

Calculations beyond one loop

Gudrun Heinrich

Max Planck Institute for Physics, Munich

Stress-testing the Standard Model at the LHC

Kavli Institute for Theoretical Physics

May 25, 2016



Calculations beyond one loop



methods

Gudrun Heinrich

Max Planck Institute for Physics, Munich

Stress-testing the Standard Model at the LHC

Kavli Institute for Theoretical Physics

May 25, 2016



Calculations beyond one loop



methods

tools



Gudrun Heinrich

Max Planck Institute for Physics, Munich

Stress-testing the Standard Model at the LHC

Kavli Institute for Theoretical Physics

May 25, 2016



Calculations beyond one loop



methods



tools



applications

Gudrun Heinrich

Max Planck Institute for Physics, Munich

Stress-testing the Standard Model at the LHC

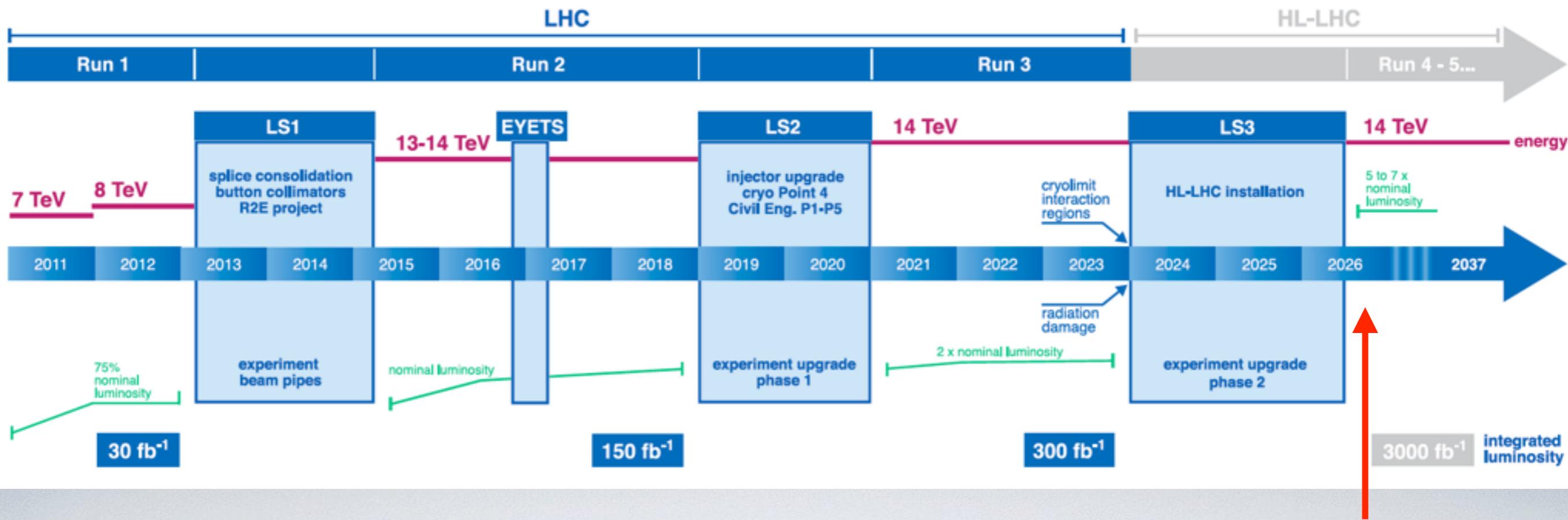
Kavli Institute for Theoretical Physics

May 25, 2016



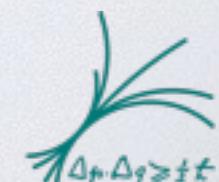
The experimental frontier

LHC / HL-LHC Plan



Linear Collider?

wealth of data, need to **match experimental precision with theory predictions**



The precision frontier

PDFs

PRECISION

fixed order calculations
NLO (QCD+EW), NNLO, ...

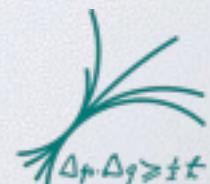
quark mass effects

parametric uncertainties
(e.g. couplings)

resummation

non-perturbative effects
(hadronisation, underlying event, ...)

parton shower
(matching/merging)



current status



MAX-PLANCK-GESELLSCHAFT



Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

current status

- NLO automation:
pretty advanced
NLO matched to parton
shower is new state of the art



current status

- NLO automation:

pretty advanced

NLO matched to parton shower is new state of the art



- NNLO: automation starts to become feasible!

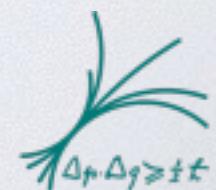


fixed order calculations



fixed order calculations

beyond one loop



Contents

methods:

- analytic
- (semi-)numerical

tools for:

- amplitude generation
- reduction
- loop integrals
- real radiation

applications:

- Higgs boson pair production at NLO with full mass dependence



Contents

methods:

- analytic
- (semi-)numerical



tools for:

- amplitude generation
- reduction
- loop integrals
- real radiation

applications:

- Higgs boson pair production at NLO with full mass dependence



Contents

methods:

- analytic
- (semi-)numerical



tools for:

- amplitude generation
- reduction
- loop integrals
- real radiation



applications:

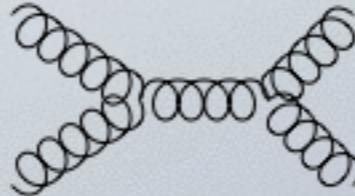
- Higgs boson pair production at NLO with full mass dependence



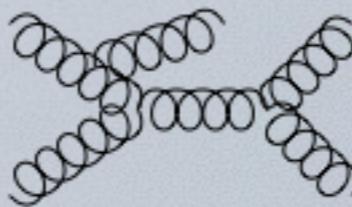
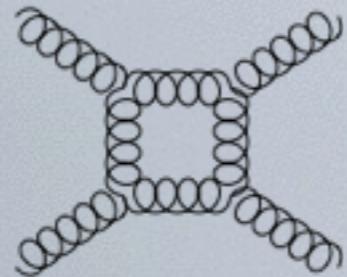
building blocks of higher order calculations

example 2 to 2 scattering

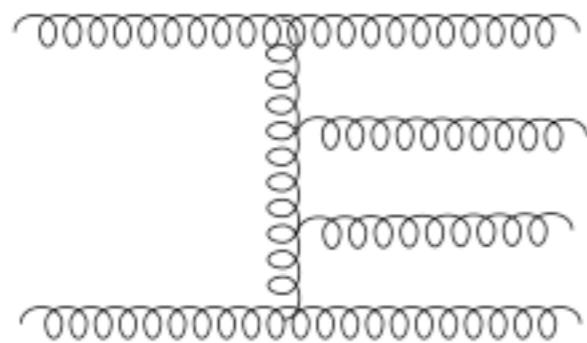
LO: usually tree level diagrams



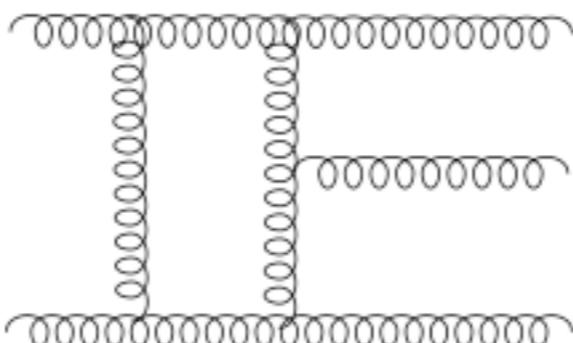
NLO: one loop (virtual) + extra real radiation + subtraction terms



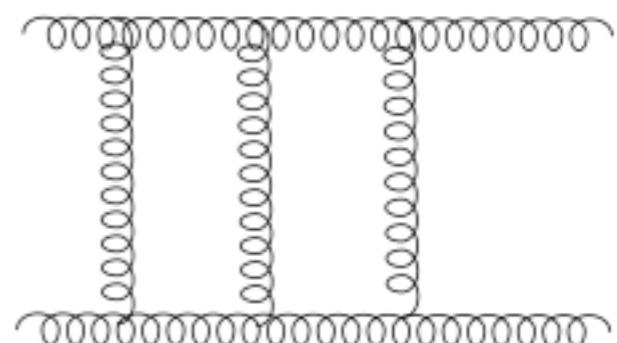
NNLO:



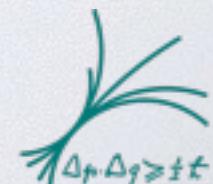
double real



1-loop virtual
⊗ single real

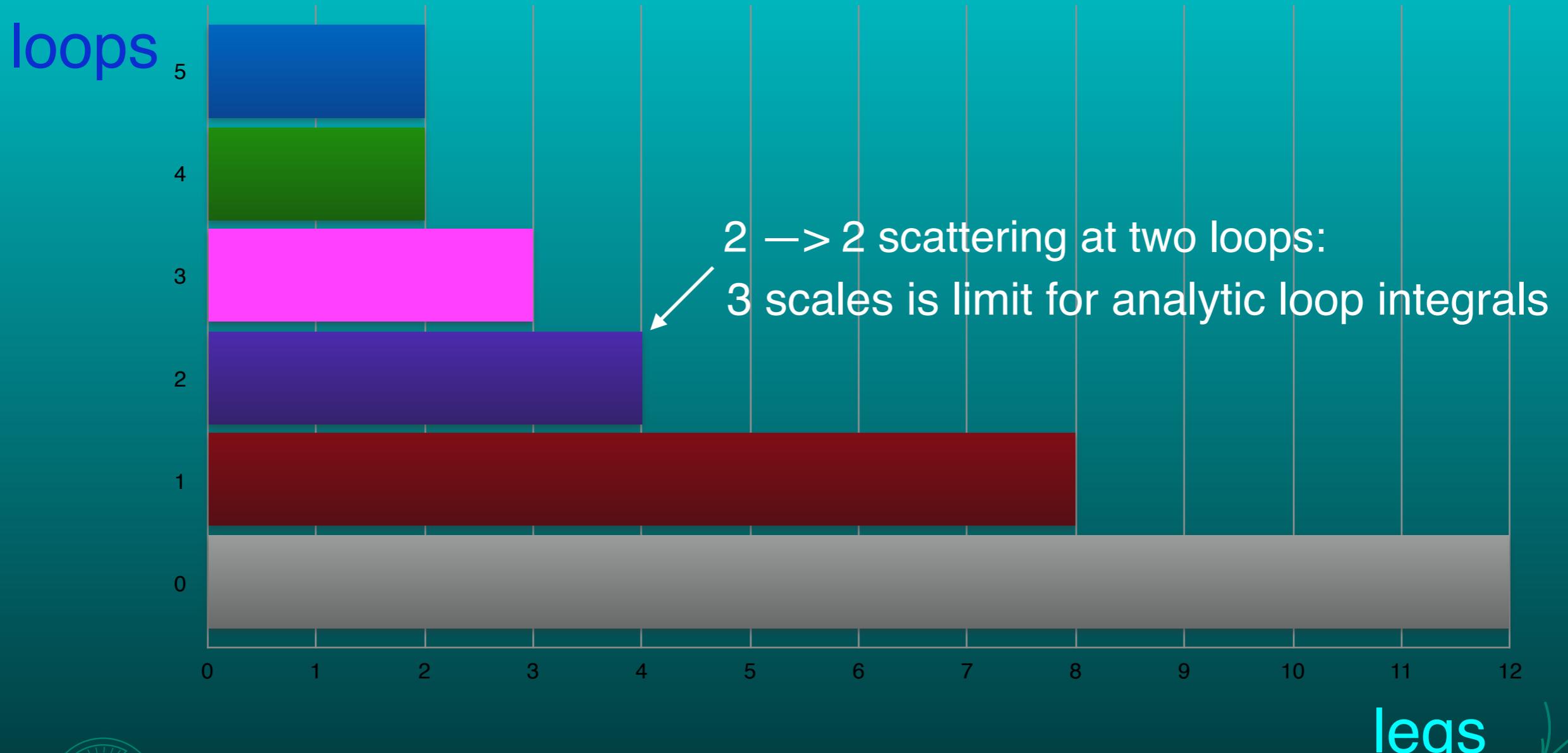


2-loop virtual



measure of complexity

#loops + #legs + #scales (masses, off-shellness)



(refers to physical results, not individual integrals)



tasks/problems beyond one loop

1. automated amplitude generation

tools e.g. QGRAF [P.Nogueira], FeynArts [T.Hahn et al.]

saturation of Lorentz/spin indices: helicity amplitudes or
projectors to form factors

2. reduction of the loop amplitudes to coefficients \otimes master integrals

reduction highly non-trivial; no unique master integral basis beyond one loop

tools e.g. Reduze [C.Studerus, A.v.Manteuffel], FIRE [A.V.Smirnov], LiteRed [R.N.Lee]

based on integration by parts (IBP) relations

$$\int d^D k \frac{\partial}{\partial k^\mu} v^\mu f(k, p_i) = 0$$

new: reduce complexity by construction of algebraic identities from numerical samples

A.v.Manteuffel, R.Schabinger '14

two-loop **integrand** reduction:

very interesting new developments, but not ready for automation yet (?)



new ideas for amplitude reduction

- integrand reduction at two loops
(generalized unitarity cuts, polynomial division)
Mastrolia, Ossola '11; Badger, Frellesvig, Zhang '12,
Mastrolia,Mirabella,Ossola,Peraro '12; Feng, Huang '12;
Papadopoulos et al.'12; Ita '15; Mastrolia, Peraro, Primo '16 ...
- maximal unitarity
Kosower, Larsen '11; Johansson, Caron-Huot, Zhang, Søgaard, ...
- colour-kinematics duality
Bern, Carrasco, Johansson '08; ...
- five-gluon two-loop helicity amplitudes in YM theory
Badger, Mogull, Ochirov, O'Connell '15
Gehrmann, Henn, Lo Presti '15; Dunbar, Perkins '16



tasks/problems beyond one loop

3. calculation of the master integrals

analytically? may not always be possible

numerically? may not always be accurate/fast enough

4. subtraction of IR divergent real radiation

lots of interesting recent developments, see previous talks

5. stable and fast Monte Carlo program



NNLO subtraction of IR divergent real radiation

main methods:

- antenna subtraction Gehrmann, Gehrmann-De Ridder, Glover 05

- qT subtraction Catani, Grazzini 07

(only final states with no jets at LO; “slicing”; use resummation results)

- N-jettiness Gaunt, Stahlhofen, Tackmann, Walsh 15
Boughezal, Focke, Liu, Petriello 15

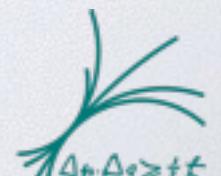
(“slicing”; use resummation results)

- sector-improved residue subtraction Czakon 10;

(numerical pole cancellation) Boughezal, Melnikov, Petriello 11

- colourful subtraction Del Duca, Somogyi, Trocsanyi 05

(so far applied only to colourless initial states)



4-particle processes at NNLO

- antenna subtraction

$e^+e^- \rightarrow 3 \text{ jets}$ [Gehrmann-DeRidder, Gehrmann, Glover, GH '07; Weinzierl '08]

$ep \rightarrow 2 \text{ jets}$ [Gehrmann, Niehues '16]

$pp \rightarrow 2 \text{ jets}$ [Currie, Gehrmann-DeRidder, Gehrmann, Glover, Pires '14,'16]

$pp \rightarrow H + \text{jet}$ [Chen, Gehrmann, Glover, Jaquier '14,'16]

$pp \rightarrow Z + \text{jet}$ [Gehrmann-DeRidder, Gehrmann, Glover, Huss, Morgan '15,'16]

- colorful subtraction subtraction

$e^+e^- \rightarrow 3 \text{ jets}$ [Kardos, Somogyi, Trocsanyi et al '16]

- qt subtraction

$pp \rightarrow HV, \quad pp \rightarrow \gamma\gamma$ [Catani, Cieri, De Florian, Ferrera, Grazzini, Tramontano '07 - '14]

$pp \rightarrow Z \gamma$ [Grazzini, Kallweit, Rathlev, Torre '13]

$pp \rightarrow VV$ [Cascioli, T.Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs '13,'14]

- N-jettiness

$pp \rightarrow H + \text{jet}$ [Boughezal, Focke, Giele, Liu, Petriello '15]

$pp \rightarrow V + \text{jet}$ [Boughezal, Focke, Liu, Petriello '15,'16]

$pp \rightarrow H + V \quad pp \rightarrow \gamma \gamma$ [Campbell, Ellis, Li, Williams '16]

- sector-improved residue subtraction

$pp \rightarrow t \bar{t}$ [Czakon, Fiedler, Mitov '13,'15,'16]

$pp \rightarrow H + \text{jet}$ [Boughezal, Caola, Melnikov, Petriello, Schulze '14]

$pp \rightarrow t + \text{jet}$ [Brucherseifer, Caola, Melnikov '14]

4-particle processes at NNLO

- antenna subtraction

$e^+e^- \rightarrow 3 \text{ jets}$ [Gehrmann-DeRidder, Gehrmann, Glover, GH '07; Weinzierl '08]

$ep \rightarrow 2 \text{ jets}$ [Gehrmann, Niehues '16]

$pp \rightarrow 2 \text{ jets}$ [Currie, Gehrmann-DeRidder, Gehrmann, Glover, Pires '14,'16]

$pp \rightarrow H + \text{jet}$ [Chen, Gehrmann, Glover, Jaquier '14,'16]

$pp \rightarrow Z + \text{jet}$ [Gehrmann-DeRidder, Gehrmann, Glover, Huss, Morgan '15,'16]

- colorful subtraction subtraction

$e^+e^- \rightarrow 3 \text{ jets}$ [Kardos, Somogyi, Trocsanyi et al '16]

- qt subtraction

$pp \rightarrow HV, \quad pp \rightarrow \gamma\gamma$ [Catani, Cieri, De Florian, Ferro, Grazzini, Tramontano '07 - '14]

$pp \rightarrow Z \gamma$ [Grazzini, Kallweit, Rathlev, Torre '13]

$pp \rightarrow VV$ [Cascioli, T.Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs '13,'14]

- N-jettiness

$pp \rightarrow H + \text{jet}^2$ [Boughezal, Focke, Giele, Liu, Petriello '15]

$pp \rightarrow V + \text{jet}$ [Boughezal, Focke, Liu, Petriello '15,'16]

$pp \rightarrow H + V \quad pp \rightarrow \gamma \gamma$ [Campbell, Ellis, Li, Williams '16]

- sector-improved residue subtraction

$pp \rightarrow t \bar{t}$ [Czakon, Fiedler, Mitov '13,'15,'16]

$pp \rightarrow H + \text{jet}$ [Boughezal, Caola, Melnikov, Petriello, Schulze '14]

$pp \rightarrow t + \text{jet}$ [Brucherseifer, Caola, Melnikov '14]

2 to 2 NNLO results are emerging rapidly!

some methods for (multi-)loop integrals

• analytic

- direct integration [Feynman; 't Hooft, Veltman ... ; Brown '08; Panzer '13; Schnetz '13, von Manteuffel, Panzer, Schabinger '15, ...]
- Mellin-Barnes representation [Tausk '99, Smirnov '99, ...]
- differential equations [Kotikov '91; Remiddi '97, Gehrmann, Remiddi '00; **Henn '13**, ...]



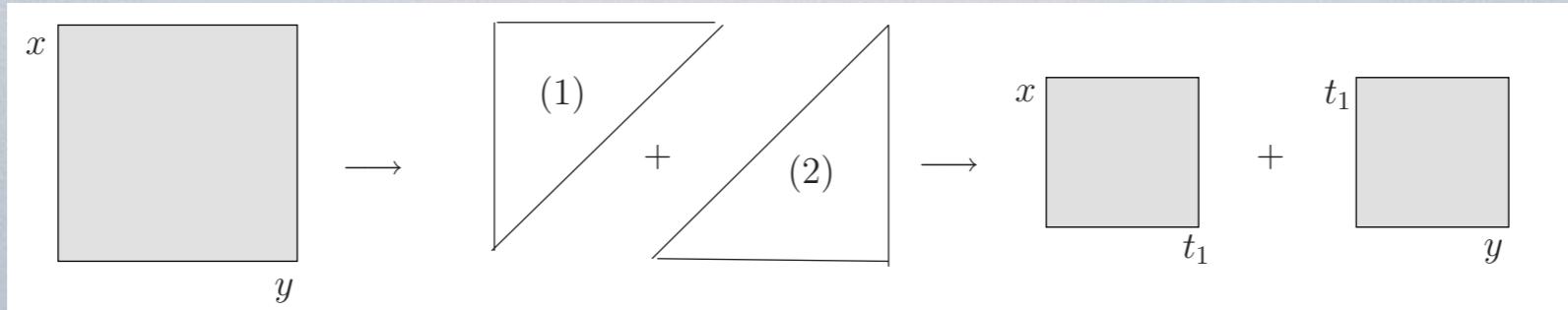
linear reducibility

• (semi-)numerical

- numerical solution of differential equations [Caffo, Czyz, Laporta, Remiddi '98; Czakon, Mitov ...]
- numerical evaluation of Mellin-Barnes representations [Czakon; ... Dubovsky, Freitas, Gluza, Riemann, Usovitsch '16]
- dispersion relations [Bauberger et al '94 ...]
- use Bernstein-Sato-Tkachov theorem [Passarino et al '01 ...]
- sector decomposition [Binoth, GH, et al '00 ...]



sector decomposition



algorithmic procedure to factorise end-point singularities

$$I = \int_0^1 dx \int_0^1 dy x^{-1-\epsilon} (a_1 x + a_2 y)^{-1} [\underbrace{\Theta(x-y)}_{(1)} + \underbrace{\Theta(y-x)}_{(2)}]$$

subst. (1) $y = x z$ (2) $x = y z$ to remap to unit cube

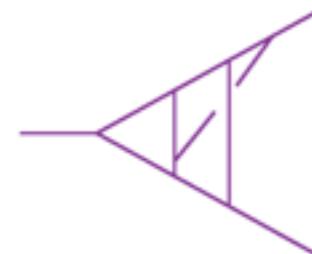
$$\begin{aligned} I &= \int_0^1 dx x^{-1-\epsilon} \int_0^1 dz (a_1 + a_2 z)^{-1} \\ &+ \int_0^1 dy y^{-1-\epsilon} \int_0^1 dz z^{-1-\epsilon} (a_1 z + a_2)^{-1} \end{aligned}$$

sector decomposition

<http://secdec.hepforge.org>

SecDec is hosted by Hepforge, IPPP Durham

- Home
- Subversion
- Tracker
- Wiki



SecDec

Sophia Borowka, Gudrun Heinrich, Stephan Jahn, Stephen Jones, Matthias Kerner, Johannes Schlenk, Tom Zirke

A program to evaluate dimensionally regulated parameter integrals numerically

[home](#) [download program](#) [user manual](#) [faq](#) [changelog](#)

NEW: Version 3.0 of the program can be downloaded as [SecDec-3.0.8.tar.gz](#).

Version 2.1.6.1 of the program can be downloaded as [SecDec-2.1.6.1.tar.gz](#).

To install the program:

- tar xvzf SecDec-3.0.8.tar.gz
- cd SecDec-3.0.8
- make

algorithm: T. Binoth, GH '00

version 1.0: J. Carter, GH '10

version 2.0: S.Borowka, J. Carter, GH '12

version 3.0: S.Borowka, GH, S.Jones, M.Kerner,
J.Schlenk, T.Zirke '15

contour
deformation



MAX-PLANCK-GESELLSCHAFT



Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

other public programs based on sector decomposition:

- **sector_decomposition** (uses Ginac) (only Euclidean region)

[Bogner, Weinzierl '07]

supplemented with **CSectors**

for construction of integrand in terms of Feynman parameters

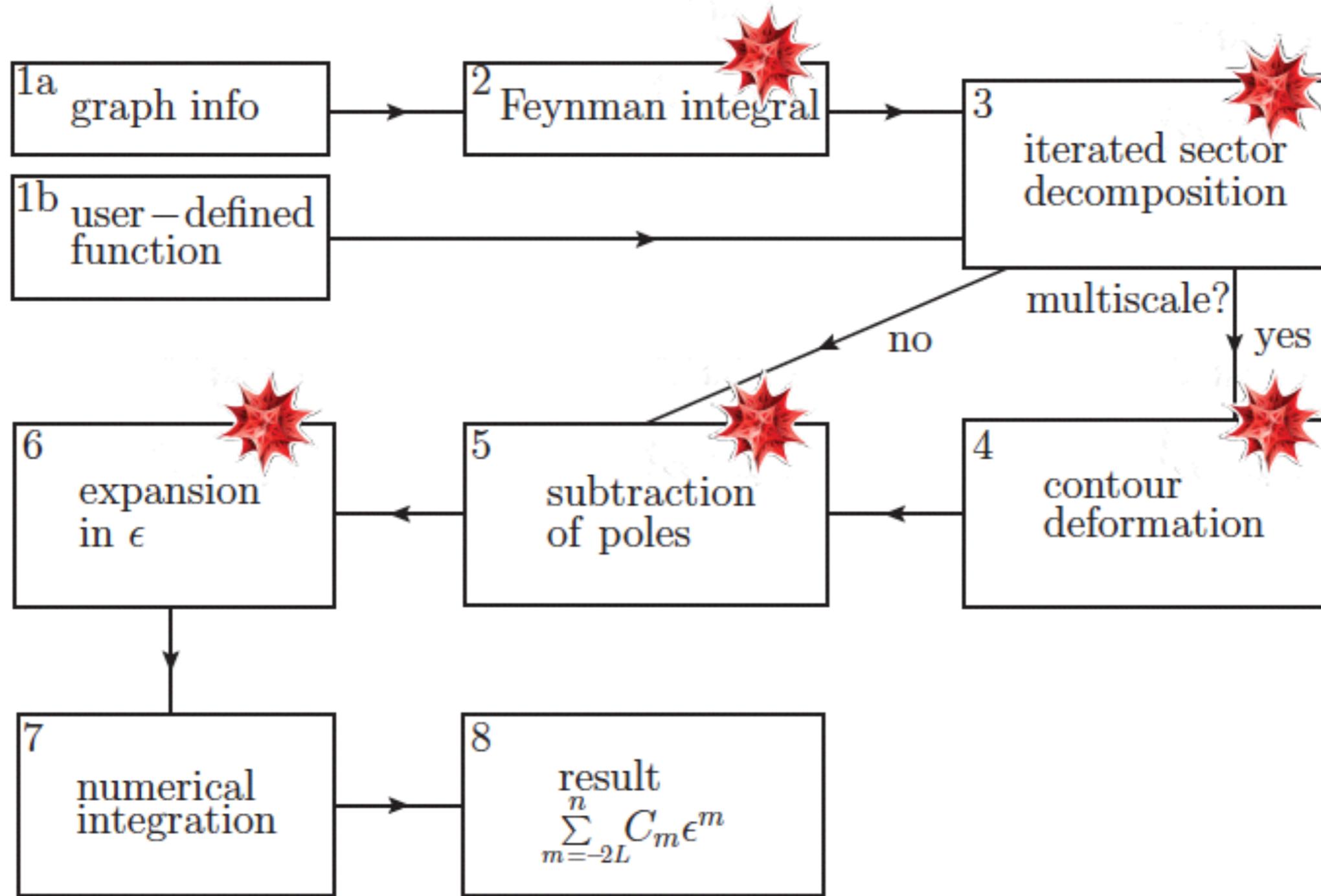
[Gluza, Kajda, Riemann, Yundin '10]

- **FESTA** (versions 1,2,3,4) (uses Mathematica, C++)

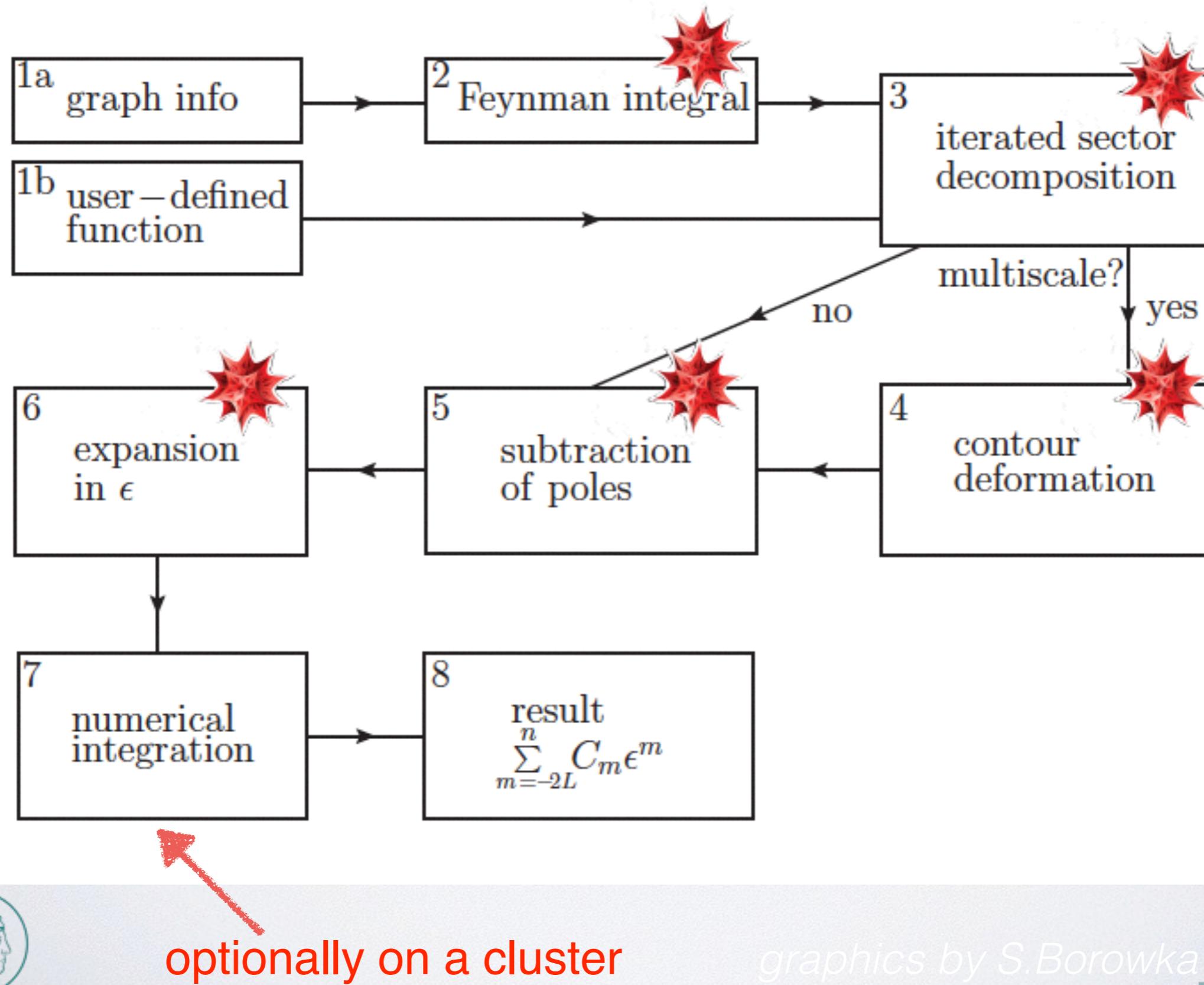
[A.Smirnov, V.Smirnov, Tentyukov, '08,'09,'13,'15]



SecDec basic workflow



SecDec basic workflow



new features in SecDec-3.0

S. Borowka, GH, S. Jahn, M. Kerner, S. Jones, J. Schlenk, T. Zirke

- improved user interface → easy input files, custom definition of kinematics
- implementation of two new decompositions strategies G1, G2 based on a geometric algorithm (**J. Schlenk**, inspired by Kaneko/Ueda '10)
→ guaranteed to stop, produces less sectors than original strategy X
- propagators with zero or negative powers are possible
→ easy interface to IBP reduction programs
- usage on a cluster facilitated
- linear propagators can be treated
- speed improvements
- option to use numerical integrators from Mathematica
- better treatment of linear divergences
- complex masses



SecDec development

- **SecDec** so far has been mainly used to check analytically calculated integrals
- important **new step**:
use SecDec like a **library** to evaluate analytically unknown master integrals within the calculation of two-loop amplitudes



SecDec development

- **SecDec** so far has been mainly used to check analytically calculated integrals

- important **new step**:

use SecDec like a **library** to evaluate analytically unknown master integrals within the calculation of two-loop amplitudes

- **pySecDec**: algebraic part in form of python modules

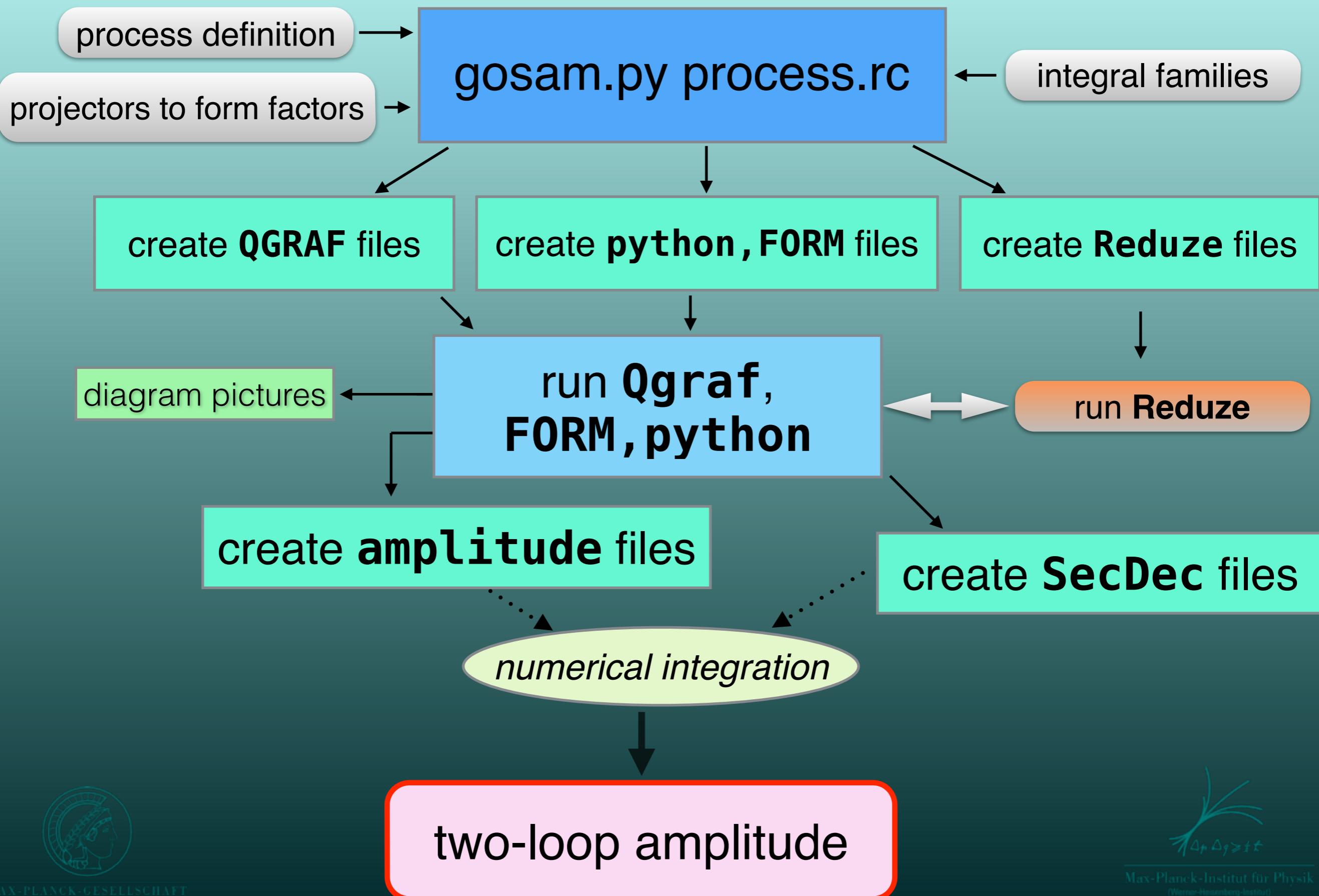


S. Jahn, S. Jones, T. Zirke et al.

coming soon



automated 2-loop amplitudes: GoSam @ 2 loops



credits

GoSam 2-loop

N.Greiner, GH, S.Jahn, S.Jones,
M.Kerner, J.Schlenk, T.Zirke

QGRAF

P. Nogueira

FORM

J. Vermaseren, J. Kuipers, T. Ueda, J. Vollinga

Reduze

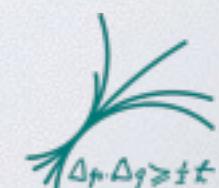
C. Studerus, A. von Manteuffel

GoSam 1-loop

T. Binoth, G.Cullen, H.van Deurzen, N.Greiner, GH,
S.Jahn, G.Luisoni, P. Mastrolia, E.Mirabella,
G. Ossola, T. Peraro, T. Reiter, J. Reichel,
J. Schlenk, J.F. von Soden-Fraunhofen, F. Tramontano

SecDec

S.Borowka, GH, S.Jahn, S.Jones, M.Kerner, J.Schlenk, T.Zirke



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)



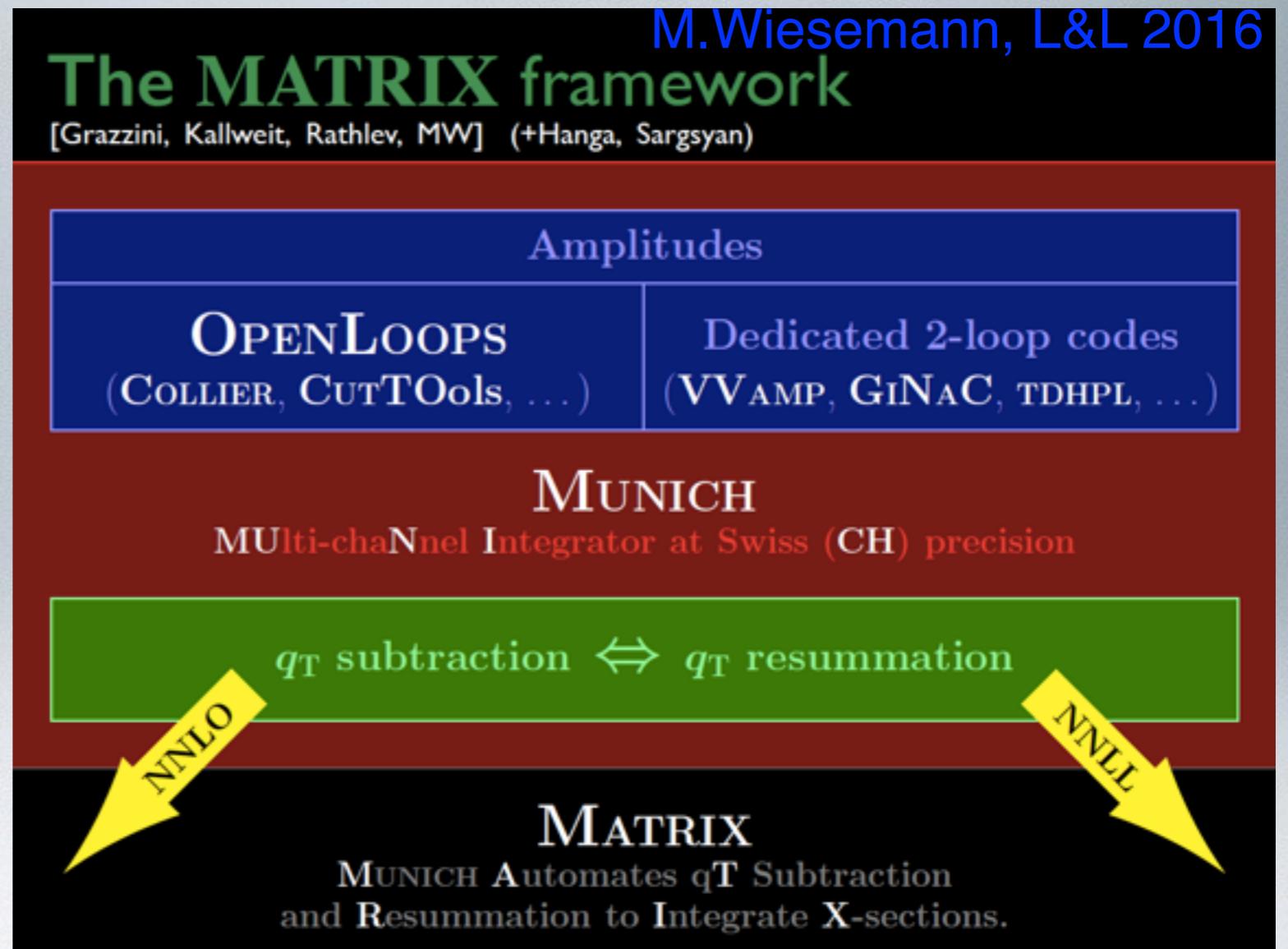
NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)
- Matrix



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)
- Matrix



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)
- Matrix
- MCFM-8.0



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)

- Matrix

- MCFM-8.0

MCFM 8.0

Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello and Williams (in prep).

Lots of work by many people upgrading MCFM to NNLO version, including MPI on top of OMP (Campbell, Giele, Ellis 14) version. We will release code very soon.

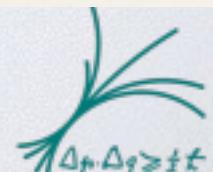
Process	v8.0 (~ weeks)	v8.x (~ months)	Calculation in MCFM framework
H	X		
W/Z	X		
HW/HZ	X		
$\gamma\gamma$	X		
$V\gamma$		X	
VV		X	
$Z + j$			[1]
$W + j$			[2]
$H + j$			[3]

[1] Boughezal et al, 1512.01291

[2] Boughezal et al, 1504.02131

[3] Boughezal et al, 1505.03893

Keith Ellis, L&L 2016



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)
- Matrix
- MCFM-8.0



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)
- Matrix
- MCFM-8.0
- NNLOJet



NNLO frameworks

- GoSam-2loop (so far focus on 2-loop **amplitudes**)

- Matrix

- MCFM-8.0

- NNLOJet

NNLOJET

X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann,
NG, A. Huss, M. Jaquier, T. Morgan, J. Niehues, J. Pires

UDUR, ETH, UZH, MPI, Peking University

Implementing NNLO corrections using Antenna subtraction for

- ✓ $pp \rightarrow H \rightarrow \gamma\gamma$ plus 0, 1, 2 jets
- ✓ $pp \rightarrow e^+e^-$ plus 0, 1 jets
- ✓ $pp \rightarrow$ dijets
- ✓ $ep \rightarrow 2(+1)$ jets
- ✓ ...

Nigel Glover, HiggsTools Annual Meeting 2016



NNLO frameworks

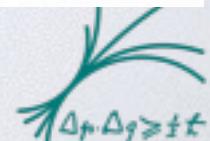
- GoSam-2loop (so far focus on 2-loop **amplitudes**)
- Matrix
- MCFM-8.0
- NNLOJet



STRIPPER - Implementation

- General purpose event generator for NNLO computation
- Based on four-dimensional formulation of the subtraction scheme
- Complete independent implementation
- SM tree-level matrix elements are included [vanHameren, Bury; '09, '15]
- **Process independent:** User has to interface the one-loop and two-loop **finite** contributions
- **Speed:** Monte Carlo over processes and polarizations
- Simultaneous computation of:
 - Different PDF sets (LHAPDF interface)
 - Different renormalization and factorizations scales
 - Different observables

Czakon, Heymes (D.Heymes, L&L 2016)



application of GoSam-2Loop



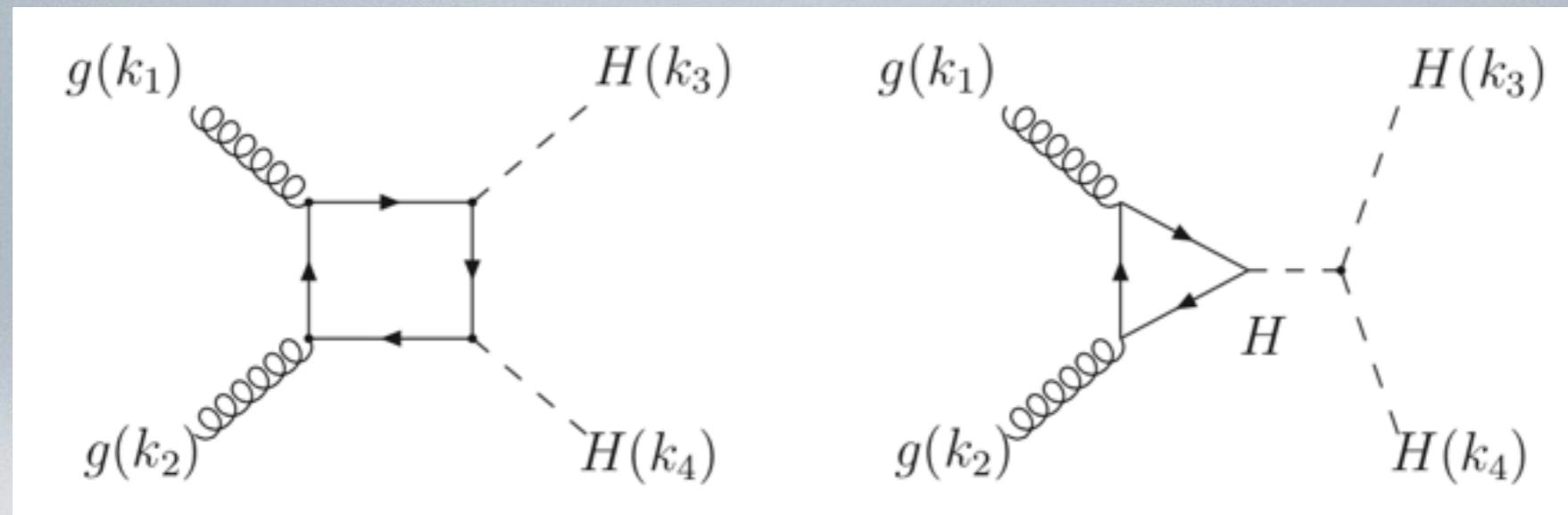
Higgs boson pair production

S.Borowka, N.Greiner, GH, S.Jones, M.Kerner, J.Schlenk, U.Schubert, T.Zirke
arXiv:1604.06447

$gg \rightarrow HH$ at NLO (full mass dependence)

4 independent scales s_{12} , s_{23} , m_H , m_t

Leading Order already involves 1-loop diagrams



NLO (= 2 loops)



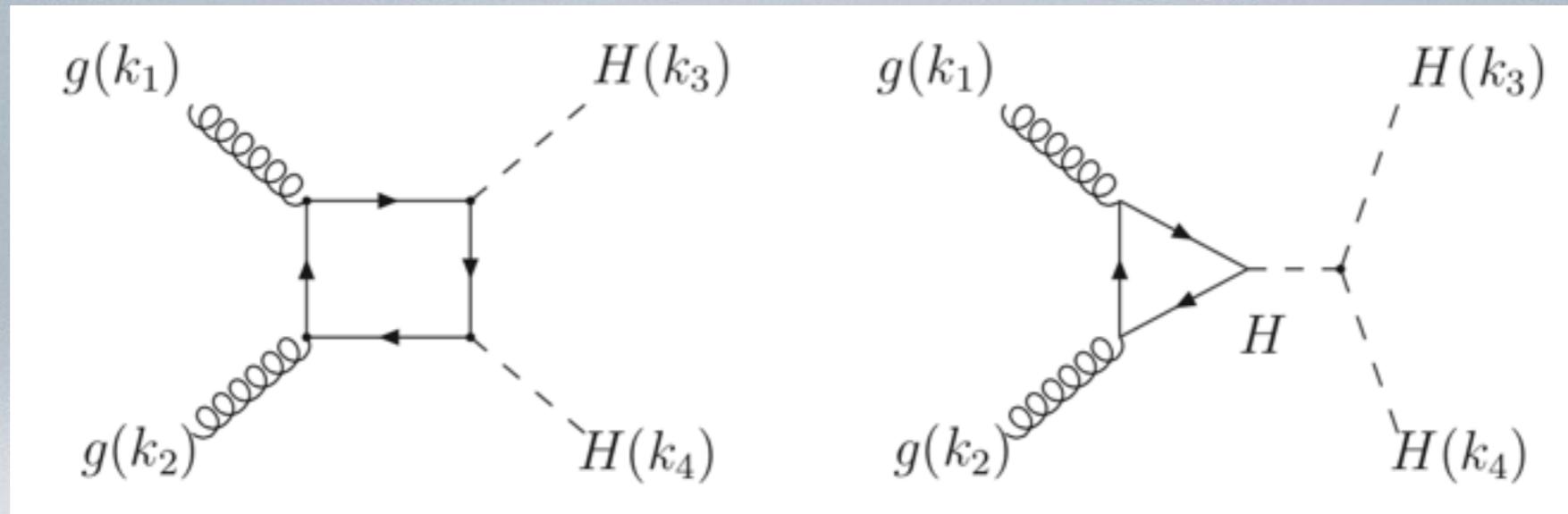
Higgs boson pair production

S.Borowka, N.Greiner, GH, S.Jones, M.Kerner, J.Schlenk, U.Schubert, T.Zirke
arXiv:1604.06447

$gg \rightarrow HH$ at NLO (full mass dependence)

4 independent scales s_{12} , s_{23} , m_H , m_t

Leading Order already involves 1-loop diagrams



NLO (= 2 loops)

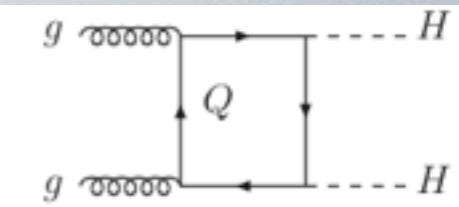
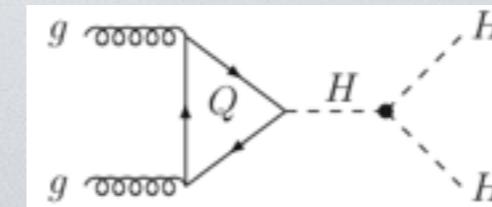
(most) 2-loop diagrams not known analytically
with full mass dependence



previous results in the literature

LO with full heavy quark mass dependence

Glover, van der Bij '88, Plehn, Spira, Zerwas '96



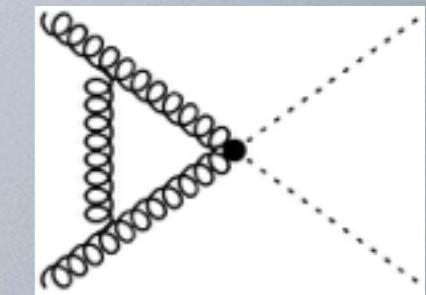
NLO in $m_t \rightarrow \infty$ limit (HEFT): Dawson, Dittmaier, Spira '98 (HPAIR)

- **supplemented with $1/m_t$ expansion:** ($\pm 10\%$)

Grigo, Hoff, Melnikov, Steinhauser '13, '15 ; Degrassi, Giardino, Gröber '16

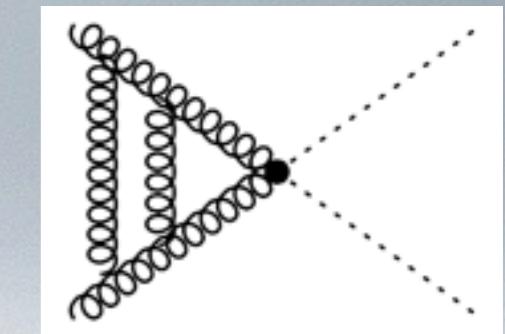
- **full mass dependence in NLO real radiation part and matching to parton shower** **-10%**

Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14;
Maltoni, Vryonidou, Zaro '14



NNLO in $m_t \rightarrow \infty$ limit: **+20%**

De Florian, Mazzitelli '13



- **including all matching coefficients** Grigo, Melnikov, Steinhauser '14

- **supplemented with $1/m_t$ expansion:** Grigo, Hoff, Steinhauser '15

- **soft gluon resummation NNLL matched to NNLO** De Florian, Mazzitelli '15

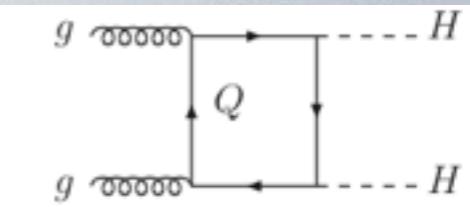
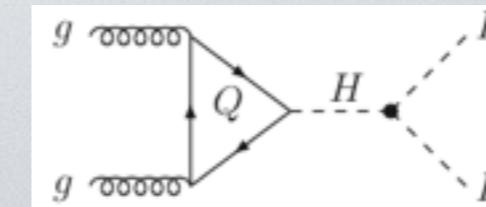
+ lots of phenomenological studies

Baglio, Barr, Dolan, Englert, Ferreira de Lima, Goncalves-Netto, Greiner, Gröber, Krauss, Maierhöfer, Maltoni, Mühlleitner, Papaefstathiou, Spannowsky, Spira, Thompson, Vryonidou, Zaro, Zurita, ...

previous results in the literature

LO with full heavy quark mass dependence

Glover, van der Bij '88, Plehn, Spira, Zerwas '96



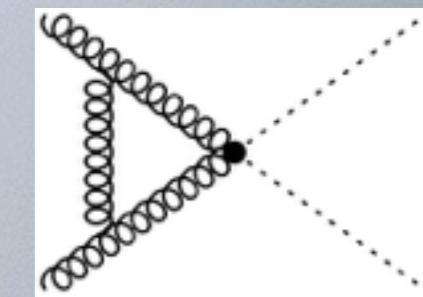
NLO in $m_t \rightarrow \infty$ limit (HEFT): Dawson, Dittmaier, Spira '98 (HPAIR)

- supplemented with $1/m_t$ expansion: ($\pm 10\%$)

Grigo, Hoff, Melnikov, Steinhauser '13, '15 ; Degrassi, Giardino, Gröber '16

- full mass dependence in NLO real radiation part and matching to parton shower -10%

Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14;
Maltoni, Vryonidou, Zaro '14



NNLO in $m_t \rightarrow \infty$ limit:

De Florian, Mazzitelli '13



- including $1/m_t^2$ coefficients Grigo, Melnikov, Steinhauser '14

- supplemented with $1/m_t$ expansion: Grigo, Hoff, Steinhauser '15

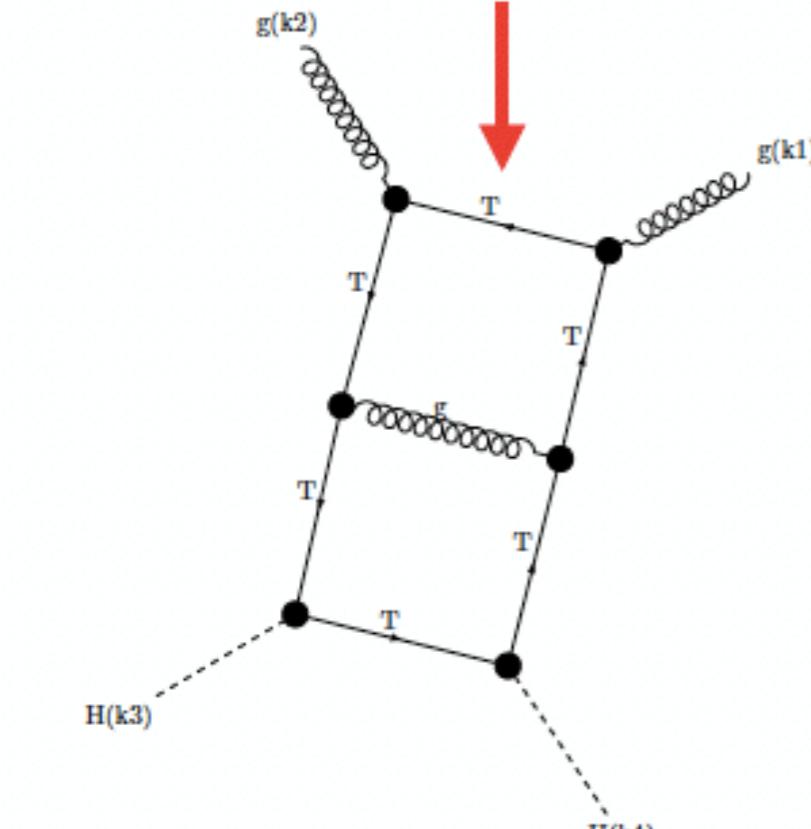
- soft gluon resummation NNLL matched to NNLO De Florian, Mazzitelli '15

+ lots of phenomenological studies

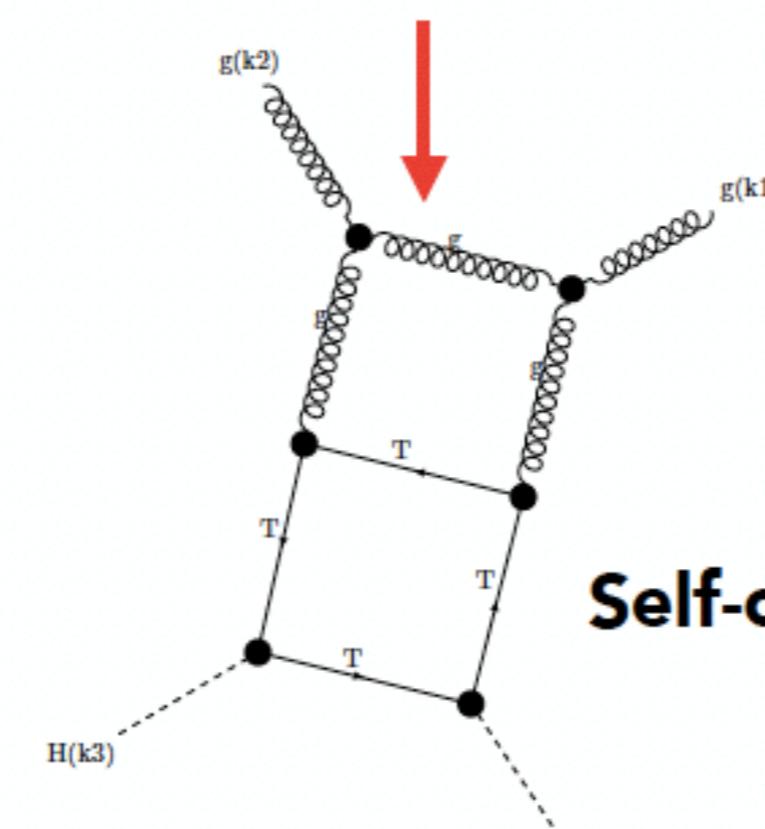
Baglio, Barr, Dolan, Englert, Ferreira de Lima, Goncalves-Netto, Greiner, Gröber, Krauss, Maierhöfer, Maltoni, Mühlleitner, Papaefstathiou, Spannowsky, Spira, Thompson, Vryonidou, Zaro, Zurita, ...

integrals contributing to Higgs boson pair production at NLO

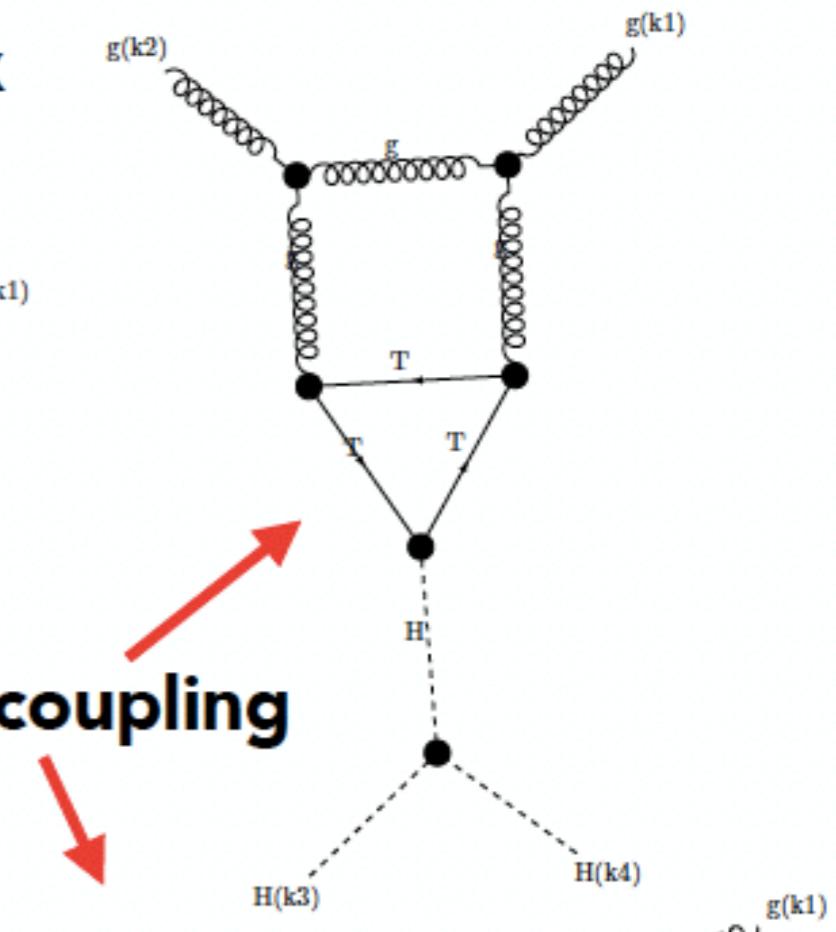
Massive Double Box



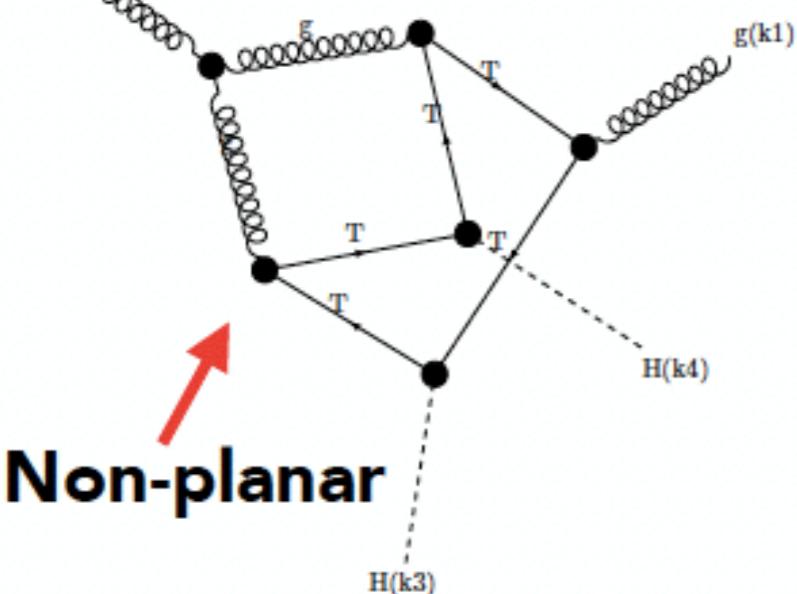
Massless/Massive Box



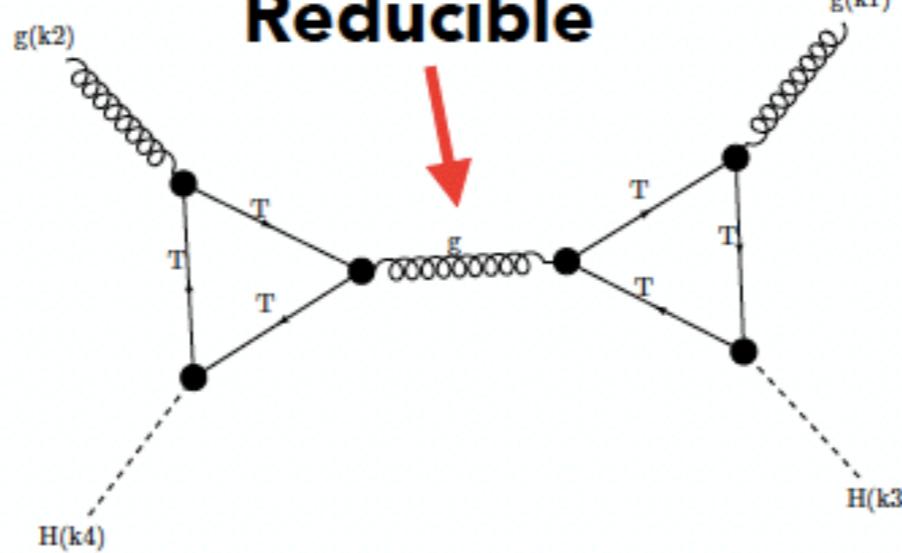
Self-coupling



Non-planar



Reducible



25 *graphics by Stephen Jones*

amplitude structure

$$\mathcal{M}_{ab} = \delta_{ab} \epsilon_1^\mu \epsilon_2^\nu \mathcal{M}_{\mu\nu}$$

define form factors F1, F2 (independent of loop order)

$$\mathcal{M}^{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_1^{\mu\nu} + F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_2^{\mu\nu}$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2} \quad T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2 (p_1 \cdot p_2)} \tilde{T}_2^{\mu\nu}$$

$$\tilde{T}_2^{\mu\nu} = \{m_h^2 p_1^\nu p_2^\mu - 2(p_1 \cdot p_3) p_3^\nu p_2^\mu - 2(p_2 \cdot p_3) p_3^\mu p_1^\nu + 2(p_1 \cdot p_2) p_3^\nu p_3^\mu\}$$

$$p_T^2 = (\hat{t}\hat{u} - m_h^4)/\hat{s}$$

diagrams containing Higgs self-coupling contribute only to F1

F2 contains only box-type diagrams



Projectors onto form factors F1, F2

$$P_1^{\mu\nu} \mathcal{M}_{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D)$$

$$P_2^{\mu\nu} \mathcal{M}_{\mu\nu} = F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D)$$

$$P_1^{\mu\nu} = \frac{1}{4} \frac{D-2}{D-3} T_1^{\mu\nu} - \frac{1}{4} \frac{D-4}{D-3} T_2^{\mu\nu}$$

$$P_2^{\mu\nu} = -\frac{1}{4} \frac{D-4}{D-3} T_1^{\mu\nu} + \frac{1}{4} \frac{D-2}{D-3} T_2^{\mu\nu}$$

form factors F1, F2 have been chosen such that

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -F_1$$

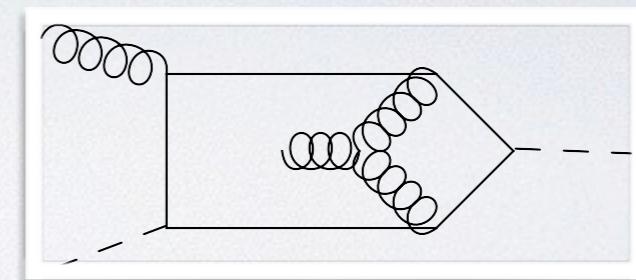
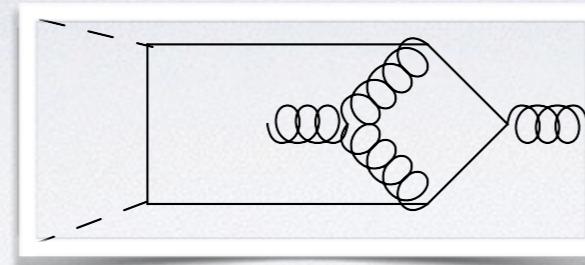
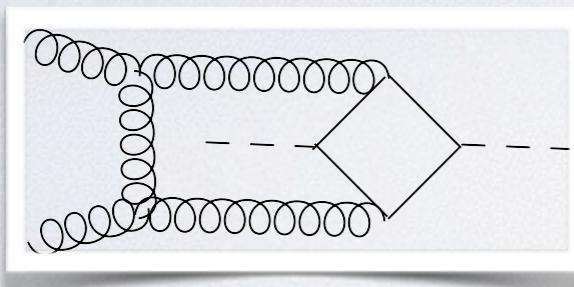
$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -F_2$$



2-loop amplitude

- define 8 integral families with 9 propagators each
- before reduction about 10000 integrals
- after employing **Reduze** about 330 integrals, up to $s=4$ inv. props.
- use of partly finite basis **Panzer 14; von Manteuffel, Panzer, Schabinger 15**
- non-planar integrals computed without full reduction
- all integrals calculated with **SecDec**
- total number of integrals after decomposition 11244,
3086 non-planar

examples



scalar 7-propagator corner integrals: 30-50 sector integrals
but also numerators, higher propagator powers, pinches



integration of the 2-loop amplitude

- parallelisation on GPU



SecDec functions:
numerous, but individual functions
do not require large memory usage



integration of the 2-loop amplitude

- parallelisation on GPU



- integration with quasi Monte Carlo method

- target accuracy set at amplitude level

(3% for F1, ~10% for F2, depending on F2/F1)

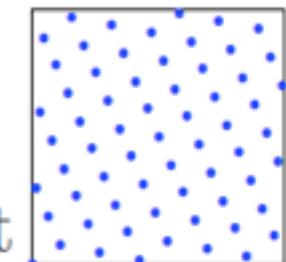
- number of sampling points dynamically set for each integral

SecDec functions:
numerous, but individual functions
do not require large memory usage

QMC rank-1 lattice rule

$$I = \int d\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^n f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$

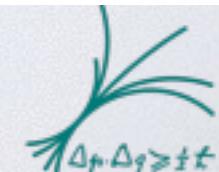


$\{\dots\}$ = fractional part

\vec{g} = generating vector

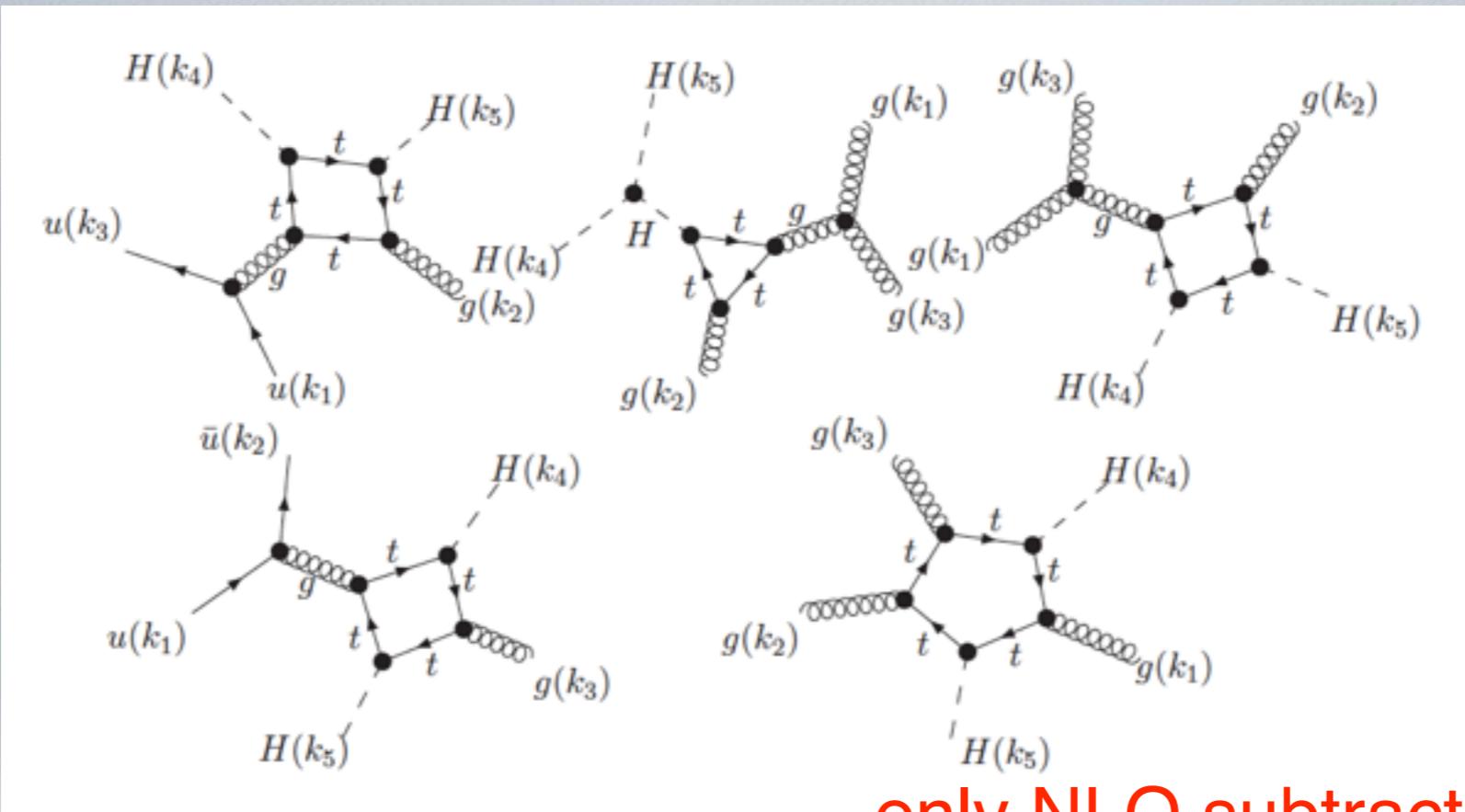
$\vec{\Delta}_k$ = randomized shift

m different estimates $I_1 \dots I_m$
 \rightarrow error estimate



real radiation

GoSam + Catani-Seymour dipole subtraction



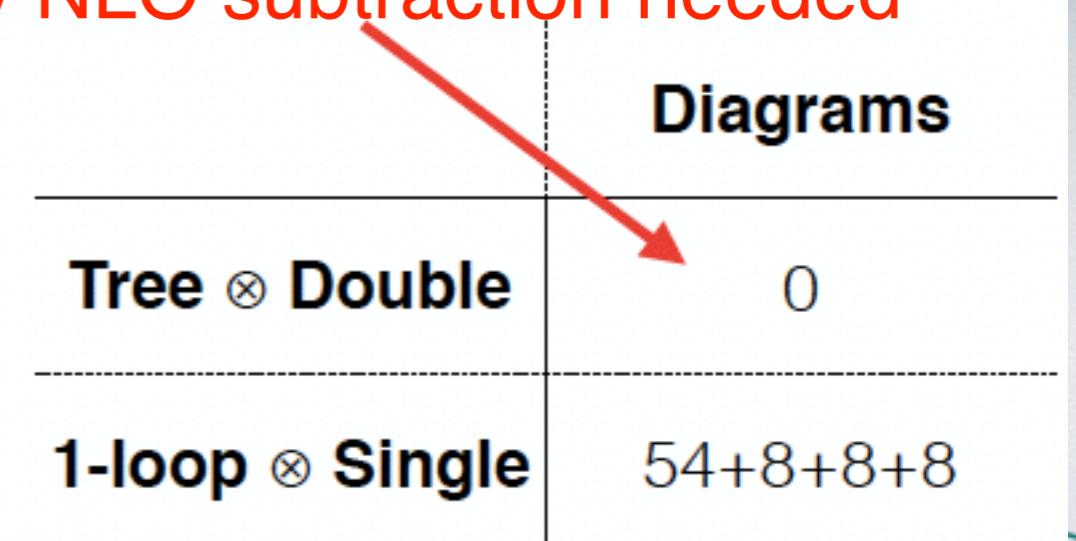
1-j Channels:

$$gg \rightarrow HH + g$$

$$gq \rightarrow HH + q \quad g\bar{q} \rightarrow HH + \bar{q}$$

$$q\bar{q} \rightarrow HH + g$$

only NLO subtraction needed



checks

- independent calculation of (unreduced) amplitude
- checked invariance under $t \leftrightarrow u$
- single H reproduced, comparison to Sushi Harlander, Liebler, Mantler 13,16
- independence of dipole α parameter
- comparison of HEFT result to MG5_aMC@NLO Maltoni, Vryonidou, Zaro 14,15
- comparison of 1/mt expansion with Jens Hoff [Grigo, Hoff, Steinhauser 15]

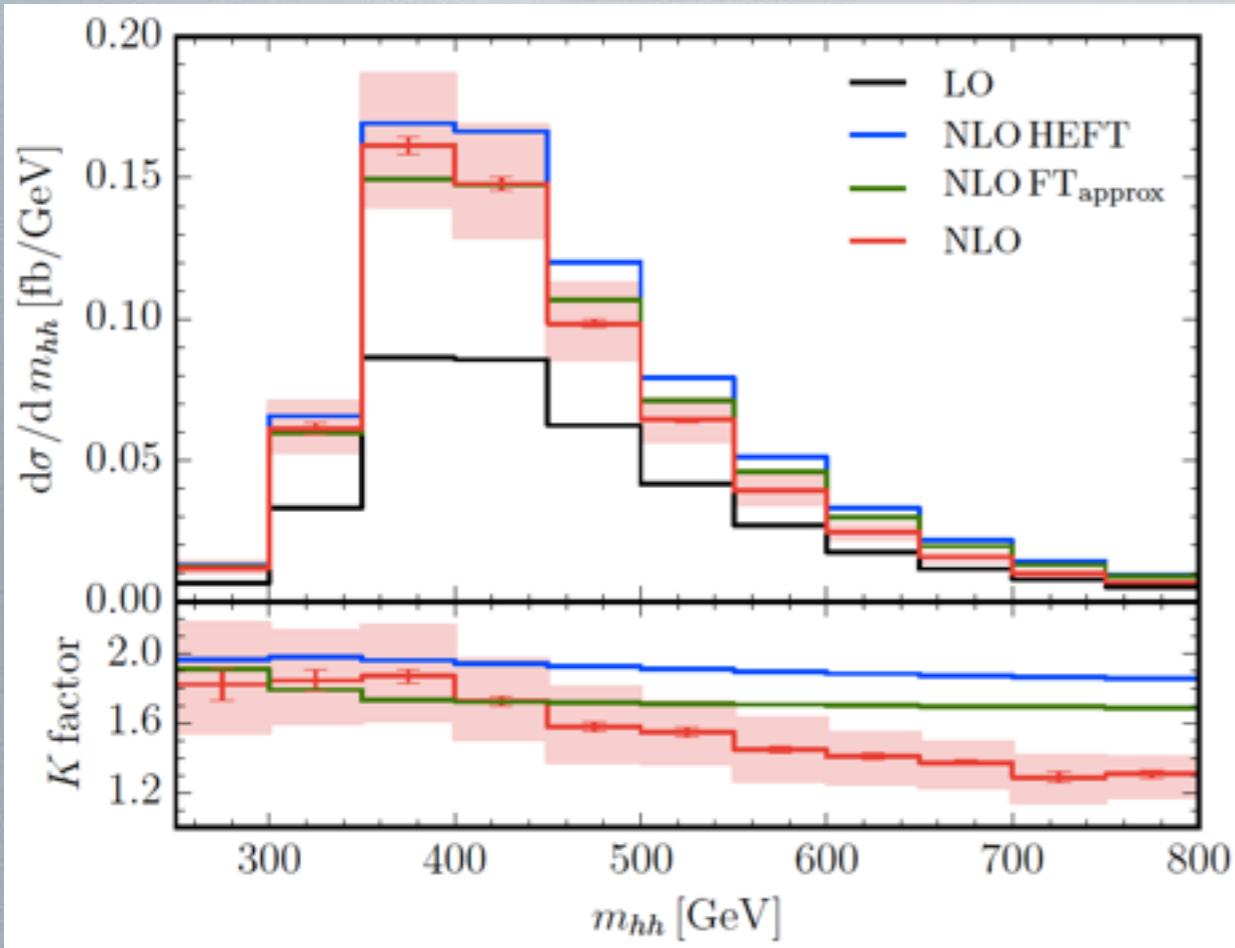


checks

- independent calculation of (unreduced) amplitude
- checked invariance under $t \leftrightarrow u$
- single H reproduced, comparison to Sushi Harlander, Liebler, Mantler 13,16
- independence of dipole α parameter
- comparison of HEFT result to MG5_aMC@NLO Maltoni, Vryonidou, Zaro 14,15
- comparison of $1/mt$ expansion with Jens Hoff [Grigo, Hoff, Steinhauser 15]



numerical results



$$\sigma^{NLO} = 32.80^{+13\%}_{-12\%} \text{ fb}$$

$\pm 0.4\%$ (stat.) $\pm 0.1\%$ (int.)

$$\sigma^{NLO}_{HEFT} = 38.32^{+18\%}_{-15\%} \text{ fb}$$

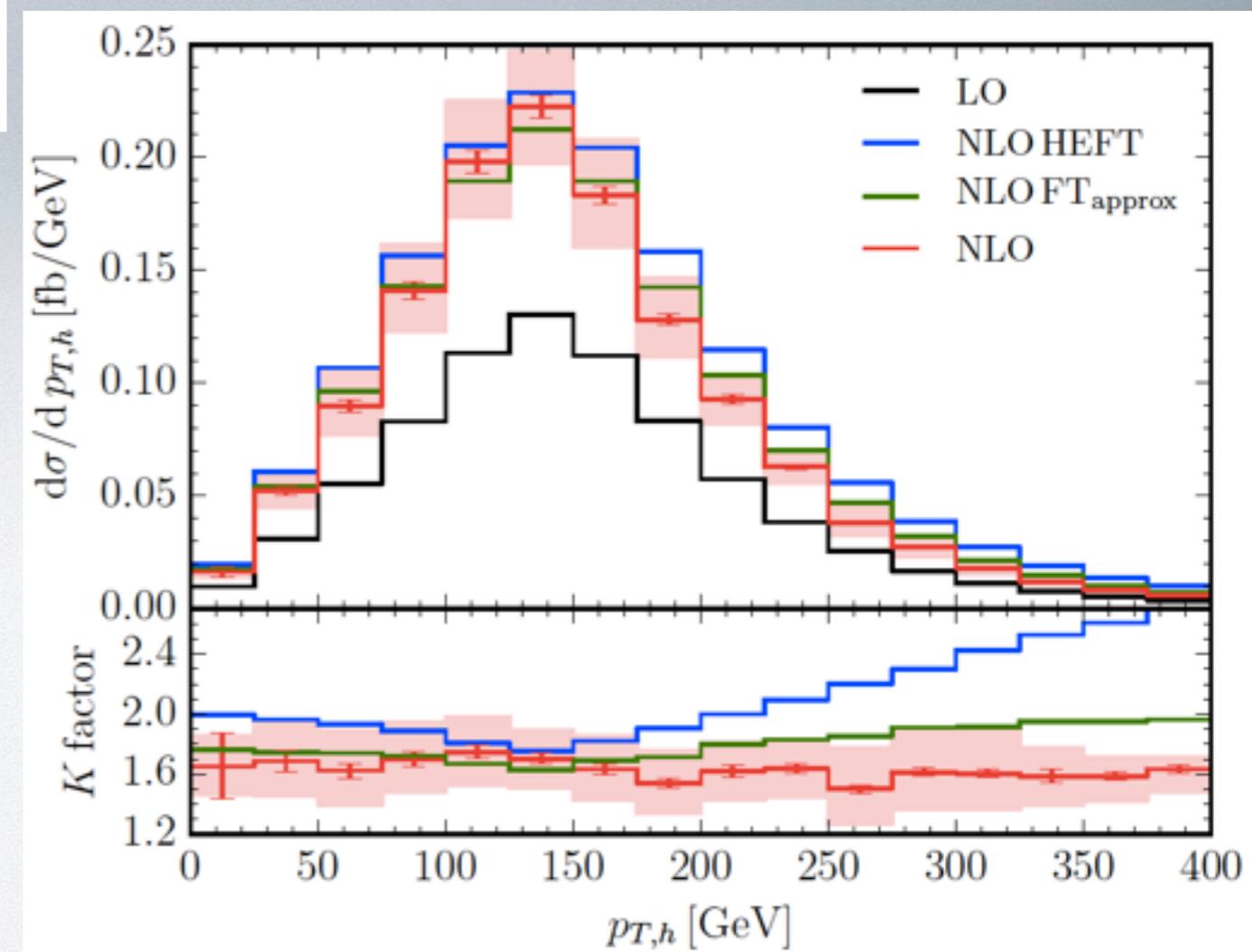
$$\sigma^{LO} = 19.85^{+28\%}_{-21\%} \text{ fb}$$



$m_h=125 \text{ GeV}, m_t=173 \text{ GeV}$

central scale $m_{hh}/2$

FTapprox: full mt dependence in real part only



new: results at various energies

\sqrt{s}	NLO HEFT	NLO FT _{approx}	NLO ^{exact}
7 TeV	$6.44^{+20.1\%}_{-16.9\%}$	$5.95^{+17.3\%}_{-15.7\%}$	$5.80^{+16.3\%}_{-15.2\%}$
8 TeV	$9.37^{+19.8\%}_{-16.5\%}$	$8.61^{+16.7\%}_{-15.1\%}$	$8.35^{+15.7\%}_{-14.6\%}$
13 TeV	$32.22^{+18.2\%}_{-15.1\%}$	$28.90^{+15.0\%}_{-13.4\%}$	$27.72^{+13.7\%}_{-12.7\%}$
14 TeV	$38.29^{+18.1\%}_{-14.8\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$32.80^{+13.4\%}_{-12.5\%}$

if we define

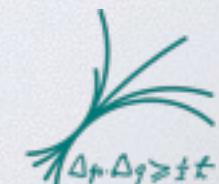
$$\sigma(gg \rightarrow hh)_{NLO}^{exact} = \sigma(gg \rightarrow hh)_{NLO}^{HEFT} (1 + \delta_t)$$

$$\delta_t(7 \text{ TeV}) = -10.04\%$$

$$\delta_t(8 \text{ TeV}) = -10.88\%$$

$$\delta_t(13 \text{ TeV}) = -13.99\%$$

$$\delta_t(14 \text{ TeV}) = -14.38\%$$



comparison to 1/mt expansion

$$V_N = \left(d\hat{\sigma}_{\text{exp},N}^{\text{virt}} + d\hat{\sigma}_{\text{exp},N}^{\text{LO}}(\epsilon) \otimes I \right) \frac{d\hat{\sigma}^{\text{LO}}(\epsilon)}{d\hat{\sigma}_{\text{exp},N}^{\text{LO}}(\epsilon)}$$

$$d\hat{\sigma}_{\text{exp},N} = \sum_{\rho=0}^N d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t} \right)^{2\rho}$$

$$V'_N = V_N \cdot \frac{\text{Born}(m_t)}{\text{Born}_N}$$

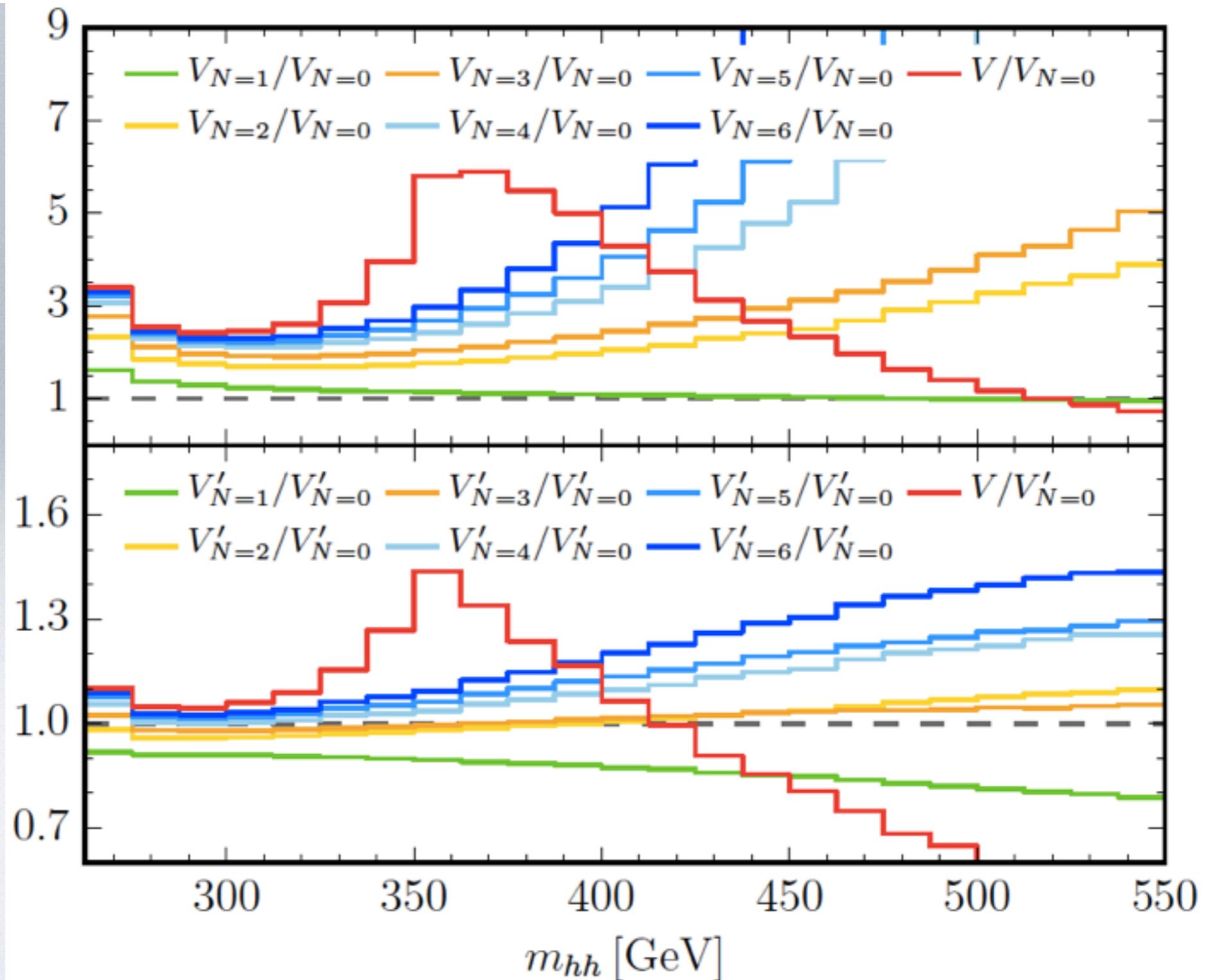
$V_{N=0}$: HEFT

$V'_{N=0}$:

Born-improved HEFT

N<4:
Tom Zirke

N=4,5,6
Jens Hoff



summary and outlook

- stress-testing the SM is a precision game!



summary and outlook

- stress-testing the SM is a precision game!
- NNLO automation is still in its infancy, but growing rapidly



summary and outlook

- stress-testing the SM is a precision game!
- NNLO automation is still in its infancy, but growing rapidly



- numerical methods for 2-loop integrals can prove very useful in cases where analytic results are not available
- method used for gg to HH 2-loop integrals applicable to a range of processes involving several mass scales (also electro-weak)

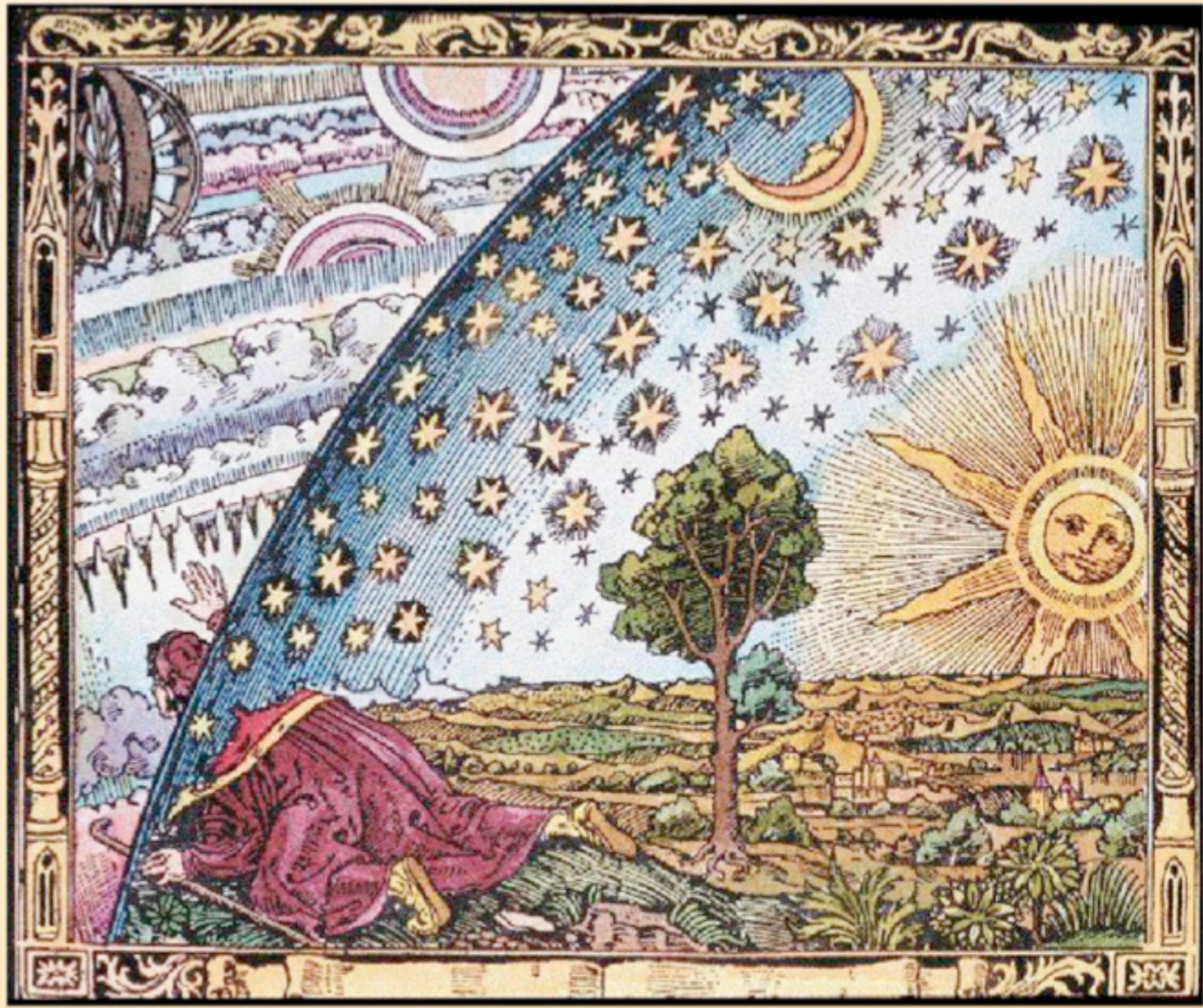




MAX-PLANCK-GESELLSCHAFT



Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)



BACKUP SLIDES

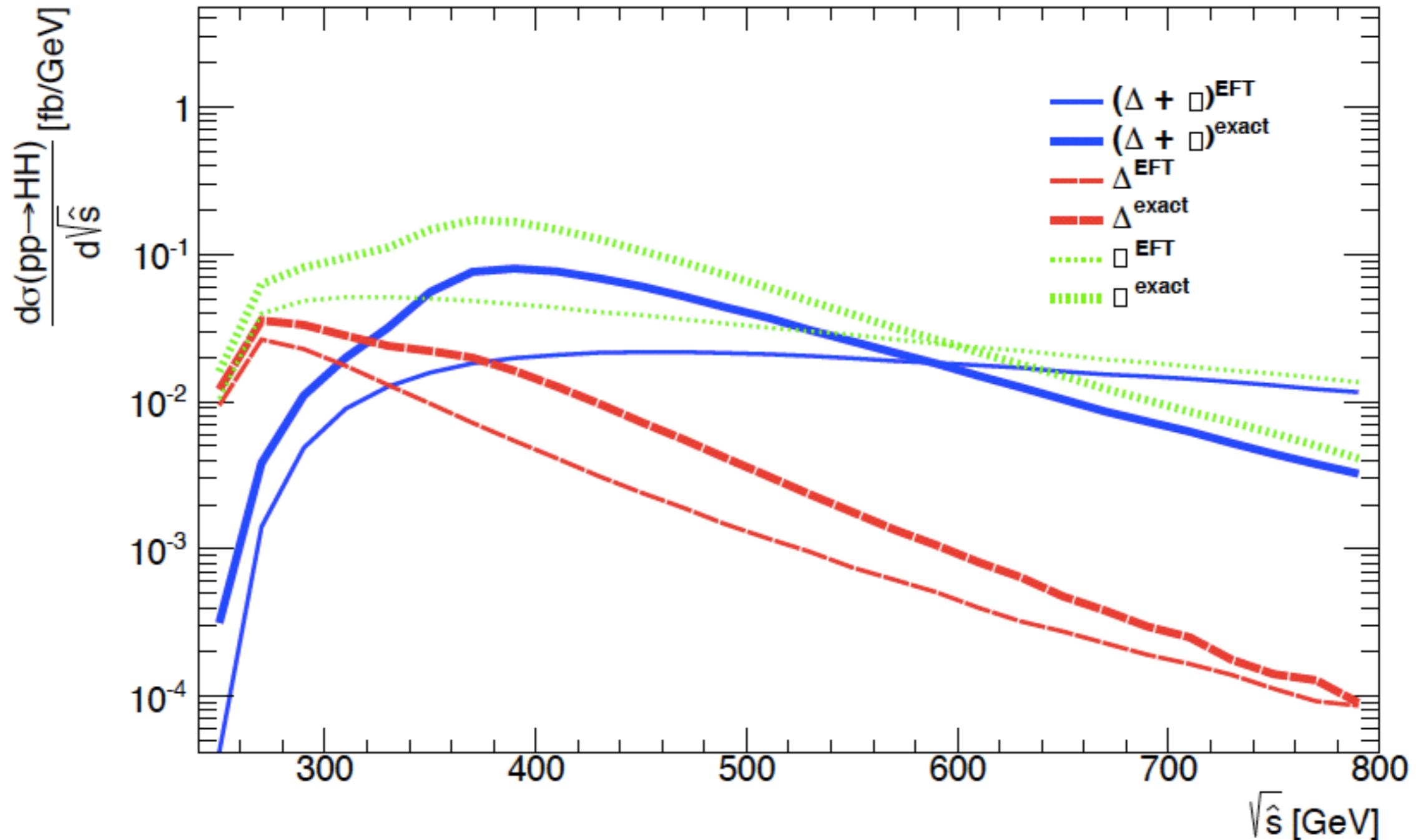


MAX-PLANCK-GESELLSCHAFT



Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

top mass effects at LO



Slawinska, van den Wollenberg, Eijk, Bentvelsen 14



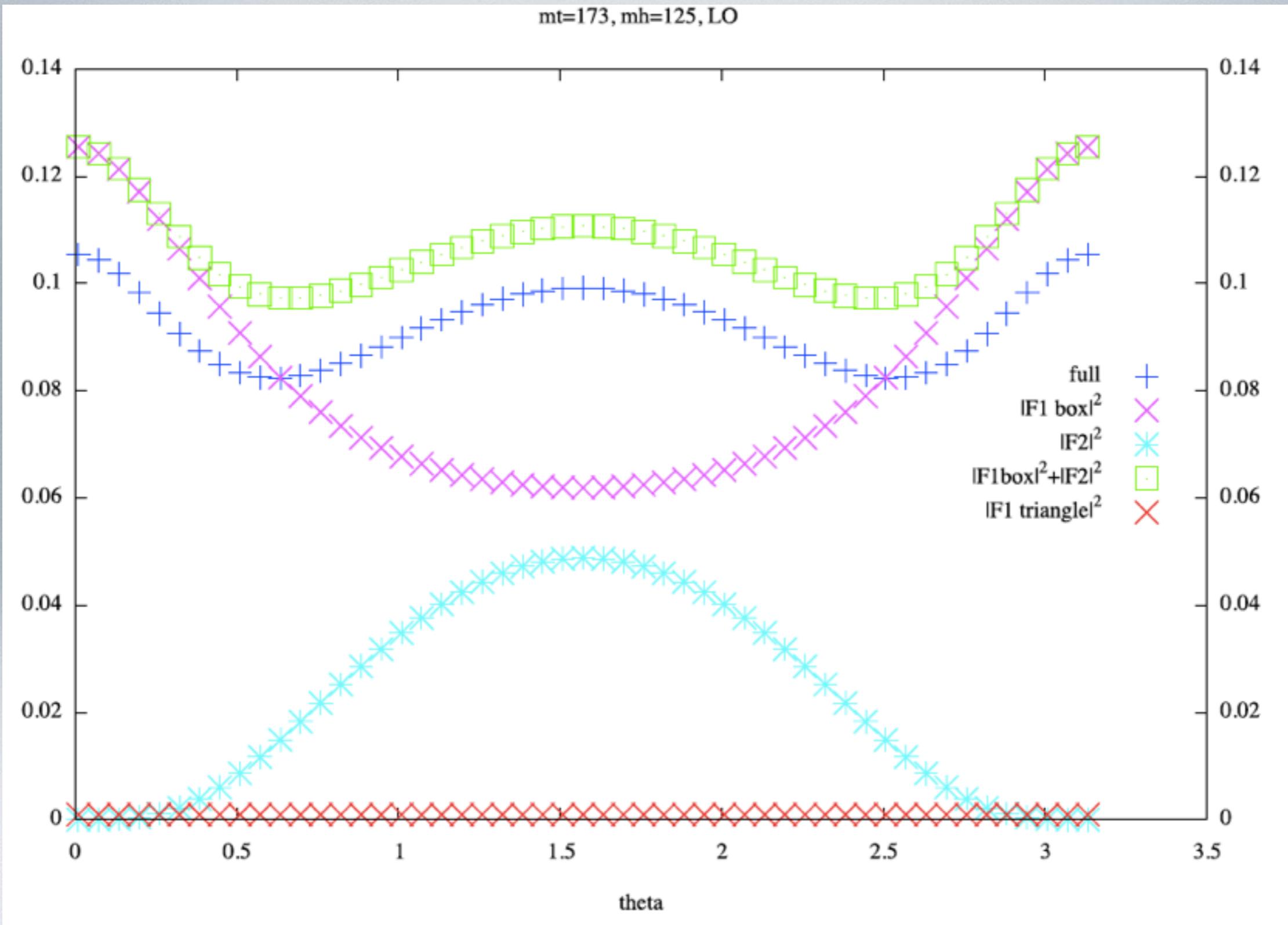
MAX-PLANCK-GESELLSCHAFT



Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

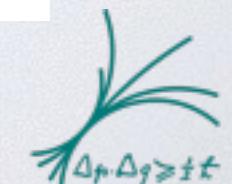
angular dependence at LO

mt=173, mh=125, LO



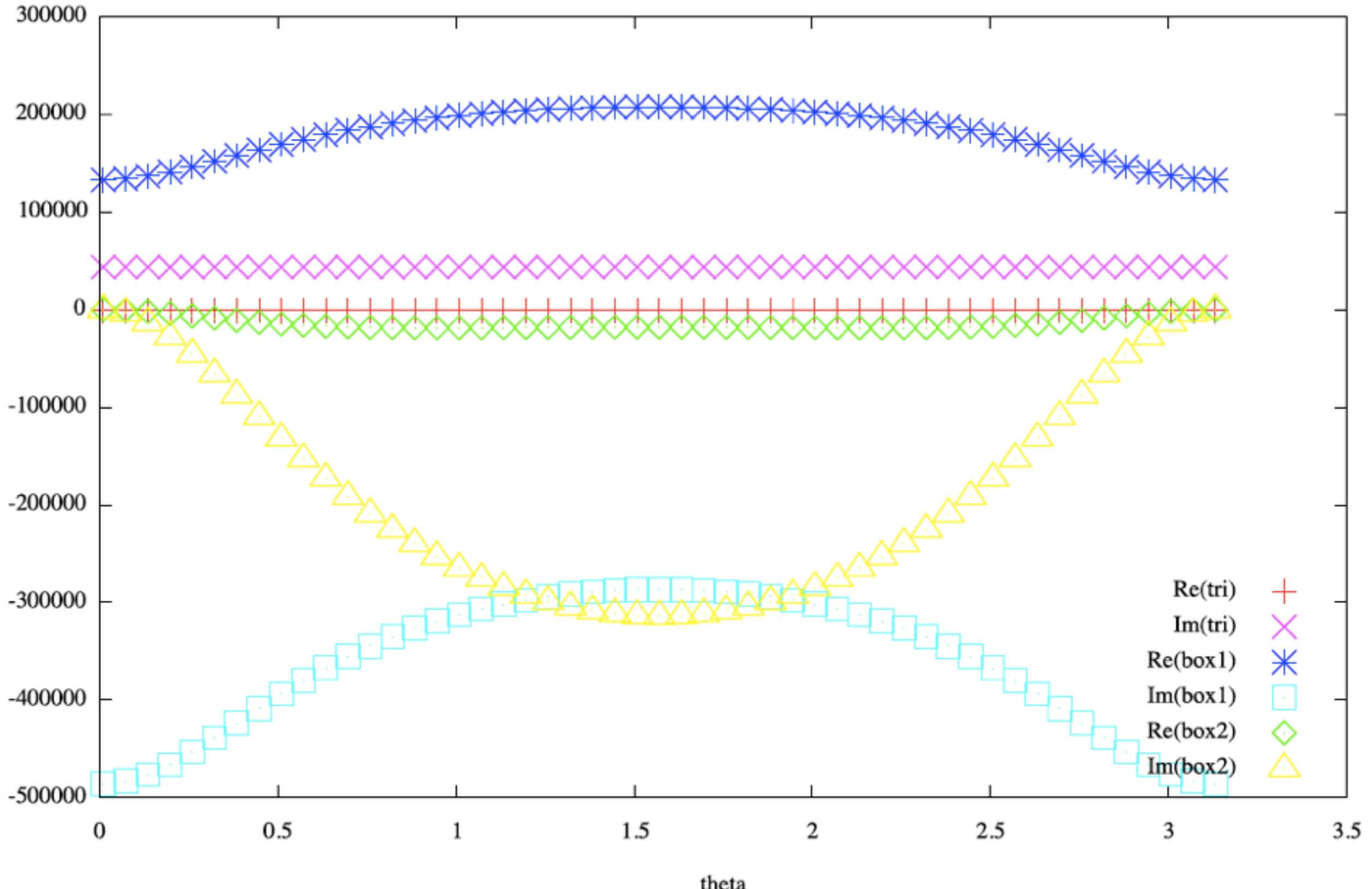
$$t = m_H^2 - \frac{s}{4} (1 - \beta \cos \theta)$$

$$\beta = \sqrt{1 - \frac{4m_H^2}{s}}$$



angular dependence at LO

mt=173, mh=125, LO



Single program to compute **all** coefficients & integrals to obtain **amplitude** to given accuracy

desired precision

list of GPUs & CPUs

```
Amplitude si(epsrel,devinds,crossings);  
  
// coeffs/coeff1.cpp  
si.addTerm(  
    string("ReduzeF1L2_230000010ord0"),  
    ReduzeF1L2_230000010ord0nfunc(),  
    crossing,  
    &ReduzeF1L2_230000010ord0Integrand,  
    &ReduzeF1L2_230000010ord0findoptlam,  
    ReduzeF1L2_230000010ord0ndim(),  
    params,  
    termCoeff1  
);
```

name & reference to integrand to integrate

$(\hat{s}, \hat{t}, m_t^2, m_h^2)$

vector of coefficients

```
// coeffs/coeff204.cpp  
si.addTerm(  
    string("ReduzeF3L2diminc2_131010100ord1"),  
    ReduzeF3L2diminc2_131010100ord1nfunc(),  
    crossing,  
    &ReduzeF3L2diminc2_131010100ord1Integrand,  
    &ReduzeF3L2diminc2_131010100ord1findoptlam,  
    ReduzeF3L2diminc2_131010100ord1ndim(),  
    params,  
    termCoeff2  
);
```

$C_{1,-2}, C_{1,-1}, \dots$ for all Form Factors, evaluated at this phase-space point

Find contour deformation (physical region) in parallel for all integrals in amplitude

```
si.optimizeLambda();  
si.integrate();
```

Computes integrals in parallel on GPUs & CPUs. Dynamically adjusts # points per sector to reduce amplitude error



thanks: Stephen Jones

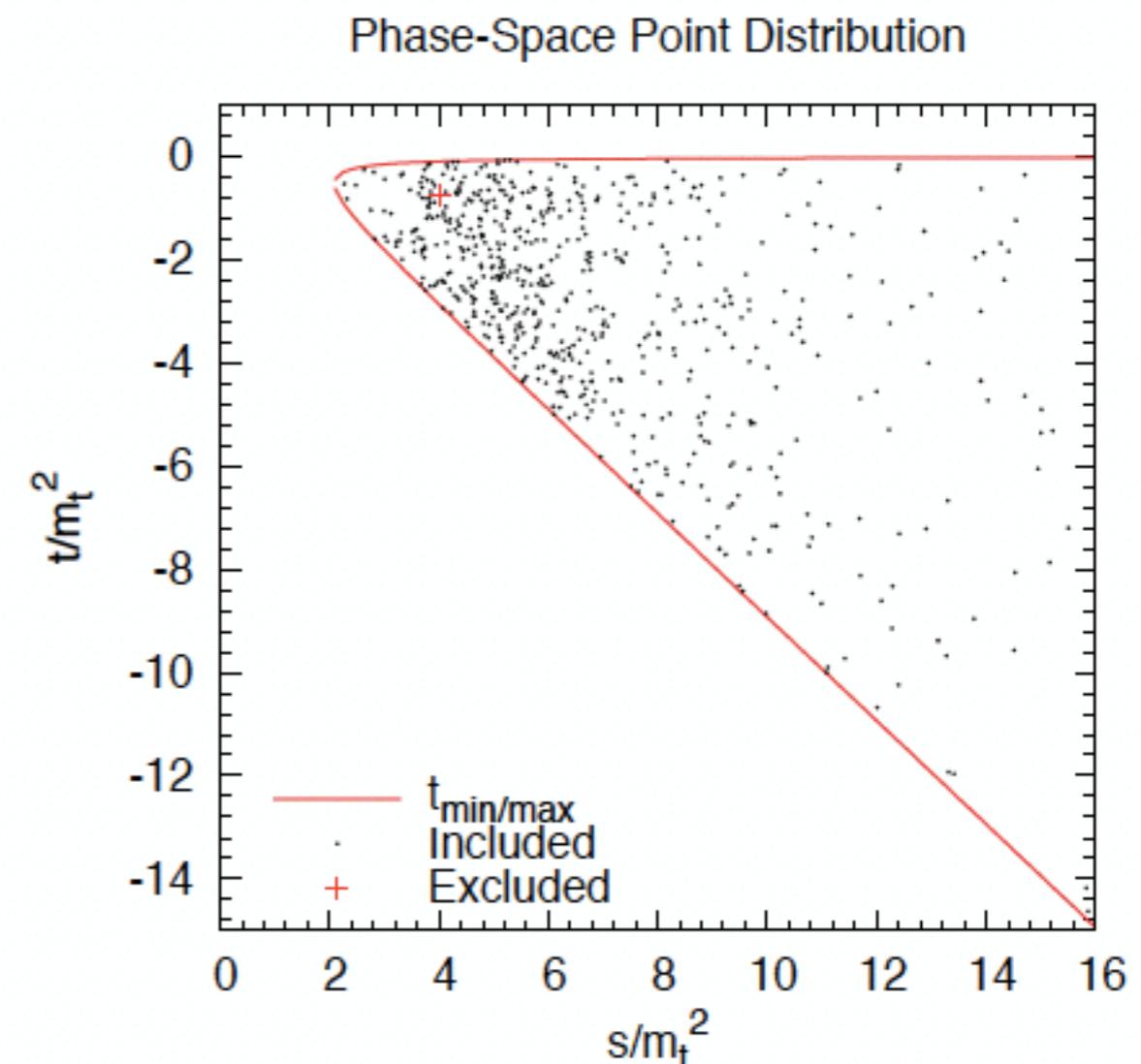


Phase-space implemented by hand

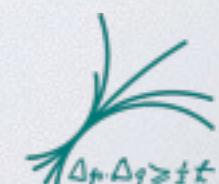
limited to 2-3 w/ 2 massive particles

Events for virtual:

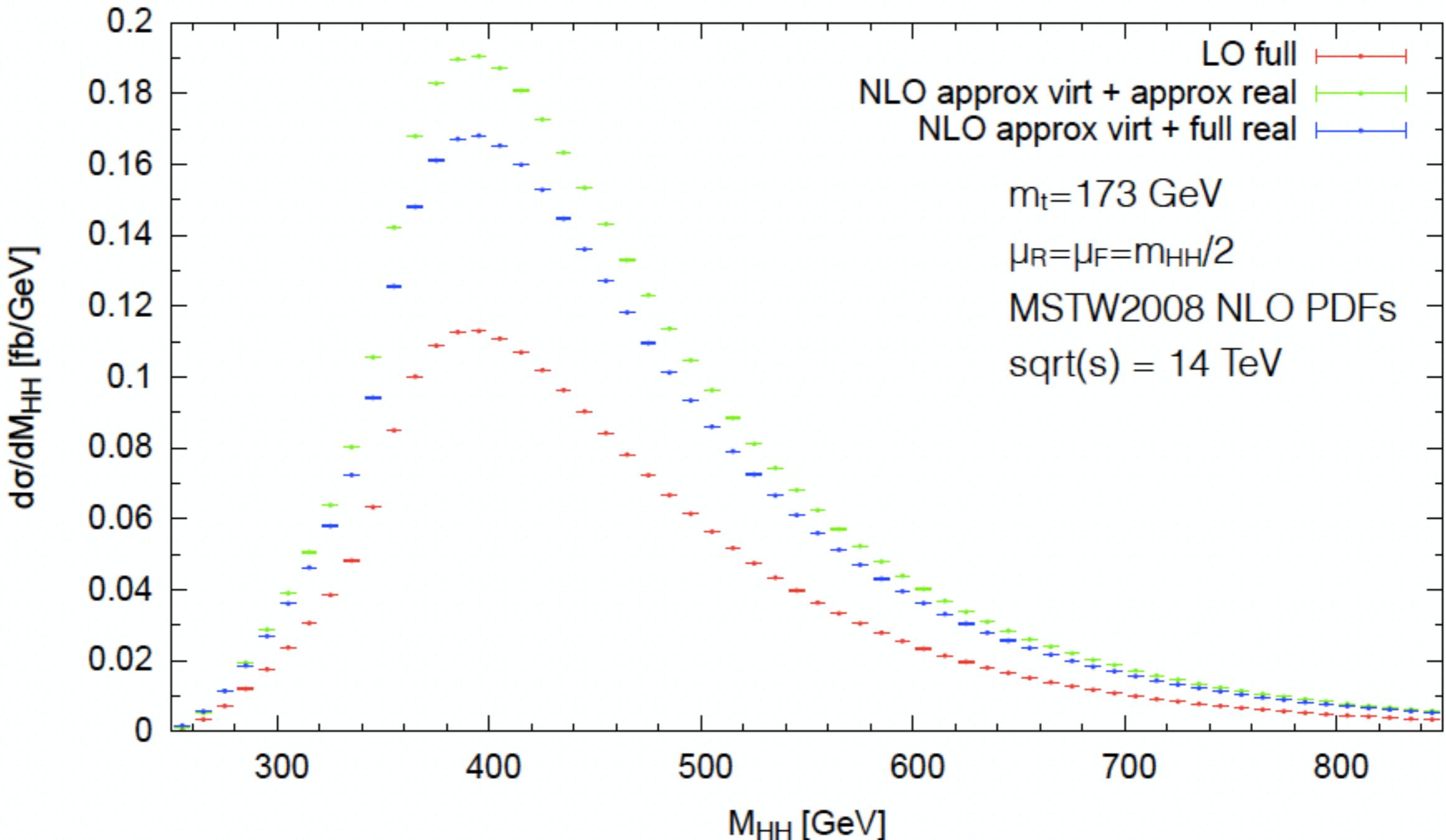
- 1) VEGAS algorithm applied to LO matrix element $\mathcal{O}(100k)$ events computed
- 2) Using LO events unweighted events generated using accept/reject method $\mathcal{O}(30k)$ events selected
- 3) Randomly select 666 Events (woops), compute at NLO, exclude 1



Note: No grids used either for integrals or phase-space



Mass effects in M_{HH} distribution (I)



- „approx“ = rescaled expansion with $N=0$
- Known negative mass effects from real radiation

Mass effects in M_{HH} distribution (II)

