Matching and merging fixed orders and parton showers

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The long road to precise Monte-Carlo simulations



NLO+PS matching – Basics

Leading-order calculation for observable O

$$\langle O \rangle = \int \mathrm{d}\Phi_B \,\mathrm{B}(\Phi_B) \,O(\Phi_B)$$

NLO calculation for same observable

$$\langle O \rangle = \int \mathrm{d}\Phi_B \left\{ \mathrm{B}(\Phi_B) + \tilde{V}(\Phi_B) \right\} O(\Phi_B) + \int \mathrm{d}\Phi_R \,\mathrm{R}(\Phi_R) \,O(\Phi_R)$$

Parton-shower result (zero and one emission)

$$\begin{split} \langle O \rangle &= \int \mathrm{d}\Phi_B \,\mathrm{B}(\Phi_B) \bigg[\Delta^{(\mathrm{K})}(t_c) \,O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \,\mathrm{K}(\Phi_1) \,\Delta^{(\mathrm{K})}(t(\Phi_1)) \,O(\Phi_R) \bigg] \\ &\xrightarrow{\mathcal{O}(\alpha_s)} \int \mathrm{d}\Phi_B \,\mathrm{B}(\Phi_B) \bigg\{ 1 - \int_{t_c} \mathrm{d}\Phi_1 \,\mathrm{K}(\Phi_1) \bigg\} O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_B \,\mathrm{d}\Phi_1 \,\mathrm{B}(\Phi_B) \,\mathrm{K}(\Phi_1) \,O(\Phi_R) \bigg\} \end{split}$$

 $\begin{array}{l} \mbox{Phase space: } \mathrm{d}\Phi_1 = \mathrm{d}t\,\mathrm{d}z\,\mathrm{d}\phi\,J(t,z,\phi) \\ \mbox{Splitting functions: } \mathrm{K}(t,z) \rightarrow \alpha_s/(2\pi t)\sum \mathrm{P}(z)\,\Theta(\mu_Q^2-t) \\ \mbox{Sudakov factors: } \Delta^{(\mathrm{K})}(t) = \exp\Big\{-\int_t\mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\Big\} \end{array}$

[Frixione,Webber] hep-ph/0204244

▶ Subtract $\mathcal{O}(\alpha_s)$ PS terms from subtracted NLO result $(t_c \rightarrow 0)$ 1/N_c corrections faded out in soft region by smoothing function

$$\begin{split} \bar{\mathbf{B}}^{(\mathrm{K})}(\Phi_B) &= \mathbf{B}(\Phi_B) + \tilde{\mathbf{V}}(\Phi_B) + \mathbf{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \left[\mathbf{S}(\Phi_R) - \mathbf{B}(\Phi_B) \, \mathbf{K}(\Phi_1) \right] f(\Phi_1) \\ \mathbf{H}^{(\mathrm{K})}(\Phi_R) &= \left[\mathbf{R}(\Phi_R) - \mathbf{B}(\Phi_B) \, \mathbf{K}(\Phi_1) \right] f(\Phi_1) \end{split}$$

 \blacktriangleright Add parton shower, described by generating functional $\mathcal{F}_{\rm MC}$

$$\langle O \rangle = \int \mathrm{d}\Phi_B \,\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) \,\mathcal{F}_{\mathrm{MC}}^{(0)}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R \,\mathrm{H}^{(\mathrm{K})}(\Phi_R) \,\mathcal{F}_{\mathrm{MC}}^{(1)}(t(\Phi_R), O)$$

Expansion of matched result up to first emission

$$\begin{aligned} \langle O \rangle &= \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) \bigg[\Delta^{(\mathrm{K})}(t_c) O(\Phi_B) \\ &+ \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \Delta^{(\mathrm{K})}(t(\Phi_1)) O(\Phi_r) \bigg] + \int \mathrm{d}\Phi_R \,\mathrm{H}^{(\mathrm{K})}(\Phi_{n+1}) O(\Phi_R) \end{aligned}$$

NLO+PS matching – POWHEG

[Nason] hep-ph/0409146 [Frixione,Nason,Oleari] arXiv:0709.2092

▶ Replace $BK \to R \Rightarrow H^{(R)}$ zero, $\bar{B}^{(R)}$ positive in physical region

$$\begin{split} \langle O \rangle &= \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B) \Bigg[\Delta^{(\mathrm{R})}(t_c, s_{\mathrm{had}}) \, O(\Phi_B) \\ &+ \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R})}(t(\Phi_1), s_{\mathrm{had}}) \, O(\Phi_R) \Bigg] \end{split}$$

- ▶ µ_Q² changed to hadronic centre-of-mass energy squared, s_{had}, to cover full phase space for real-emission correction
- ► Absence of hard events → enhanced high-p_T region (K = B/B) Formally beyond NLO, but often sizeable → Avoid by split R → R^s + R^f

$$\begin{aligned} \langle O \rangle &= \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R}^{\mathrm{s}})}(\Phi_B) \bigg[\Delta^{(\mathrm{R}^{\mathrm{s}})}(t_c, s_{\mathrm{had}}) O(\Phi_B) \\ &+ \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}^{\mathrm{s}}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R}^{\mathrm{s}})}(t(\Phi_1), s_{\mathrm{had}}) O(\Phi_R) \bigg] + \int \mathrm{d}\Phi_R \, \mathrm{R}^{\mathrm{f}}(\Phi_R) \end{aligned}$$

- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
- Parton shower populates soft domain
- ▶ Need criterion to define "hard" & "soft" → jet measure Q and corresponding cut, Q_{cut}



Parton-shower histories

- \blacktriangleright Start with some "core" process for example $e^+e^- \to q\bar{q}$
- ► This process is considered inclusive It sets the resummation scale µ²_Q
- Higher-multiplicity ME can be reduced to core by clustering
- Clustering algorithm uniquely defined by requiring exact correspondence between ME & PS
 - Identify most likely splitting according to PS emission probability
 - Combine partons into mother according to PS kinematics
 - Continue until no clustering possible





[Catani,Krauss,Kuhn,Webber] hep-ph/0109231 [Lönnblad] hep-ph/0112284, arXiv:1211.7204

- Higher-multiplicity MEs that can be reduced to core process are included in core's inclusive cross section (unitarity of PS)
- ► Sudakov suppression factors needed to make inclusive MEs exclusive
- Most efficiently computed with pseudo-showers
 - Start PS from core process
 - ► Evolve until predefined branching ↔ truncated parton shower
 - Emissions producing additional hard jets lead to event veto/weight

$$\Delta^{(\mathrm{K})}(t; >Q_{\mathrm{cut}}) = \exp\left\{-\int_{t} \mathrm{d}\Phi_{1} \,\mathrm{K}(\Phi_{1}) \,\Theta(Q-Q_{\mathrm{cut}})\right\}$$



▶ ME \oplus PS for 0+1-jet in MC@NLO notation

$$\begin{split} \langle O \rangle &= \int \mathrm{d}\Phi_B \mathbf{B}(\Phi_B) \Bigg[\Delta^{(\mathrm{K})}(t_c) \, O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathbf{K}(\Phi_1) \, \Delta^{(\mathrm{K})}(t) \, \Theta(Q_{\mathrm{cut}} - Q) \, O(\Phi_R) \Bigg] \\ &+ \int \mathrm{d}\Phi_R \, \mathbf{R}(\Phi_R) \, \Delta^{(\mathrm{K})}(t(\Phi_R); > Q_{\mathrm{cut}}) \, \Theta(Q - Q_{\mathrm{cut}}) \, O(\Phi_R) + \dots \end{split}$$

- ▶ Reorder by parton multiplicity k, change notation $R_k \rightarrow B_{k+1}$
- Analyze exclusive contribution from k hard partons only $(t_0 = \mu_Q^2)$

$$\begin{aligned} O\rangle_{k}^{\text{excl}} &= \int \mathrm{d}\Phi_{k} \,\mathrm{B}_{k} \,\prod_{i=0}^{k-1} \Delta_{i}^{(\mathrm{K})}(t_{i+1}, t_{i}; \boldsymbol{>} \boldsymbol{Q}_{\text{cut}}) \,\Theta(Q_{k} - Q_{\text{cut}}) \\ &\times \left[\Delta_{k}^{(\mathrm{K})}(t_{c}, t_{k}) \,O_{k} \,+\, \int_{t_{c}}^{t_{k}} \mathrm{d}\Phi_{1} \,\mathrm{K}_{k} \,\Delta_{k}^{(\mathrm{K})}(t_{k+1}, t_{k}) \,\Theta(\boldsymbol{Q}_{\text{cut}} - \boldsymbol{Q}_{k+1}) \,O_{k+1} \right] \end{aligned}$$

ME+PS merging – Unitarization

[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467

Unitarity condition of PS:

$$1 = \Delta^{(K)}(t_c) + \int_{t_c} d\Phi_1 K(\Phi_1) \Delta^{(K)}(t)$$

 ME+PS(@NLO) violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

$$K(\Phi_1) \rightarrow \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)}$$

Can be corrected by explicit subtraction

$$1 = \underbrace{\left\{ \Delta^{(\mathrm{K})}(t_c) + \int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1) - \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)} \right] \Theta(Q - Q_{\mathrm{cut}}) \Delta^{(\mathrm{K})}(t) \right\}}_{\text{unresolved emission / virtual correction}} + \underbrace{\int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1)\Theta(Q_{\mathrm{cut}} - Q) + \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)}\Theta(Q - Q_{\mathrm{cut}}) \right] \Delta^{(\mathrm{K})}(t)}_{\text{resolved emission}}$$



 \blacktriangleright Analyze exclusive contribution from k hard partons

$$\begin{split} \langle O \rangle_{k}^{\text{excl}} &= \int \mathrm{d}\Phi_{k} \,\bar{\mathrm{B}}_{k}^{(\mathrm{K})} \prod_{i=0}^{k-1} \Delta_{i}^{(\mathrm{K})}(t_{i+1}, t_{i}; > Q_{\text{cut}}) \,\Theta(Q_{k} - Q_{\text{cut}}) \\ &\times \left(1 + \frac{\mathrm{B}_{k}}{\bar{\mathrm{B}}_{k}^{(\mathrm{K})}} \sum_{i=0}^{k-1} \int_{t_{i+1}}^{t_{i}} \mathrm{d}\Phi_{1} \mathrm{K}_{i} \,\Theta(Q_{i} - Q_{\text{cut}}) \right) \\ &\times \left[\Delta_{k}^{(\mathrm{K})}(t_{c}, t_{k}) \,O_{k} + \int_{t_{c}}^{t_{k}} \mathrm{d}\Phi_{1} \,\mathrm{K}_{k} \,\Delta_{k}^{(\mathrm{K})}(t_{k+1}, t_{k}) \,\Theta(Q_{\text{cut}} - Q_{k+1}) \,O_{k+1} \right] \\ &+ \int \mathrm{d}\Phi_{k+1} \,\mathrm{H}_{k}^{(\mathrm{K})} \,\Delta_{k}^{(\mathrm{K})}(t_{k}; > Q_{\text{cut}}) \,\Theta(Q_{k} - Q_{\text{cut}}) \,\Theta(Q_{\text{cut}} - Q_{k+1}) \,O_{k+1} \end{split}$$

- $\blacktriangleright \ \ \mathsf{Born} \ \mathsf{matrix} \ \mathsf{element} \to \mathsf{NLO}\text{-weighted} \ \mathsf{Born}$
- Add hard remainder function
- Subtract $\mathcal{O}(\alpha_s)$ terms contained in truncated PS

ME+PS merging – Practical implementations

► LO schemes

Method	Shower Generator	Unitary	References
MLM	Herwig/Pythia	No	[Mangano,Moretti,Pittau] hep-ph/0108069 [Alwall et al.] arXiv:0706.2569
CKKW	Apacic	No	[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
CKKW-L	Ariadne/Pythia	No	[Lönnblad] hep-ph/0112284 [Lönnblad,Prestel] arXiv:1109.4829
METS	Sherpa CSS	No	[Krauss,Schumann,Siegert,SH] arXiv:0903.1219
CKKW'	Herwig++	No	[Hamilton,Richardson,Tully] arXiv:0905.3072
UMEPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.4827 [Plätzer] arXiv:1211.5467

NLO schemes

Method	Shower Generator	Unitary	References
NL^3	Ariadne/Pythia	No	[Lavesson,Lönnblad] arXiv:0811.2912
MEPS@NLO	Sherpa CSS	No	[Krauss,Schönherr,Siegert,SH] arXiv:1207.5030 [Gehrmann,Krauss,Schönherr,Siegert,SH] 5031
FxFx	Herwig(++)/Pythia	No	[Frederix, Frixione] arXiv:1209.6215
UNLOPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.7278

NNLOPS

[Hamilton,Nason,Zanderighi] arXiv:1212.4504 [Hamilton,Nason,Re,Zanderighi] arXiv:1309.0017

- Based on MINLO procedure [Hamilton,Nason,Zanderighi] arXiv:1206.3572
- Extended to NNLL resummation and reweighted to NNLO differentially in Born phase space

UN²LOPS

[Li,Prestel,SH] arXiv:1405.3607

- Based on UNLOPS merging
 [Lönnblad,Prestel] arXiv:1211.7278
- q_T-cutoff technique for NNLO, combined with subtracted MC@NLO for 1-jet contribution





Comparison of approaches



[Les Houches SM WG] arXiv:1605.04692

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[Les Houches SM WG] arXiv:1605.04692

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[Les Houches SM WG] arXiv:1605.04692



NNLO+PS for WH production



- Using NNLOPS technique
- Small corrections on Born-type observables, but visible effects e.g. on jet veto xs



[Frederix,Hamilton] arXiv:1512.02663

- First method to combine NNLO matched simulation with higher-multiplicity NLO matched simulation
- NNLOPS for H/HJ and modified MINLO for HJJ production



Electroweak merging in Sherpa



[Kallweit,Lindert,Maierhöfer,Pozzorini,Schönherr] arXiv:1511.08692



- QCD parton shower merged with QCD+EW matrix elements
- Implemented in Sherpa +OpenLoops framework

Electroweak merging in Pythia

[Christiansen, Prestel] arXiv:1510.01517

- ▶ QCD+EW parton shower implemented in Pythia 8
- Consistently merged to QCD+EW LO matrix elements



NLO merging in Herwig++

[Bellm] PhD thesis

Unitarized LO and NLO merging implemented in Herwig++



Matrix-element corrections in Vincia

[Fischer, Prestel, Ritzmann, Skands] arXiv:1605.06142

- ► Alternative to merging → iterative matrix-element corrections
- Implemented in Vincia for both final- and initial-state shower



- Matching & merging a very active field
- Most methods come with uncertainty estimates
- Number of tools allows to eliminate "theory bias"
- ► There is no perfect method just yet ...