Disordered horizons and strongly coupled metals

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JoeFest - Feb. 2014

Joe at the eV scale ...

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Effective Field Theory and the Fermi Surface

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Low-energy dynamics of the spinon–gauge system

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Outline

(1) Coherent metals and

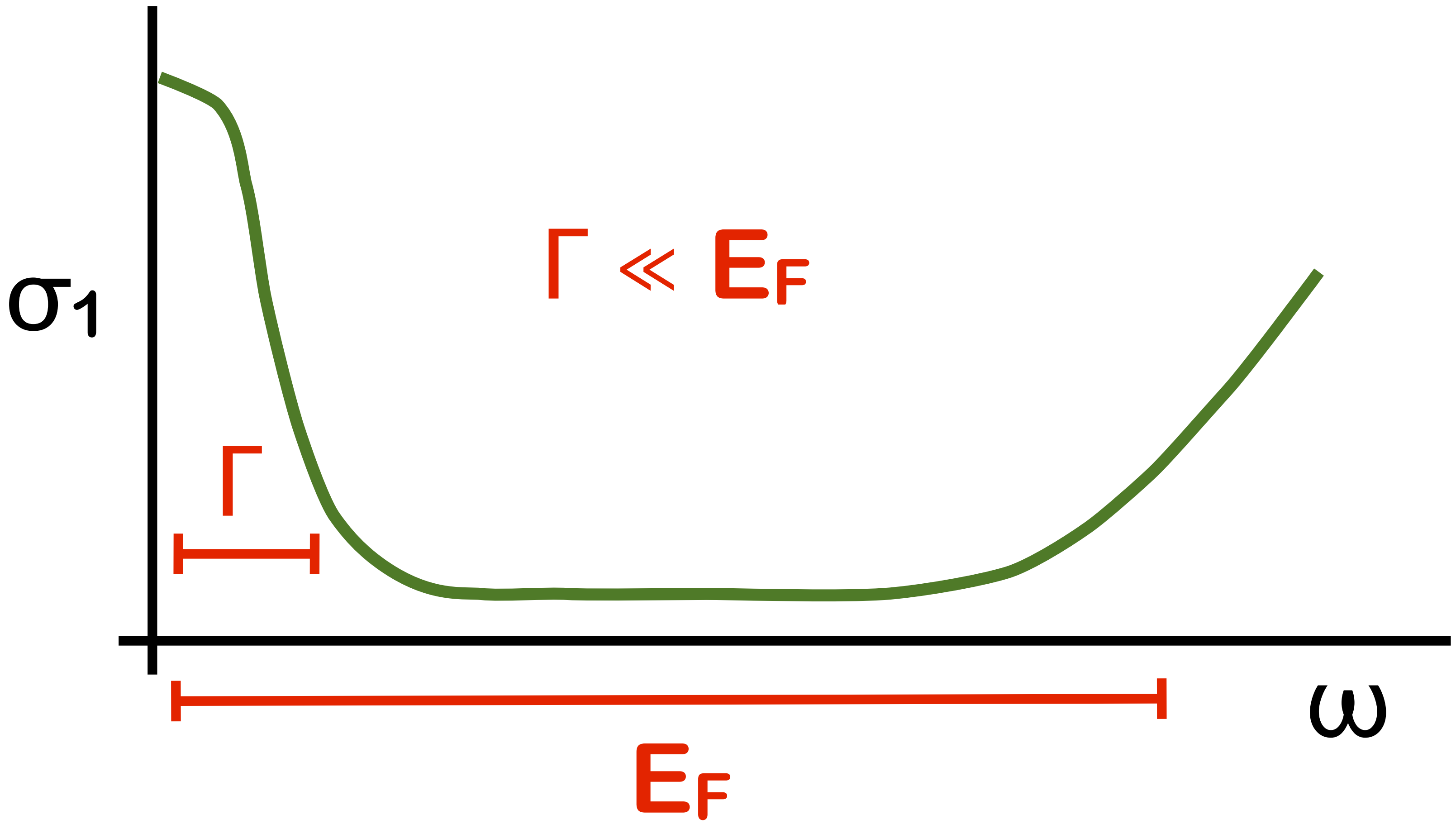
the Wiedemann-Franz law.

(2) Disordered fixed points in holography.

(1) Coherent metals

Coherent metals

- In a coherent metal, there exist long-lived quantities whose slow decay controls the d.c. conductivity.



Quasiparticles or not?

- Two simple scenarios allow excitations to live longer than natural 'high energy' scale:

  (1) Low energy interactions are weak
  → Quasiparticles!
  → ∞ many long lived densities δn_k.

  (2) Interactions are strong at low energies (no quasiparticles!) but effects of disorder and lattice (umklapp) are weak:
  → Total momentum P relaxes slowly.

- **With quasiparticles: formalism for transport is Boltzmann equation.**
- **Transport controlled by relaxation of the total momentum only: right formalism is the memory matrix.**

$$\sigma(\omega) = \frac{\chi_{PJ}^2}{\chi_{PP}} \frac{1}{-i\omega + \Gamma}$$

current-momentum susceptibilities.

momentum relaxation rate.

$$\Gamma = \frac{V_0^2}{\chi_{PP}} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2} k_x^2 \frac{\mathrm{Im} G^R_{\mathcal{O}\mathcal{O}}(\omega, k)}{\omega}$$

- **Some steps in the derivation:**

$$H = H_0 + \int d^2x V(x) \mathcal{O}(x,t)$$

$$\begin{aligned} \langle \dot{P} \dot{P} \rangle &\sim \langle [H,P][H,P] \rangle \\ &\sim \langle [P,H][P,H] \rangle \\ &\sim (\nabla V)^2 \langle \mathcal{O}\mathcal{O} \rangle \end{aligned}$$

- **Details:**

[Hartnoll-Hofman @ 1201.3917;
Hartnoll-Herzog @ 0801.1693;
Hartnoll-Kovtun-Muller-Sachdev @ 0706.3215]

Holographic metals

- Transport in all (coherent) holographic metals is controlled by momentum relaxation.

- This has a robust, experimentally accessible, consequence for thermal transport...

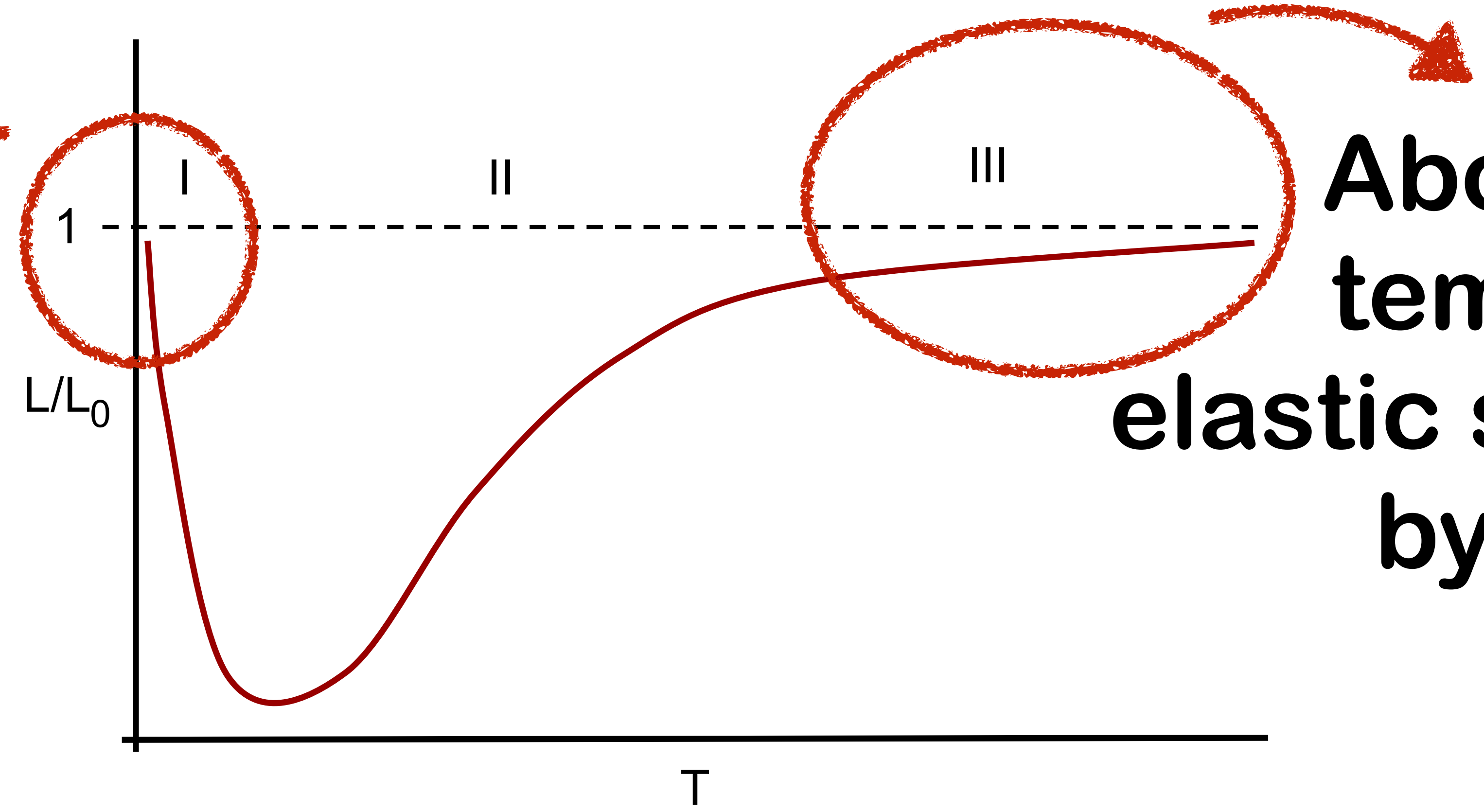
(II) Wiedemann-Franz law

Wiedemann-Franz law

- In a Fermi liquid with purely elastic scattering:

$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2}{3}$$

Elastic impurity scattering



Above Debye temperature, elastic scattering by phonons.

Wiedemann-Franz law

[Mahajan-Barkeshli-Hartnoll, PRB 88 (2013) 125107]

- Conventional discussion of violations of WF says:

  (i) $L > L_0$ → Extra neutral excitations.

  (ii) $L < L_0$ → Inelastic scattering.

- Without quasiparticles, L_0 is not the right reference point!

Lorenz ratio in hydrodynamic metals

- ‘Quasi-hydrodynamic’ systems:

$$\sigma = \frac{\chi_{JP}^2}{\chi_{PP}}\frac{1}{\Gamma}, \qquad \overline{\kappa} = \frac{\chi_{QP}^2}{\chi_{PP}}\frac{1}{T}\frac{1}{\Gamma}, \qquad \alpha = \frac{\chi_{QP}\chi_{JP}}{\chi_{PP}}\frac{1}{T}\frac{1}{\Gamma}$$

- The usual open circuit thermal conductivity:

$$\kappa = \overline{\kappa} - \frac{T\alpha^2}{\sigma} \qquad \begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \alpha T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -\frac{1}{T}(\nabla T) \end{pmatrix}$$

- Cancellation!:

$$L \sim \frac{\kappa}{\sigma T} \sim 0$$

Explanation:

(i) open circuit boundary conditions $\Rightarrow$ $J = 0$.

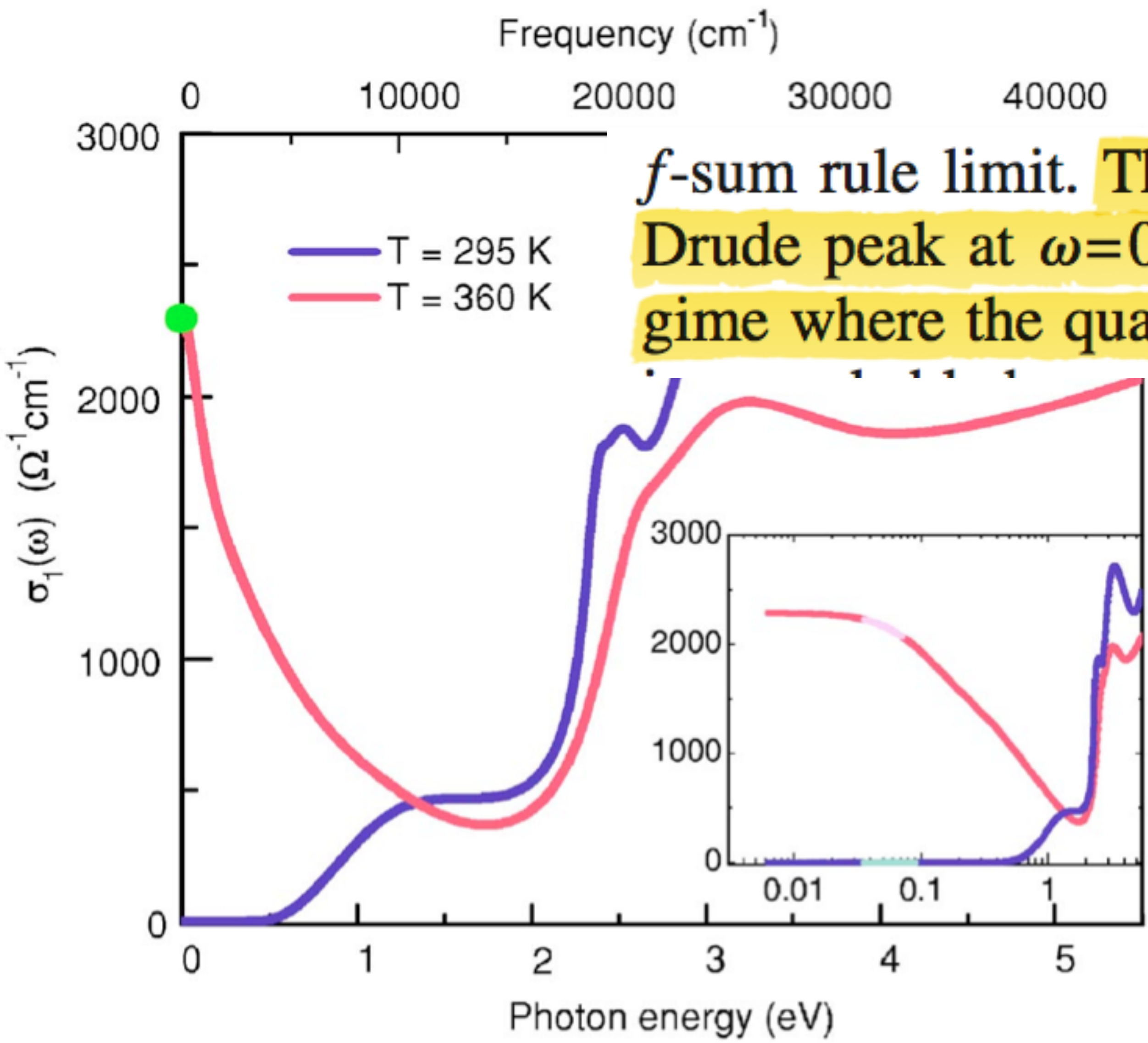
(ii) $J = 0$ $\Rightarrow$ $P = 0$.

(iii) P only long-lived quantity $\Rightarrow$ Q decays quickly.

(iv) $\Rightarrow$ $L \ll 1$.

In terms of thermopower, Fermi Liquids have $L \gg S^2$, quasi-hydrodynamic metals have $L \ll S^2$.

A candidate: VO_2



f-sum rule limit. The coherent part of the conductivity (the Drude peak at ω=0) persists beyond the IRM limit in a regime where the quasiparticle picture is no longer valid. This

Qazilbash et al. PRB ’06

- **Just above the MIT: A possible Drude peak without quasiparticles!**

A candidate: VO_2

- A forthcoming work by the group of Junqiao Wu has measured the thermal conductivity in VO_2 just above the MIT (using single crystal VO_2 nanobeams).
- From the jump at the MIT, they are able to isolate the electronic contribution to the thermal conductivity.
- They have found $L \ll 1$!

(III) Disordered horizons

Disordered fixed points in holography

[Hartnoll-Santos, 1402.0872]

- The relevance of a disordered coupling

$$\int dt d^d x \, h(x)\mathcal{O}(t,x)$$

- Is determined by the 'Harris criterion':

$$\Delta > \frac{d+2}{2}$$

- If relevant, need to follow the flow to IR.

Relevant disorder

- Relevant disorder is difficult in QFT. E.g. Wilson-Fisher theory perturbed by a random mass term: Statistical physics model (no time) has an IR disordered fixed point, accessible in ε-expansion. Quantum critical model (with time) does not.

- Solvable lattice models in 1+1 show 'infinite disorder' fixed points with 'activated scaling': $\omega \sim e^{-1/p^{\alpha}}$ (z = ∞!)

Disordered fixed points in holography

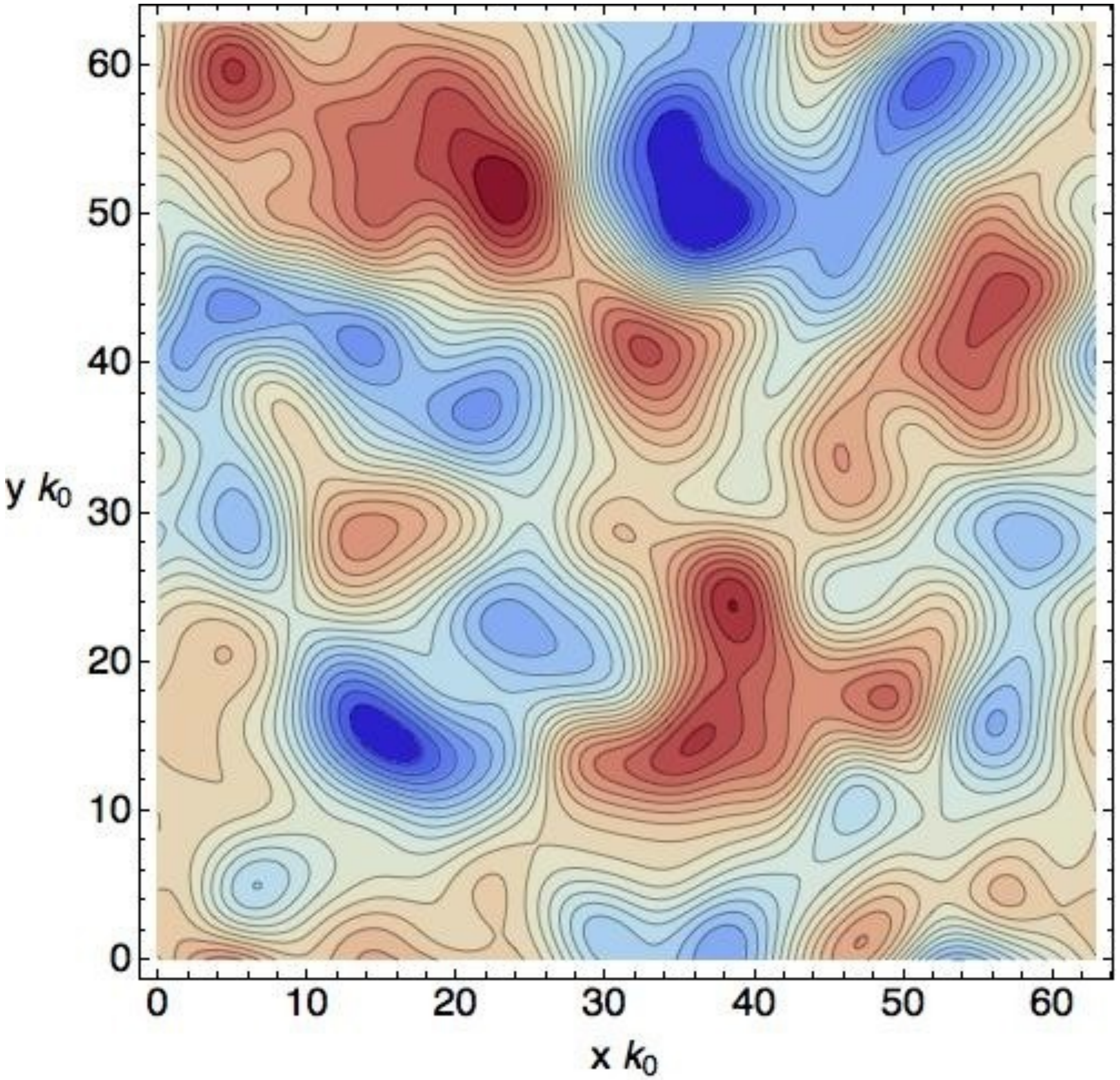
- A random coupling is generated by

$$h(x) = \bar{V} \sum_{n=1}^{N-1} 2\sqrt{\Delta k}\, \cos(n\Delta k\, x + \gamma_n)$$

- Solved Einstein-scalar bulk theory with a marginally relevant random source.
- Resummed the logarithmic growth to find stable disordered IR fixed points.
- Confirmed and extended results via numerical simulation of full disorder.

Disordered fixed points in holography

- **Zero temperature IR geometry**

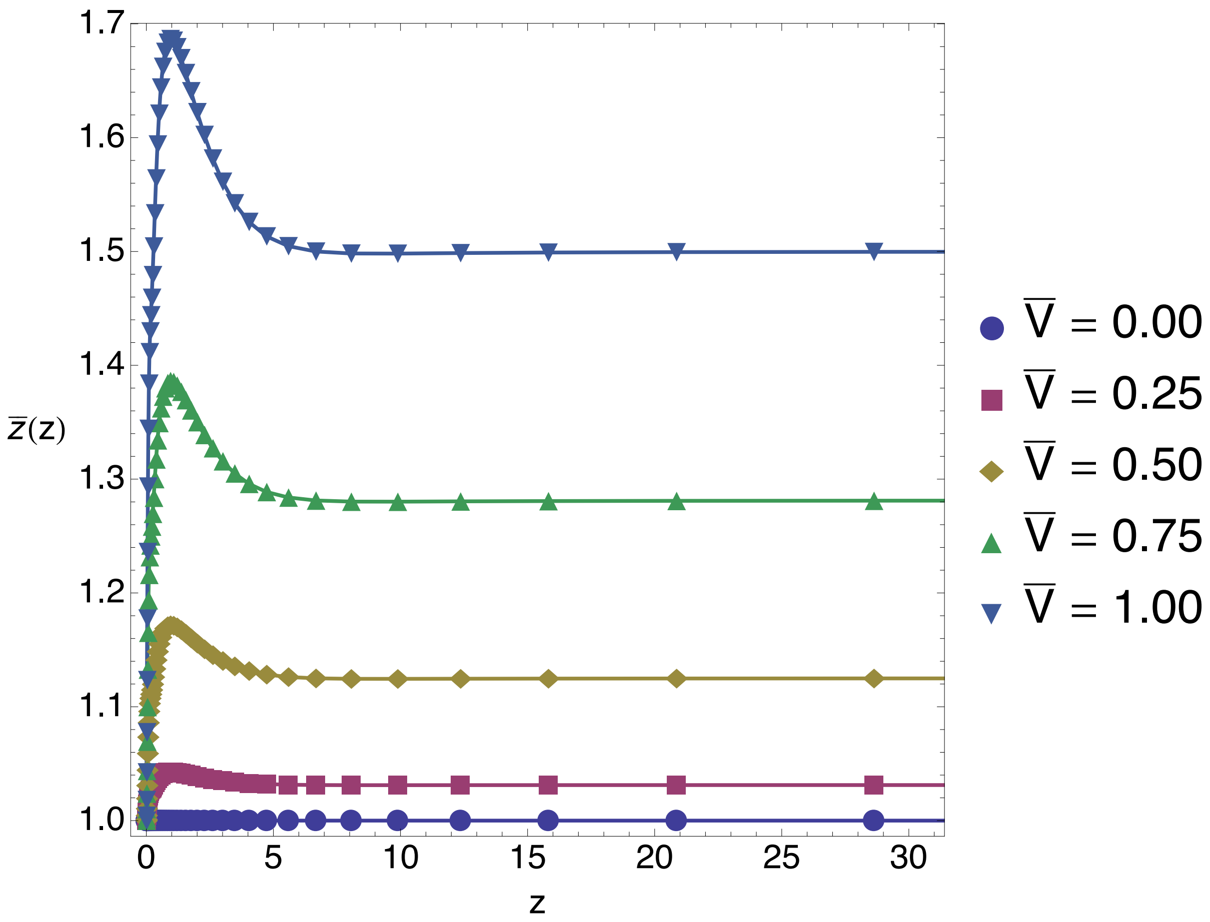


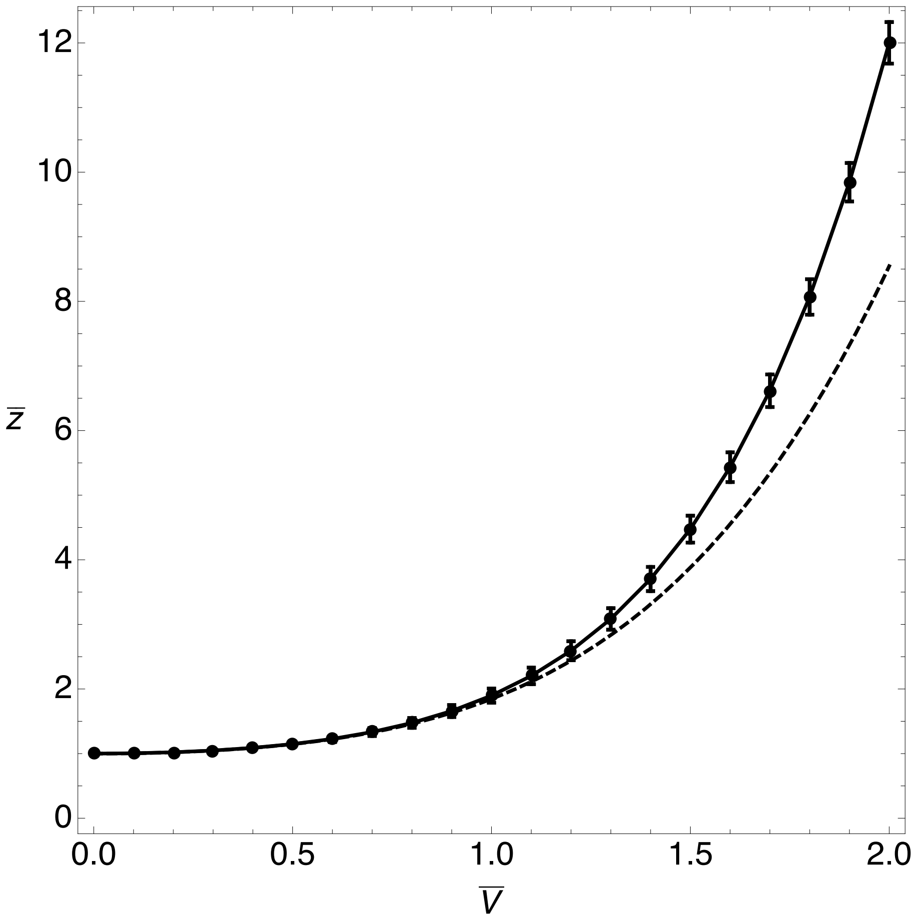
$$ds^2_{\mathrm{IR}} = \frac{1}{z^2}\left(-\frac{dt^2}{z^{2\beta(\bar{V})}} + dx^2 + dz^2\right)$$

Disordered fixed points in holography

$$\bar{z} = 1 + \frac{\pi^{(d-1)/2}}{2}\Gamma\left(\frac{d+1}{2}\right)\bar{V}^2 + \cdots$$

$$\bar{z} = 1 + \tfrac{1}{2}\bar{V}^2 + \tfrac{1}{2}(\ln 2)\bar{V}^4 + \cdots .$$





Disordered fixed points in holography

- The results suggest that $z \to \infty$ for strictly relevant disorder.
- $z = \infty$ is known to arise in 'infinite disorder' fixed points of solvable one dimensional models [D.S. Fisher '92].
- Work in progress …

Conclusions

- Coherent metals: slow decay rate Γ.
- Non-quasiparticle coherent metals: transport controlled by momentum relaxation.
- Dramatic WF violation predicted. Potentially observed in VO_2.
- Constructed zero temperature finite disorder fixed points in holography.

Happy Birthday Joe!