY QUESTION:

Can ETH breakdown in fully Quantum Many-Body systems?

Maybe....

Describe: (T. Grover, MPAF 2013)



Jim Garrison



Ryan Mishmash

- **New Wavefunction Diagnostics**, beyond entanglement entropy:
local partial-measurement, and then post-measurement entanglement
- **Quantum Disentangled Liquids**: New “Thermal” states with “hidden” locality

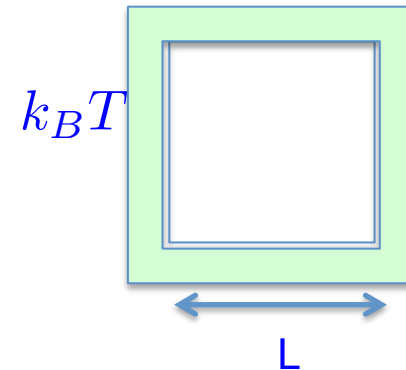
Thermal and entanglement entropy

Thermal entropy, system w/ heat bath:

Canonical ensemble $\hat{\rho}_{th} = \frac{1}{Z} e^{-\beta \hat{\mathcal{H}}}$

Thermal entropy: Number of states, extensive for $T > 0$

$$S_{th} \sim L^d$$



Entanglement Entropy: Isolated, single eigenstate $|\psi\rangle$

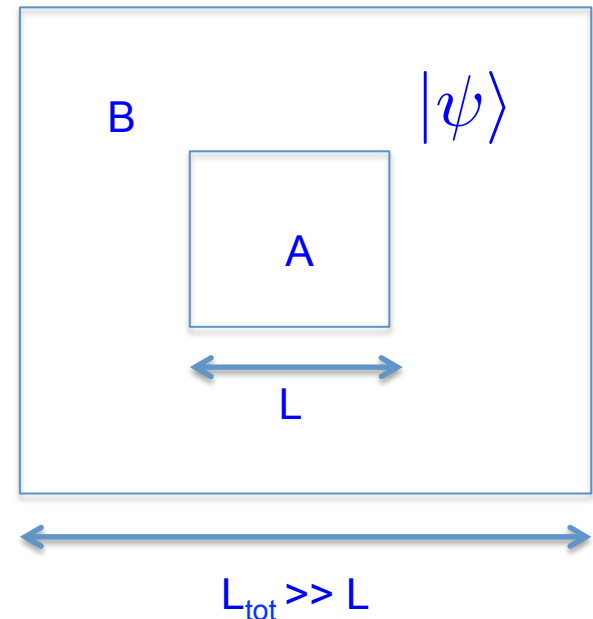
$$\hat{\mathcal{H}}|\psi\rangle = E|\psi\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

Spatial partition: Regions A and B

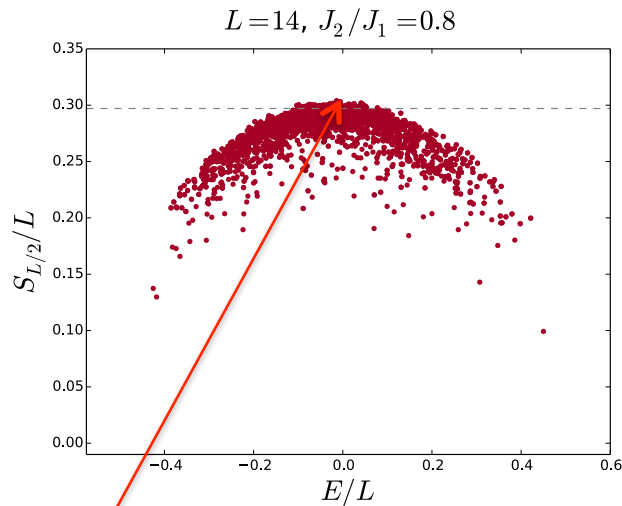
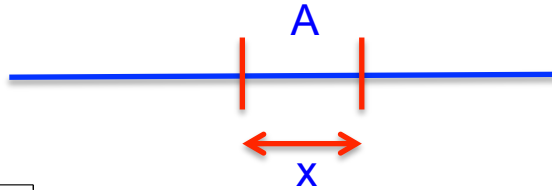
Reduced density matrix $\hat{\rho}_A = \text{Tr}_B(\hat{\rho})$

Entanglement entropy: $S_A(L) = -\text{Tr}_A(\hat{\rho}_A \ln \hat{\rho}_A)$



Entanglement entropy: 1d spin-chain

1d $s=1/2$ J_1 - J_2 chain



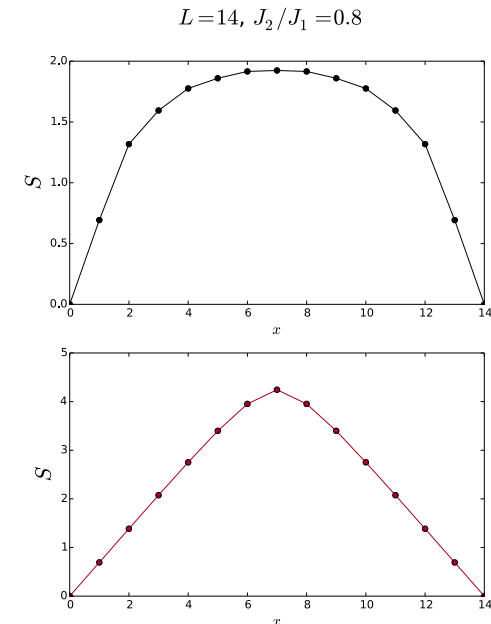
Maximum entropy state,
minimal (zero) local information $S_A/L_A \approx \ln 2$

Ground state,
Area-law:

$$S_A \sim O(1)$$

“High-energy” state,
Volume law:

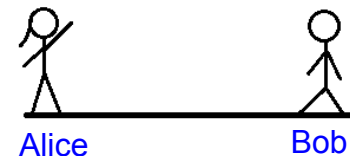
$$S_A \sim |x - L/2|$$



Non-locality of volume-law states

Maximum-entropy (volume-law) eigenstate
possesses no “local” information,

(Entropy: Classical - choose to “ignore” local information
Quantum - Local information **does not exist**)

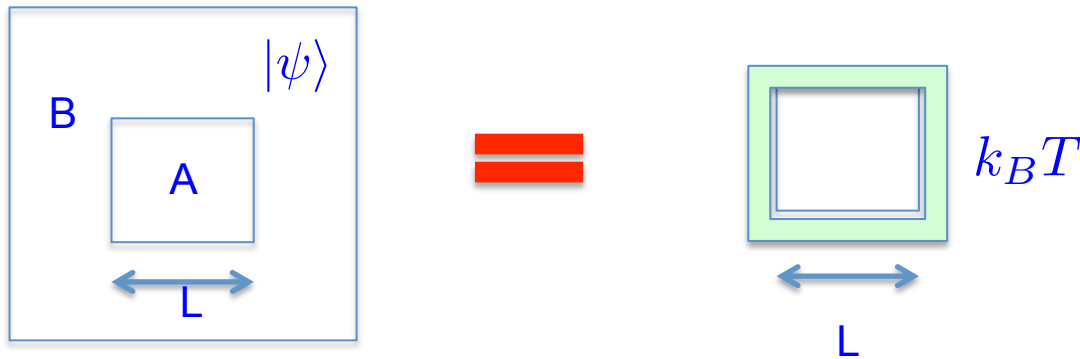


cf EPR pair, neither Alice nor Bob have
any local information about their spin

Eigenstate Thermalization Hypothesis (ETH)

Josh Deutsch, Mark Srednicki

ETH = Microcanonical ensemble for $E/L^d > 0$ eigenstate
equivalent, in region A, to canonical ensemble w/ heat bath



Eigenstates w/ (nearly) same energy have
identical correlations inside A

Equivalence of Thermal and entanglement entropies

$$S_A/L^d = S_{th}/L^d; \quad L \rightarrow \infty$$

Can ETH Break Down?

YES: In “**Many-Body-Localized**” (MBL) phase

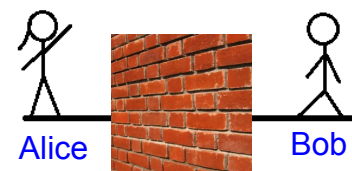
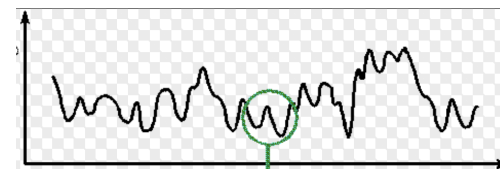
Isolated, interacting, quantum particles in **random potential** can be in a Many-Body-Localized (MBL) phase

Eigenstates in MBL have area law entanglement entropy (even at $E/L^d > 0$) while thermal entropy is extensive

$$S_A \sim L^{d-1} \quad S_{th} \sim L^d$$

- Lack of thermalization: Particles don't serve as own thermal bath
- Conserved quantities (eg charge, or even energy) do not propagate, zero conductivity
- **Entanglement does not “propagate”, local quantum information cannot be transmitted**

I.V. Gornyi, A.D. Mirlin, D.G. Polyakov (2005)
D. Basko, I. Aleiner, B. Altshuler (2006)
V. Oganesyan, D. Huse (2007)





Alice + Bob embedded in MBL,
completely isolated, quantum info
cannot be transmitted

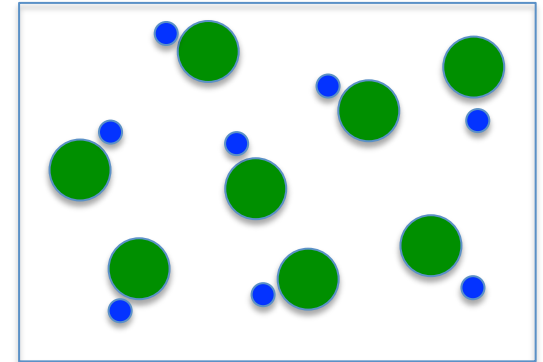
But... The random potential constitutes classical
(frozen) set of degrees of freedom...

Can ETH Breakdown in fully Quantum system?

When might this happen?

Neutral atoms, isolated in box, prepared in many-body eigenstate, energy E

Nucleus: 
Electron: 

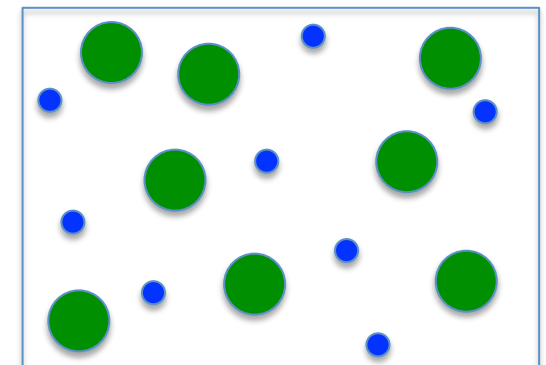


Atomic-Liquid

Vary energy E (the “temperature”)

- $T \sim 10$ - $1,000$ K **atomic-liquid**
- $T > 10$ eV, **ionized-plasma**

Question: Are eigenstates of **atomic-liquid** qualitatively the same as the **ionized-plasma** eigenstates?



Ionized-Plasma

Atomic-Liquid vs ionized-plasma?

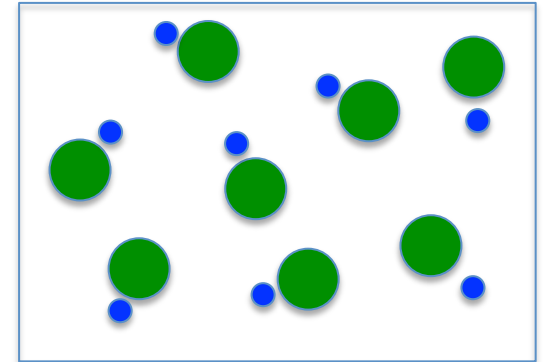
Canonical answer: Atomic-liquid and ionized plasma are “equivalent”

- Atomic-liquid and ionized-plasma eigenstates both volume-law entangled
- Atoms weakly ionized (even at low “ T ”)
- Low density of ionized electrons “should” thermalize

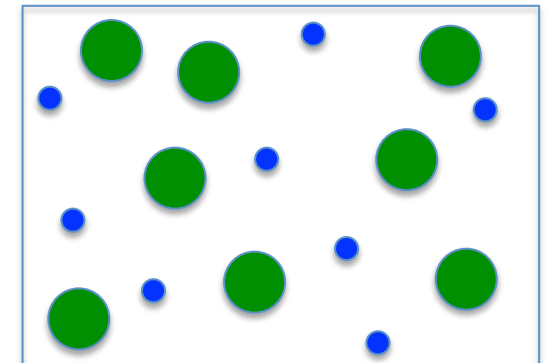
But...

How does one (or can one) define “ionized”?

How to characterize thermalization of “ionized” electrons?



Atomic-Liquid



Ionized-Plasma

Plan:

- Describe new wavefcn diagnostic (to characterize ionization?)
- Propose new quantum phases of $T > 0$ multi-component fluids:

“Quantum Disentangled Liquids”

New wavefcn diagnostic: “Post-measurement entanglement”

Consider ($E > 0$) volume-law wavefcn,

$$\Psi(R, r)$$

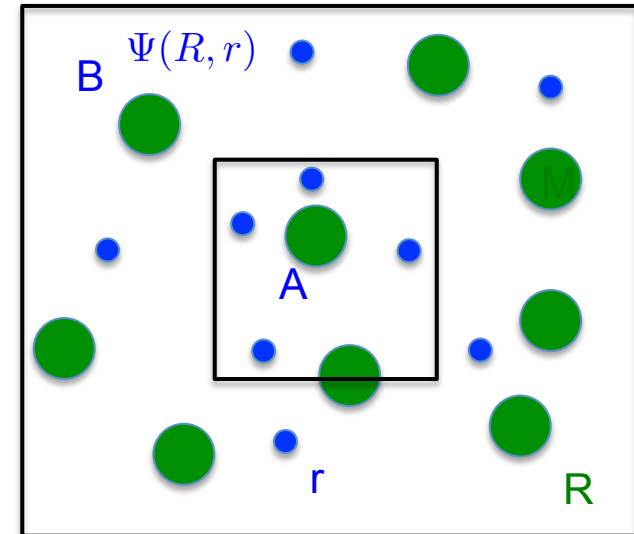
Diagnostic:

- (i) **Measure** positions of nuclei, but not electrons, (partial measurement) find \underline{R}
- (ii) Consider wavefcn of (unmeasured) electrons,

$$\psi_e(r) \sim \Psi(\underline{R}, r)$$
- (iii) Compute (spatial) bi-partition entanglement entropy of electron wavefcn (w/ fixed nuclei coordinates, \underline{R})
- (iv) Repeat (i)-(iii) and average, gives post-measurement entanglement entropy

$$S_A^{r/R}$$

Can repeat w/ $R \leftrightarrow r$ $S_A^{R/r}$



(R, r denote nuclei/electron coordinates)

Interest: Size scaling of post-measurment entanglement entropies?

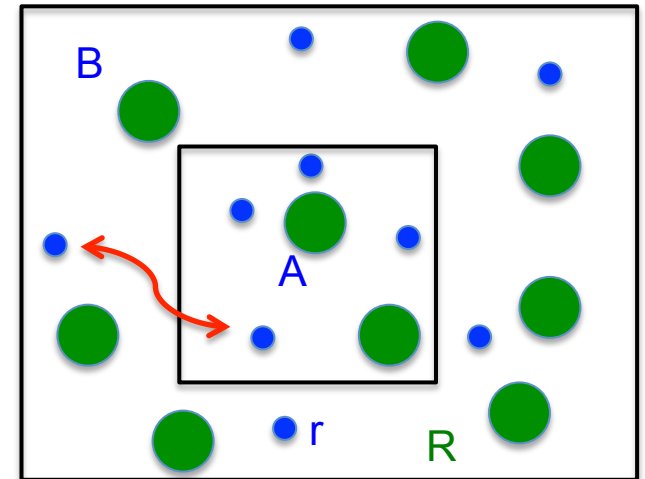
Scaling of post-measurement entanglement entropy?

Volume Law: $S_A^{r/R} \sim L^d$

Electrons entangled (thermalized) even after measuring nuclei positions,

Fully Thermalized Eigenstate

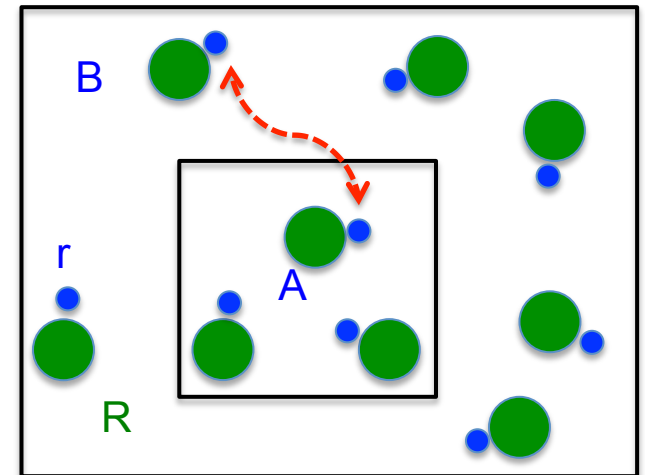
ionized-pasma, fully
thermal w/ volume laws



Area Law: $S_A^{r/R} \sim L^{d-1}$

Measuring nuclei positions has disentangled (“localized”) electrons

“Quantum Disentangled Eigenstate”



Atomic Fluid with NO ionization will have area law,
but what about w/ weak ionization?

Quantum Disentangled State for atomic fluid

An atomic fluid eigenstate is Quantum-Disentangled if,

Measuring (heavy) nuclei, disentangles (light) electrons, **Area law:** $S_A^{r/R} \sim L^{d-1}$

Measuring (light) electrons does not disentangle (heavy) nuclei, **Volume law:** $S_A^{R/r} \sim L^d$



Quantum Disentangled States: General defn

Volume law entangled eigenstate ($E > 0$) $S_A^\Psi \sim L^d$

Perform (some) **local partial-measurement**, wf of un-measured d.o.f. $|\Psi\rangle \rightarrow |\psi\rangle$

Compute entanglement-entropy of post-measurement wf

$$|\psi\rangle \xrightarrow{\text{red arrow}} S_A^\psi$$

Quantum Disentangled State: post-measurement wf has area law $S_A^\psi \sim L^{d-1}$

Local Partial Measurement induces full locality

Thermal State: wf post-measurment still entangled (non-local, volume law) $S_A^\psi \sim L^d$

Do Quantum Disentangled States exist?

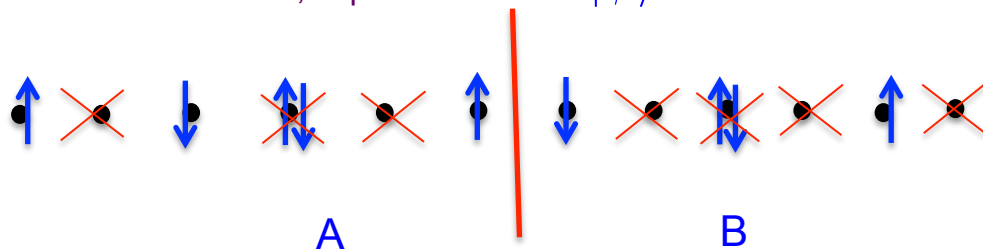
(as eigenstates of generic Hamiltonians)

Fermion Hubbard models:

Partial-measurement on volume law ($E > 0$) eigenstate $|\Psi\rangle$

Measure Charge on each site, but not spin $|\Psi\rangle \rightarrow \mathcal{P}|\Psi\rangle = |\psi\rangle$

After measurement, “spin-wavefcn” $|\psi\rangle$



Post-measurement entanglement entropy of “spin-wf” $S_{s/c}$

Do Quantum-disentangled eigenstates exist with area law “spin-wf”? $S_{s/c} \sim L^{d-1}$

Numerics: 1d “Hubbard” chain

J. Garrison, R. Mishmash, T. Grover, MPAF (in progress)

$$\hat{\mathcal{H}} = -t \sum_{i,\sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{i+1\sigma} + h.c.) - |U| \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

- Attractive inter, $U < 0$, J destroys integrability
- Exact diagonalization, full entanglement entropy
- **Measure Charge but not Spin**, to get entanglement entropy of “spin-wf”

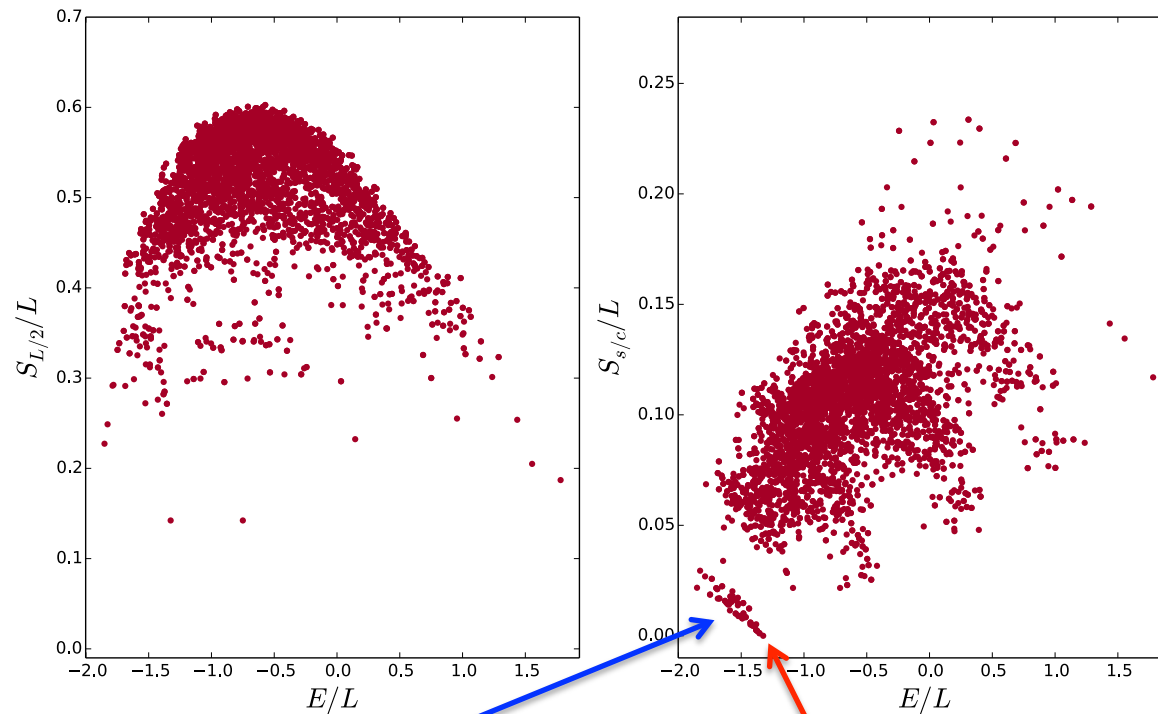
Full and post-measurement entanglement entropy

L=8 site Hubbard chain
4 up, 4 down fermions

Full Entanglement entropy

Post-measurement entropy of “spin-wf”

$$L=8, N_{\uparrow}=N_{\downarrow}=4, J=-3, U=-2.650$$



“Spin-disentangled band”

- Post-measurment “spin-wf” low entanglement
- 70 (= 8 choose 4) states in “band” = number of fully-Paired states

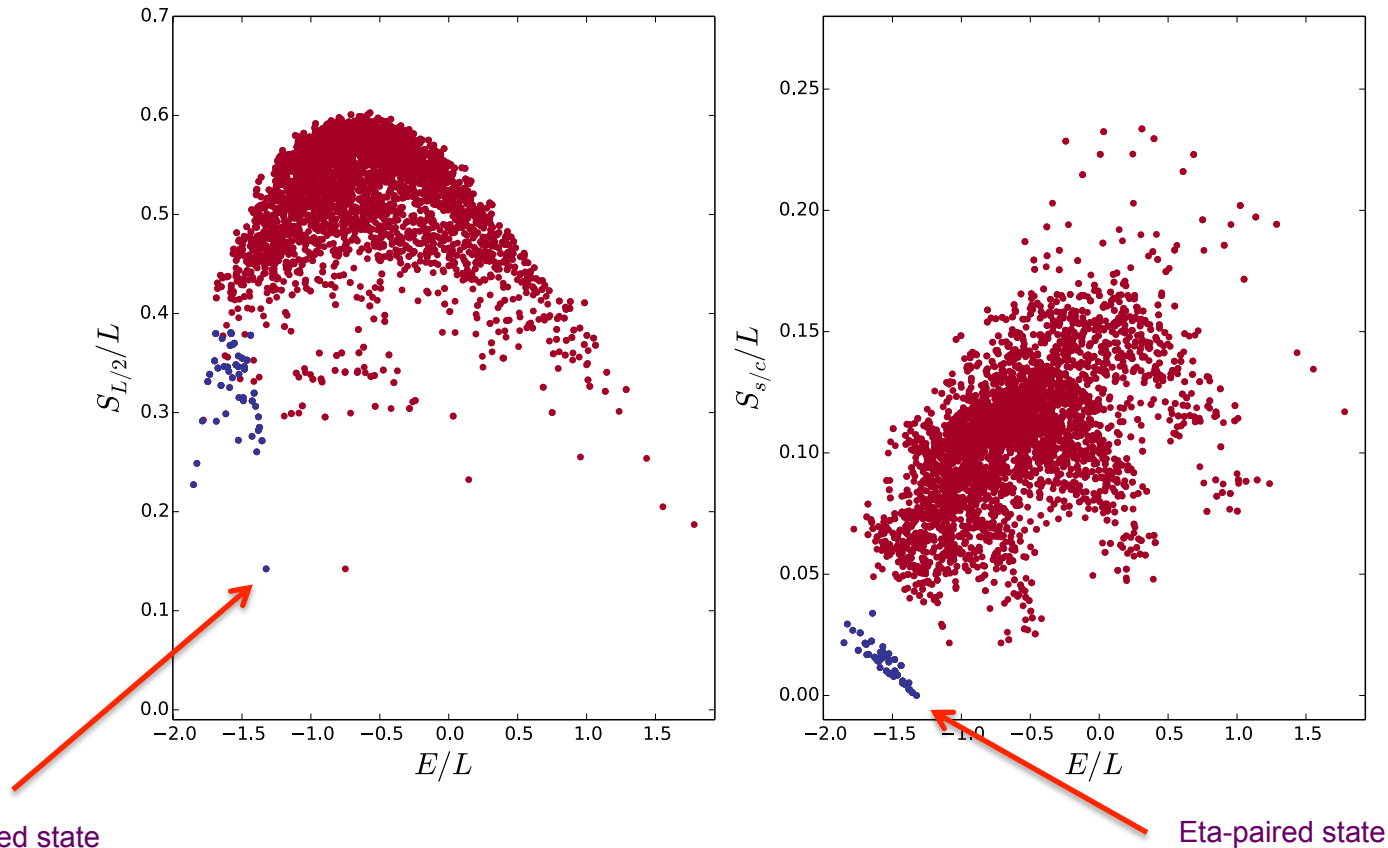
Eta-paired state, pair-condensate at momentum π
Exact $E>0$ eigenstate w/ zero entanglement

CN Yang, PRL (1989)



“Spin-disentangled” versus other states

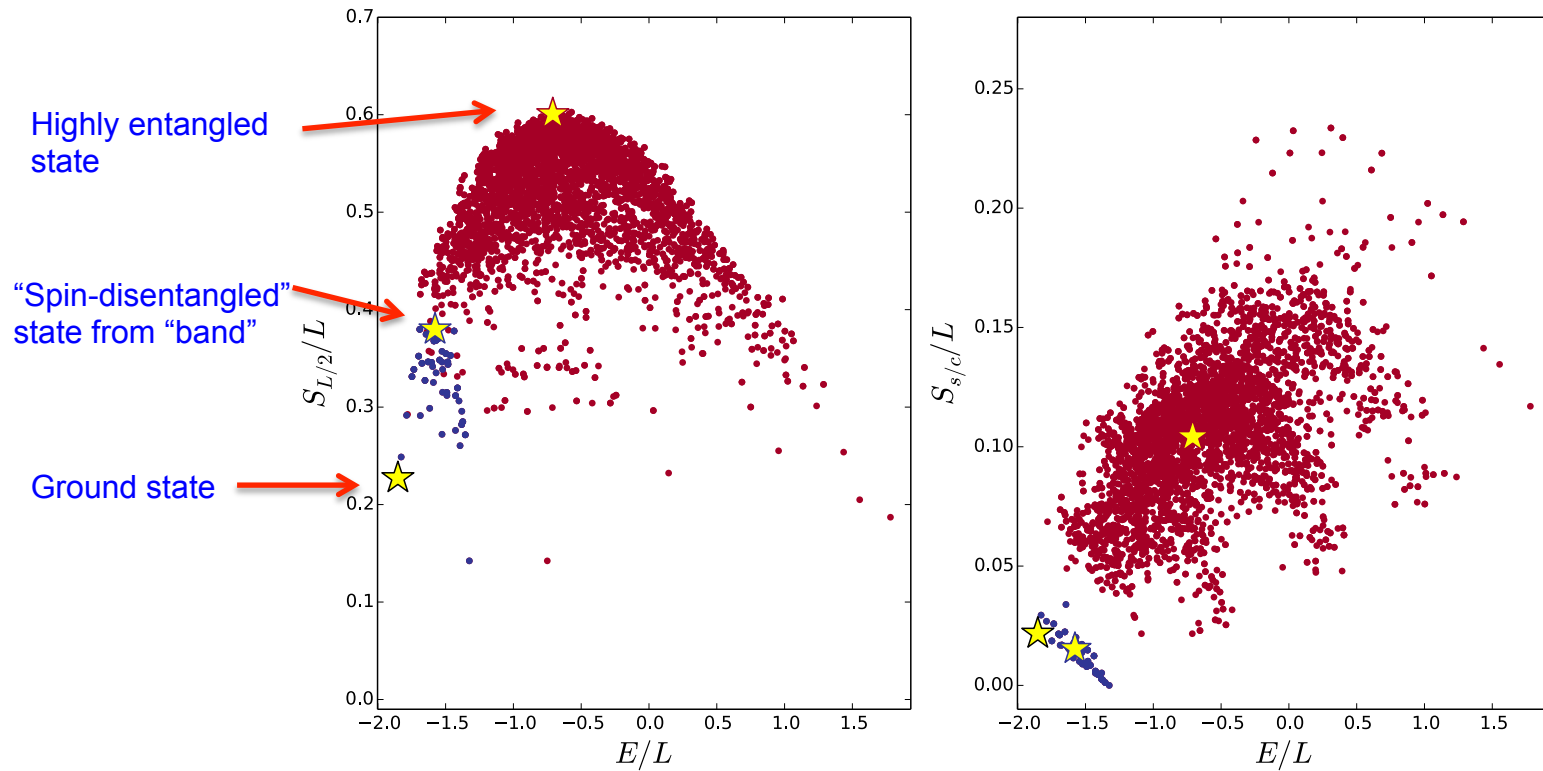
$$L=8, N_{\uparrow}=N_{\downarrow}=4, J=-3, U=-2.650$$



Possible breakdown of ETH, since wavefns at same energy have very different properties...

Focus on 3 states

$$L=8, N_{\uparrow}=N_{\downarrow}=4, J=-3, U=-2.650$$



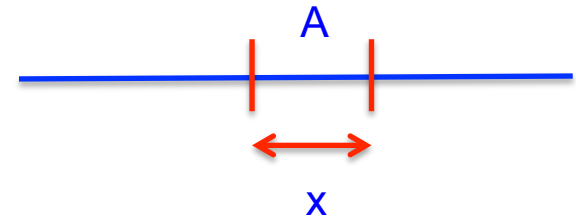
Entropy scaling for 3 eigenstates

Area-law

Full Entropy

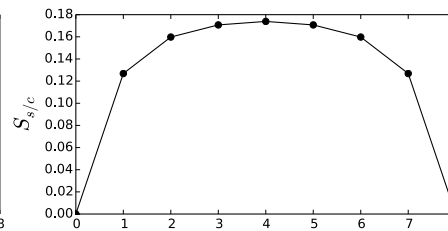
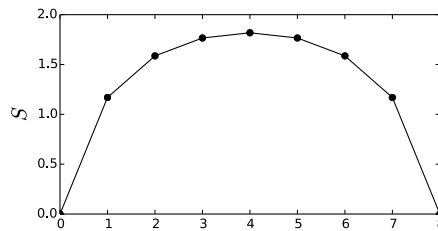
Post-measurement
Spin entropy

$$L=8, N_{\uparrow}=N_{\downarrow}=4, J=-3, U=-2.650$$



Ground state

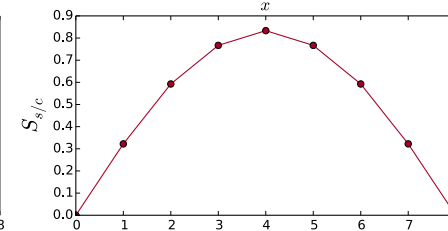
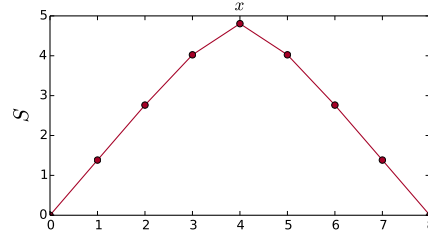
Area-law



Area-law

E>0 eigenstate,
not in “band”

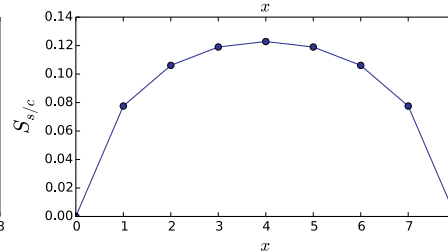
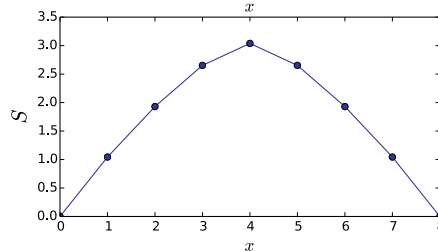
Volume-law



Volume-law

E>0 Eigenstate
from “band”

Volume-law



Area-law?

Possible band of Quantum-disentangled states, “liquid of pairs”
w/ area-law post-measurement spin-entropy?

Summary

Possible “Hidden” Locality in (some) “Thermal” eigenstates

New diagnostic for many-body wf: **Partial-local-measurement**, followed by post-measurement entanglement entropy of un-measured dof

Quantum Disentangled States: Volume-law eigenstates w/ post-measurement area-law entanglement entropy – “**induced-locality**”

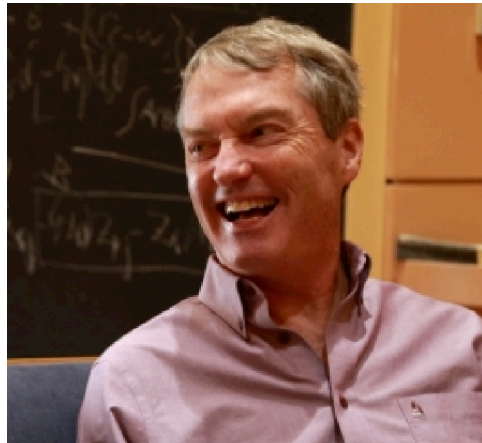
Future:

Identify Hamiltonians which exhibit Quantum-disentangled eigenstates?

“Finite-temperature QFT” w/ very-heavy and very-light fields, Quantum-Disentangled eigenstates?

Other diagnostics to reveal hidden structure in volume-law entangled states?

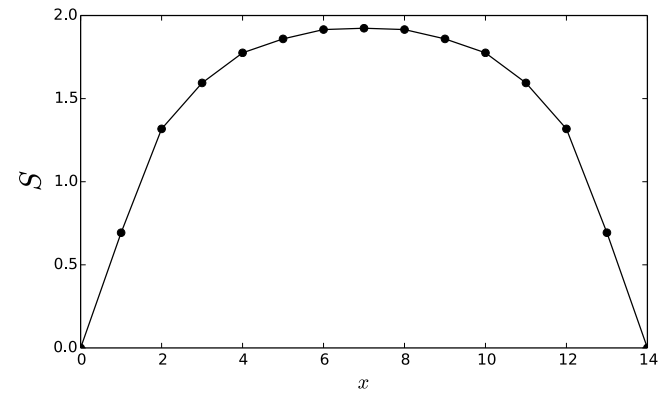
Happy Birthday Joe!



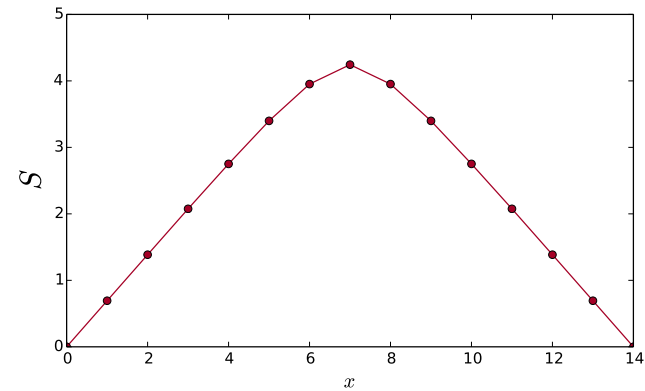
$S(x)$ -vs- x for J1-J2 $s=1/2$ model

$$L=14, J_2/J_1=0.8$$

Ground state (VBS state)



Maximally entangled state



Quantum Disentangled Eigenstates

Consider finite-energy density eigenstate of a many-body Hamiltonian, which has a volume law entanglement entropy

$$S_A^\Psi \sim L^d$$

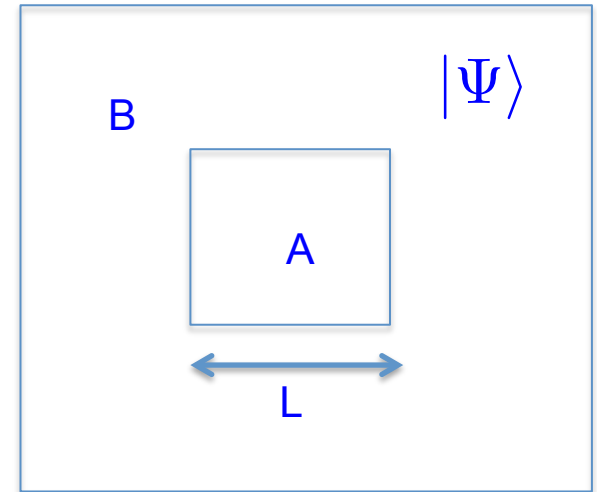
Perform (some) local **partial** measurement, and obtain projected wavefunction for unmeasured degrees of freedom

$$|\Psi\rangle \rightarrow |\psi\rangle$$

Compute spatial entanglement entropy of post-measurement wavefunction

$$S_A^\psi$$

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle$$



“**Conventional Thermal state**”: Post-measurement entanglement is still **volume** law

$$S_A^\psi \sim L^d$$

“**Quantum Disentangled state**”: **Area** law post-measurement entanglement

$$S_A^\psi \sim L^{d-1}$$

In Quantum Disentangled Liquid a *partial* local measurement induces *full* locality

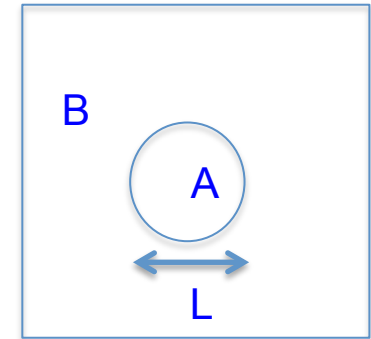
Full measurement in spatially extended system: Disentangles

Heisenberg model with $s=1/2$ $\hat{\mathcal{H}} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$

Hilbert space: Direct product of up/down spin on each site $|\{s_j\}\rangle = \prod_{j=1}^N |s_j\rangle_j \quad s_j = \pm 1$

Finite energy-density eigenstate: $|\Psi\rangle = \sum_{\{s_j\}} \phi(\{s_j\}) |\{s_j\}\rangle$
 Volume law entanglement,

$$\hat{\rho}_A = \text{Tr}_B |\Psi\rangle\langle\Psi| \quad S_A = -\text{Tr}_A [\hat{\rho}_A \ln \hat{\rho}_A] = sL^d$$



Make Full (Local) Measurement: Measure spin on each and every site, find $\{\tilde{s}_j\}$

$$|\Psi\rangle \rightarrow |\Psi'\rangle = |\{\tilde{s}_j\}\rangle \quad (\text{a direct product state})$$

After measurement: Wavefcn fully disentangled

$$S_A = sL^d \rightarrow S'_A = 0$$

Local Measurement collapses “non-local” volume law state, inducing “locality”

Local Partial-Measurements

Hubbard model for electrons $\hat{\mathcal{H}} = -t \sum_{ij} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + u \sum_i n_{i\uparrow} n_{i\downarrow} + \dots$

On-site Hilbert space: Up/down spin, empty, doubly-occupied $|\uparrow\rangle, |\downarrow\rangle, |0\rangle, |\uparrow\downarrow\rangle$

Partial-Measurement: Measure spin on each site

Single-site wf: $|\psi\rangle = \phi_\uparrow |\uparrow\rangle + \phi_\downarrow |\downarrow\rangle + \phi_0 |0\rangle + \phi_{\uparrow\downarrow} |\uparrow\downarrow\rangle$

$$|\psi\rangle \rightarrow \begin{cases} \mathcal{P}_\uparrow |\psi\rangle = \phi_\uparrow |\uparrow\rangle \\ \mathcal{P}_\downarrow |\psi\rangle = \phi_\downarrow |\downarrow\rangle \\ \mathcal{P}_0 |\psi\rangle = \phi_0 |0\rangle + \phi_{\uparrow\downarrow} |\uparrow\downarrow\rangle \end{cases} \quad \leftarrow$$

Spin projection operators

$$\mathcal{P}_\uparrow = |\uparrow\rangle\langle\uparrow|$$

$$\mathcal{P}_\downarrow = |\downarrow\rangle\langle\downarrow|$$

$$\mathcal{P}_0 = |0\rangle\langle 0| + |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$$

After measuring spin, get a charge wavefunction (doublon/holon)

Partial-Measurement: Measure charge on each site

$$|\psi\rangle \rightarrow \begin{cases} \mathcal{P}_0 |\psi\rangle = \phi_0 |0\rangle \\ \mathcal{P}_1 |\psi\rangle = \phi_\uparrow |\uparrow\rangle + \phi_\downarrow |\downarrow\rangle \\ \mathcal{P}_2 |\psi\rangle = \phi_{\uparrow\downarrow} |\uparrow\downarrow\rangle \end{cases} \quad \leftarrow$$

Charge projection operators:

$$\mathcal{P}_0 = |0\rangle\langle 0|$$

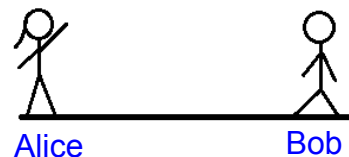
$$\mathcal{P}_1 = |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|$$

$$\mathcal{P}_2 = |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$$

After measuring charge, get a spin wavefunction (up/down)

Entanglement, locality, information and measurement

Alice and Bob, each with a $s=1/2$ particle



Direct product: $|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B$

Bob's measurement, no effect on Alice.
Alice has full **local quantum information**.

Singlet state: $|\psi\rangle = \frac{1}{\sqrt{2}} [|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B]$

Bob's measurement affects Alice. Alice has **no local quantum information** about "her" spin - two spins are "entangled"

Entanglement entropy quantifies local quantum information

Direct product state – complete local info: $S_A = S_B = 0$

Entangled Singlet state – no local info: $S_A = S_B = \ln(2)$

Local Measurement: Causes spatial disentanglement

For singlet state Bob **measures** his spin:

$$|\psi\rangle = \frac{1}{\sqrt{2}} [|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B] \longrightarrow \left\{ \begin{array}{l} |\psi'\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \\ |\psi'\rangle = |\downarrow\rangle_A \otimes |\uparrow\rangle_B \end{array} \right.$$

After measurement, direct product state

$$S_A = S_B = \ln(2) \longrightarrow S'_A = S'_B = 0$$

Measuring spin disentangles charge

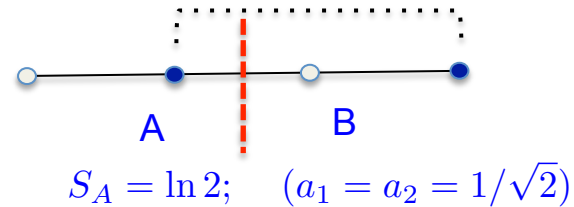
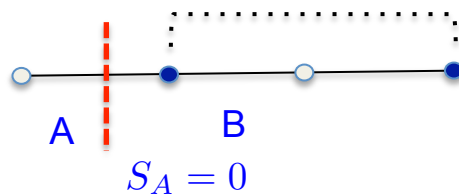
4-site Hubbard, 2 up, 2 down spin electrons

Measure spin on each site: $|\Psi\rangle \rightarrow |\Psi'\rangle$

Suppose spin measurement gives (+1,0,-1,0),
wf after measurement

$$|\Psi'\rangle = a_1 |\uparrow, 0, \downarrow, \uparrow\downarrow\rangle + a_2 |\uparrow, \uparrow\downarrow, \downarrow, 0\rangle$$

Bipartition and compute entanglement entropy



Measuring spin (partially) disentangles charge

Measuring charge (partially) disentangles spin $|\Psi\rangle \rightarrow |\Psi'\rangle$

Suppose charge measurement gives (1,0,1,2) leaving spin wf $|\Psi'\rangle = c_1 |\uparrow, 0, \downarrow, \uparrow\downarrow\rangle + c_2 |\downarrow, 0, \uparrow, \uparrow\downarrow\rangle$

Bipartition, entanglement entropy of spin wf

