New Results in planar QCD

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Rutgers

References:

- J. Kiskis, R. Narayanan, H. N., Phys. Rev. D66 (2002) 025019.
- R. Narayanan, H. N., Phys. Rev. Lett. 91 (2003) 081601.
- J. Kiskis, R.Narayanan, H. N., Phys. Lett. B574 (2003) 65.
- R. Narayanan, H. N., Nucl. Phys. B696 (2004) 107.
- R. Narayanan, H. N., hep-lat/0503033.

Plan

There are too many papers, and a review of all of them would degenerate into a sequence of expanded abstracts.

Therefore, I shall concentrate on the first part of this project, as it provides the basis for the rest.

Why work on large N ?

- Contribute to the search for a string representation.
- There is a shortcut to N=1 : **reduction.**
- Even for **massless quarks** quenching OK

(At finite N the quenched massless theory is divergent – despite several massless extrapolation that were "successfully carried out'' !
At infinite N the order of limits has to be: $\lim_{m_q \to 0} \lim_{N \to \infty}$)

- This is a feasible problem for PC clusters of today; full QCD with correct quarks is not yet feasible.

Main basis of new developments: A new form of large N reduction.

RESULTS

- ❑ Pure YM at N=1 on a **finite** l^4 Euclidean torus has a rich phase structure, including a finite temperature transition . Reduction: for l large enough there is no dependence on l at all.
- ❑ SχSB and RMT; $\psi^*\psi/N$ and F_π^2/N at N=1.
- ❑ m_π^2 (m_q) at N=1.

I'll focus on reduction because the term is often used too loosely.

Large N lattice loop Equations

- Lattice loop equations at infinite N (schematic):

$$\frac{\delta W}{\delta C} = \lambda_{lattice}^{'t\ Hooft} \sum_{C=C'+C''} W(C')W(C'')$$
$$W(C) = Tr < \mathcal{P}exp(i \oint_C A \cdot dx) > /N$$

- Precise, unambiguous on lattice, but not in continuum.
- At λ=1, W(C)=0, and this solution is unique.
- Circle around λ=1 is walled off from λ=0 – continuum.

1982: Eguchi-Kawai Reduction

- Loop equations on L^4 **lattice** do not depend on L!
- L=1. **EK reduced model**: a single site is equivalent to an infinite lattice.
- Correct in some small circle around λ=1 .

1982 Bhanot-Heller-N: I

- There is an integral rep. with λ dependence; so can expand formally around $\lambda=0$.
- Q: Where does 4D space-time in Feynman diagrams come from ?
- A: Eigenvalues of U_μ populate a moduli space of vacua that needs to be averaged over and then reproduce flat $d^4 p_1 .. d^4 p_l$

1982 Bhanot-Heller-N: II

- ⇨ Explicit identification of mechanism by which large N reduction can work also outside the strong coupling expansion.
- EK works by the basic object transported round the Wilson loop carrying Z(N) central charge which can be used to keep track of location along the loop.

1982 Bhanot-Heller-N: III

- Q: Does the mechanism of momentum space generation out of group space work when λ! 0 ?
- A: No: $U_\mu \rightarrow exp(\imath \frac{2\pi}{N} k_\mu) U_\mu$ breaks spontaneously when λ is decreased and flatness of d^4 p is lost.
- Q: Can one fix the 1^4 model to keep reduction ?
- A: Yes. Use quenching to preserve $Z(N)^4$.

Gonzalez-Arroyo – Okawa: twisting

- Avoid quenching and still maintain reduction by “twisting”.
- Lead **Gonzalez-Arroyo and Korthals-Altes** to **non-commutative field theory.**

Summary of old Reduction, both quenching and twisting

- Keep L^4 **lattice**, hence deal with a matrix model (consists of a finite number of infinite matrices)
- Reduction happens in the lattice theory, where momentum space stays compact - the continuum limit is to be taken in the usual way.

Gross-Kitazawa: Continuum Reduction

- Q: Is there a quenched four matrix model that fully reproduces all continuum Feynman diagrams ?
- A: Yes. It is the **naïve** continuum limit of the quenched Eguchi Kawai model of BHN.
- ⇒ Cannot reproduce instanton effects
- ⇒ When probed with fermions, cannot reproduce anomalies.

New continuum reduction, I: 2003: Kiskis, Narayanan, N

- Naïve attempts to decompactify momentum space in reduced models have failed in a subtle way, as we have seen.
- Q: Does there exist a **continuum remnant** of lattice large N reduction that has **no** known problems ?
- A: Yes. Reduction holds on a **continuum** torus of size l^4, so long as $l>l_c$.

New continuum reduction, II

- Consider YM at N=1 on a Euclidean torus of size l^4 (hypercubic symmetry group – just as a lattice).
- There exists a physical length, l_c, such that, so long as $l>l_c$, all W(C) are exactly l independent at N=1. Hence, the finite l looks infinite.
- For $l>l_c$, the entire $Z(N)^4$ is unbroken and so is the toric hypercubic symmetry group.

New continuum reduction, III

- On the continuum torus, momentum space is **noncompact**, but discrete, with gaps $2\pi/l$.
- The equally spaced ($Z^4(N)$ OK) eigen-angles of Polyakov loops, θ_μ^i , contribute distinguishable momenta $\Delta p_\mu^i = 2\pi k_\mu / (N*l)$, where $k_\mu = 0,..,N-1$.
- So long as the θ_μ^i are uniformly distributed, the Δp_μ^i exactly fill the $2\pi/l$ gaps.
- The θ_μ^i will be uniformly distributed if the $Z^4(N)$ remains unbroken.

The first transition, at $l=l_c$

- As l is lowered to l_c, one of the four directions is picked randomly and the associated Z(N) gets spontaneously broken. The transition is 1st order.
- The eigenvalue spectrum of the Polyakov loop opens a gap at a random location.
- The infinite N system suddenly "discovers" that one of the four directions is finite, but it still "believes" that the others are infinite.
- This has a precise meaning: the N=1 system represents finite temperature planar QCD, $T>l_c$.

String features of first transition, Atick-Witten

Independence of l above the transition and order of transition are compatible with a dual of free strings:

- Degrees of freedom given by maps from a 2d sphere to a T^4 can't depend on l as they can't detect the topology of T^4.
- The string coupling g_s goes to zero as N!1, so higher genus worldsheets are subleading (expectation values of Wilson loop probes – vertex insersions in wordsheet).
- Free energy viewpoint: transition caused by winding mode going tachyonic, but becomes first order due to a 3-point coupling which (?) also breaks hypercubic symmetry.

Phase Structure on a **finite** Torus: Lattice and Continuum at N=1

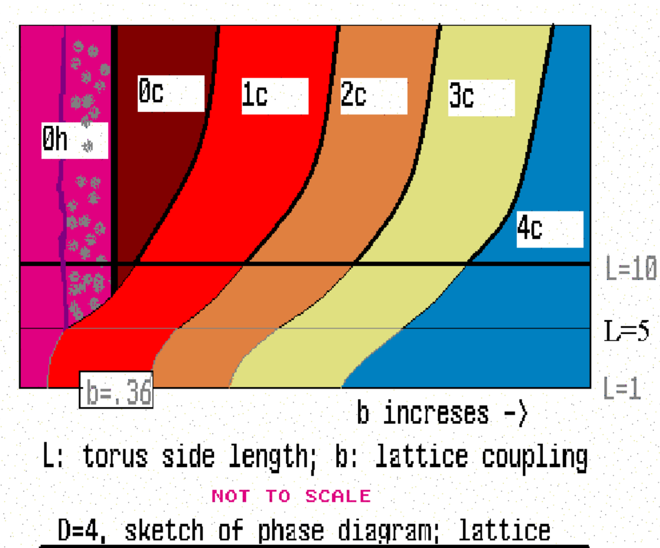
<u>Figure is about the lattice in 4D</u>

In the <u>continuum</u>, 0h disappears and boundaries obeying AF become of critical size (exponentials) values: take a b=constant line at some large b.

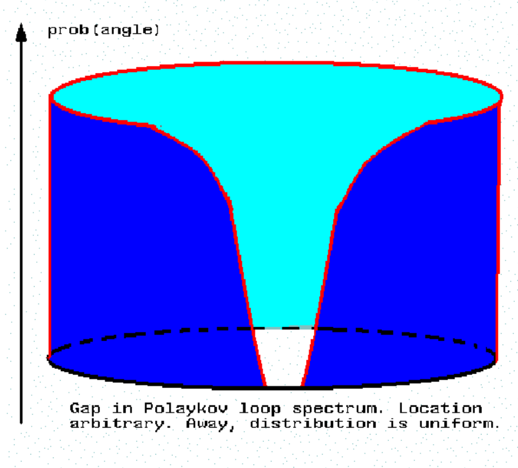
L=1: no 0c. Situation at higher L was missed in past work on reduction.

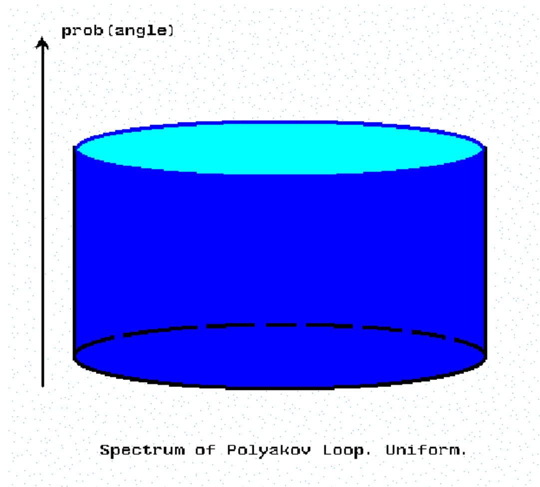
0c extends by metastability into 0h

3D: similar situation.



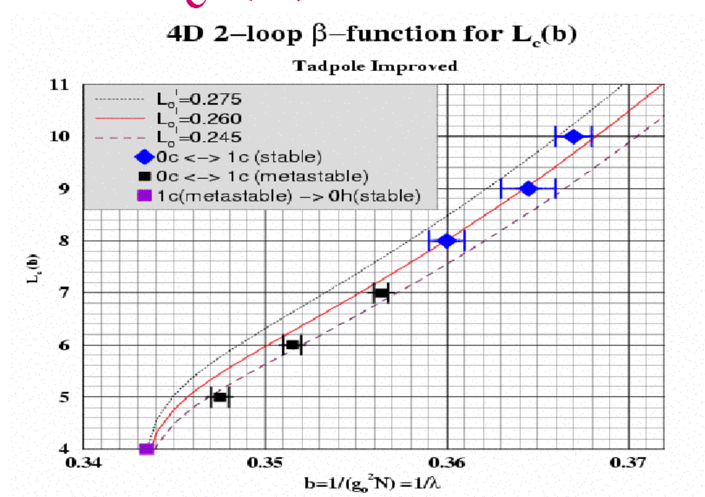
How the transitions occur: Polyakov Loop Spectra





In 0c all Polyakov Loops are Uniform. In Xc some open gaps.

AF of L_c (b) in 4D



Data for 0c →1c

AF using "Tadpole Improvement".

$$b_I = b\frac{b^2-0.58960b+0.08467}{b^2-0.50227b+0.05479} \approx b - b_o + \mathcal{O}\left(\frac{1}{b}\right)$$

$$L_c(b) = (0.260 \pm 0.015)\left(\frac{11}{48\pi^2 b_I}\right)^{\frac{51}{121}} e^{\frac{24\pi^2 b_I}{11}}$$

Questions (open)

- Do all Xc -> (X+1)c transitions have distinct continuum limits ?
- For those that do, does there exist an extension of Atick-Witten explaining their sequential occurrence ?
- Comment: Much of this extends to 3D, where nonperturbative issues are different.

What about the GK problems ?

- Planar QCD is NOT replaced by a matrix model and ordinary space-time is kept continuous at short distances. One never has an exact representation by a model consisting of a finite number of infinite matrices.
- The UV procedure defining the theory is the ordinary lattice cutoff ! (GK-s procedure only worked at infinite N). At finite UV cutoff we can use just a finite number of matrices.
- Topology and anomalies work at finite UV cutoff by employing new chiral lattice fermions (impossible in 1982).

Summary: EK Reduction in the Continuum

- Traces of Wilson Loops at $N_c=\infty$ on a torus are independent of torus size, l, if properly renormalized.
- Limitations on l, d=3,4:
  - In QCD: $l>l_c$.
  - Is $l_c=0$ in SUSY ? Even if yes, this does no guarantee equivalence to a simple matrix model since L might still need to go to infinity to get to the continuum limit. $l_c=0$ could reflect just L going to infinity slower than by AF.

Fermions

Open string probes ("probe" is a concept dual to quenching – no back reaction on gauge fields). Mesons as a tool to learn more about the gauge background (closed string background)

Fermionic Results: General

- SχSB at N=1 is described by the random matrix model (RMT) of **Shuryak and Verbaarschot.**
- The lattice version of the **GK** "momentum force feeding'' prescription for mesons works in the continuum limit. (The lattice version was introduced by **Levine and N**. in 1983)

Fermionic Results: Numerical

- The chiral condensate $<\psi^*\psi>/N$ seems to have small 1/N corrections relatively to N=3 (~ 10%).
- The pion decay constant F^2_π/N seems to have large 1/N corrections relatively to N=3 (~50%).
- $M_\pi^2\ (m_q)$ has a very simple behavior in the planar limit in 4D (well approximated by a parabola).

Exact chiral sym. on the lattice: D_0

SχSB breaking can be studied independently of the approach to the continuum limit.

- $D_0 = \frac{1+V}{2}$; V is unitary and not sparse.
- $A^{-1} = \frac{1-V}{1+V}$ & anticommutes with γ_5.
- A^{-1} can be used as valence quark propagator.
- A is gauge covariant.
- When $N_c \to \infty$ A can be modeled by RMT.

D_o modeled by a Random Matrix

In the chiral basis: $\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $A = \begin{pmatrix} 0 & C \\ -C^\dagger & 0 \end{pmatrix}$

where C is complex and there is an invariance under (vectorial) gauge transformations: $C \to U^\dagger C U$

Make C random, with enhanced symmetry:

$$p(C) d^{2n^2} C \propto e^{-\kappa^2 n \mathrm{Tr}(C^\dagger C)} d^{2n^2} C$$

n is proportional to $L^d N_c$, where d is the dimension.

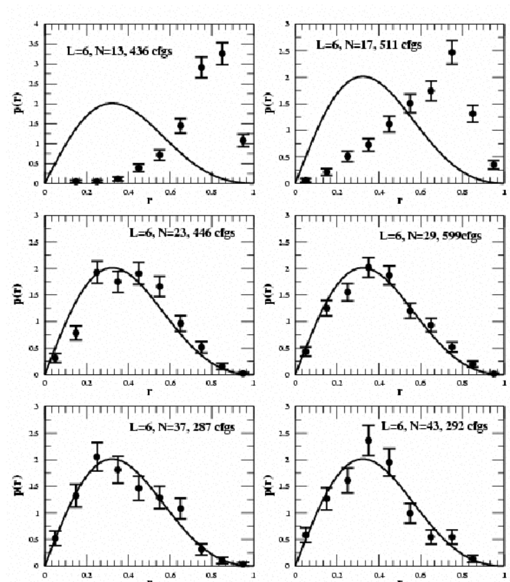
Spontaneous Chiral Symmetry Breaking

By numerical simulation it was determined that the distribution of the two lowest eigenvalues of $-A^2$ indeed is given by RMT with a nonzero κ, at finite L and infinite N_c. κ is independent of L. This establishes spontaneous chiral symmetry breaking and large N_c reduction for the chiral condensate:

$$\frac{1}{N_c}\langle\bar{\psi}\psi\rangle = \kappa.$$

Checking RMT: ratio distribution

Ratio of smallest to next smallest eigenvalue vs RMT prediction – comparison does not depend on the κ parameter. RMT holds when N_c is large enough for eigenvalue repulsion to dominate. The ‘t Hooft coupling is about 3 (b=0.35).



m_π^2 (m_q) in 2D at N=1

- In the 2D ‘t Hooft model (planar QCD in 2D) m_π^2 (m_q) contains enough information to almost construct the full meson equation of ‘t Hooft.
- That equation can be “localized” by adding extra degrees of freedom.
- Broadly speaking, an extra dimension also localizes meson wave equations in AdS duals.

2D: Details, I

’t Hooft’s equation:

$$\gamma \frac{1}{x(1-x)}\phi(x) - P\int_0^1 \frac{\phi(y)-\phi(x)}{(y-x)^2}dy = \mu^2\phi(x)$$

is equivalent to

$$-\sum_{n\neq 0}\frac{p^2}{p^2+n^2}\Psi(s) + \frac{\mu^2}{4\cosh^2\frac{s}{2}}\Psi(s) = \gamma\Psi(s)$$

2D: Details II

The 't Hooft equation has a local approximation that suggests to replace μ^2 by Δ (both measured in terms of the dimensionful coupling). γ is the quark mass squared.

$$\frac{\pi^2}{3}\frac{d^2}{ds^2}\Psi(s) + \frac{\mu^2}{4\cosh^2\frac{s}{2}}\Psi(s) = \gamma\Psi(s)$$

$$\mu^2 = 4\Delta(\Delta + \frac{\pi}{2\sqrt{3}})$$

For the pion, $\Delta = \sqrt{\gamma}$. This approximation is good to 10% for all γ.

2D: Details III

Full eq. can be localized at the expense of the introduction of an auxiliary field $\chi(s,\sigma)$

The equation leads to a natural paramterization $\mu^2 = 4\Delta(\Delta + \frac{\pi}{2\sqrt{3}})$ with $\Delta = \sqrt{\gamma} - \frac{\pi\sqrt{3}}{45}\gamma \ldots$.

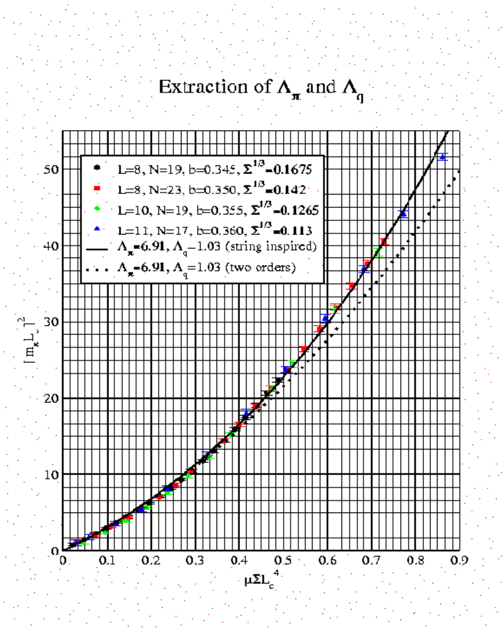
The series for $\Delta(\sqrt{\gamma})$ could be used to reconstruct entire nonlocality, if leading local term is given. Also, the leading term works very well through the entire mass range, much better than chiral perturbation theory could justify.

Question/Speculation

- Assume that 4D N=1 QCD is described by a dual with a warp factor, one scalar real function of one scalar real variable (Polyakov).
- Could a meson probe determine the unknown warp function by something akin to inverse scattering, given $m_\pi^2(m_q)$ in the planar limit ?

4D: Quark mass versus Pion mass

(different presentation from paper)



$$l_c \approx \frac{1}{264\ MeV}$$

$$m_\pi^2 = 4\Delta(\Delta + \Lambda_\pi)$$

$$\Delta = m_q + \frac{m_q^2}{\Lambda_q}$$

$$\Lambda_\pi \approx 630\ MeV$$

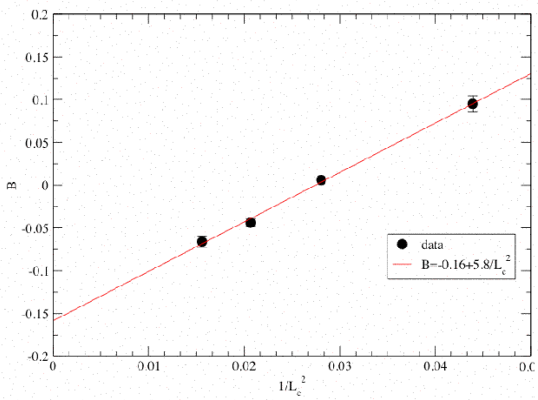
$$(QCD : \Lambda_\pi \sim 2\ GeV)$$

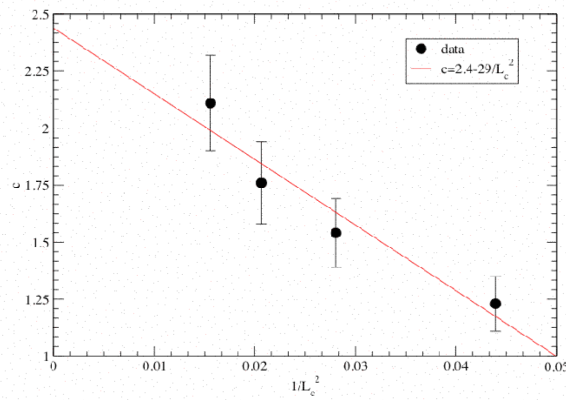
$$\Lambda_q \approx 1600\ MeV$$

4D: Approach to continuum limit

(different presentation from paper)

$$B = \Lambda_q l_c \qquad\qquad c = \Lambda_\pi l_c$$





Summary

- In the planar limit RMT and FT universalities meet and seem to coexist.
- Many new phase transition appear at $N_c=\infty$.
- Simulations at infinite N_c seem feasible on small PC clusters. IT IS THE RIGHT TIME FOR LATTICE STUDY OF PLANAR QCD !
- Barring unpleasant surprises a wide spectrum of new problems will be addressed by numerical means in the large N_c limit.