

# The Dynamics of Flavour in External Fields

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Primarily reporting on:

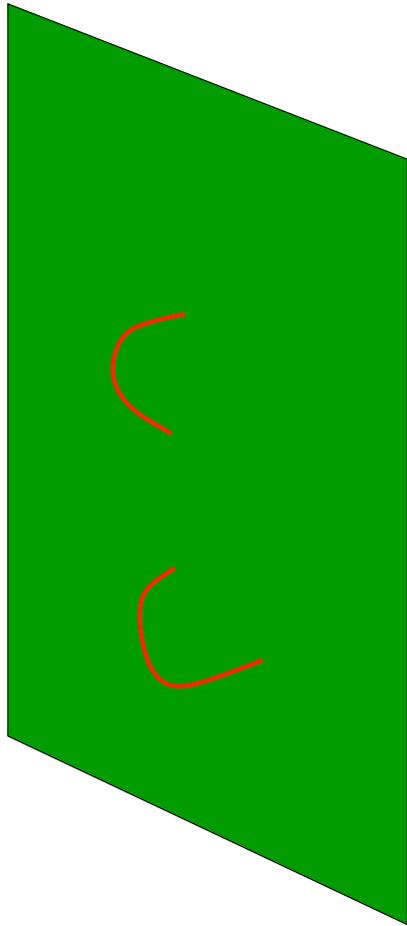
hep-th/0605088, arXiv:0709.1547, arXiv:0709.1554

by Tameem Albash, cvj, Arnab Kundu, Veselin Filev

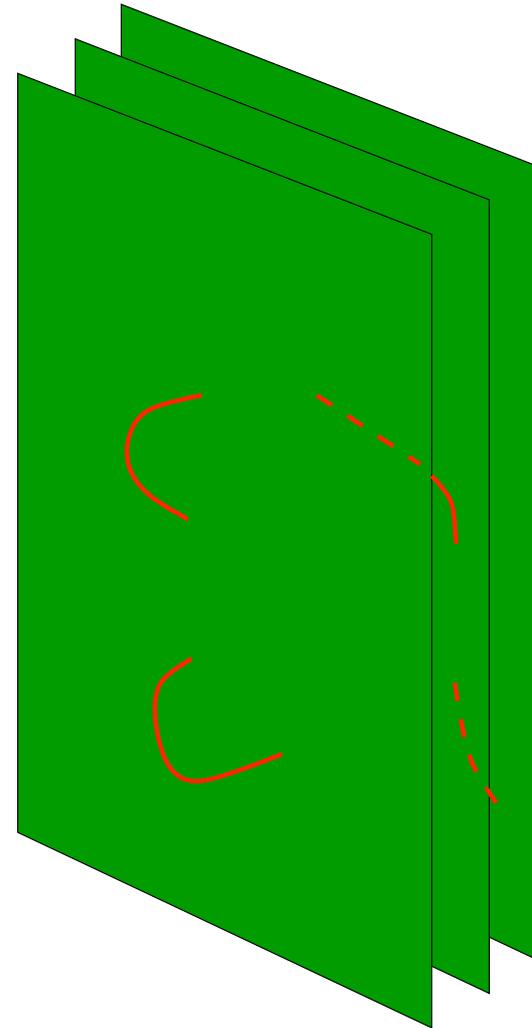
hep-th/0701001, Veselin Filev, cvj, R. C. Rashkov and, K. S. Viswanathan

# The AdS of AdS/CFT

## The Correspondence

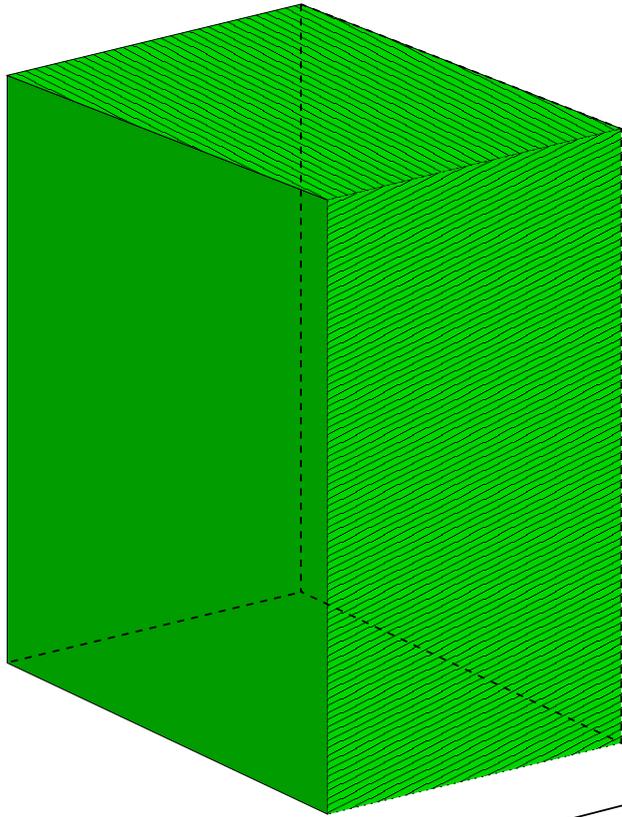


A D3-brane is 3-dim hypersurface that moves in time. Strings can end on it. Their physics captures aspects of its dynamics. The physics is  $U(1)$  susy gauge theory.



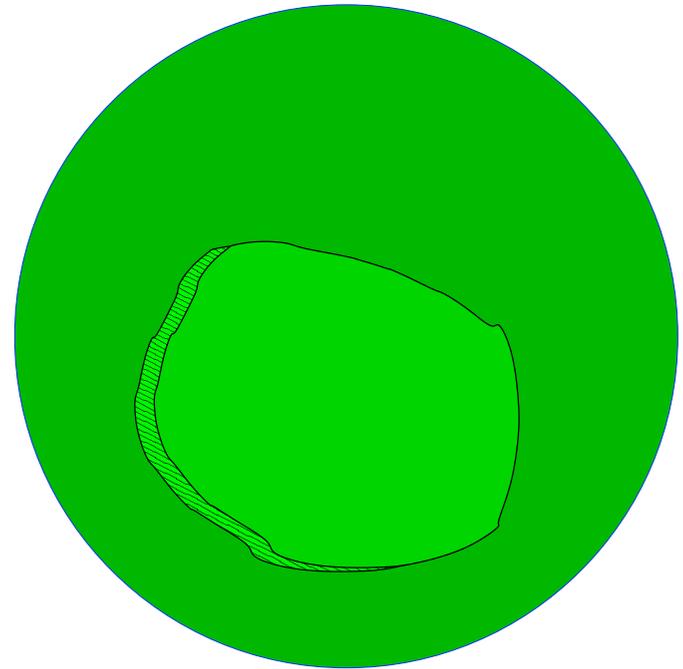
$U(N)$  for  $N$  branes.

Brane also has a gravitational footprint. Decoupling the gauge theory from the rest of the stringy physics yields “near-horizon” geometry:  $AdS_5 \times S^5$



local AdS

$\infty$   
 $u$



Shot of “global” AdS  
(gauge theory in a box)

$$ds^2 = \frac{u^2}{R^2}(-dt^2 + d\vec{x} \cdot d\vec{x}) + \frac{R^2}{u^2}du^2 + R^2(d\theta^2 + \cos^2 \theta d\Omega_3^2 + \sin^2 \theta d\phi^2)$$

$$R^2 = \sqrt{4\pi g_s N \alpha'} ; \quad 2\pi g_s = g_{\text{YM}}^2 \quad N \text{ large}; \quad g_s \text{ small}; \quad \lambda = g_{\text{YM}}^2 N \text{ large}$$

# The Dual Gauge Theory

$\mathcal{N} = 4$  Supersymmetric  $SU(N)$  Yang Mills

Superconformal invariance

$$\sim SO(4, 2)$$

isometries

global R-symmetry

$$A_\mu, 4\lambda, 6\phi$$

gauge multiplet

$$SU(4) \sim SO(6)$$

isometries

$$AdS_5 \times S^5$$

# Toward QCD

Adding fundamental flavours.

One approach is inspired by role of D3-D7 strings:

D7-brane

D3-branes

A string endpoint transforms in the fundamental.

How does this look in AdS/CFT?

Take near horizon limit of  $N$  D3-branes

$N_f$  D7-branes

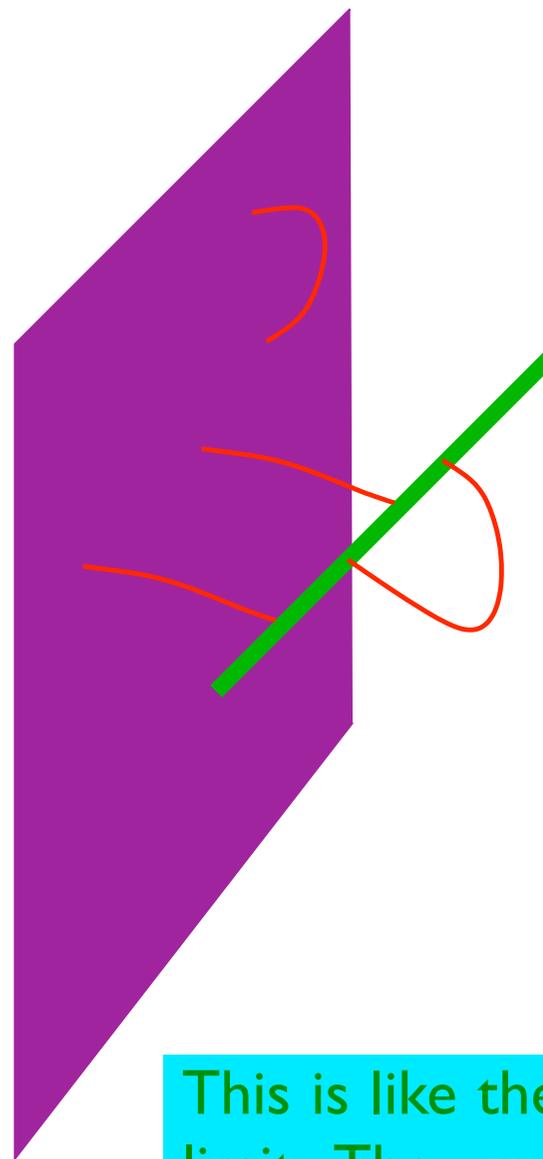
Get a sensible, controllable geometry when:

$N$  large

$$g_{YM}^2 N = \lambda = \text{finite}$$

$$g_{YM}^2 N_f = 0$$

This gives limit in which D7s are simply probes of the AdS geometry.



This is like the “quenched” limit. The quarks do not back react on the physics.

Put D7-brane at a point in  $(u, \phi)$ .

In limit, it will spread out to fill (part of)  $u$ , and have non-trivial embedding  $\theta(u)$

The D7-brane sees on its worldvolume:

$$ds^2 = \frac{u^2}{R^2}(-dt^2 + d\vec{x} \cdot d\vec{x}) + \frac{R^2}{u^2}du^2 + R^2 \left( \frac{u^2 - L^2}{u^2} \right) d\Omega_3^2$$

$$L = u \sin \theta$$

Large  $u$  is just  $\text{AdS}_5 \times S^3$

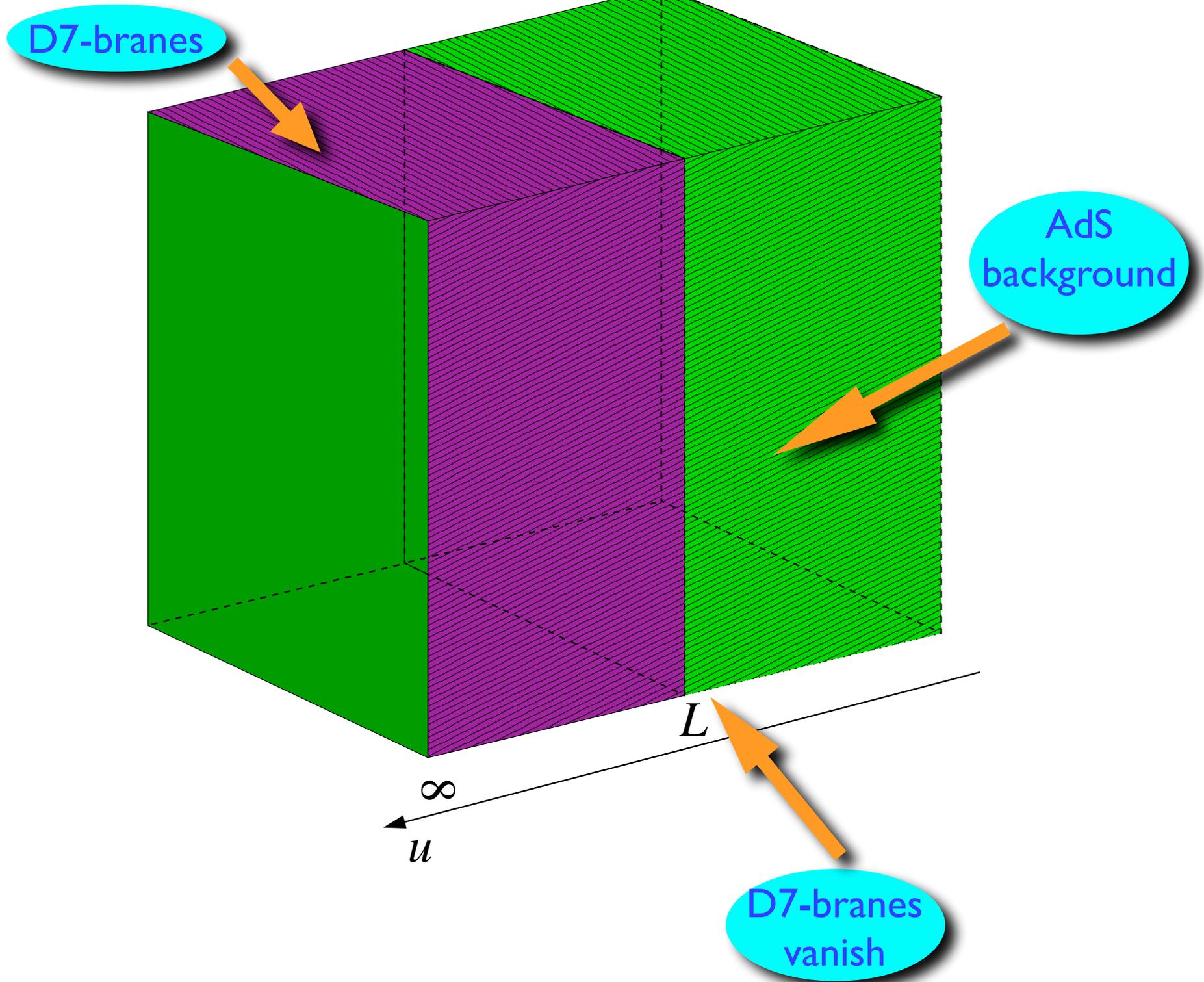


The D7-brane fills AdS and wraps  $S^3 \subset S^5$

But when  $u=L$ , only have AdS part.



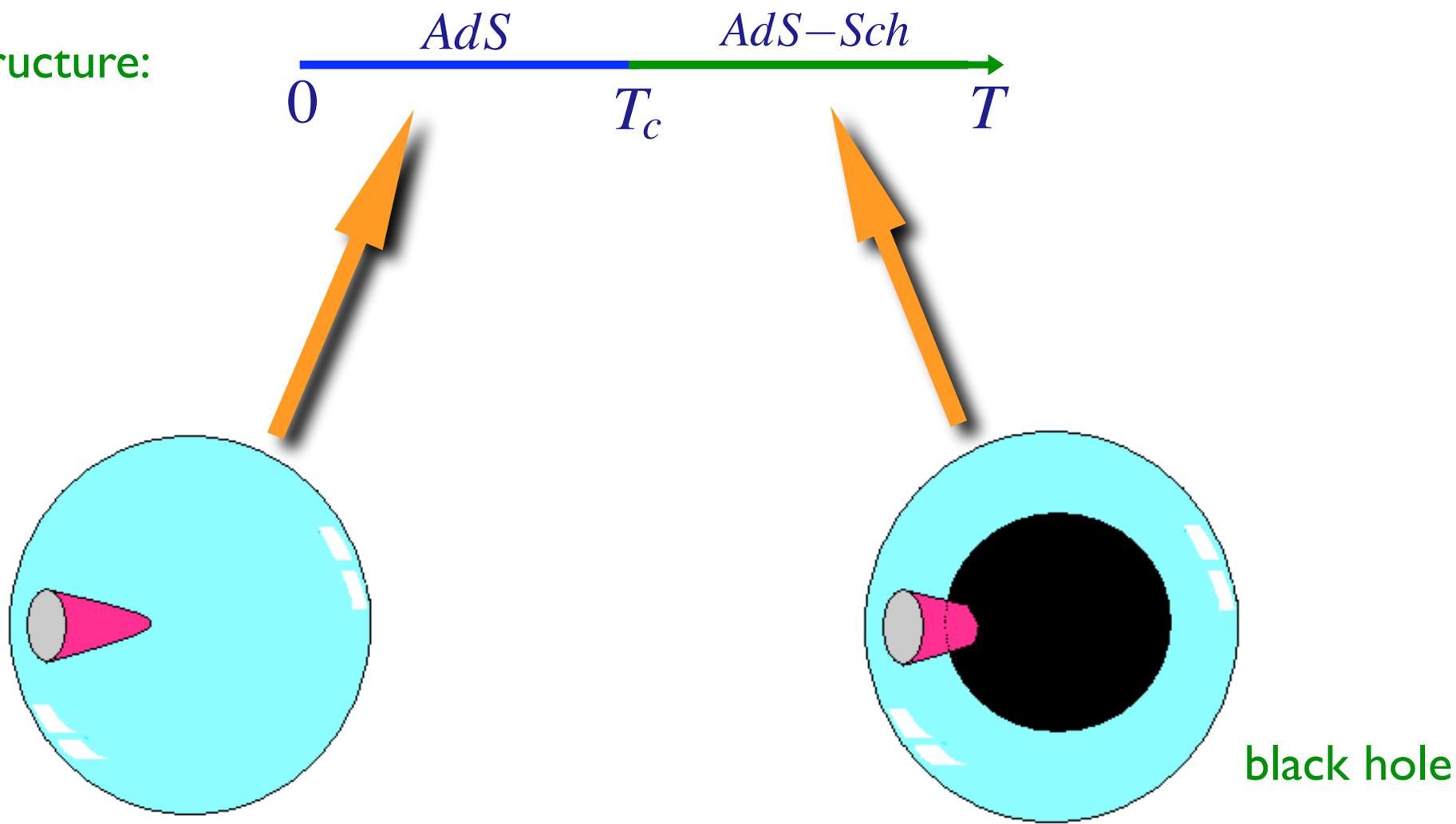
The D7-brane dissolves away!



Witten, '98  
(Hawking-Page, '83)

# Finite Temperature

Phase structure:



Here, theory is in a box, and so transition at non-zero temp. If not in box, any non-zero temperature makes the transition to “deconfined” or “plasma” phase.

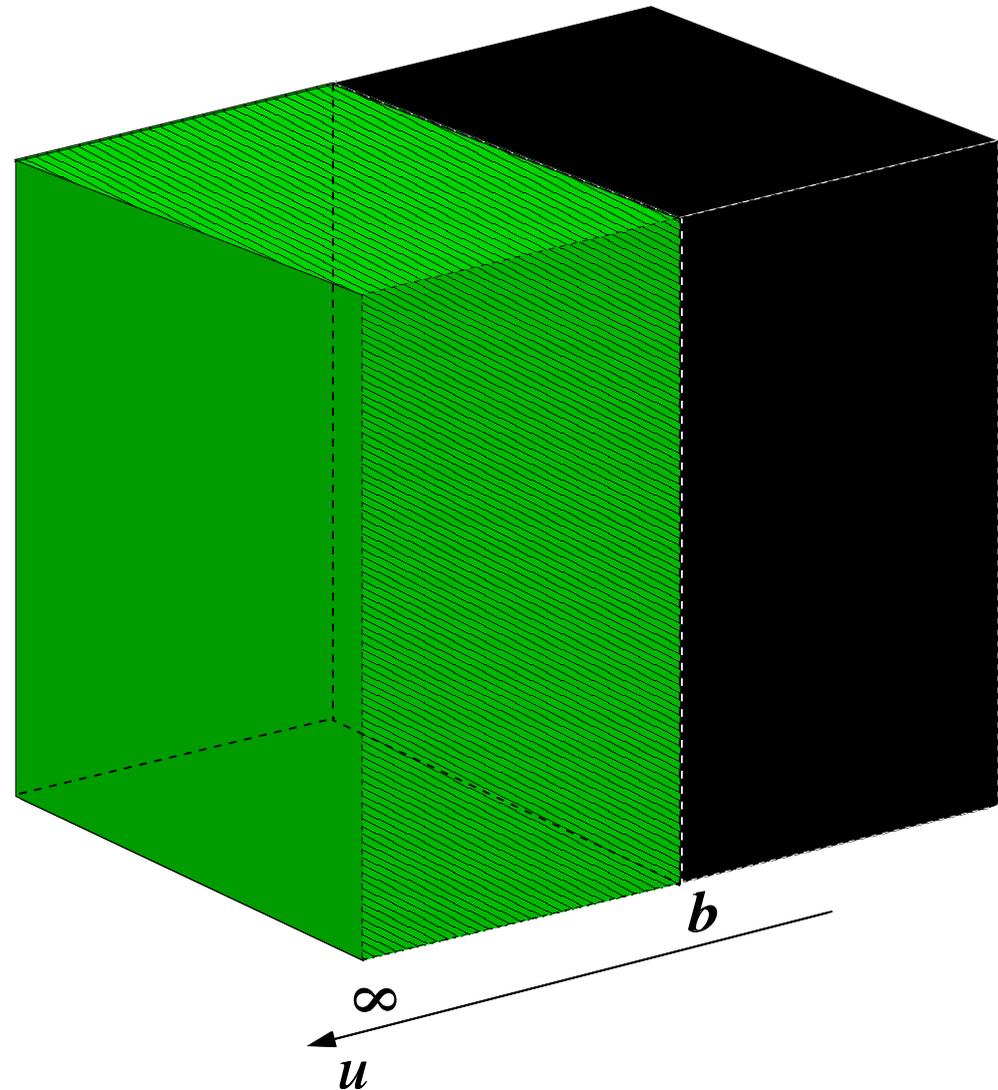
# Finite Temperature

AdS-Schwarzschild:

$$ds^2 = -\frac{f(u)}{R^2}dt^2 + \frac{R^2}{f(u)}du^2 + \frac{u^2}{R^2}d\vec{x} \cdot d\vec{x} + R^2(d\theta^2 + \cos^2\theta d\Omega_3^2 + \sin^2\theta d\phi^2)$$

$$f(u) = u^2 - \frac{b^4}{u^2}$$

$$b^2 = \frac{8G_5 m_{\text{b.h.}}}{3\pi}$$

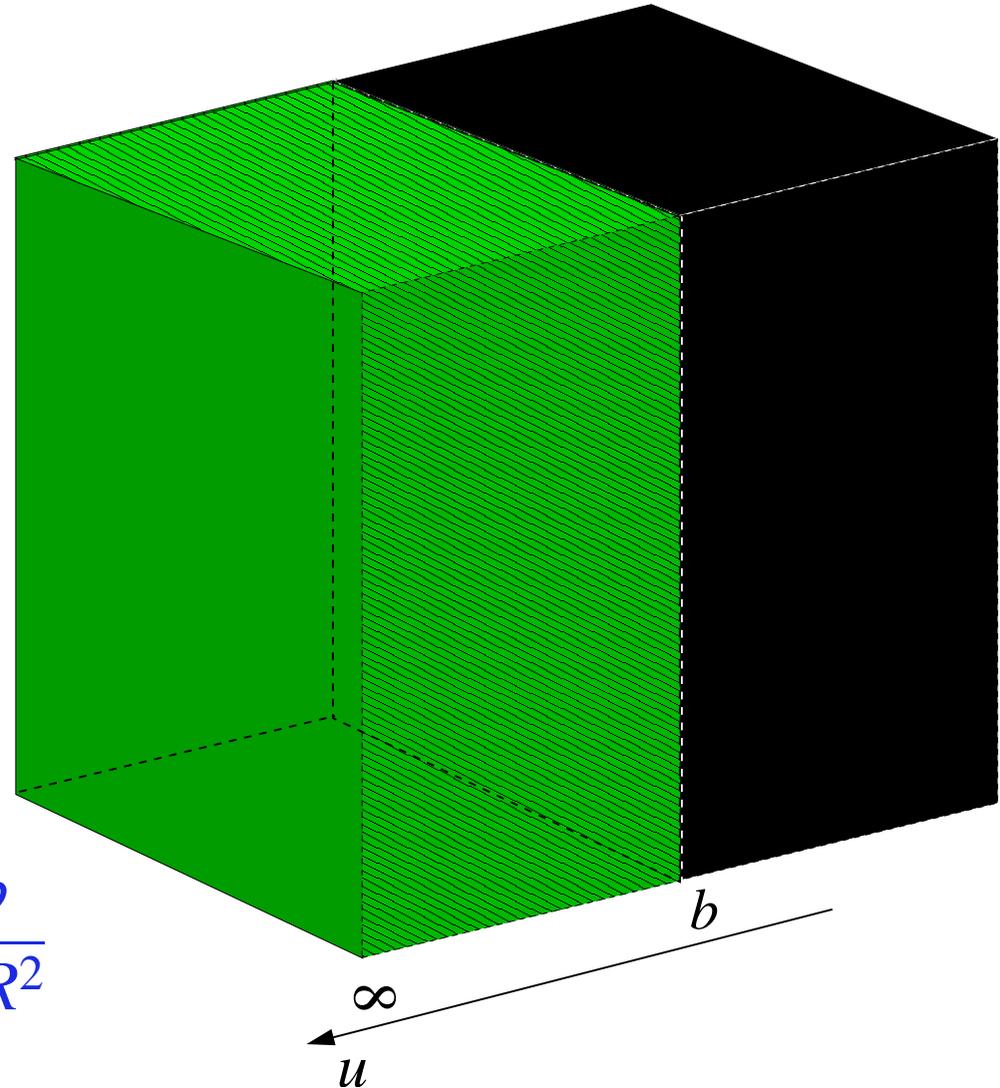


## AdS-Schwarzschild:

$$ds^2 = -\frac{f(u)}{R^2}dt^2 + \frac{R^2}{f(u)}du^2 + \frac{u^2}{R^2}d\vec{x} \cdot d\vec{x} + R^2(d\theta^2 + \cos^2\theta d\Omega_3^2 + \sin^2\theta d\phi^2)$$

$$f(u) = u^2 - \frac{b^4}{u^2}$$

$$b^2 = \frac{8G_5 m_{\text{b.h.}}}{3\pi}$$



$$-dt^2 \implies d\tau^2 \implies \beta^{-1} = \frac{b}{\pi R^2}$$

# Finite Temperature, Flavoured

AdS-Schwarzschild with D7-brane probe:

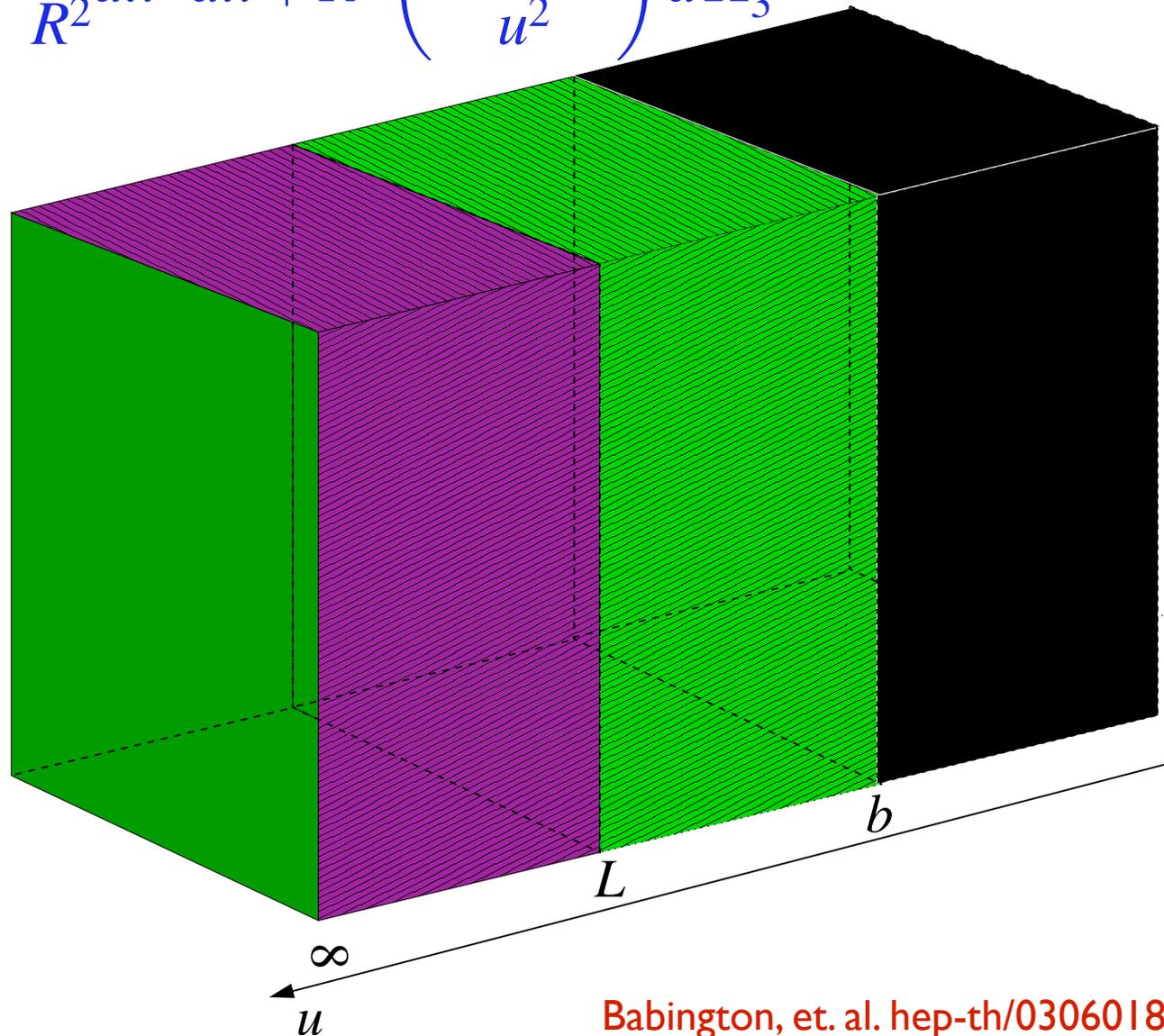
$$ds^2 = -\frac{f(u)}{R^2} dt^2 + \frac{R^2}{f(u)} du^2 + \frac{u^2}{R^2} d\vec{x} \cdot d\vec{x} + R^2 \left( \frac{u^2 - L^2}{u^2} \right) d\Omega_3^2$$

$$f(u) = u^2 - \frac{b^4}{u^2}$$

$$L = u \sin \theta$$

$$b^2 = \frac{8G_5 m_{\text{b.h.}}}{3\pi}$$

$$\beta^{-1} = \frac{b}{\pi R^2}$$



AdS-Schwarzschild with D7-brane probe:

$$ds^2 = \frac{f(u)}{R^2} d\tau^2 + \frac{R^2}{f(u)} du^2 + \frac{u^2}{R^2} d\vec{x} \cdot d\vec{x} + R^2 \left( \frac{u^2 - L^2}{u^2} \right) d\Omega_3^2$$

$$f(u) = u^2 - \frac{b^4}{u^2}$$

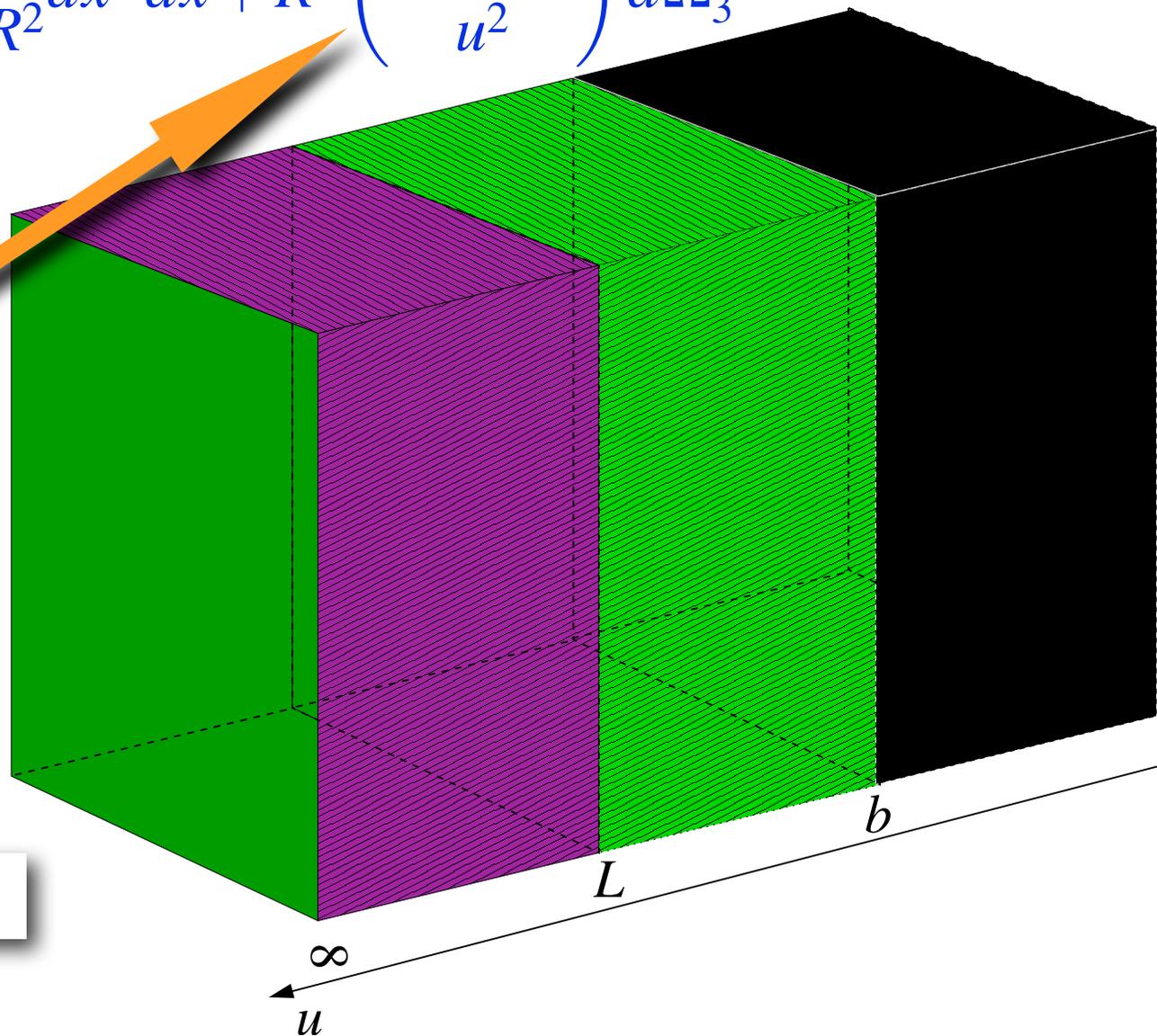
The D7-brane fills AdS and wraps  $S^3 \subset S^5$

and also wraps  $S^1$

Brane can end by either compact space collapsing...

$L > b$  ends outside horizon

$L < b$  falls into horizon



Compute the effective action for the probe in the background, and solve the equations of motion.

$$\phi = 0; \quad \theta = \theta(u)$$

$$\frac{d}{du} \left( \frac{u^2 (u^4 - b^4) \theta'(u) \cos^3(\theta(u))}{\sqrt{u^2 + (u^4 - b^4) \theta'(u)^2}} \right) + 3u^2 \cos^2 \theta(u) \sin \theta(u) \sqrt{u^2 + (u^4 - b^4) \theta'(u)^2} = 0$$

Asymptotically, we have, for large  $u$ :

$$\frac{d}{du} (u^5 \theta'(u)) + 3u^2 \theta(u) = 0, \quad \longrightarrow \quad \theta(u) = \frac{1}{u} \left( m + \frac{c}{u^2} \right) .$$

$$\lim_{u \rightarrow \infty} L(u) = m + \frac{c}{u^2} + \dots$$

$$m = 2\pi\alpha' m_q; \quad -c \simeq \langle \bar{\Psi} \Psi \rangle$$

Look for solutions and read off these values...

# Chiral Symmetry and Geometry

Chiral Symmetry here is a  $U(1)$  phase rotation of the quarks.

$$\begin{aligned}\psi &\longrightarrow e^{i\phi}\psi \\ \bar{\psi} &\longrightarrow e^{-i\phi}\bar{\psi}\end{aligned}$$

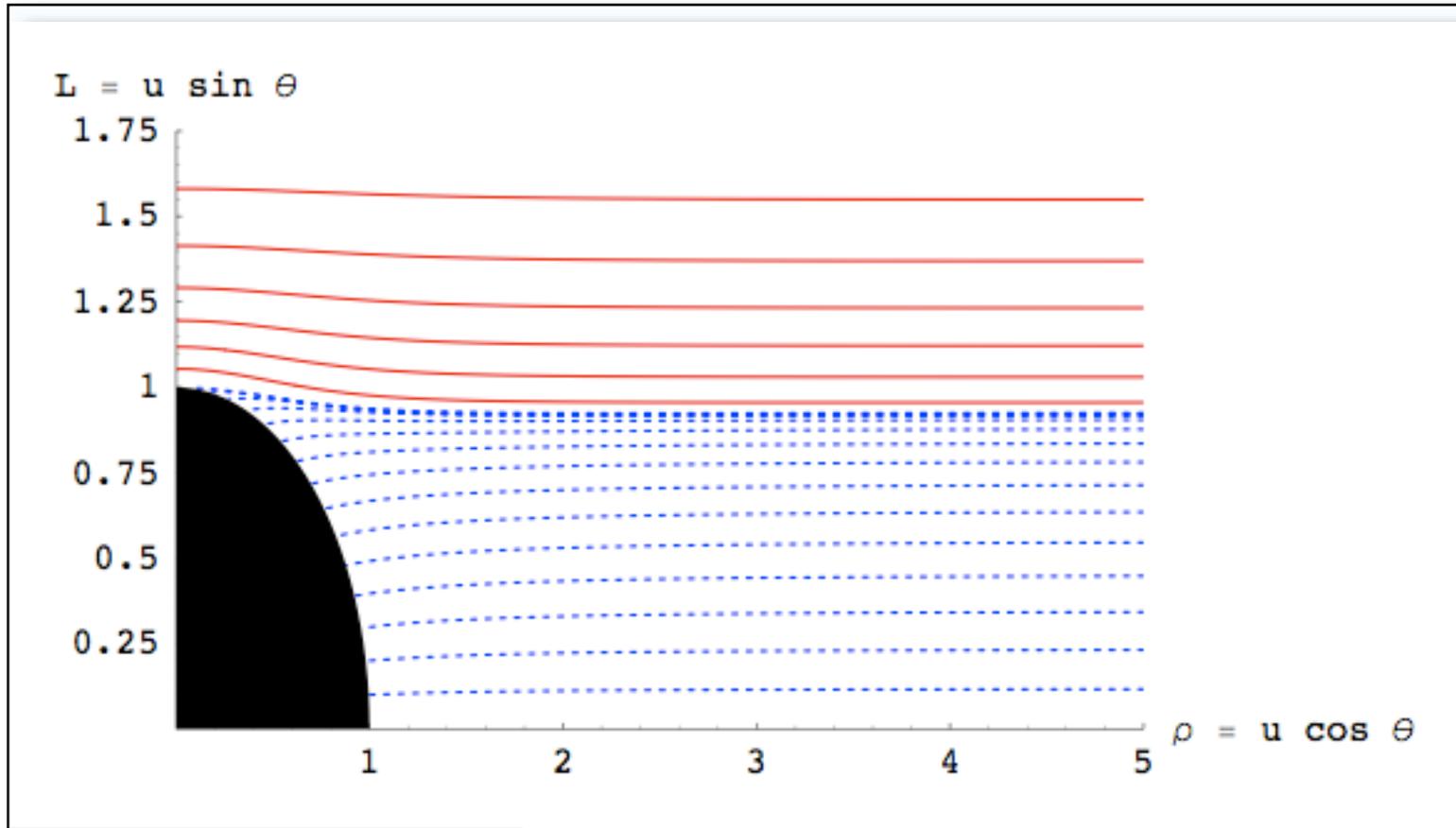
Geometrically, it is simply the  $U(1)$  rotation in the coordinate  $\phi$

This is the global symmetry corresponding to rotations around the D7 in the  $(u, \phi)$  plane.

A vev for  $\langle \bar{\psi}\psi \rangle$  corresponds to a profile of the D7-brane that breaks this symmetry.

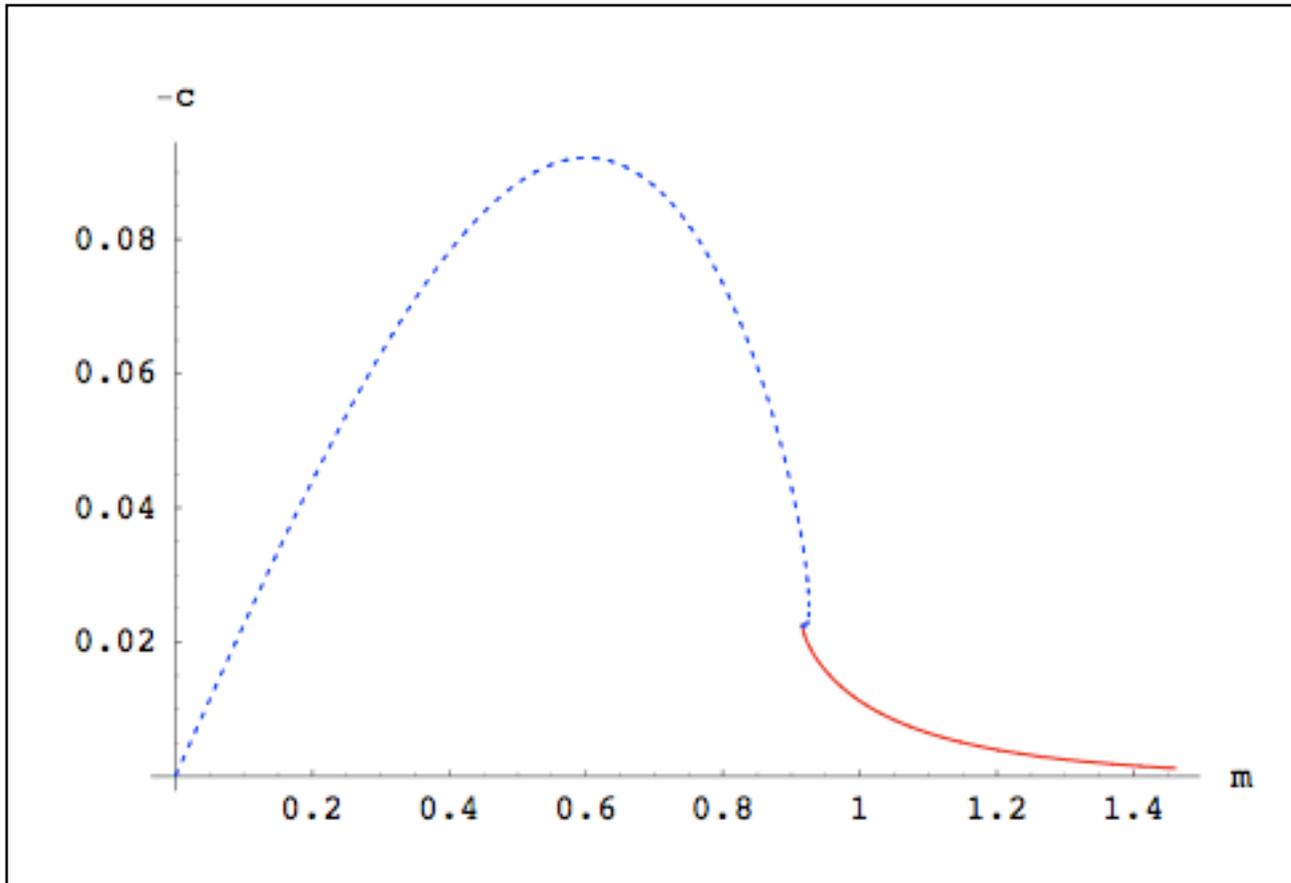
# Embedding Solutions

Here are some of the solutions we found numerically:



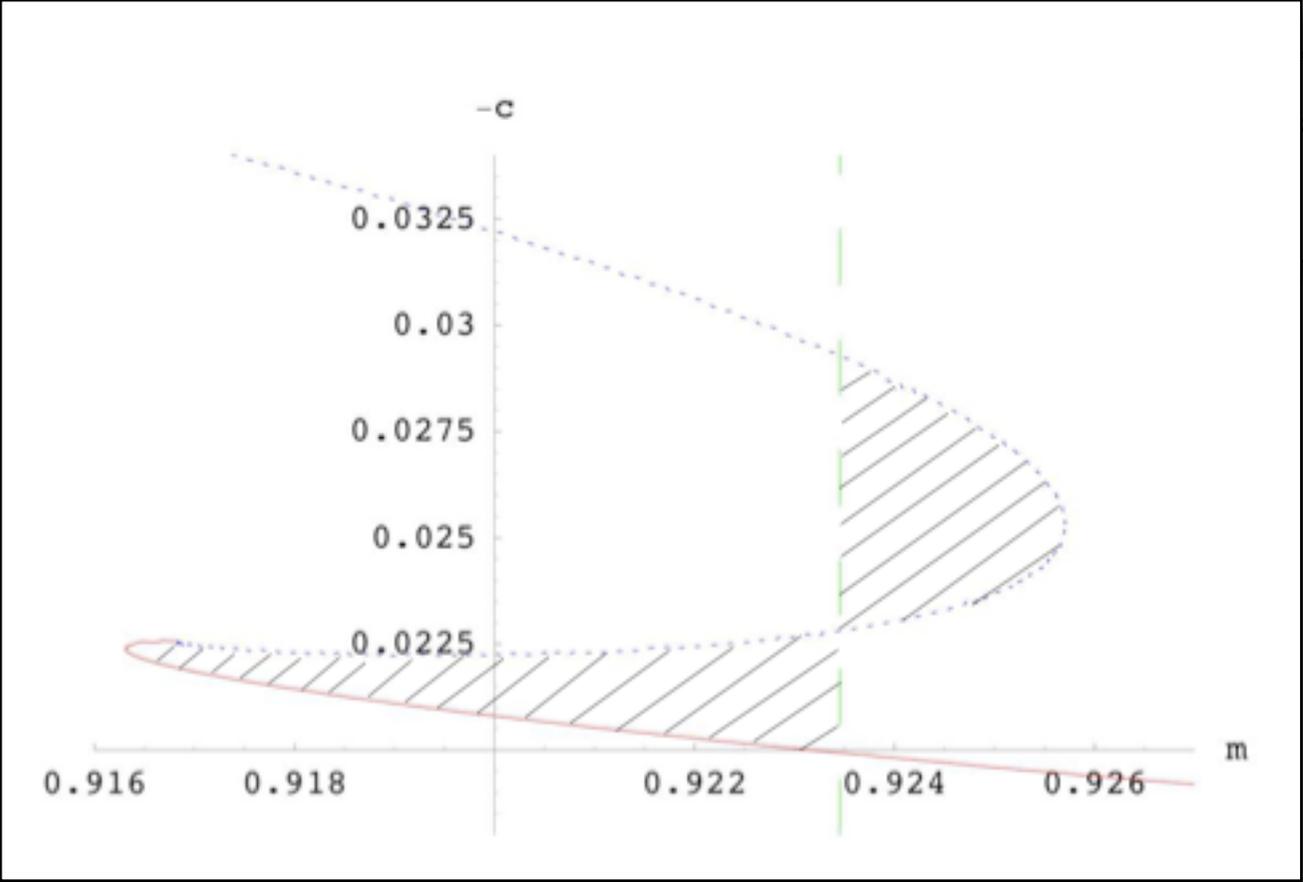
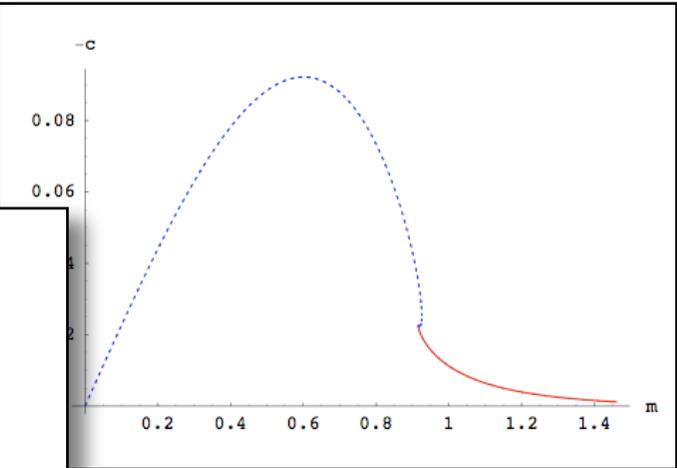
Key difference from what went before: Use shooting technique starting at horizon. Much less delicate process than starting from infinity.

Read off the Asymptotia\* from each solution:



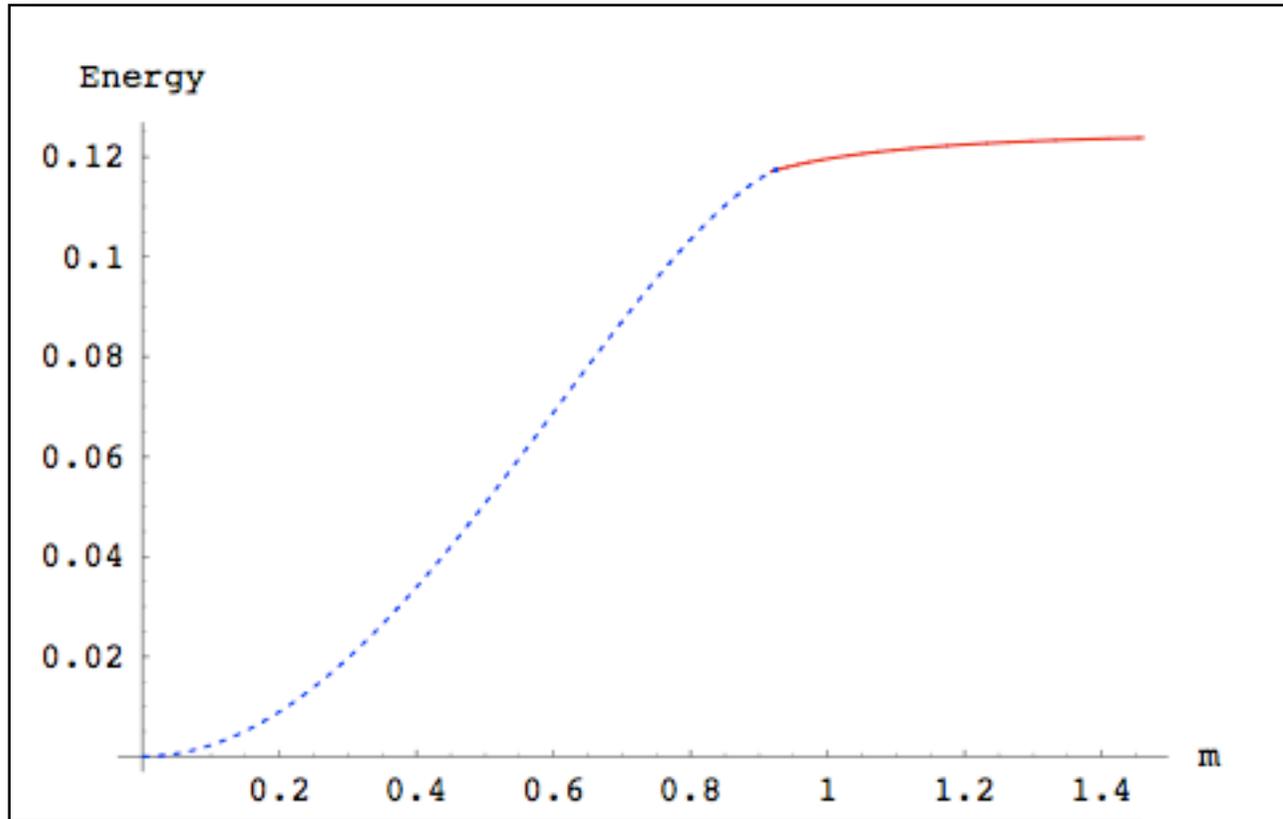
\*sorry... could not resist.

A closer look, showing some multivaluedness:

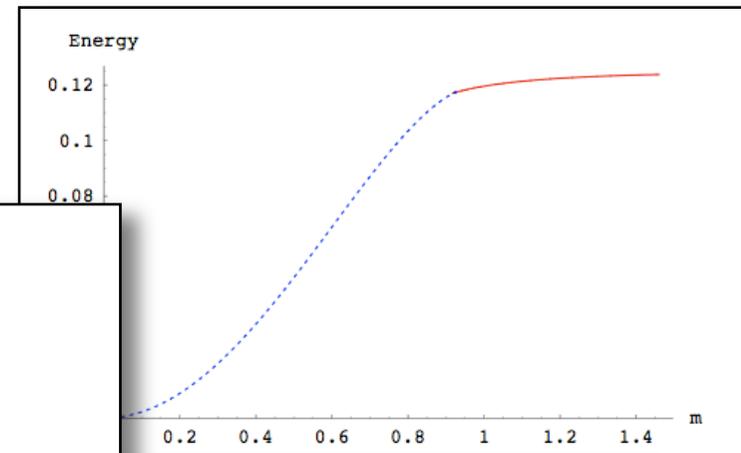
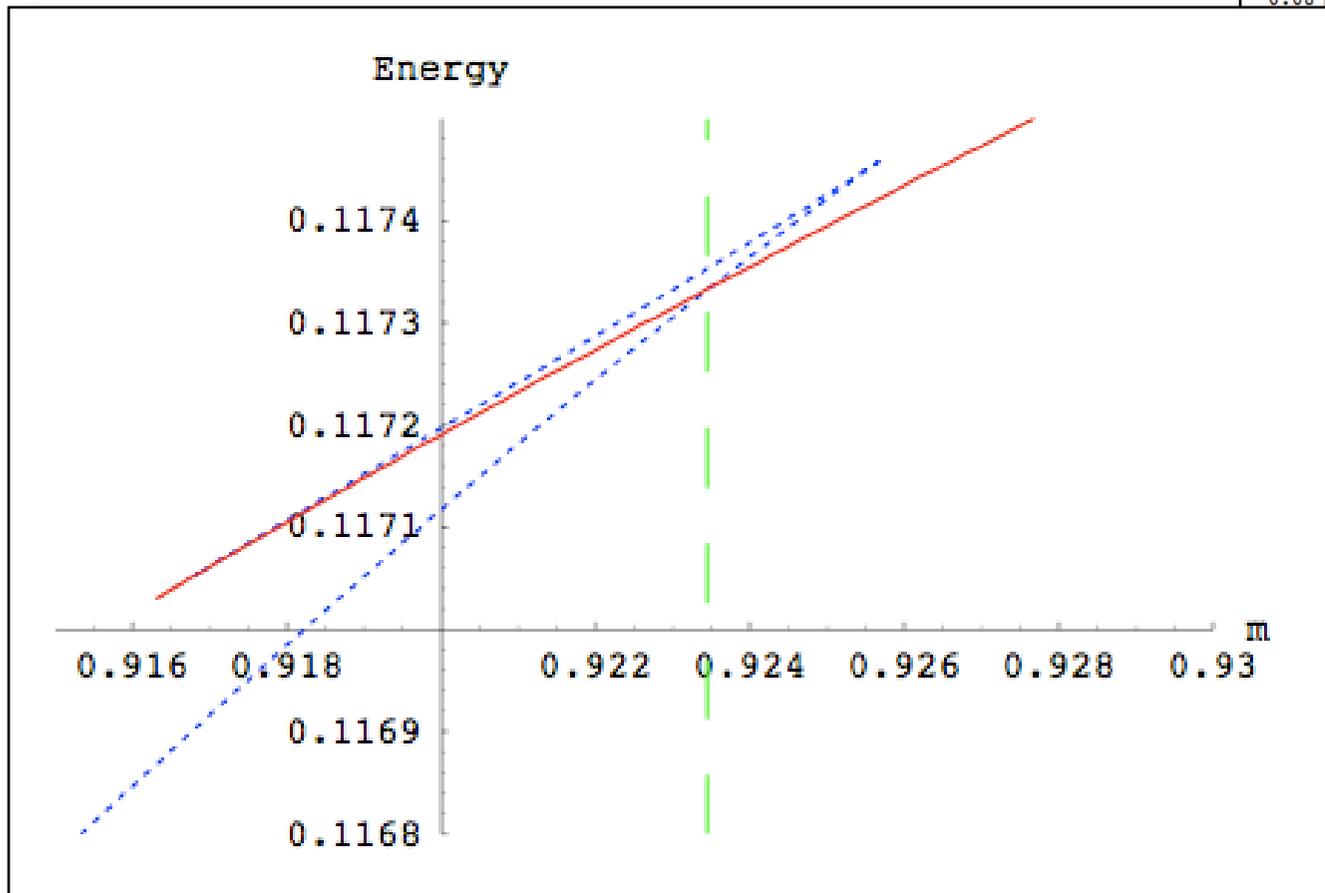


Physics will choose. Compute the free energy, using:

$$\mathcal{F} = \beta^{-1} \tau_7 \int d^4x d\Omega_3 du \sqrt{-\det g}$$



A closer look, showing the crossover:



There is a first order phase transition from one class of embedding to another at some critical value of the bare mass.

This is equivalent to a phase transition at some critical temperature (above the deconfinement temperature)

What is the nature of this transition?

Below the transition, probes are in a phase similar to that at zero temperature. Brane ends before the horizon. There are mesons, etc.

Above the transition, probes are in a new phase. Branes have fallen into the horizon. New physics.

# The Meson Spectrum

Meson spectrum studied by looking at fluctuations of D7-brane probe about  $\theta_{(0)} = \theta(u)$ ,  $\phi_{(0)} = 0$

$$\phi(z, t) = 0 + \delta\phi(z, t) = f(z)e^{-i\omega t} \quad z = u^{-2}$$

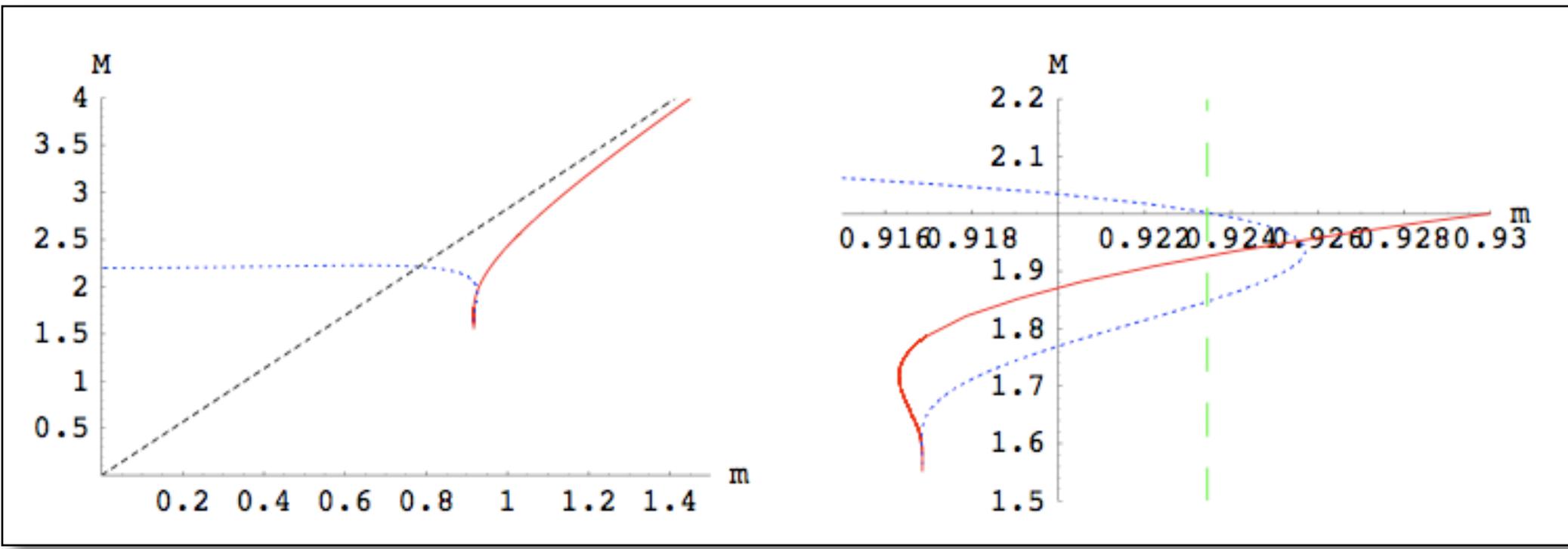
At second order, get, for the horizon solutions:

$$f''(z) + \frac{f'(z)}{z - b^{-2}} + \frac{R^4\omega^2}{16b^2} \frac{f(z)}{(z - b^{-2})^2} = 0.$$

$$f(z) = a(1 - zb^2)^{i\frac{R^2\omega}{4b}} + b(1 - zb^2)^{-i\frac{R^2\omega}{4b}}.$$

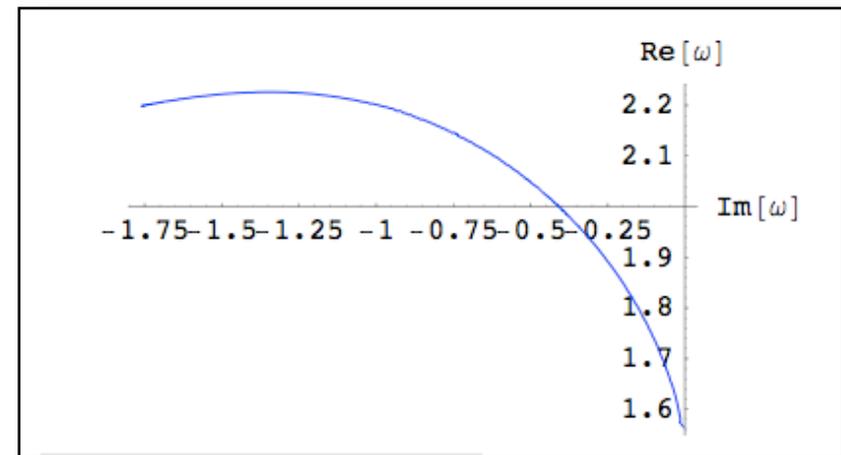
The in-falling solutions are the physical ones, (“quasi-normal modes”) and we search for solutions that are normalizable and out-going at infinity.

Result: Discrete spectrum of stable mesons for the Minkowski-type solutions,  
 Continuous spectrum of massive excitations that decay for horizon solutions.



$M, \tau$  - mass, lifetime

$$\text{Re}[\omega] = M, \quad \text{Im}[\omega] = (2\tau)^{-1}$$



What is the nature of this transition?

So it is a melting or dissociation phase transition for the mesons at  $T_{\text{melt}} > T_{\text{deconf}}$

N. Evans et. al.; others...

# Magnetic Fields and Chiral Symmetry Breaking

Another very important example can be studied by placing the gauge theory in an external magnetic field.

This is easy to do.

Add a pure gauge B-field to AdS background.  
This does not change equations of motion.

$$B = H dx^2 \wedge dx^3$$

However, through interaction  $B_{ab} + \alpha' F_{ab}$  on world-volume of probes it is equivalent to non-trivial B-field (magnetic) seen by the quarks.

Supersymmetry for probe sector is broken by B-field.

# Equations of Motion

Equation of motion for  $L_0(\rho)$

( $\rho$  is the radial coordinate here.)

$$\partial_\rho \left( \rho^3 \frac{L'_0}{\sqrt{1 + L_0'^2}} \sqrt{1 + \frac{R^4 H^2}{(\rho^2 + L_0^2)^2}} \right) + \frac{\sqrt{1 + L_0'^2}}{\sqrt{1 + \frac{R^4 h^2}{(\rho^2 + L_0^2)^2}}} \frac{2\rho^3 L_0 R^4 H^2}{(\rho^2 + L_0^2)^3} = 0$$

Asymptotically, we have:

$$\partial_\rho \left( \rho^3 \frac{L'_0}{\sqrt{1 + L_0'^2}} \right) = 0 \quad \longrightarrow \quad L_0(\rho) = m + \frac{c}{\rho^2} + \dots$$

Can read off same information as before about the condensate and mass.

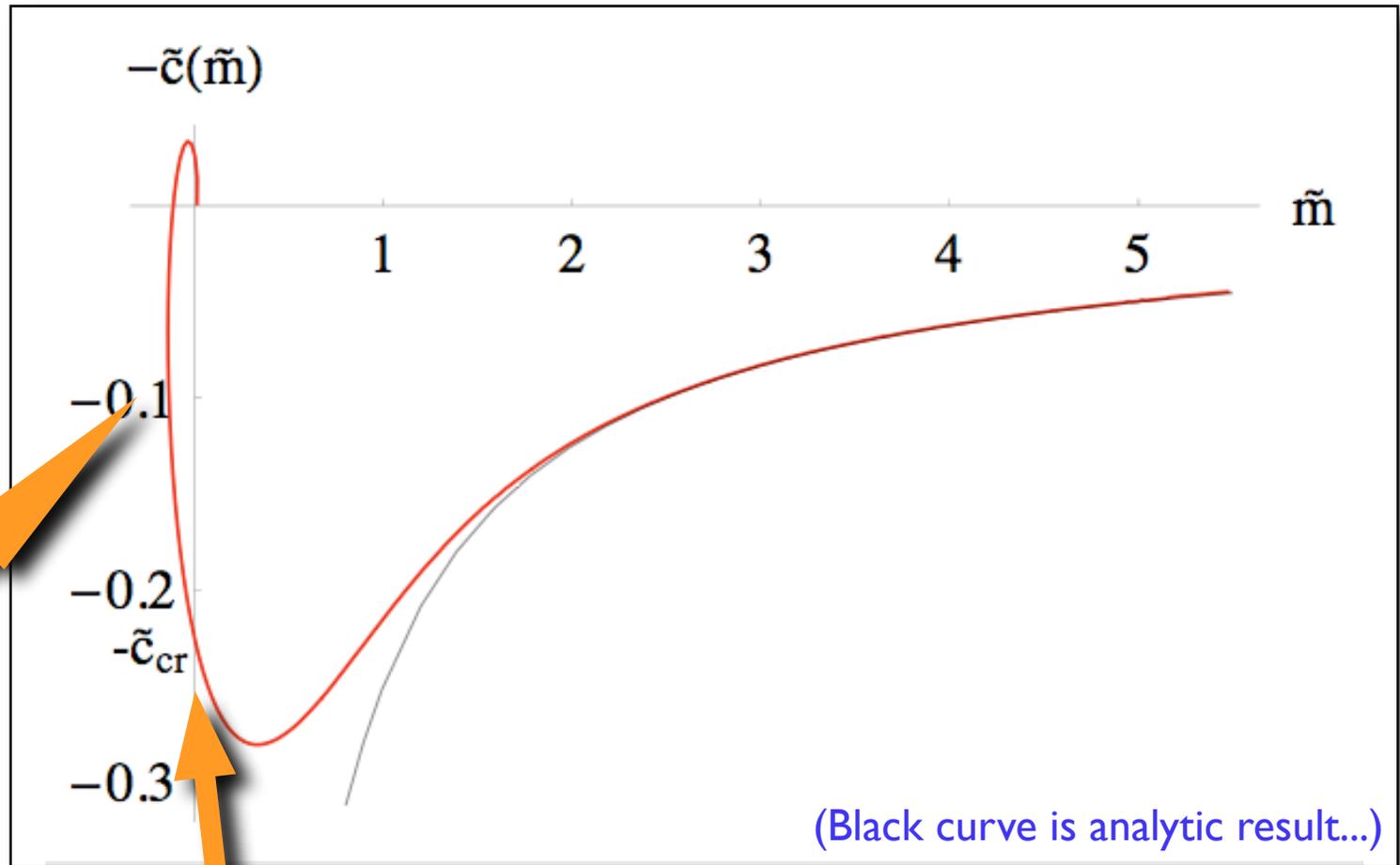
Proceed both numerically and analytically (details in paper)...

# Embedding Solutions

Can already see analytically that for large quark mass or small  $H$ , have a condensate:

$$\langle \bar{\psi}\psi \rangle \propto -c = -\frac{R^4}{4m} H^2$$

Numerically:



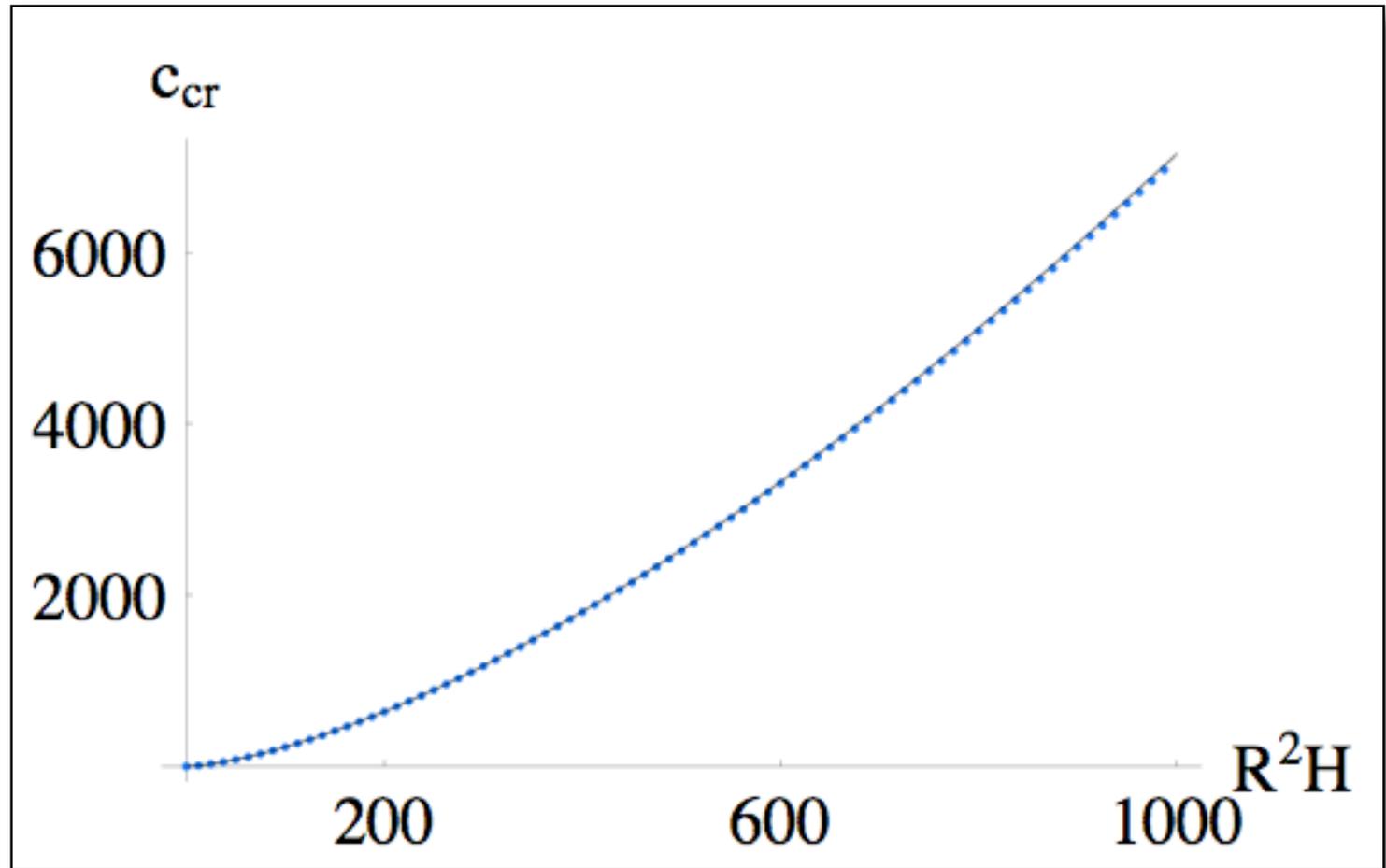
spiral structure

Non-zero condensate at vanishing mass - spontaneous chiral symmetry breaking!

# Spontaneous Chiral Symmetry Breaking

Analytically, can work out the dependence:  $c_{cr} = 0.226R^3H^{3/2}$

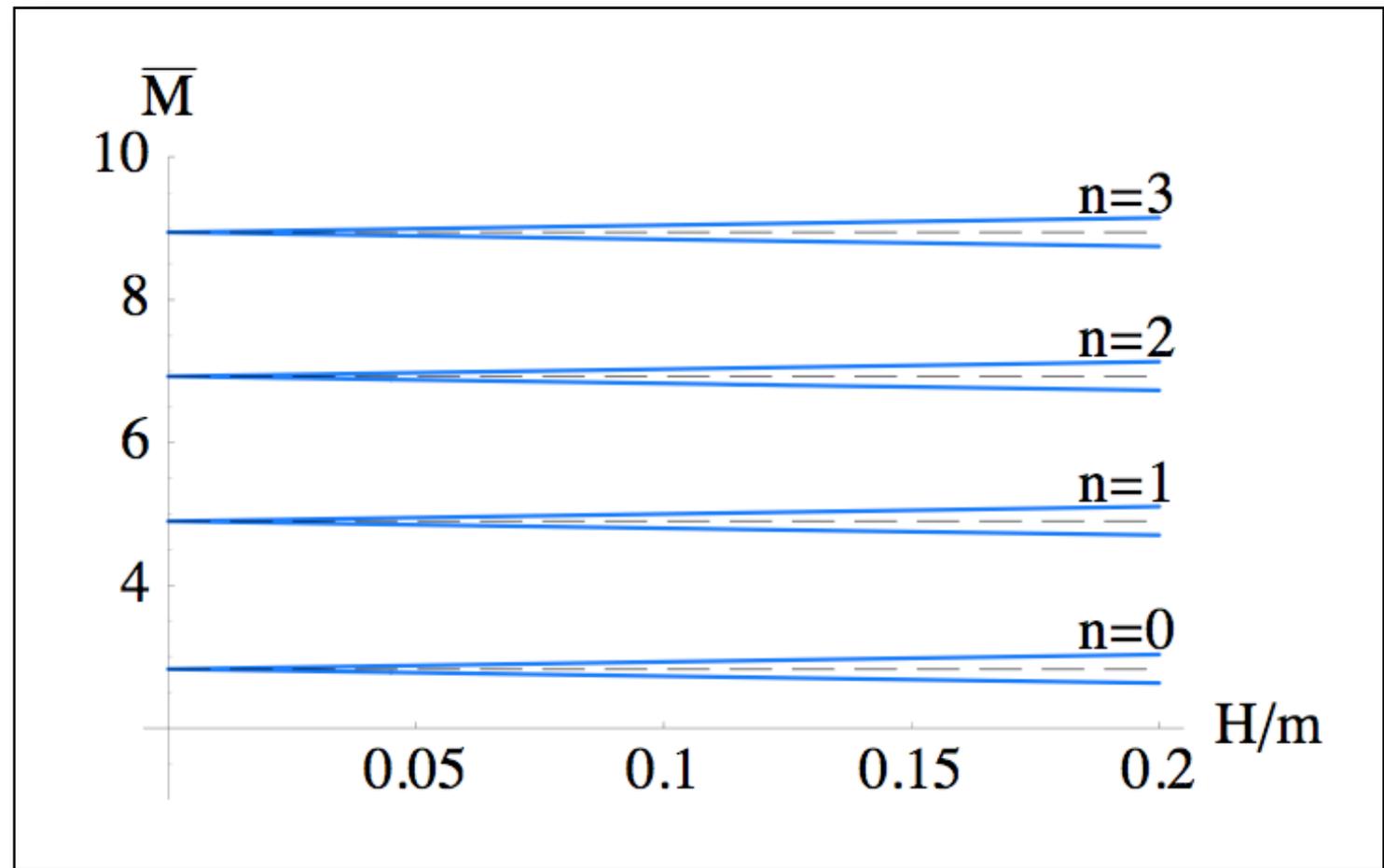
Numerically:



(Black curve is analytic result...)

# Zeeman Effect

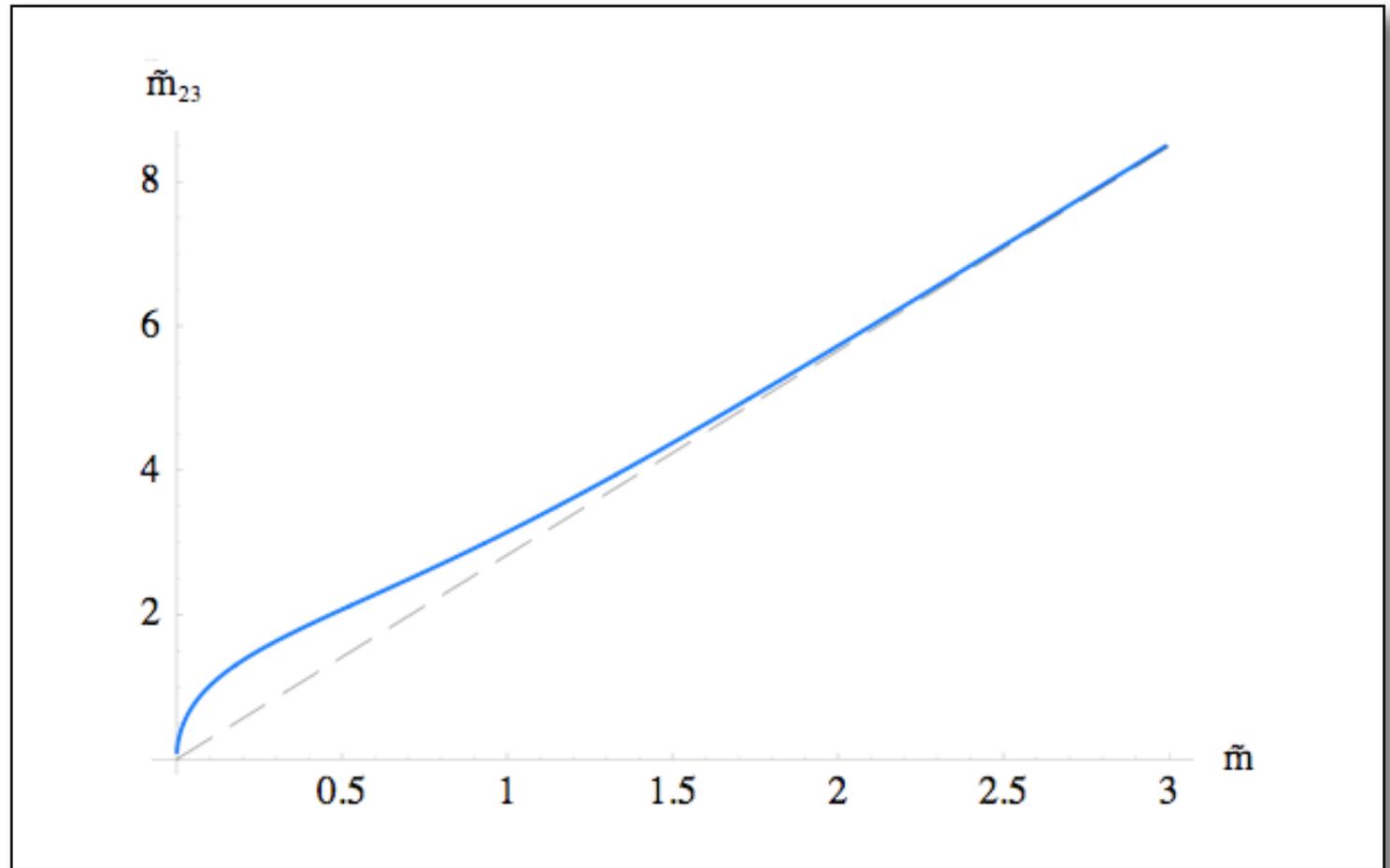
Discrete, gapped, spectrum in 0-1 plane (weak fields):



(Black lines are pure AdS result.)

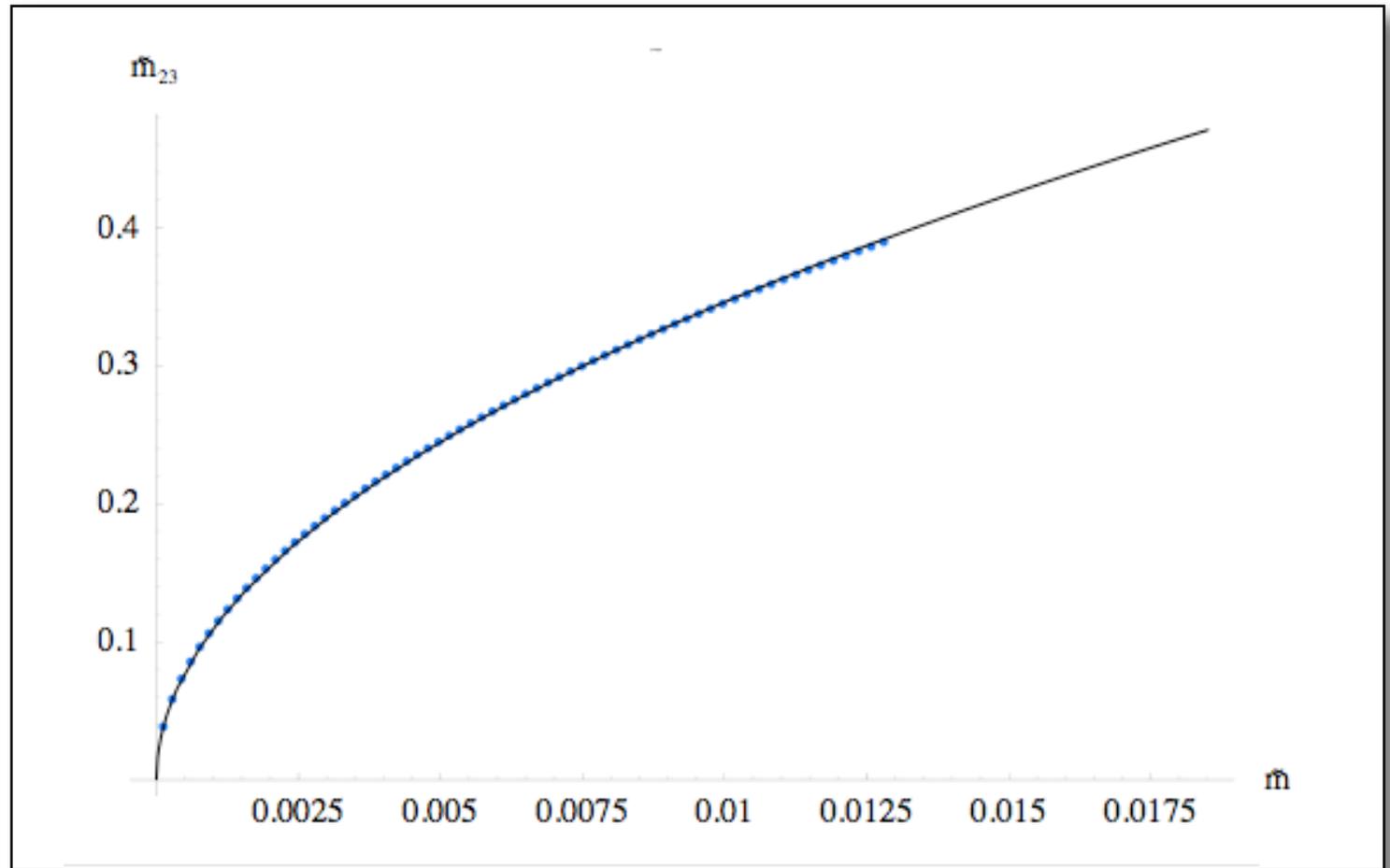
# Meson Spectrum

Goldstone in 2-3 plane:

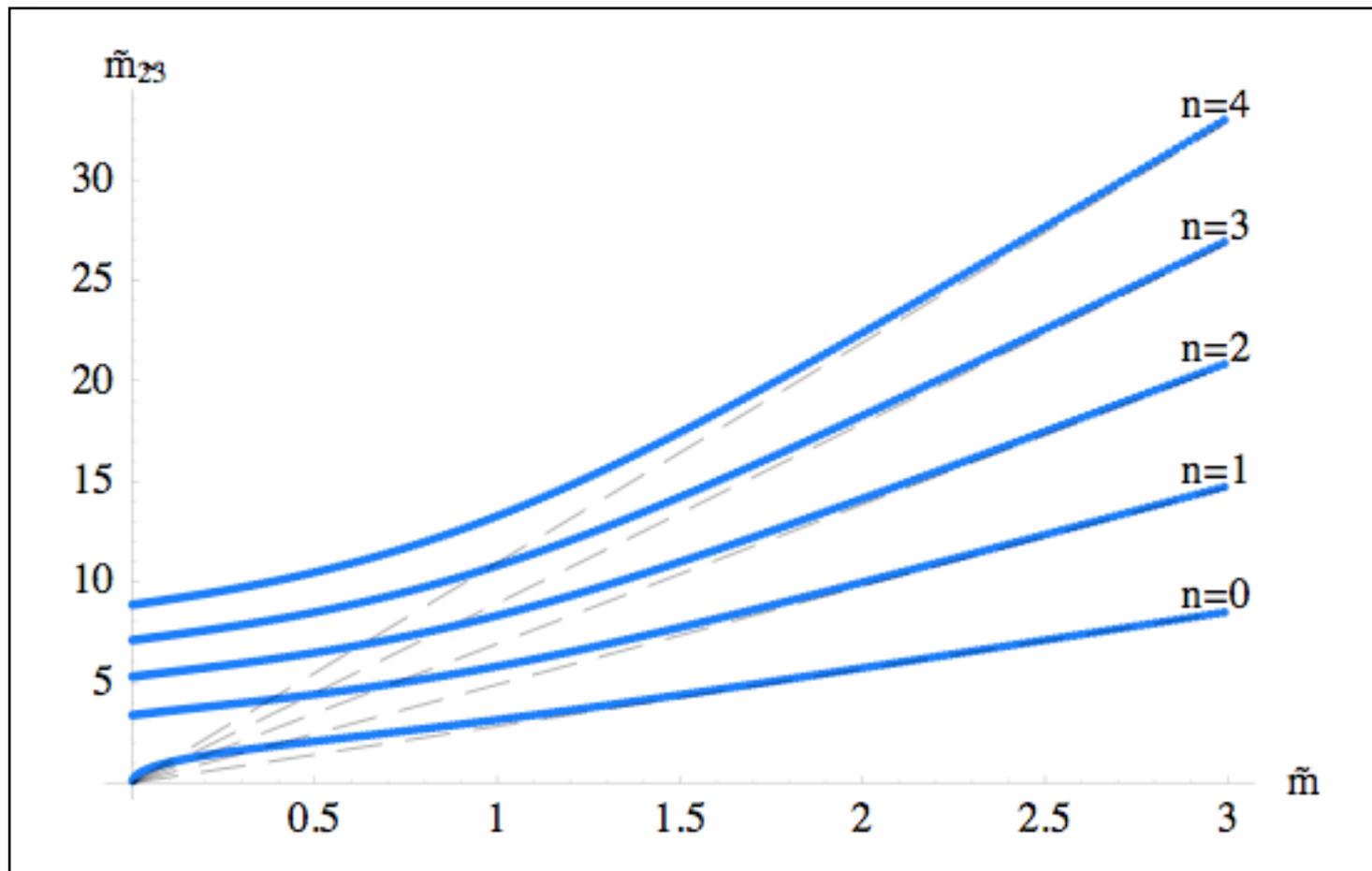


(Curve is pure AdS result.)

Closeup to see Gell-Mann, Oakes, Renner behaviour:



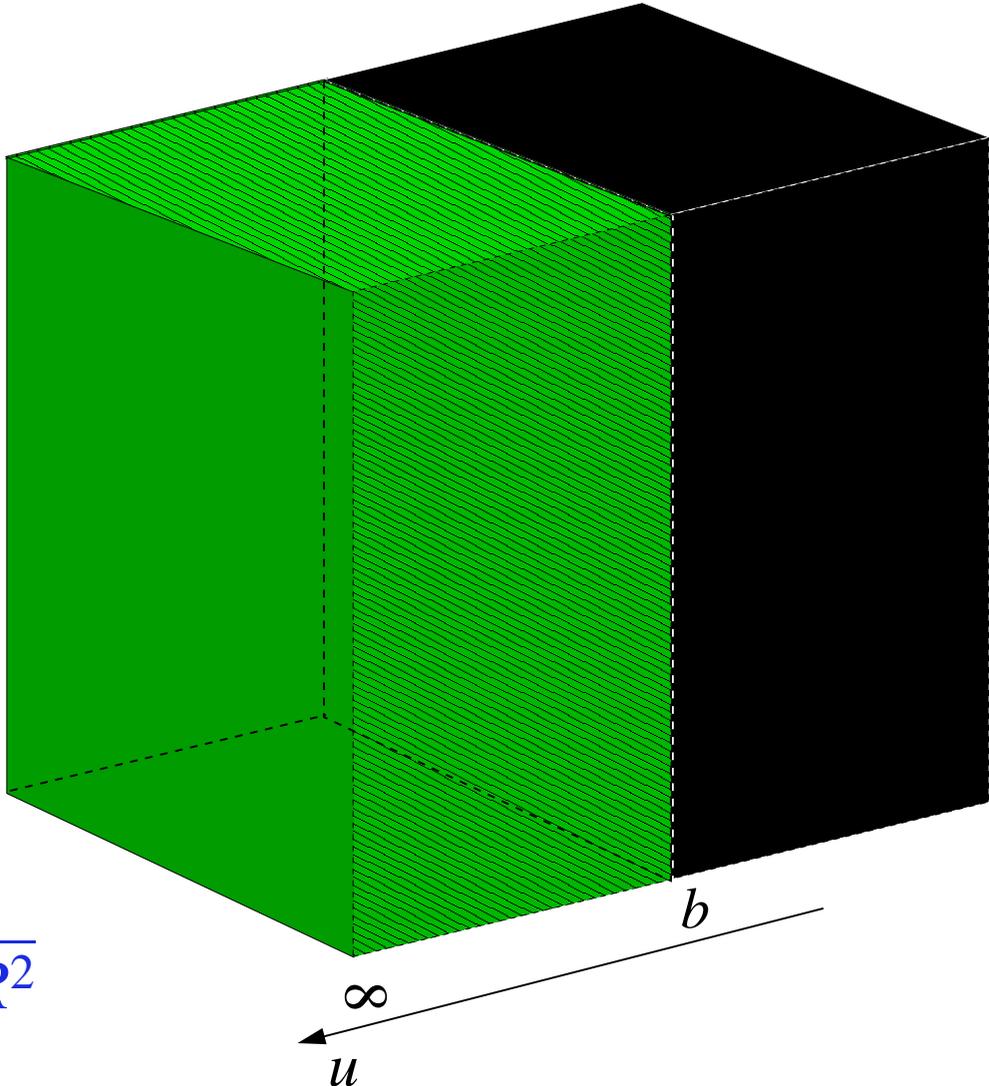
Some more of the 2-3 spectrum:



# Magnetic Field and Temperature

Do it all again, but with a black hole. The metric is

$$ds^2 = -\frac{f(u)}{R^2}dt^2 + \frac{R^2}{f(u)}du^2 + \frac{u^2}{R^2}d\vec{x} \cdot d\vec{x} + R^2(d\theta^2 + \cos^2\theta d\Omega_3^2 + \sin^2\theta d\phi^2)$$


$$f(u) = u^2 - \frac{b^4}{u^2}$$

$$b^2 = \frac{8G_5 m_{\text{b.h.}}}{3\pi}$$


$$-dt^2 \implies d\tau^2 \implies \beta^{-1} = \frac{b}{\pi R^2}$$

Useful change of variables:

$$r^2 = \frac{1}{2}(u^2 + \sqrt{u^4 - b^4}) = \rho^2 + L^2$$

$$\rho = r \cos \theta, \quad L = r \sin \theta.$$

and so we have:

$$ds^2/\alpha' =$$

$$- \left( \frac{(4r^4 - b^4)^2}{4r^2 R^2 (4r^4 + b^4)} \right) dt^2 + \frac{4r^4 + b^4}{4R^2 r^2} d\vec{x}^2 + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\phi^2)$$

Our probe embedding ansatz will be:

$$\phi \equiv \text{const}, \quad L \equiv L(\rho)$$

And our pure gauge B-field will be:

$$B = H dx^2 \wedge dx^3$$

The resulting probe Lagrangian for finite temperature and magnetic field is:

$$\mathcal{L} = -\rho^3 \left( 1 - \frac{b^8}{16 (\rho^2 + L(\rho)^2)^4} \right) \left\{ 1 + \frac{16H^2 (\rho^2 + L(\rho)^2)^2 R^4}{(b^4 + 4 (\rho^2 + L(\rho)^2)^2)^2} \right\}^{\frac{1}{2}} \sqrt{1 + L'(\rho)^2}$$

Again this asymptotes nicely:

$$\mathcal{L} \approx -\rho^3 \sqrt{1 + L'(\rho)^2} \quad \rho \gg b$$

With the same solution allowing us to read off the physics

$$L(\rho) = m + \frac{c}{\rho^2} + \dots$$

$$\langle \bar{\psi} \psi \rangle \propto -c \quad m_q = m/2\pi\alpha'$$

Again, it is straightforward to extract some interesting results by hand. For large mass or weak magnetic field:

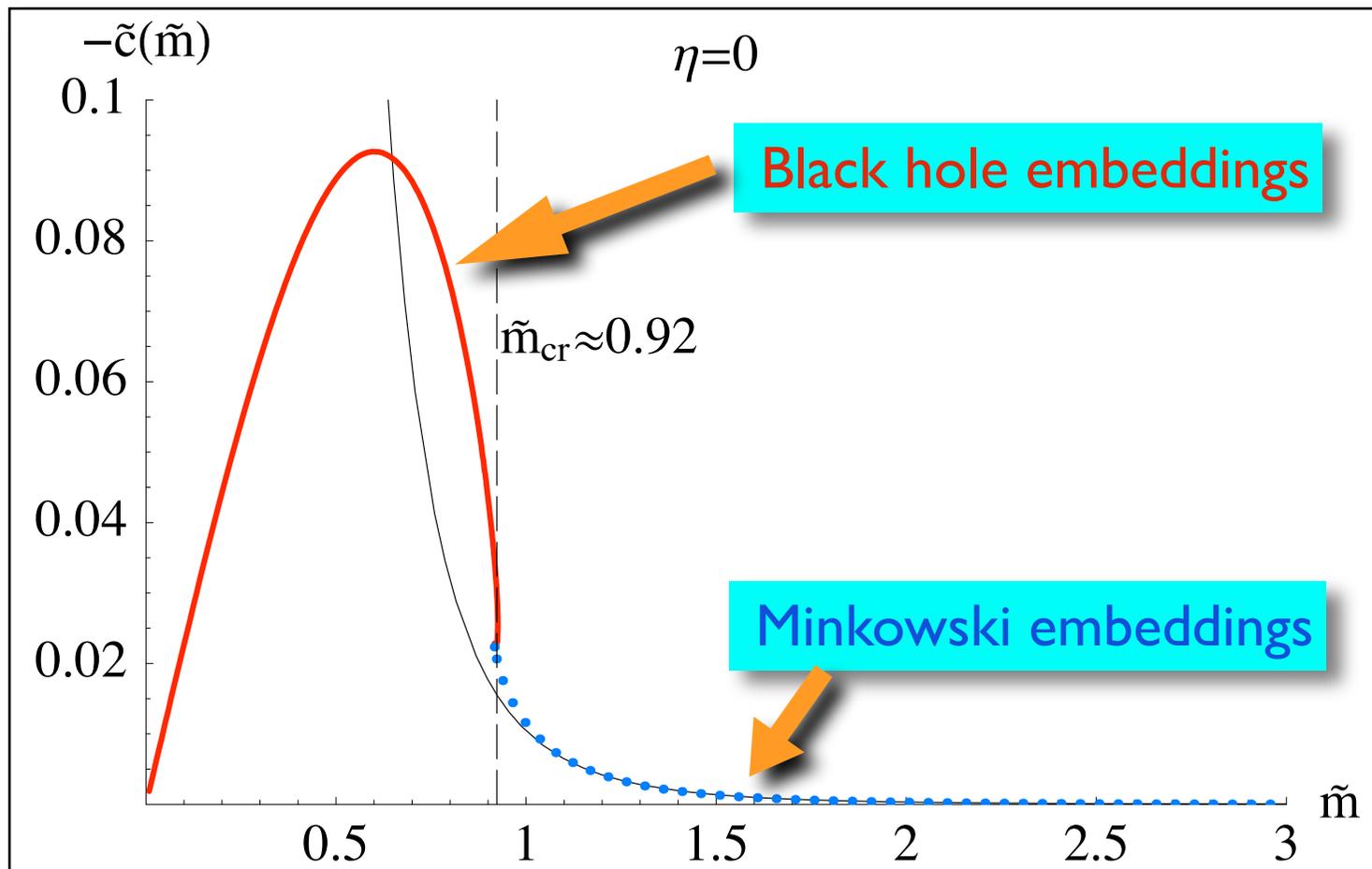
$$\langle \bar{\psi}\psi \rangle \propto -c = -\frac{R^4}{4m}H^2 + \frac{b^8 + 4b^4R^4H^2 + 8R^8H^4}{96m^5}$$

For large enough mass, the condensate vanishes from below.

We can use numerical methods to see what happens at low values, and as a function of temperature.

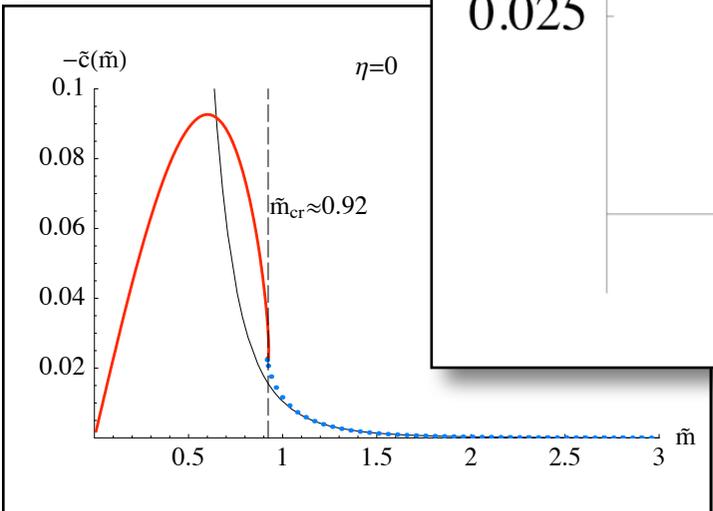
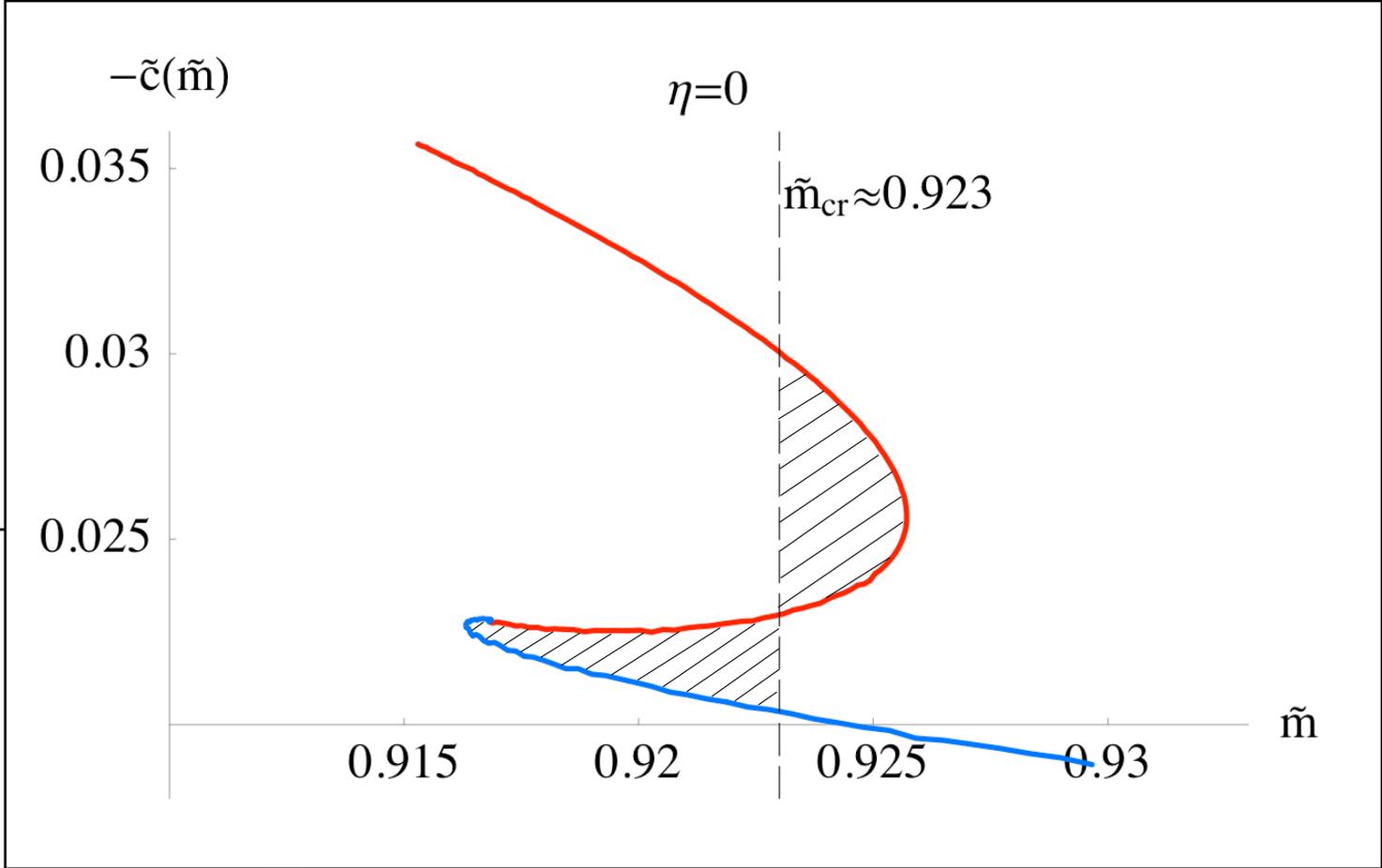
Some dimensionless parameters:

$$\tilde{\rho} = \frac{\rho}{b}, \quad \eta = \frac{R^2}{b^2} H, \quad \tilde{m} = \frac{m}{b},$$
$$\tilde{L}(\tilde{\rho}) = \frac{L(b\tilde{\rho})}{b} = \tilde{m} + \frac{\tilde{c}}{\tilde{\rho}^2} + \dots$$



The familiar finite temperature, zero field case.

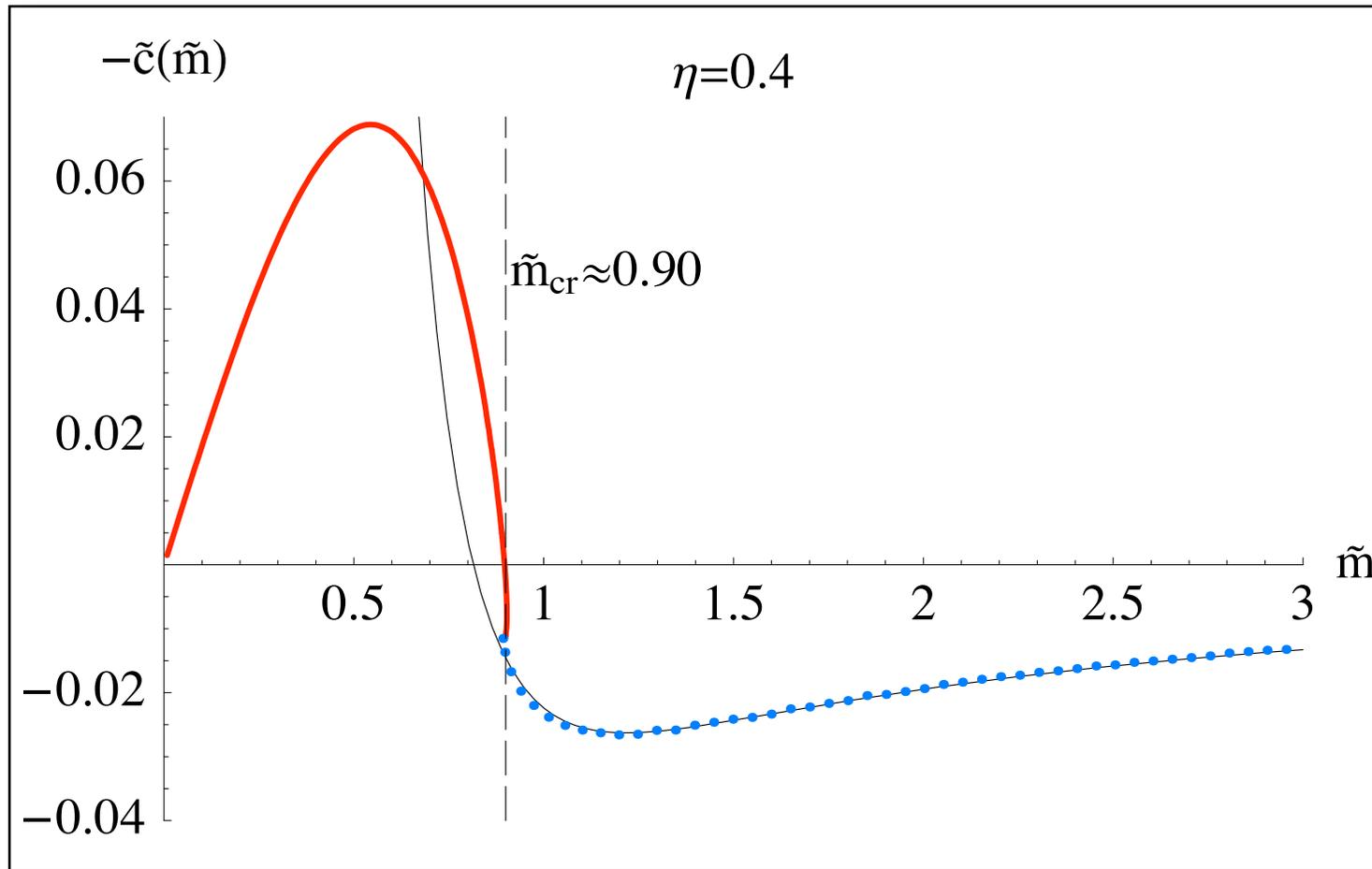
There's the melting transition region blown up:



The familiar finite temperature, zero field case.

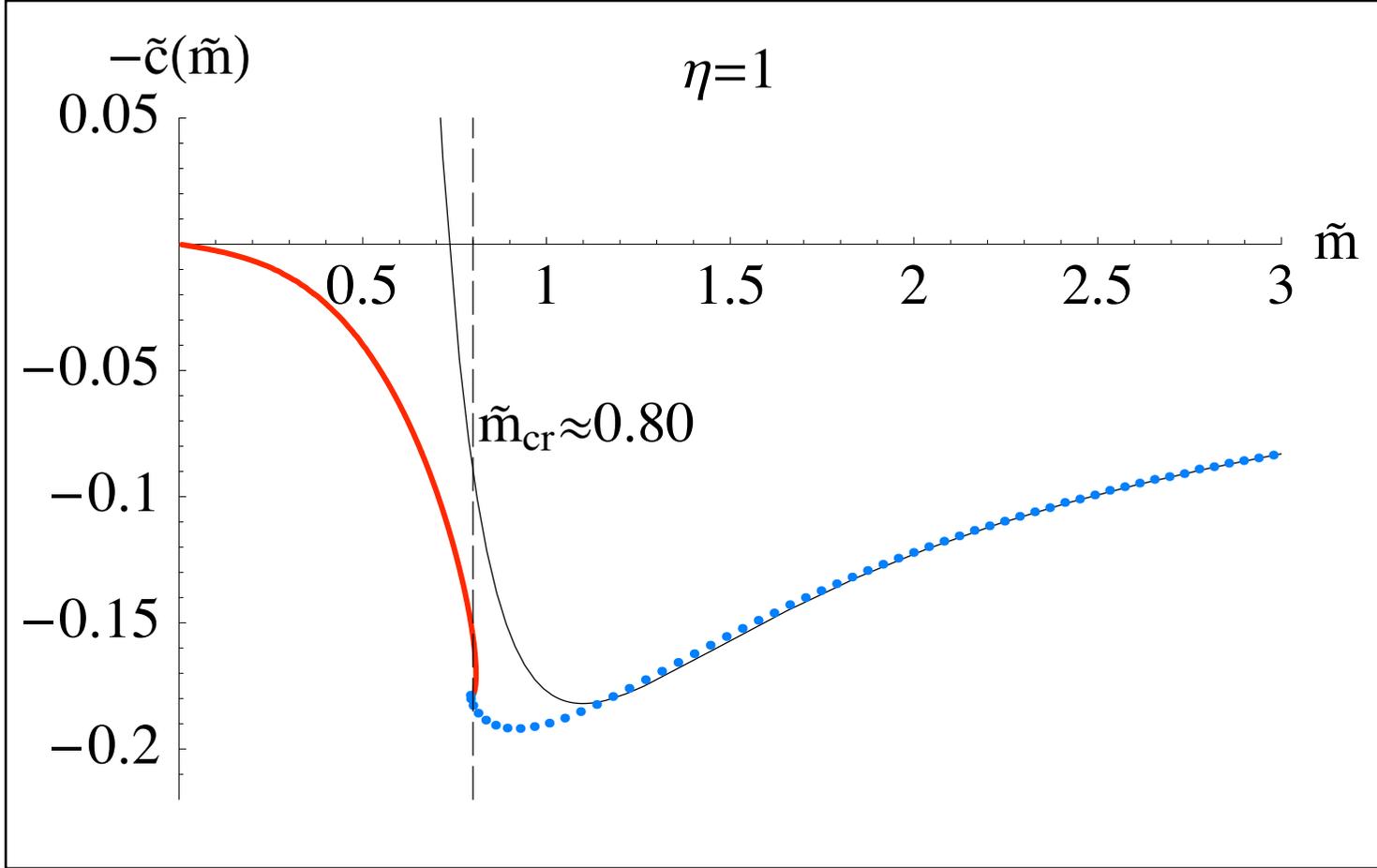
Now turn on the magnetic field:

$$\eta \equiv \frac{R^2}{b^2} H$$



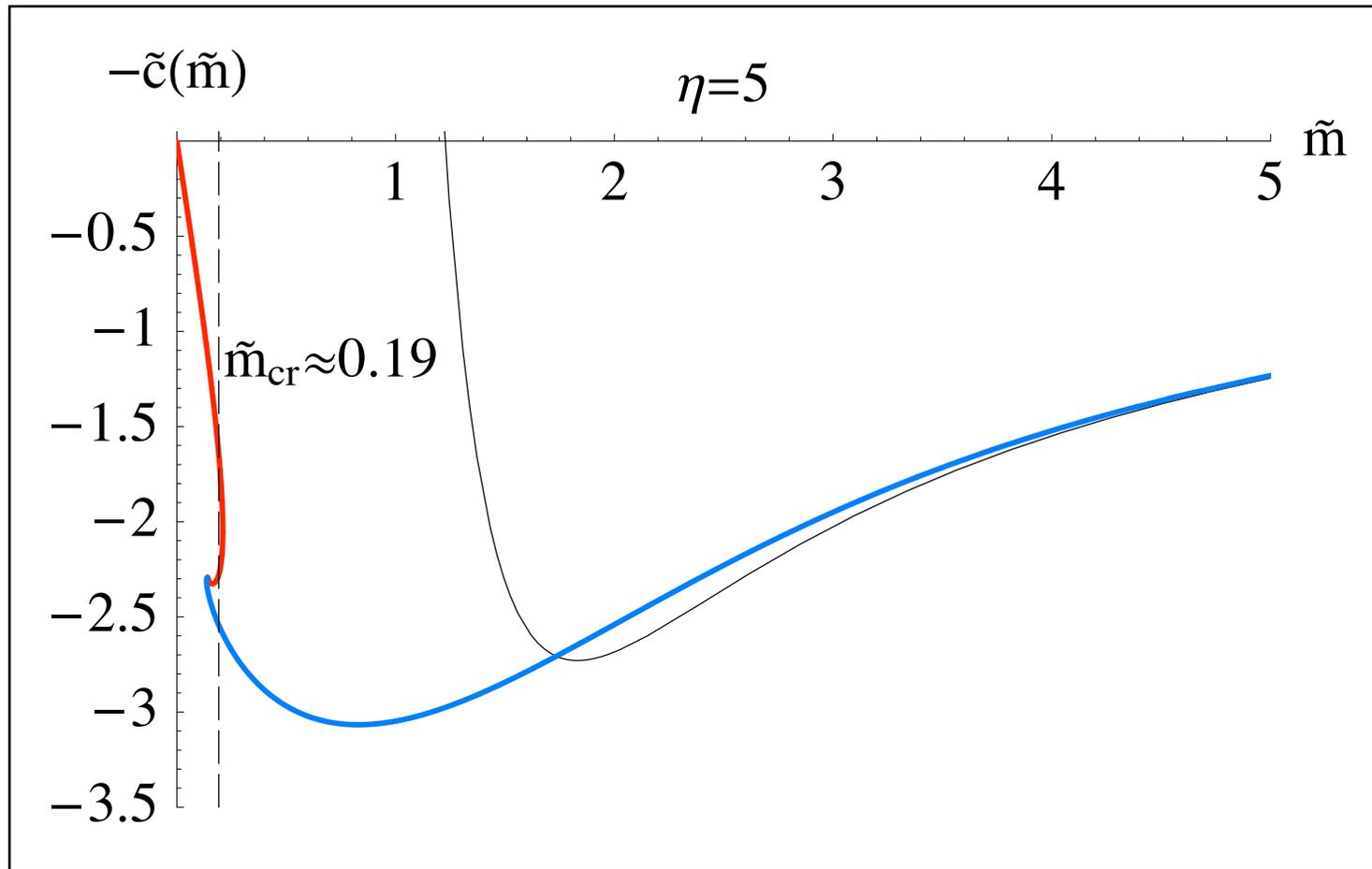
transition mass decreases (melting temperature increases)

Turn up the magnetic field:



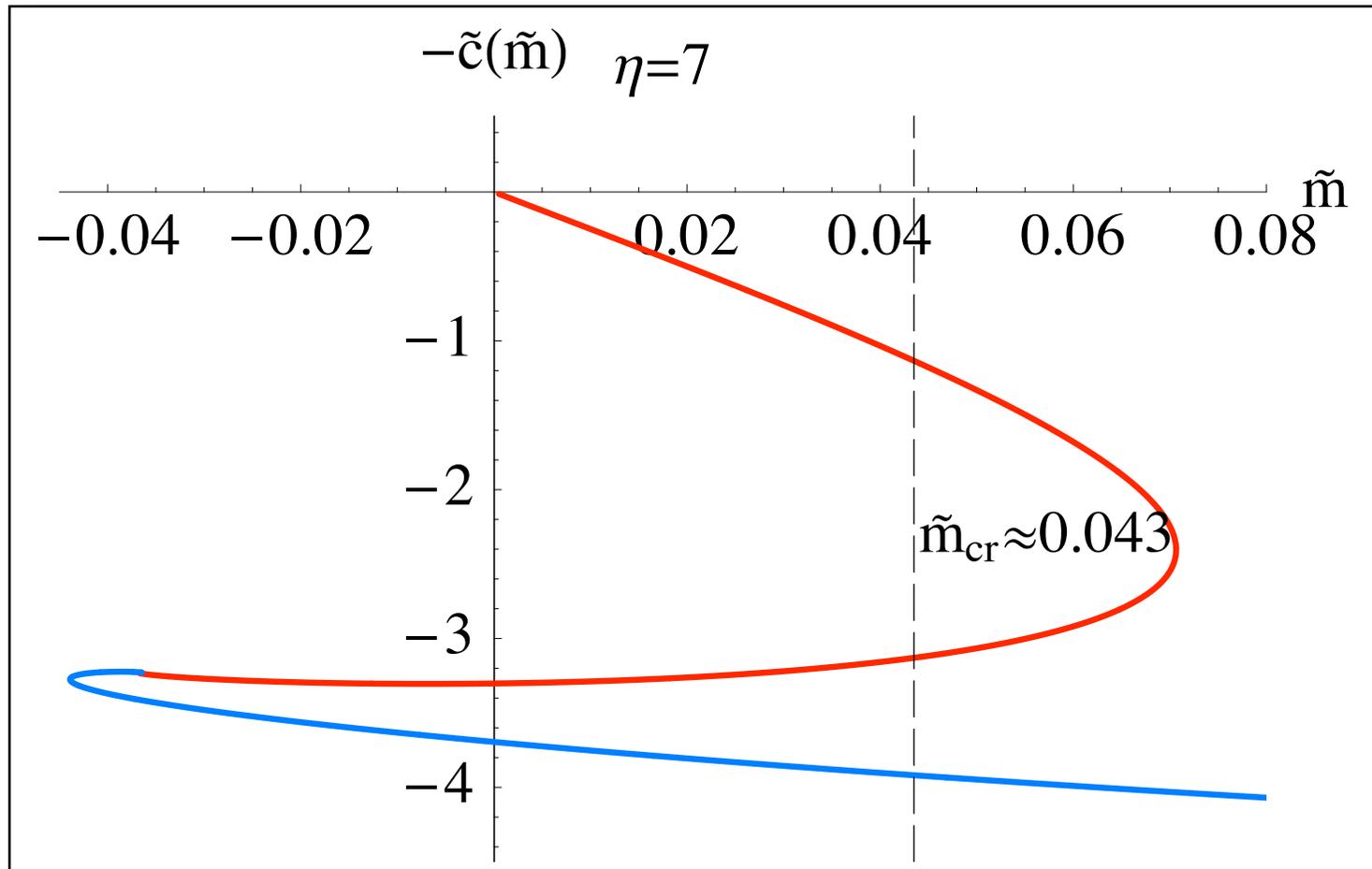
transition mass decreases further (melting temperature increases)

Keep your eye on the transition region...



transition mass decreases further (melting temperature increases)

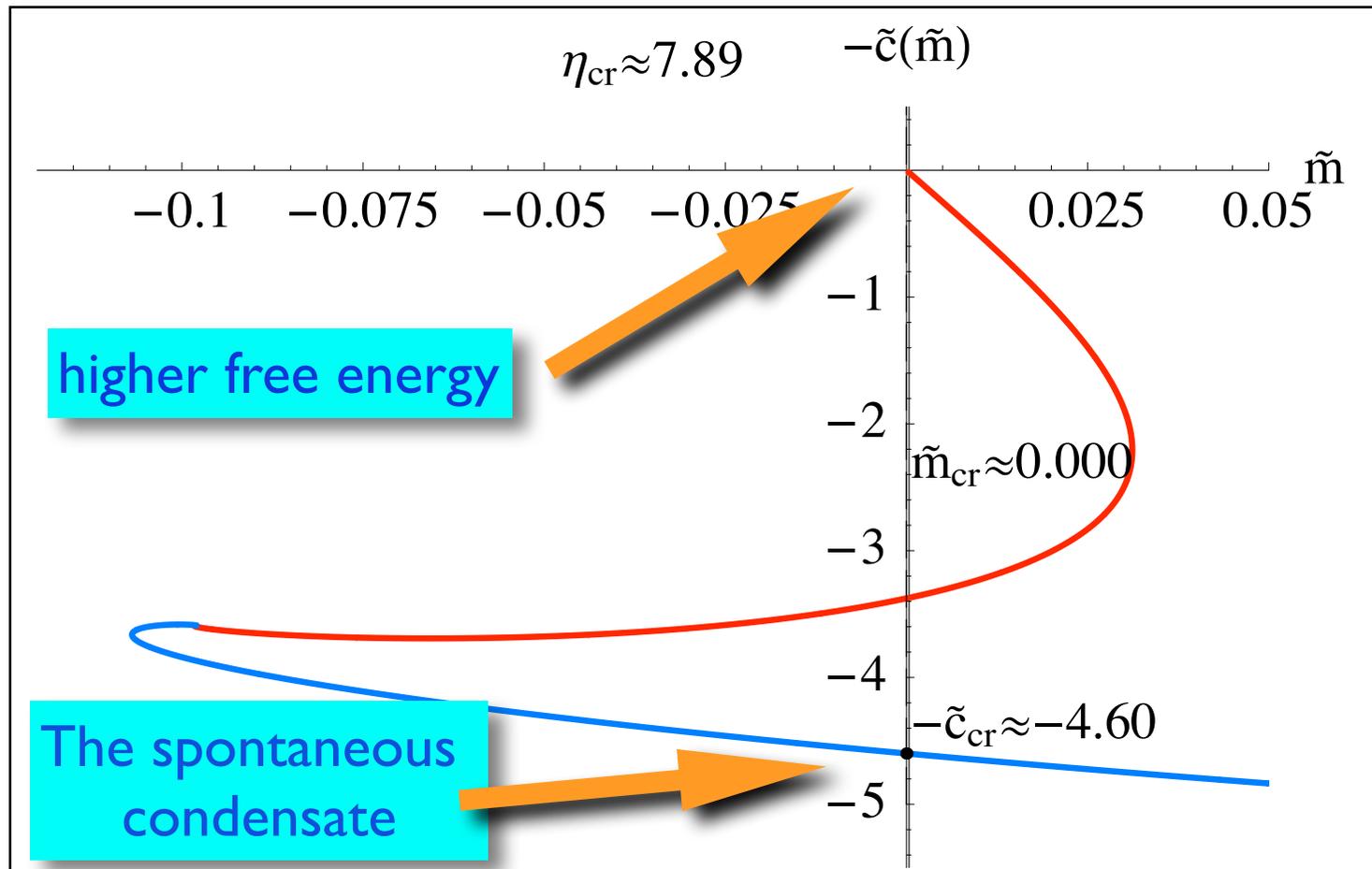
Keep your eye on the transition region...



transition mass decreases further (melting temperature increases)

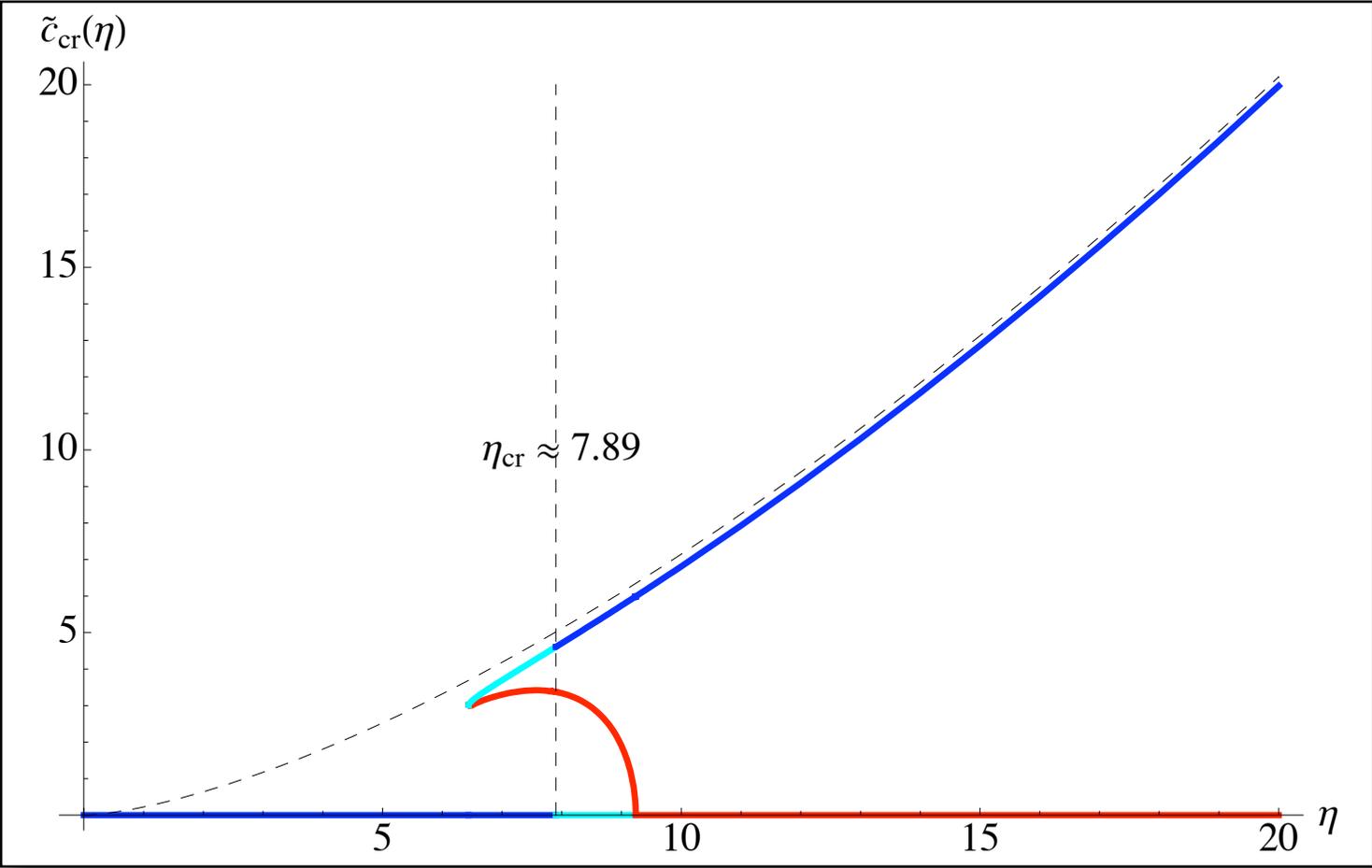
# Spontaneous Chiral Symmetry Breaking

Eventually, at some critical field, the transition mass is zero!



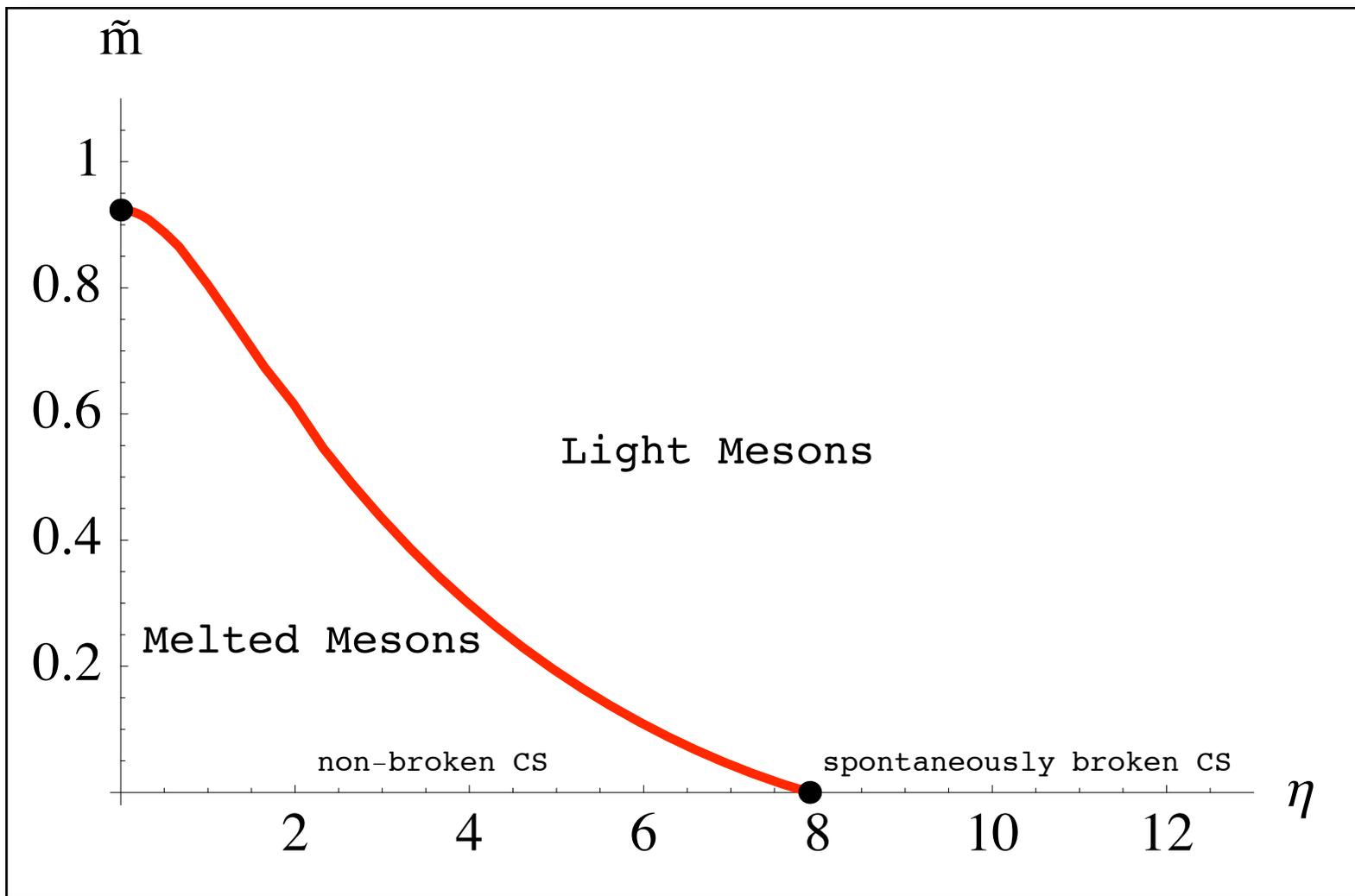
There is no longer any melting...black hole embeddings gone...  
There is chiral symmetry breaking.

The values of the condensate:



# Phase Structure

Finally, the rather lovely phase diagram of all of this:



# Electric Fields and Metal-Insulator Transitions

Filev et. al.

Let's now study an external electric field.

This is also easy to do.

Consider the following ansatz for the world-volume gauge field:

$$A_1(r) = -Et + B(r)$$

Karch & O'Bannon

This will give an electric field and a current, as we shall see:

$$F_{t1} = E \quad J^1$$

Keeping terms up to second order, we get for the DBI:

$$\begin{aligned}\tilde{\mathcal{L}} &= \frac{\cos^3 \theta(\tilde{r})}{16\tilde{r}^5} \left[ \tilde{g}^2 \left( g^2 - 16\tilde{r}^4 \tilde{E}^2 \right) \left( 1 + \tilde{r}^2 \theta'(\tilde{r})^2 \right) + 4\tilde{r}^4 g^2 \tilde{g} \tilde{B}'(\tilde{r})^2 \right]^{1/2} \\ g &= 4\tilde{r}^4 - 1, \quad \tilde{g} = 4\tilde{r}^4 + 1.\end{aligned}$$

where our embedding ansatz was:

$$\theta \equiv \theta(r)$$

and I've introduced dimensionless parameters *via*:

$$r = b\tilde{r}, \quad B(r) = \frac{b}{2\pi\alpha'} \tilde{B}(\tilde{r}), \quad E = \frac{b^2}{2\pi\alpha' R^2} \tilde{E}, \quad \theta(r) = \theta(\tilde{r})$$

Solving in the asymptotic region, we find the normalizable solution:

$$\lim_{\tilde{r} \rightarrow \infty} \tilde{B}(\tilde{r}) = \frac{\tilde{T}}{2\tilde{r}^2} + \dots \quad \text{and} \quad \langle J^1 \rangle = \langle \bar{\psi} \gamma^1 \psi \rangle = 4\pi^3 \alpha' b^3 V N_f T_{D7} \tilde{T}$$

We can convert (via Legendre transform) the full solution for  $B(r)$  into a solution for the current:

$$\tilde{T}^2 = \frac{(4\tilde{r}_*^4 - 1)^2 (4\tilde{r}_*^4 + 1)^3 \cos^6 \theta(\tilde{r}_*)}{64\tilde{r}_*^6 (4\tilde{r}_*^4 + 1)^2}$$

where

$$\tilde{r}_*^2 = \frac{\tilde{E} + \sqrt{\tilde{E}^2 + 1}}{2}$$

defines a “vanishing locus” or “pseudo-horizon”.

Notice that the current vanishes at vanishing electric field, and also that it vanishes for the Minkowski embeddings, for which  $\theta = \pi/2$

We can solve for the embedding. The equation is a mess, so I'll spare you.

Asymptotically

$$\frac{d}{d\tilde{r}} (\tilde{r}^5 \theta'(\tilde{r})) + 3\tilde{r}^3 \theta(\tilde{r}) = 0$$

giving:

$$\theta(\tilde{r}) = \frac{\tilde{m}}{\tilde{r}} + \frac{\tilde{c}}{\tilde{r}^3}$$

From which we can again read off the mass and condensate:

$$m_q = \frac{b \tilde{m}}{2\pi\alpha'}, \quad \langle \bar{\psi}\psi \rangle = -8\pi^3 \alpha' V N_f T_{D7} b^3 \tilde{c}.$$

Notice again that for large mass we can derive:

$$\langle \bar{\psi}\psi \rangle \propto -c = \frac{R^4 E^2}{4m} + \frac{b^8 + 4b^4 R^4 E^2 + 8R^8 E^4}{96m^5} + O\left(\frac{1}{m^7}\right).$$

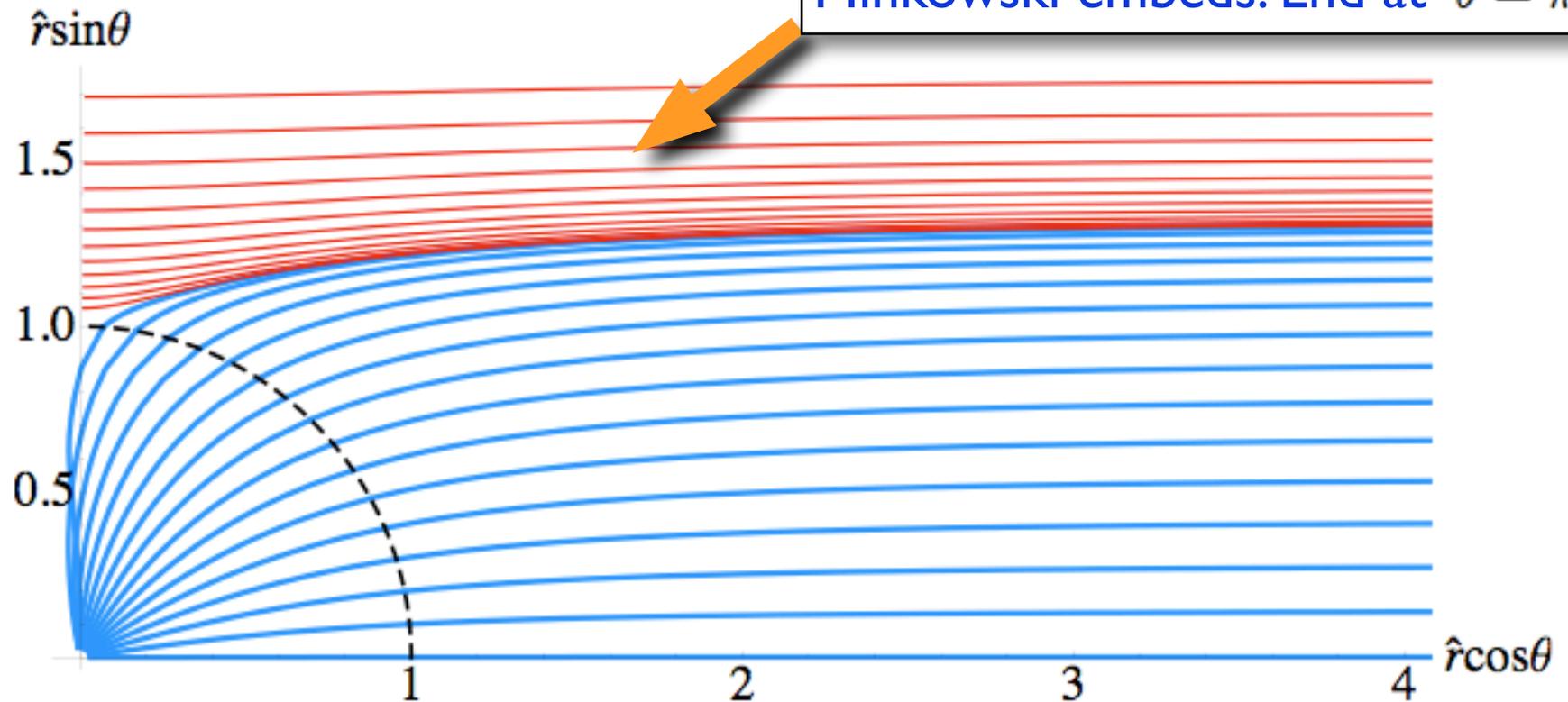
...condensate  
vanishes, but  
from above.

# Zero Temperature

Useful to use a new set of dimensionless coordinates  $\hat{r}$  and  $\hat{m}$

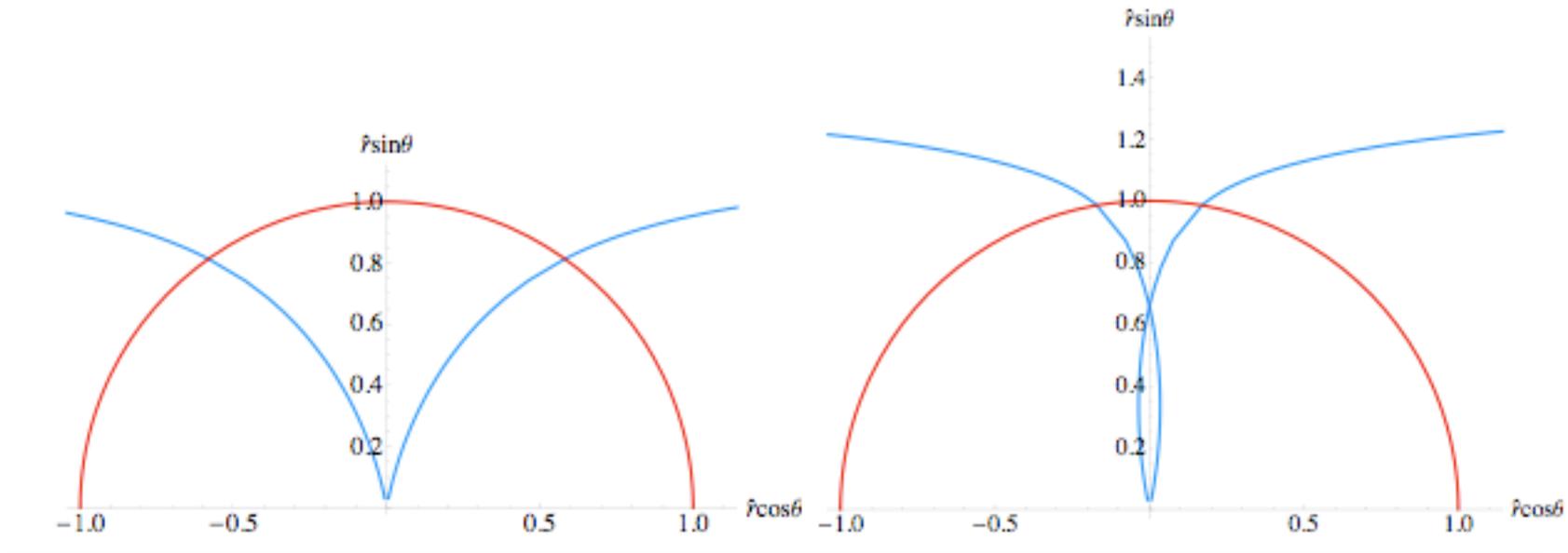
$$r = R\sqrt{E}\hat{r} ; \quad m = R\sqrt{E}\hat{m}$$

Resort to pictures, in view of time:



Three types of embedding to consider, even at zero temperature...

In addition to the Minkowski embeddings, there are two types that pass the pseudo-horizon:



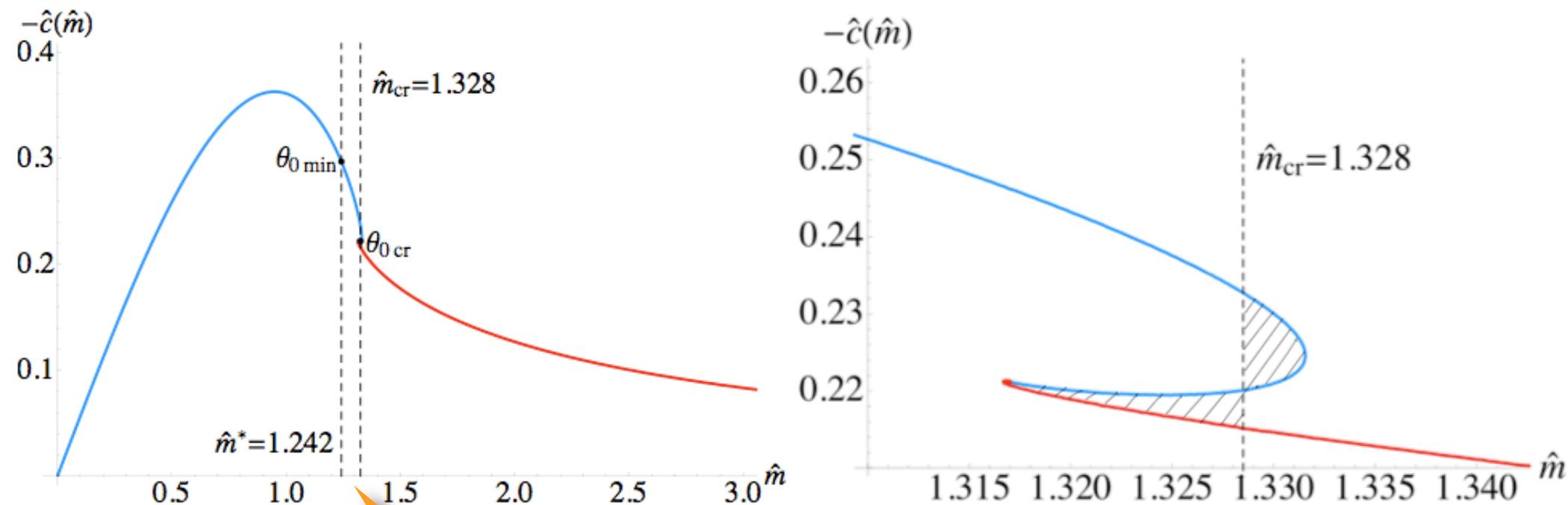
smooth

Conical singularities present

We must keep an eye on these conical solutions...

# A Metal-Insulator Transition

Now look at the mass vs condensate, and see that there's a new transition:

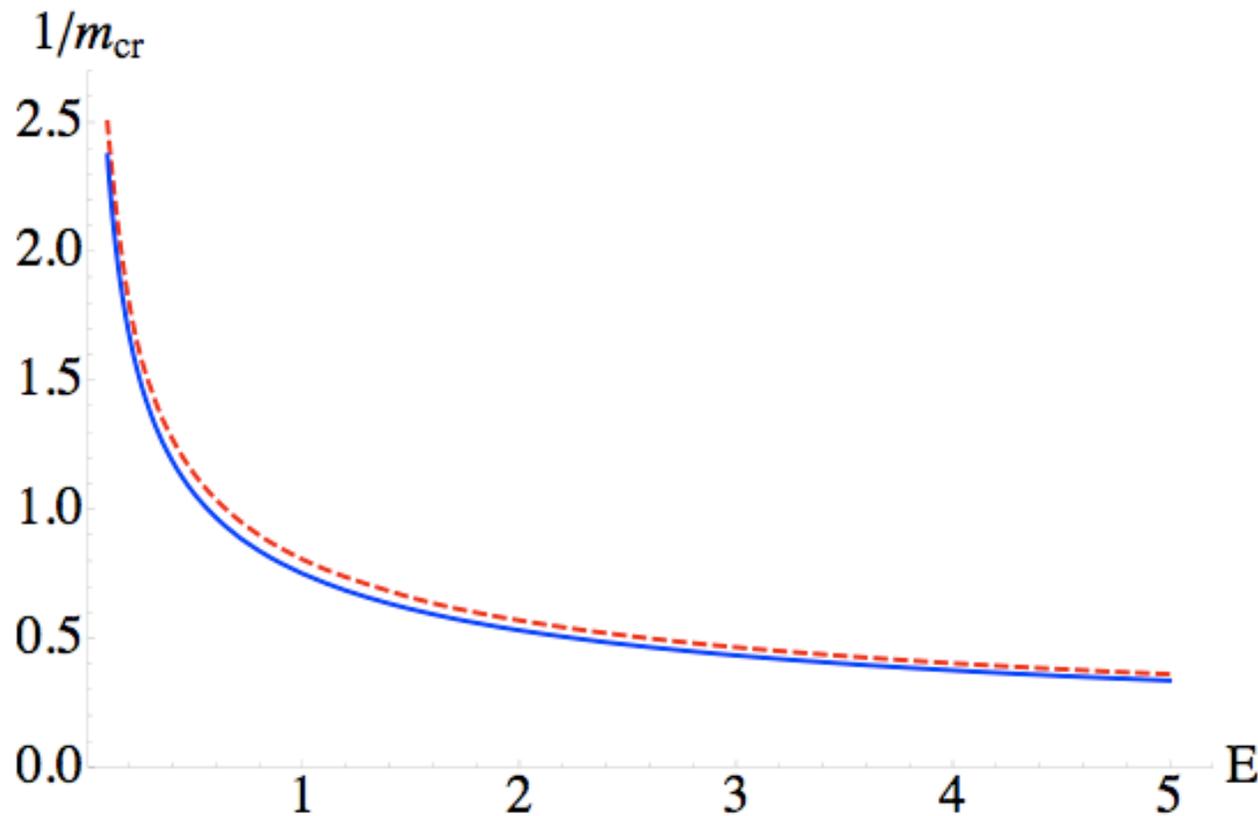


Transition between Minkowski (mesons, vanishing current) and non-Minkowski (quarks, non-vanishing current)

Window where conical singularities present

# A Metal-Insulator Transition

Inverse critical mass vs field:

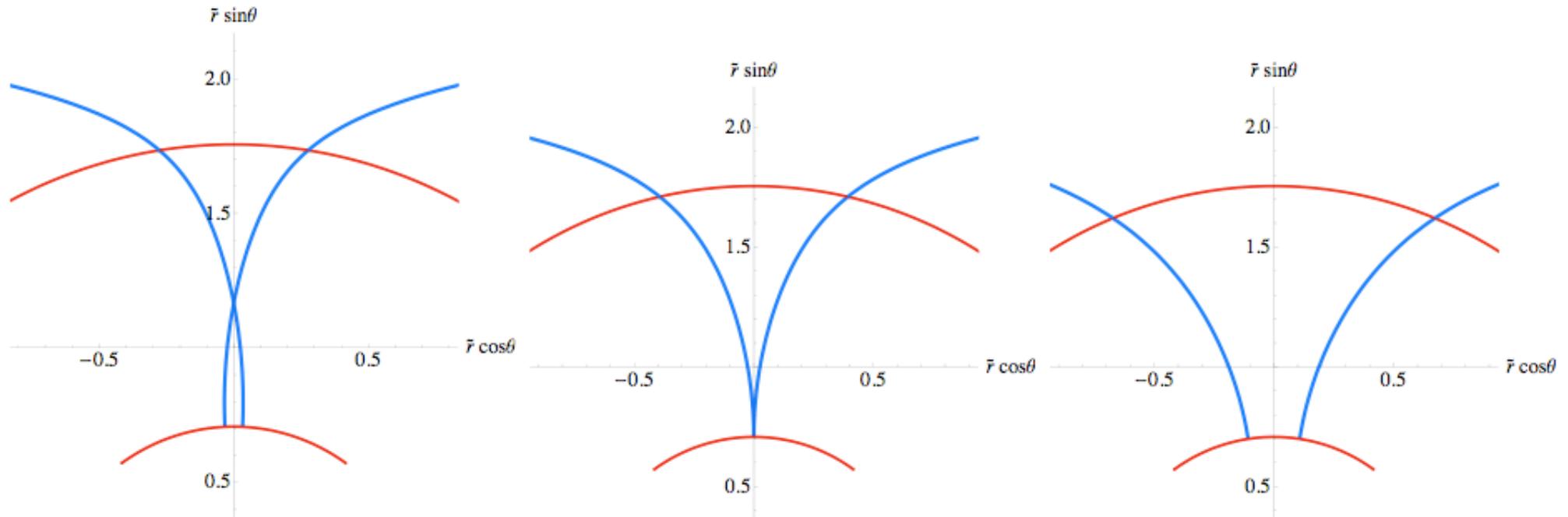


Binding energy of mesons proportional to quark mass - So a strong enough  $E$  will unbind (dissociate) the mesons into constituent quarks. (red line)

Note: Region where conical singularities present between red and blue curves

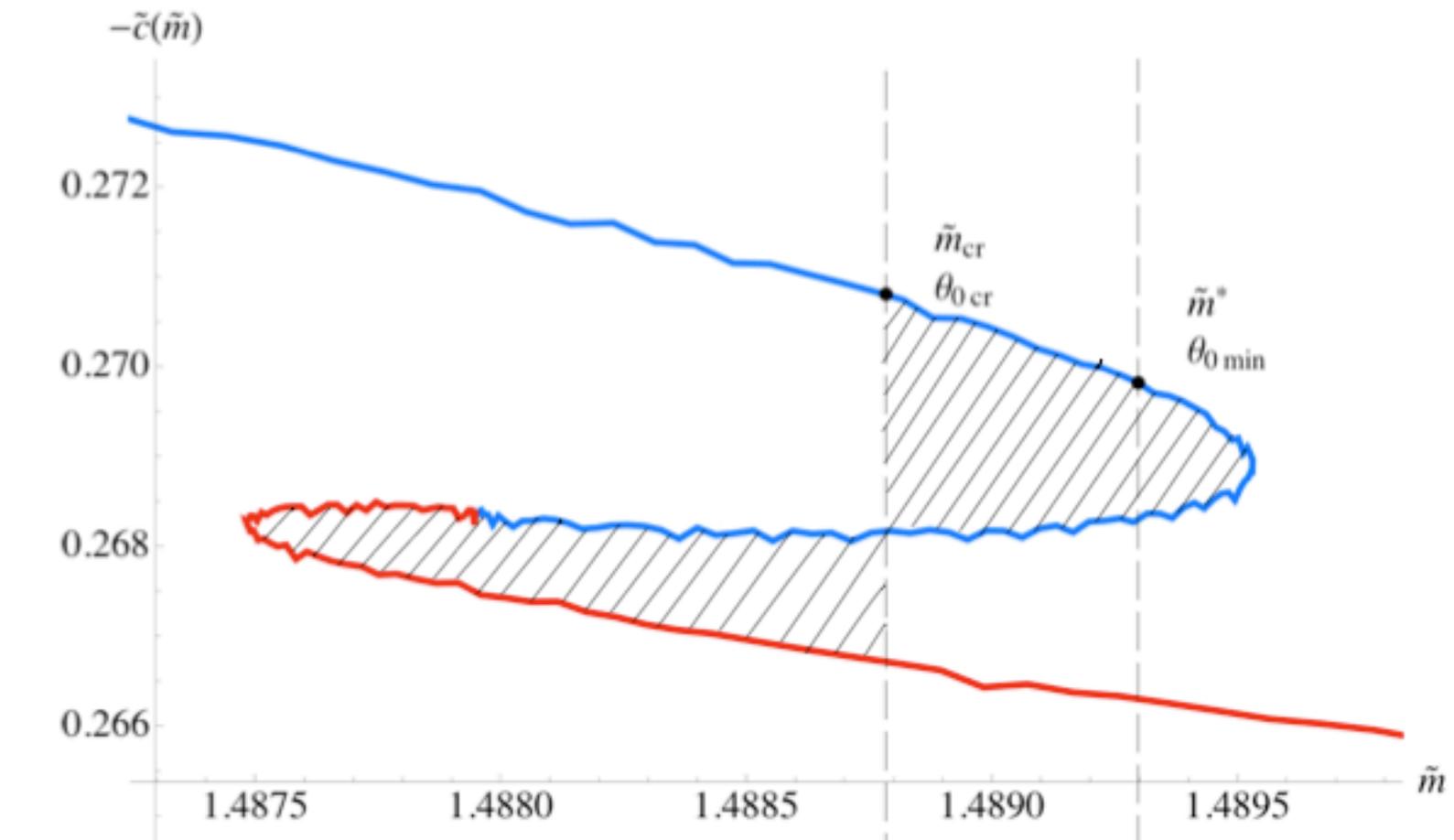
# Finite Temperature

The different types of non-Minkowski embedding when there's an horizon:



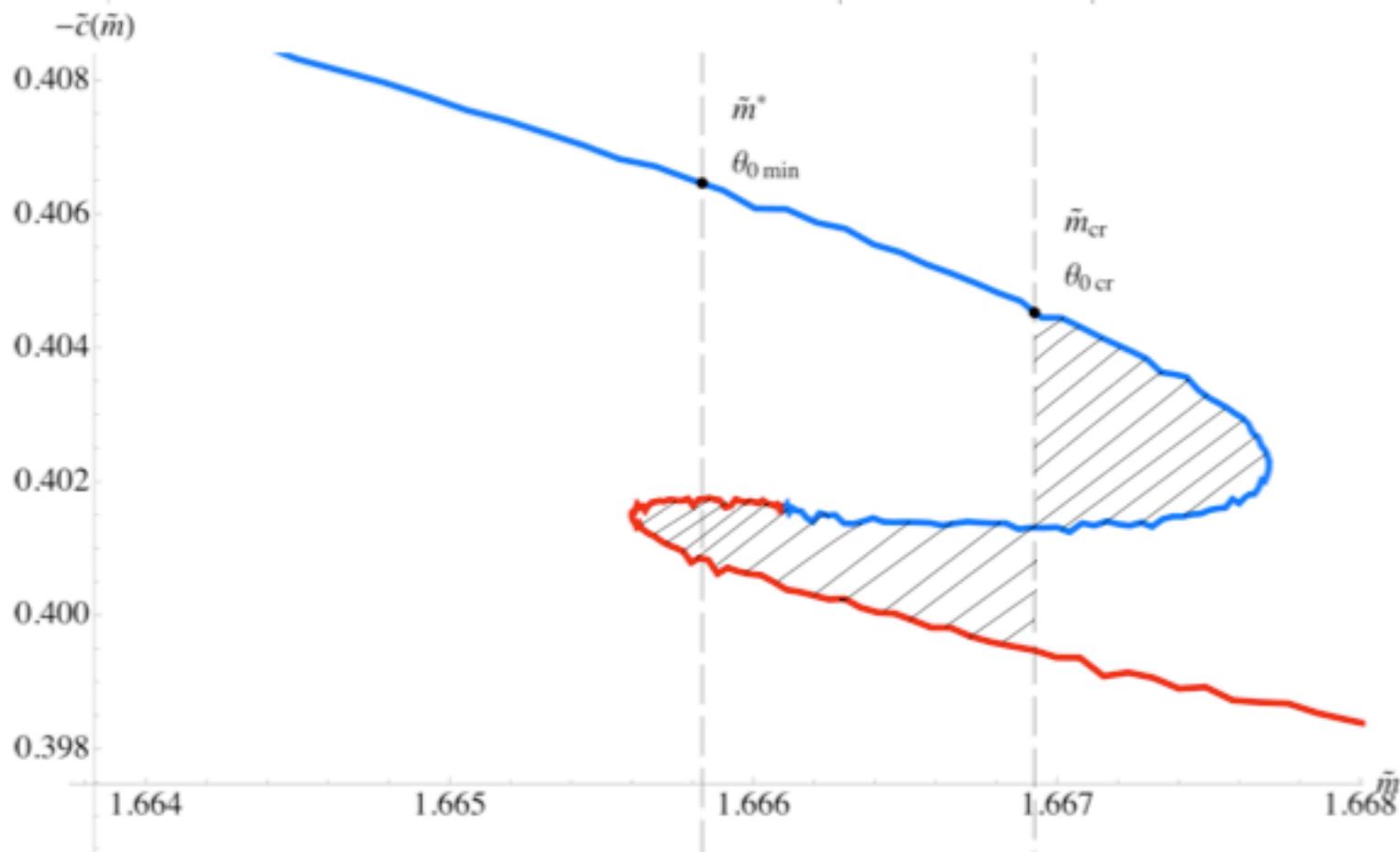
# Finite Temperature

Key difference: For small enough electric field, the conical singular solutions are skipped by the transition:



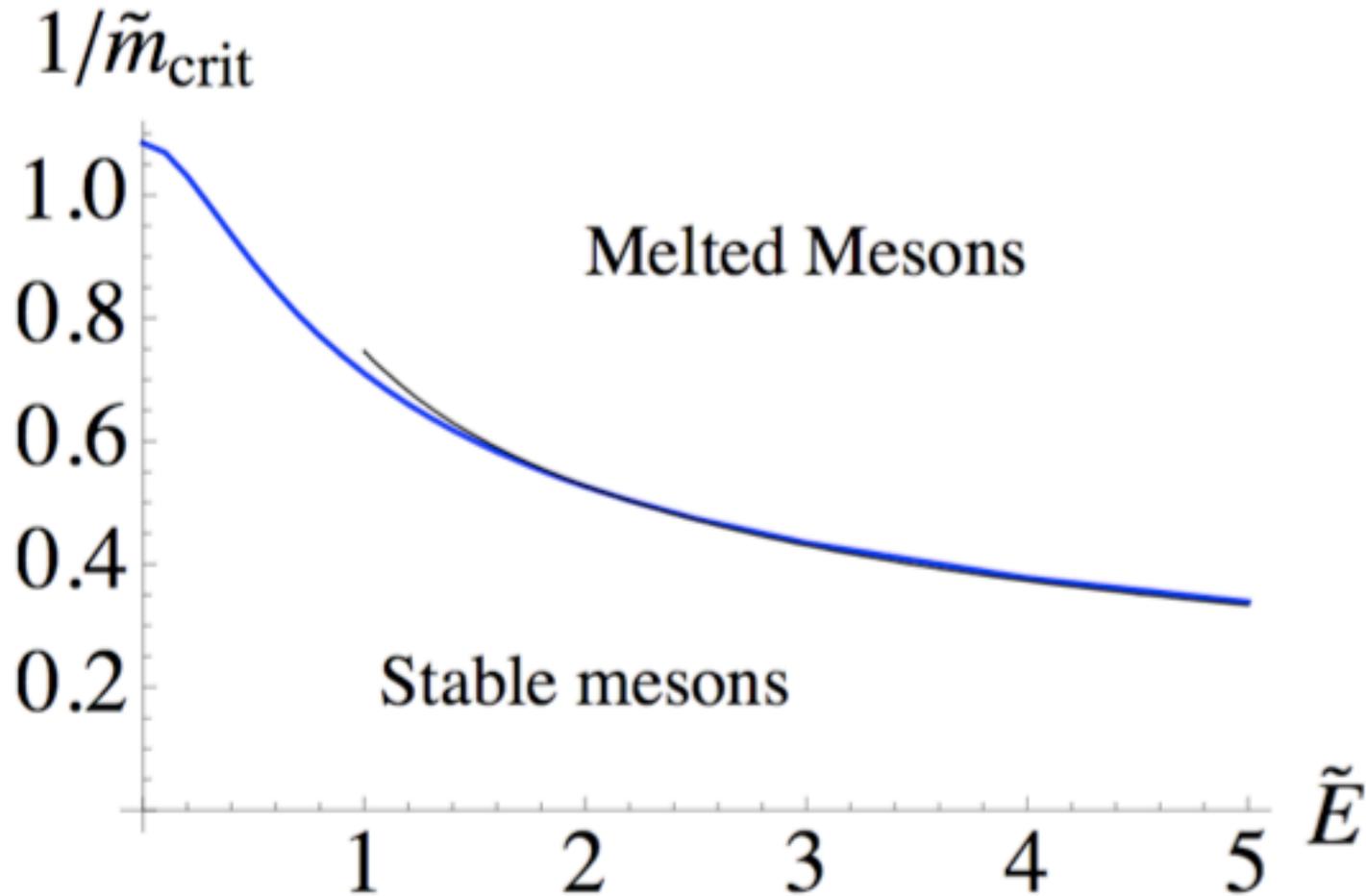
# Finite Temperature

Beyond a critical value of the electric field, the conical singular solutions are not skipped by the transition:



# Finite Temperature

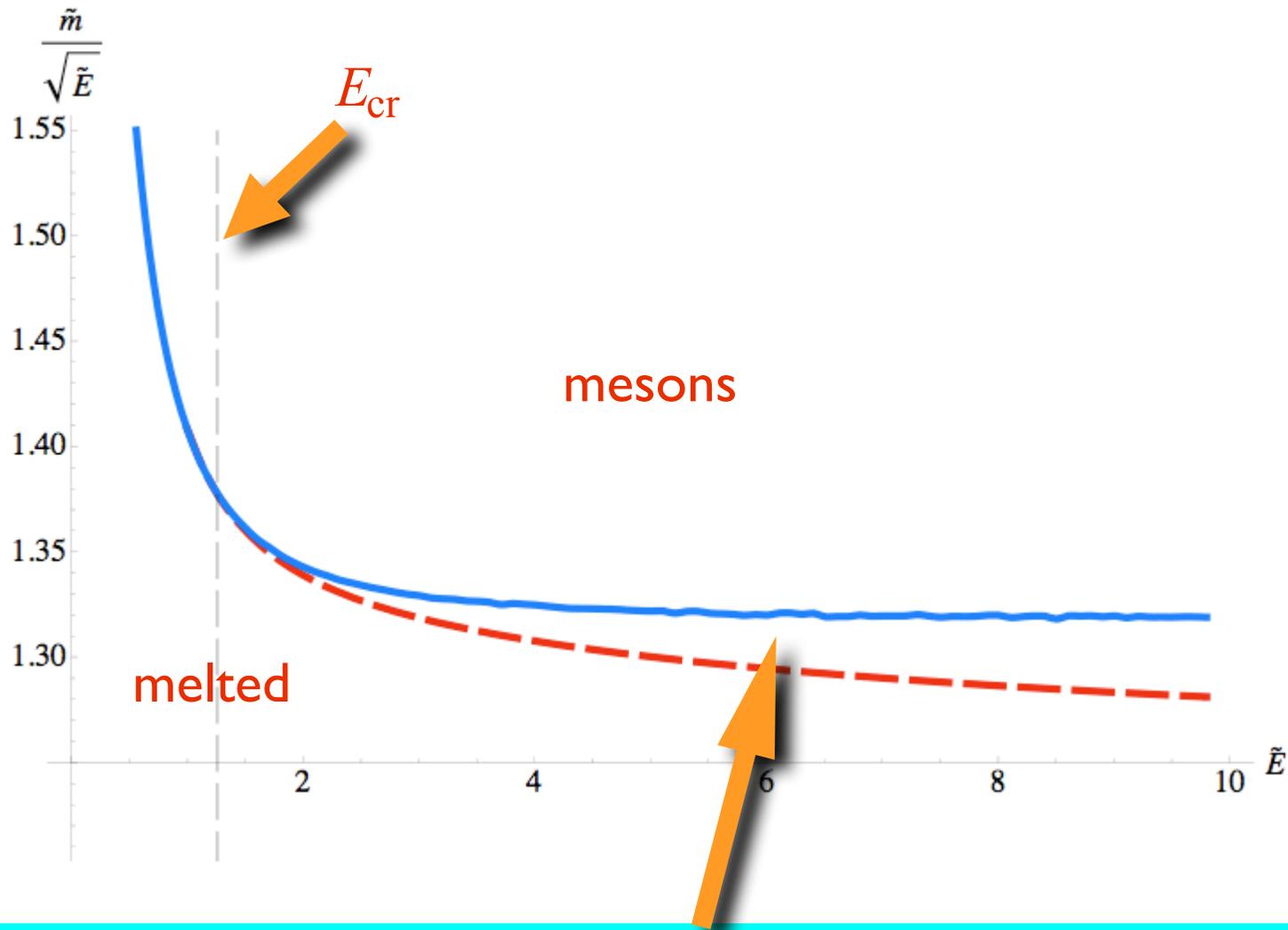
Phase diagram (refined overleaf):



Need to keep eye on the conical solutions however....

# Finite Temperature

Phase diagram:



Note: Region where conical singularities present between red and blue curves

# Conclusions

These studies provide very clean probes of important phenomena.

Related results in:  
Erdmenger et.al., arXiv:0709.1551

Nice complement and extension (in magnetic case), of field theory literature on “magnetically catalyzed SSB”.

Review:  
Miransky, hep-th/0208180

Much more can be done to construct other instructive examples.

Going beyond the probe limit is a major challenge.

Important to complete the story of the conical singular solutions. Are they repaired by stringy physics in the interior? Are there completely new solutions to study, modifying the phase diagram?



...Just because it's Santa Barbara!