Techniques for NNLO precision

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Stress-testing the Standard Model at the LHC

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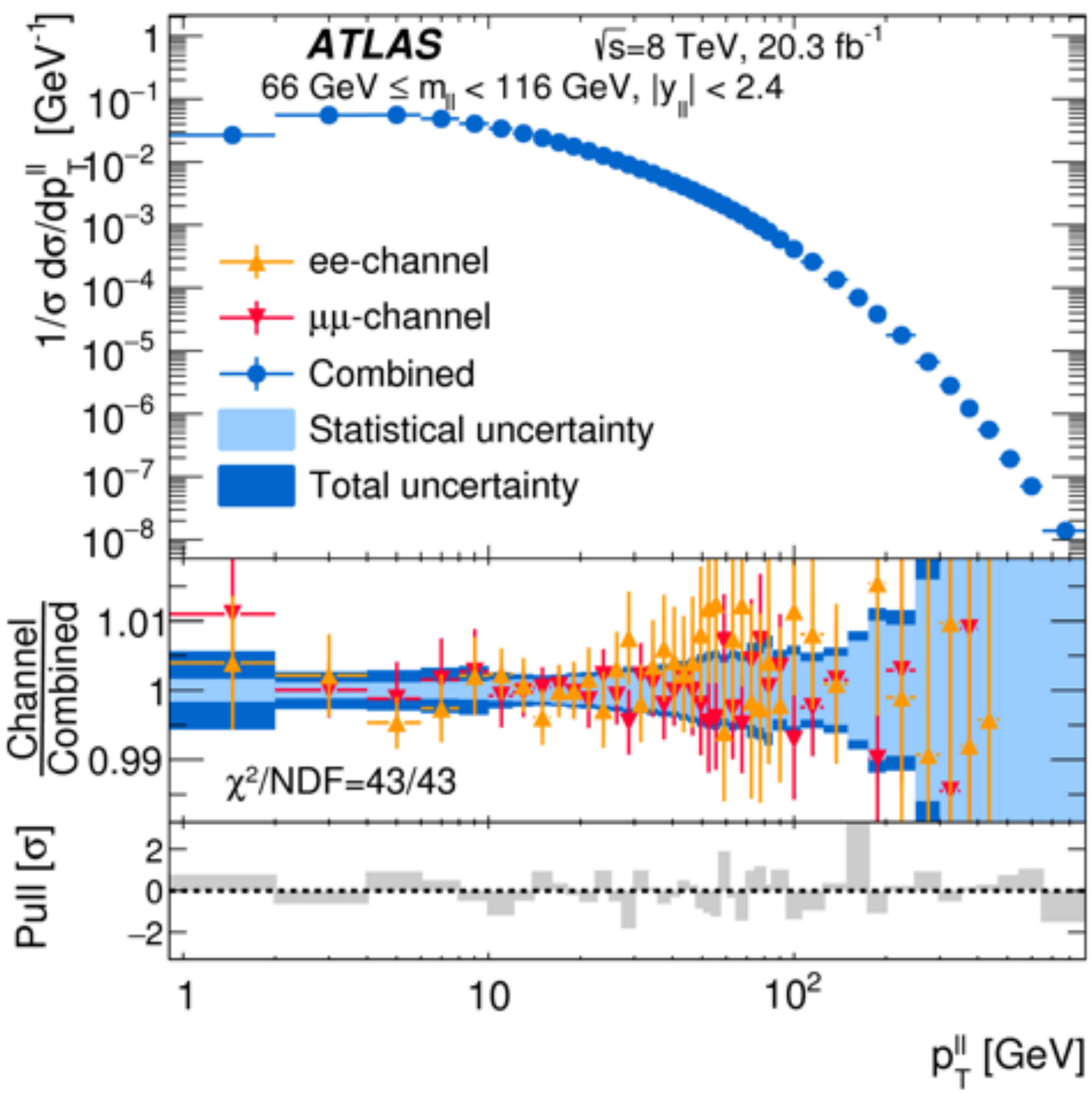
Outline

- Introduction: the need for NNLO precision
- The components of a NNLO calculation
- Subtraction schemes at NNLO
- N-jettiness as a subtraction scheme
- Future directions

Why we need NNLO precision

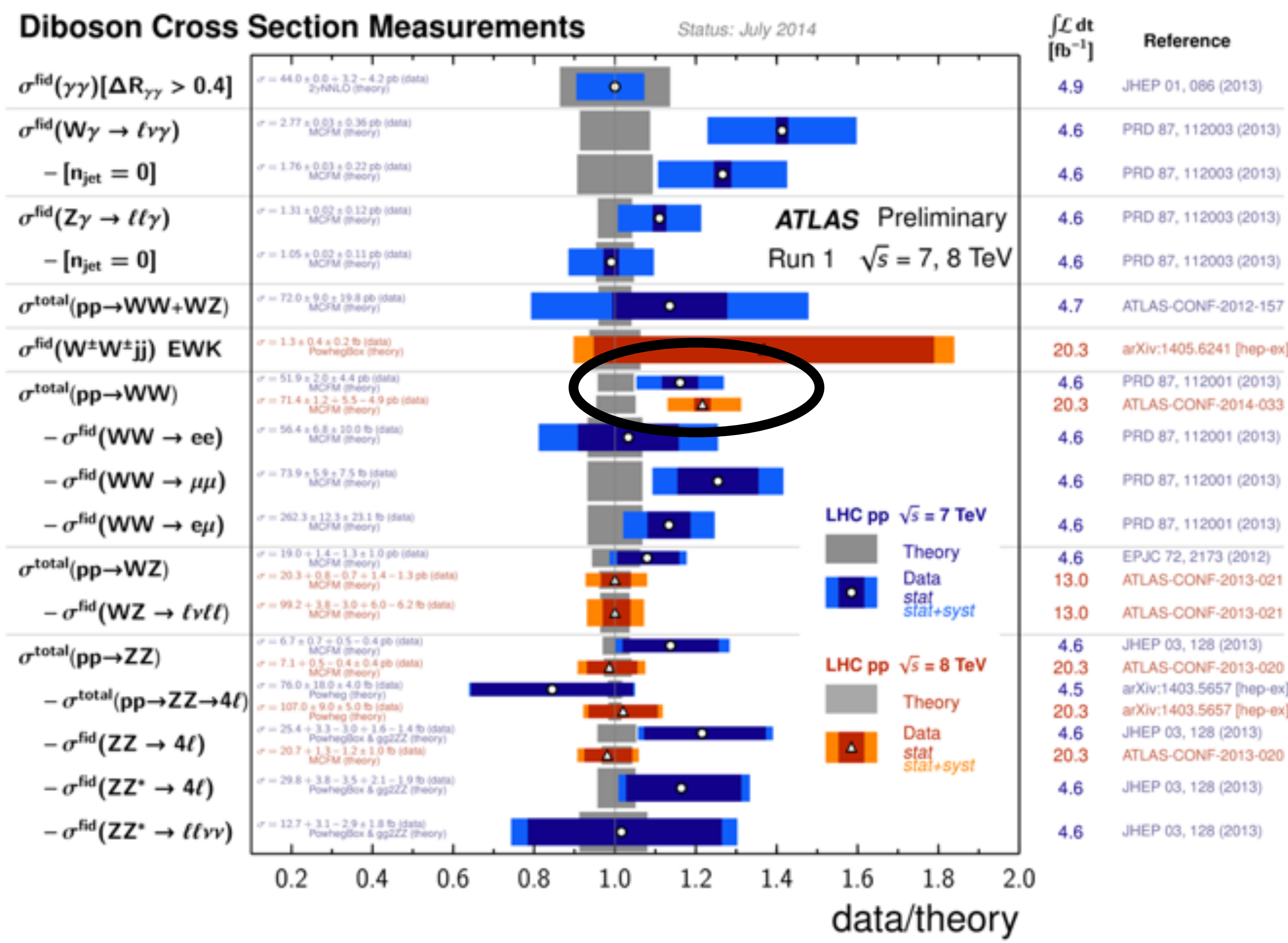
- The experimental precision achieved at the LHC in multiple channels is challenging our ability to calculation precisely enough to compare theory to data



- Experimental errors on inclusive p_T spectrum of lepton pairs in Z-production under 1%
- Important input into PDF fits; detector calibration
- Scale uncertainties at NLO at the 10% level

Why we need NNLO precision

- NNLO is needed to avoid misinterpretation of experimental data
- An example: the WW cross section

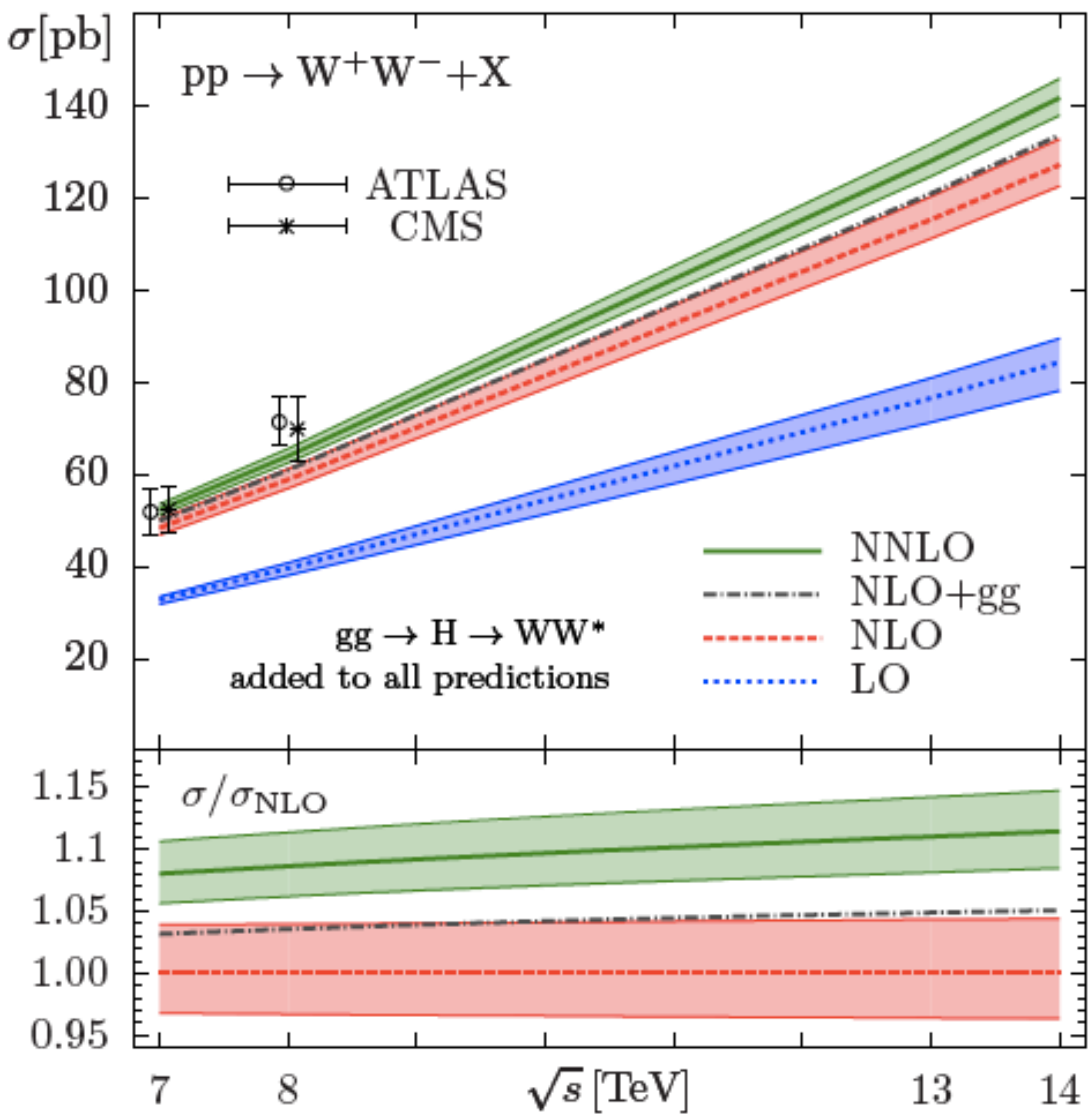


Could it be light charginos?

(Curtin, Jaiswal, Meade 1206.6888, and others)

The WW cross section now

- Sizable NNLO QCD corrections! Theory within 1σ agreement of ATLAS and CMS for both CM energies   Gehrmann, Grazzini, Kallweit, Maierhofer, von Manteuffel, Pozzorini, Rathlev, Tancredi (2014)

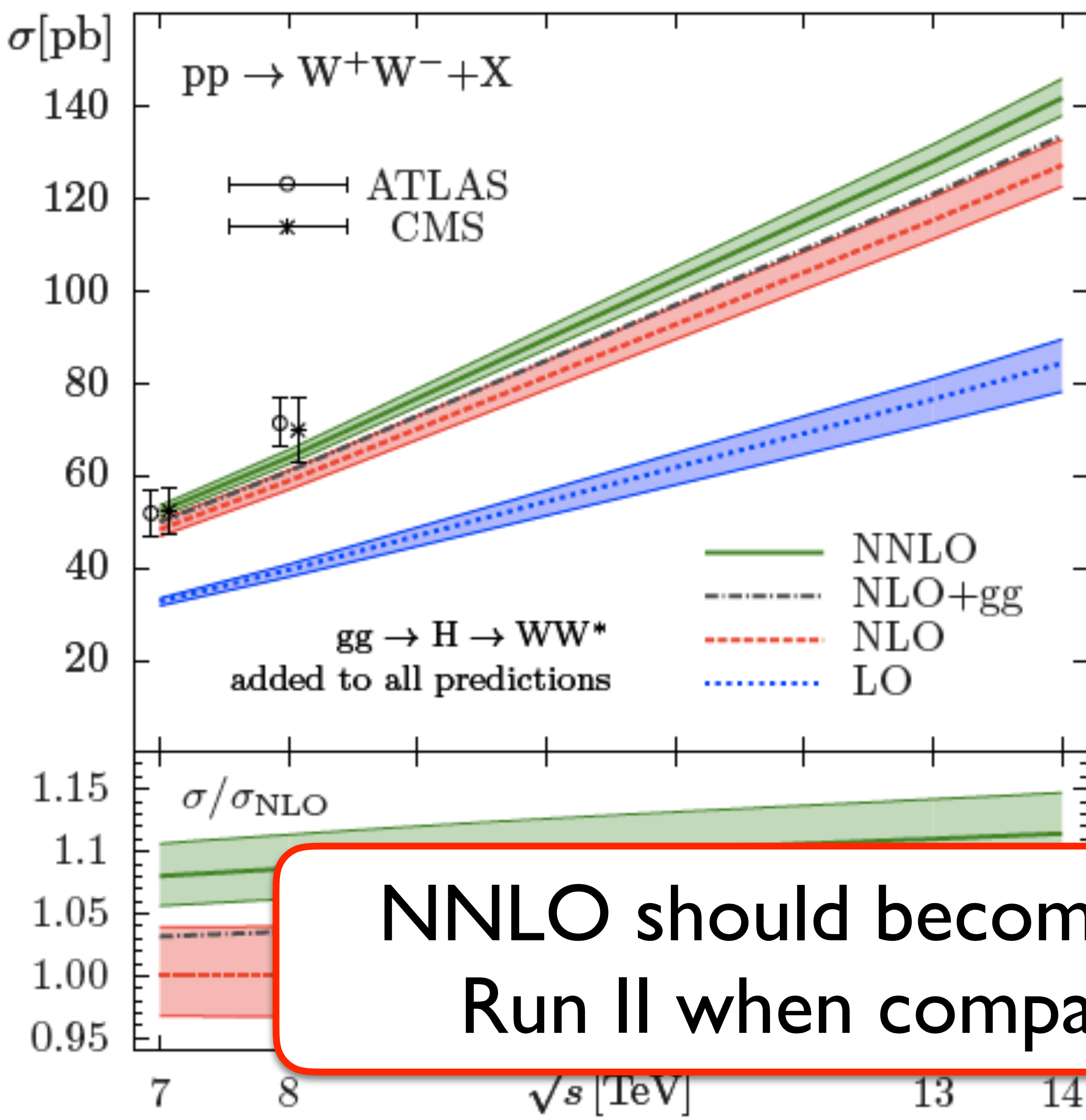


- Enhancement of theory is expected when the extrapolation from the fiducial region is properly modeled, further improving agreement

Monni, Zanderighi (2014) (see also Jaiswal, Okui, Curtin, Meade, Tien (2014))

The WW cross section now

- Sizable NNLO QCD corrections! Theory within 1σ agreement of ATLAS and CMS for both CM energies Gehrmann, Grazzini, Kallweit, Maierhofer, von Manteuffel, Pozzorini, Rathlev, Tancredi 1408.5243



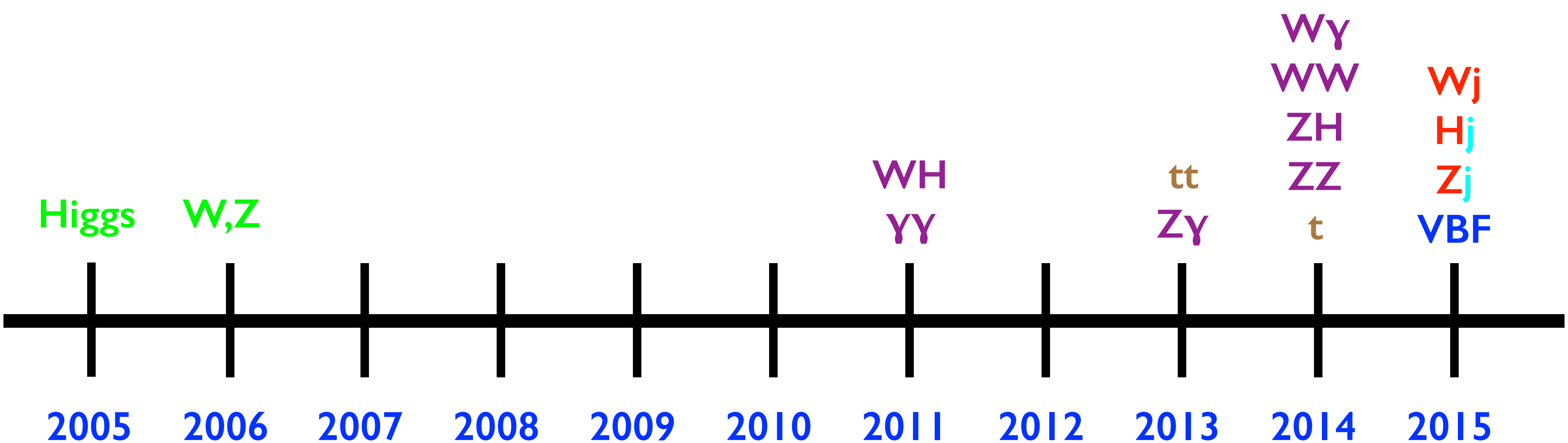
- Enhancement of theory is expected when the extrapolation from the fiducial region is properly modeled, further improving agreement

Monni, Zanderighi 1410.4745 (see also Jaiswal, Okui, 1407.4537; Curtin, Meade, Tien 1406.0848)

NNLO should become the standard during Run II when comparing theory to data

A timeline of NNLO hadron collider cross sections

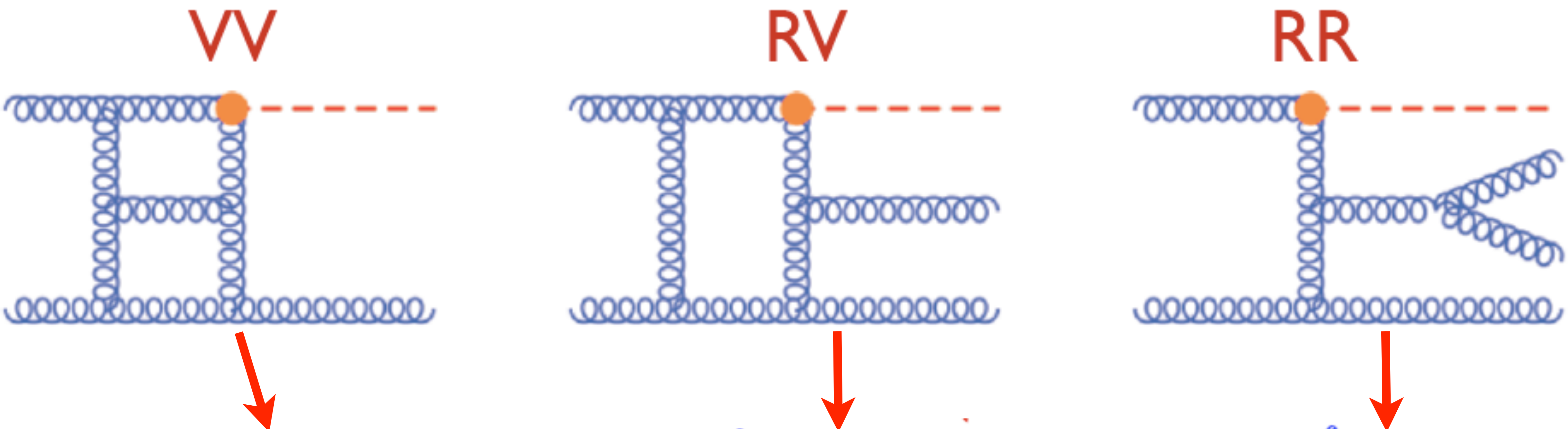
- Complete NNLO hadron-collider cross sections with control over kinematics:



NNLO becoming available for 2→2 scattering over the past few years

Fixed-order cross sections at NNLO

- Need the following ingredients for NNLO cross sections:



$$\int \left[\frac{vv_4}{\epsilon^4}+\frac{vv_3}{\epsilon^3}+\frac{vv_2}{\epsilon^2}+\frac{vv_1}{\epsilon}+vv_0\right]\, d\Phi_2 \qquad \int \left[\frac{rv_2}{\epsilon^2}+\frac{rv_1}{\epsilon}+rv_0\right]\, d\Phi_3 \qquad \int [rr_0]\, d\Phi_4$$

- In principle this is straightforward: draw all diagrams and calculate. In practice, it is complicated by by the implicit poles in the real radiation corrections that only appear after integration over phase space.

- A generic, working procedure to extract infrared singularities in the presence of final-state jets was unknown for many years.

Double virtual corrections

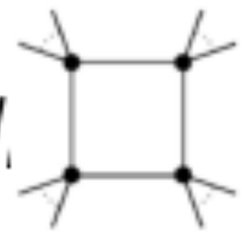
- While still computed process-by-process, many results needed for LHC phenomenology are known. Recent new insight into how to cleverly organize such calculations.

See talks by Heinrich, Weinzierl

- 2→2 massless parton scattering: Bern, Dixon, Kosower; Anastasiou, Glover, Oleari, Tejeda-Yeomans (2000-2002)
- qqbar→V+jet: Garland, Gehrmann, Koukoutsakis, Glover (2001-2002); Gehrmann, Tancredi (2011)
- gg→H+jet: Gehrmann, Jaquier, Koukoutsakis, Glover (2011); see also Becher, Bell, Lorentzen, Marti (2013)
- qqbar→VV, V*V*: Gehrmann, Manteuffel, Tancredi (2014-2015); Caola, Henn, Melnikov, Smirnov (2014-2015)
- qqbar→ttbar, gg→ttbar: Czakon, Fiedler, Mitov (2013); Bonciani, Ferroglia, Gehrmann, Manteuffel, Studerus (2008-2011)
- gg→ggg: Badger, Mogull, Ochirov, O'Connell (2015); Gehrmann, Henn, Lo Presti (2015)

Real-virtual corrections

- An appropriate basis for one-loop integrals is well-known.

$A = \sum d_i$  $+ \sum c_i$  $+ \sum b_i$  $+ R$

- Numerical libraries exist for the evaluation of the scalar basis integrals
- Both traditional reduction and unitarity-based methods can obtain the coefficients of the basis integrals.

Must evaluate these one-loop corrections in the singular limit of the external legs. Stability of existing NLO tools must be investigated when used for NNLO RV.

Infrared organization at NLO

- Well-honed techniques for calculating and combining real+virtual at NLO
- To deal with IR singularities of real emission, subtraction schemes are normally used (dipole subtraction (Catani, Seymour), FKS subtraction (Frixione, Kunszt, Signer), antennae subtraction (Kosower; Campbell, Cullen, Glover))

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO}\right]$$

Simple enough to integrate analytically so that 1/ε poles can be cancelled against virtual corrections

Approximates real-emission matrix elements in all singular limits so this difference is numerically integrable

Natural to attempt to extend this approach to NNLO

Subtraction at NNLO

- The generic form of an NNLO subtraction scheme is the following:

$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} \,,
\end{aligned}
$$

- Maximally singular configurations at NNLO can have two collinear, two soft singularities
- Subtraction terms must account for all of the many possible singular configurations: triple-collinear ($p_1||p_2||p_3$), double-collinear ($p_1||p_2,p_3||p_4$), double-soft, single-soft, soft +collinear, etc.

- The factorization of the matrix elements in all singular configurations is known in the literature

The triple-collinear example

- To illustrate the problems that occur when trying to use these formulae, consider the triple-gluon collinear limit. The factorization of the matrix element squared in this limit is the following.

$$|\mathcal{M}(\ldots,p_1,p_1,p_3)|^2 \approx \frac{4g_s^4}{s_{123}^2}\mathcal{M}^{\mu}(\ldots,p_1+p_2+p_3)\mathcal{M}^{\nu *}(\ldots,p_1+p_2+p_3)\,P^{\mu\nu}_{g_1g_2g_3}$$

$z_i = E_i/(\sum E_j)$

$$\begin{aligned}
\hat{P}^{\mu\nu}_{g_1g_2g_3} &= C_A^2\left\{\frac{(1-\epsilon)}{4s_{12}^2}\left[-g^{\mu\nu}t^2_{12,3}+16s_{123}\frac{z_1^2z_2^2}{z_3(1-z_3)}\left(\frac{\widetilde{k}_2}{z_2}-\frac{\widetilde{k}_1}{z_1}\right)^{\mu}\left(\frac{\widetilde{k}_2}{z_2}-\frac{\widetilde{k}_1}{z_1}\right)^{\nu}\right]\right.\\
&- \frac{3}{4}(1-\epsilon)g^{\mu\nu}+\frac{s_{123}}{s_{12}}g^{\mu\nu}\frac{1}{z_3}\left[\frac{2(1-z_3)+4z_3^2}{1-z_3}-\frac{1-2z_3(1-z_3)}{z_1(1-z_1)}\right]\\
&+ \frac{s_{123}(1-\epsilon)}{s_{12}s_{13}}\left[2z_1\left(\widetilde{k}_2^{\mu}\widetilde{k}_2^{\nu}\frac{1-2z_3}{z_3(1-z_3)}+\widetilde{k}_3^{\mu}\widetilde{k}_3^{\nu}\frac{1-2z_2}{z_2(1-z_2)}\right)\right.\\
&+ \frac{s_{123}}{2(1-\epsilon)}g^{\mu\nu}\left(\frac{4z_2z_3+2z_1(1-z_1)-1}{(1-z_2)(1-z_3)}-\frac{1-2z_1(1-z_1)}{z_2z_3}\right)\\
&+ \left.\left.\left(\widetilde{k}_2^{\mu}\widetilde{k}_3^{\nu}+\widetilde{k}_3^{\mu}\widetilde{k}_2^{\nu}\right)\left(\frac{2z_2(1-z_2)}{z_3(1-z_3)}-3\right)\right]\right\}+(5\text{ permutations})\ .
\end{aligned}$$

Catani, Grazzini (1999)

Overlapping singularities

- To illustrate the problems that occur when trying to use these formulae, consider the triple-gluon collinear limit. The factorization of the matrix element squared in this limit is the following.

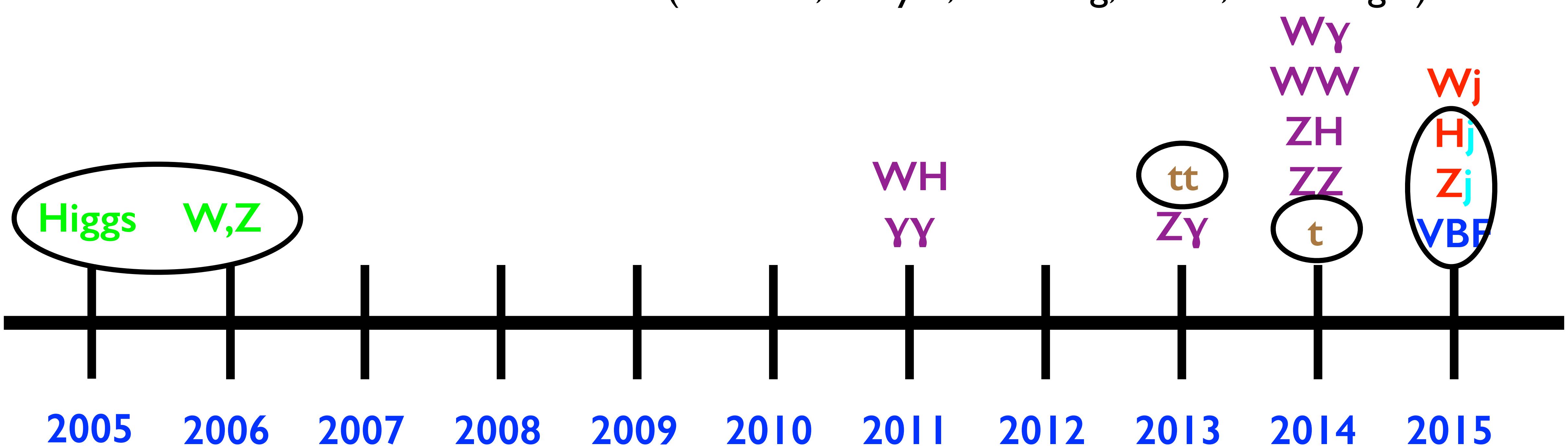
$$|\mathcal{M}(\ldots,p_1,p_1,p_3)|^2 \approx \frac{4g_s^4}{s_{123}^2}\mathcal{M}^{\mu}(\ldots,p_1+p_2+p_3)\mathcal{M}^{\nu *}(\ldots,p_1+p_2+p_3)\,P^{\mu\nu}_{g_1 g_2 g_3}$$

- When one introduces an explicit parameterization: $s_{123} \sim E_1E_2(1-c_{12})+E_1E_3(1-c_{13})+E_2E_3(1-c_{23})$
- What goes to zero quicker? $E_1, E_2, E_3, (1-c_{12}), (1-c_{13})$, or $(1-c_{23})$?

Need a systematic technique for handling such singularities

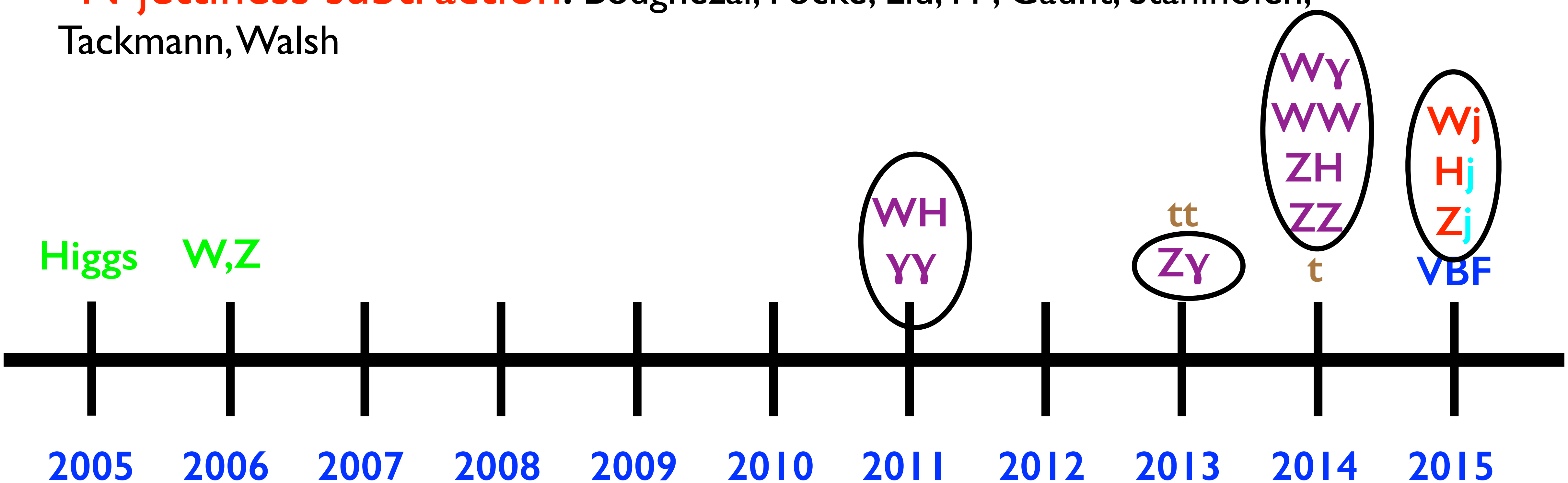
Subtraction methods at NNLO

- **Sector decomposition**: an algorithm to disentangle the overlapping singularities and extract poles as a Laurent expansion. Coefficients of the poles are computed numerically (Binoth, Heinrich; Anastasiou, Melnikov, FP)
- **Sector-improved residue subraction**: a combination of sector decomposition and FKS-style partitioning. The necessary sectors become process-independent (Czakon; Boughezal, Melnikov, FP)
- **Antennae subtraction**: uses physical matrix elements that encapsulate the singular limits of matrix elements; needs an azimuthal averaging in the final state (Kosower; Gehrmann, Gehrmann-de Ridder, Glover)
- Colorful subtraction: fully-local scheme with analytic determination of poles (Del Duca, Somogyi, Trocsanyi)
- **Projection-to-Born**: specialized technique for VBF that relies upon knowledge of the inclusive structure function at NNLO (Cacciari, Dreyer, Karlberg, Salam, Zanderighi)



RG-assisted methods at NNLO

- Resummation-assisted techniques: leverage knowledge of analytic resummation to remove double-real emission singularities.
  - q_T-subtraction: Catani, Grazzini
  - N-jettiness subtraction: Boughezal, Focke, Liu, FP; Gaunt, Stahlhofen, Tackmann, Walsh



RG-assisted methods are responsible for significant recent progress and numerous phenomenological results (see Boughezal's talk on Monday). Will be the focus of the remainder of the talk.

First realization: q_T-subtraction

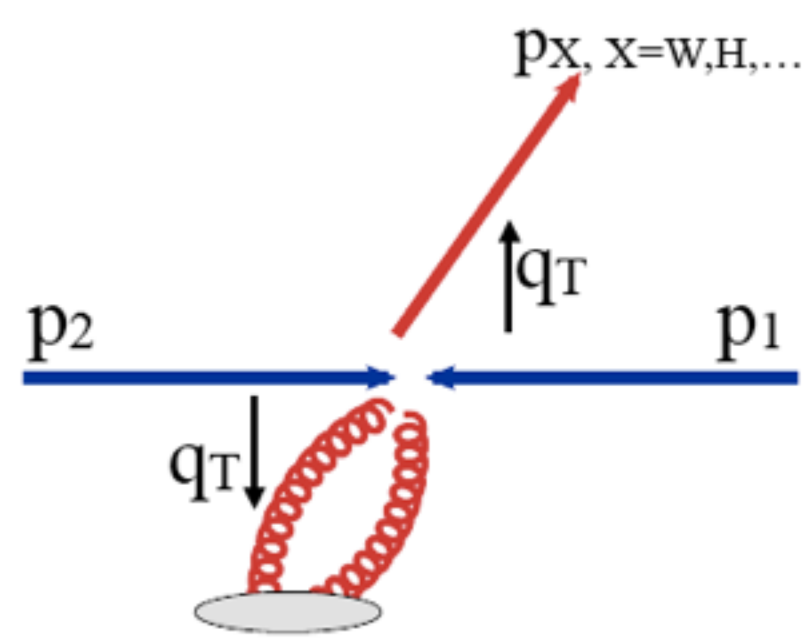
- For color-neutral final states, the transverse momentum completely determines the double-unresolved singular limit of the final state
Catani, Grazzini (2007)

$$\sigma_{NNLO} = \int dq_T \, \frac{d\sigma}{dq_T} \, \theta(q_T^{\text{cut}} - q_T) + \int dq_T \, \frac{d\sigma}{dq_T} \theta(q_T - q_T^{\text{cut}})$$

Obtained from the Collins-Soper-Sterman factorization theorem for small-q_T

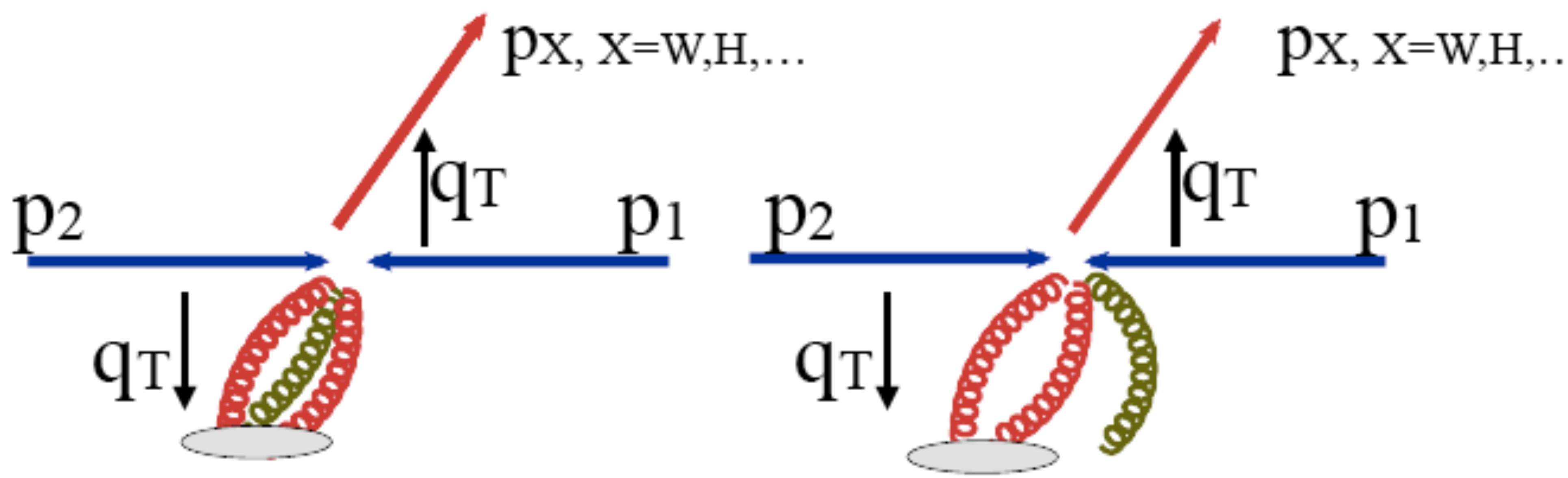
This is an NLO cross section for the color-neutral final state plus an additional jet

- Maximally reuses information from existing NLO calculations



q_T-subtraction

- Difficult to extend beyond color-neutral final states; q_T no longer separates double-unresolved limit from NLO singular limits



double-unresolved

single-unresolved

Need a different resolution parameter when final-state jets are present!

Two requirements: must separate double-unresolved limits from single-unresolved and hard regions, and must be resummable (or, must know how to analytically obtain the result below the resolution parameter)

N-jettiness

- N-jettiness, τ_N, is an event-shape variable designed to veto final-state jets Stewart, Tackmann, Waalewijn 0910.0467

$$\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}.$$

N=number of final-state jets

Momenta of the two beams and the final-state jets

Measure of the jet hardness (we take $Q_i=2E_i$)

All final-state partons

$\tau_N=0$: all radiation is either soft, or collinear to a beam/jet; at NNLO, gives the double-unresolved limit

$\tau_N>0$: at least one additional radiation is resolved; have N+1 final-state jets

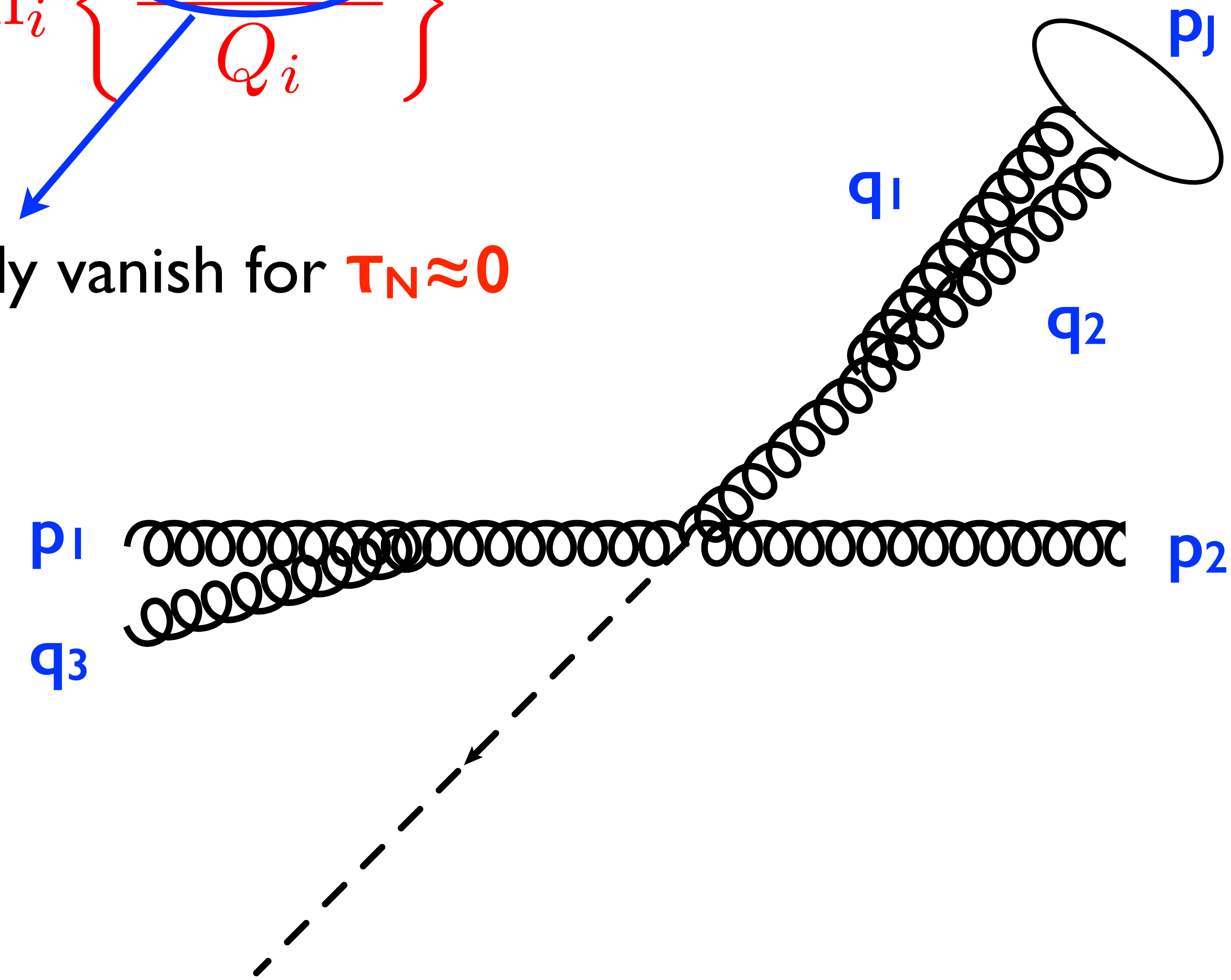
This is the resolution parameter we are looking for!

N-jettiness

$\tau_N \approx 0$: all radiation is either soft, or collinear to a beam/jet; at NNLO, gives the double-unresolved limit

$$\tau_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}$$

Must all nearly vanish for **$\tau_N \approx 0$**

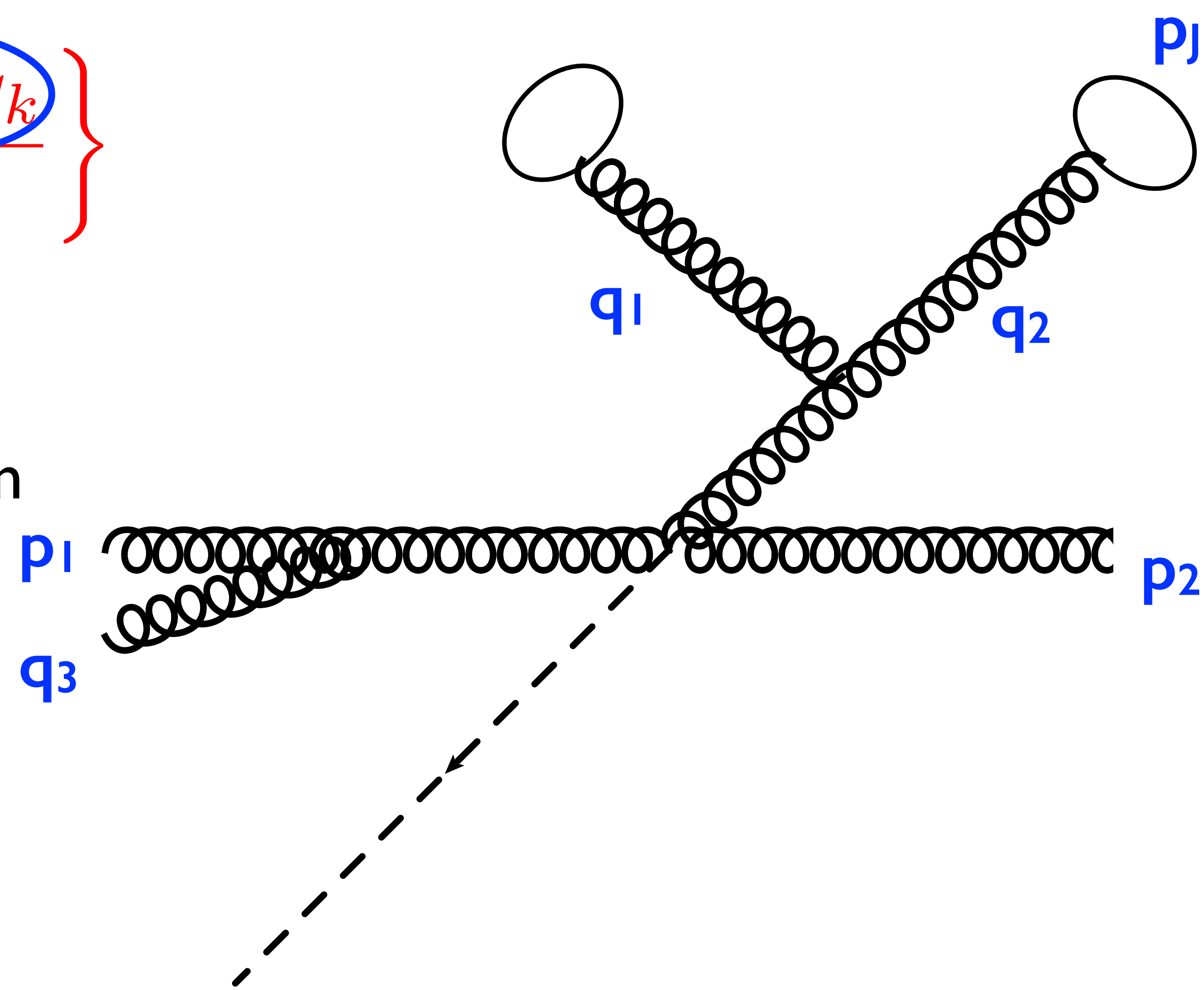


N-jettiness

$\tau_N > 0$: at least one additional radiation is resolved; have N+1 final-state jets

$$\tau_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}$$

Must have at least one dot product non-vanishing; clearly can obtain from an NLO calculation with N+1 jets



N-jettiness subtraction

- N-jettiness can be applied to obtain exact NNLO cross sections
Boughezal, Focke, Liu, FP (2015)
- Introduce τ_N^{cut} that separates the $\tau_N=0$ doubly-unresolved limit of phase space from the single-unresolved and hard regions

$$
\begin{aligned}
\sigma_{NNLO} &= \int \mathrm{d}\Phi_N \, |\mathcal{M}_N|^2 + \int \mathrm{d}\Phi_{N+1} \, |\mathcal{M}_{N+1}|^2 \, \theta_N^< \\
&+ \int \mathrm{d}\Phi_{N+2} \, |\mathcal{M}_{N+2}|^2 \, \theta_N^< + \int \mathrm{d}\Phi_{N+1} \, |\mathcal{M}_{N+1}|^2 \, \theta_N^> \\
&+ \int \mathrm{d}\Phi_{N+2} \, |\mathcal{M}_{N+2}|^2 \, \theta_N^> \\
&\equiv \sigma_{NNLO}(\mathcal{T}_N < \mathcal{T}_N^{cut}) + \sigma_{NNLO}(\mathcal{T}_N > \mathcal{T}_N^{cut})
\end{aligned}
$$

$$\theta_N^< = \theta(\tau_N^{cut} - \tau_N) \quad \text{and} \quad \theta_N^> = \theta(\tau_N - \tau_N^{cut})$$

N-jettiness subtraction

- For $\tau_N > \tau_N^{cut}$, at least one of the two additional radiations that appear at NNLO is resolved; this region of phase space contains the NLO correction to the N+1 jet process. Can be obtained from any NLO program.
- For $\tau_N < \tau_N^{cut}$, both additional radiations are unresolved. A factorization theorem giving the all-orders result for small N-jettiness was derived Stewart, Tackmann, Waalewijn 0910.0467

$$\sigma(\tau_N < \tau_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots$$

describes hard radiation; in dim-reg, coincides with the 2-loop virtual corrections

describes radiation collinear to an initial-state beam

describes soft radiation

describes radiation collinear to a final-state jet

- The ellipses denote power corrections that become negligible for small τ_N^{cut}

Ingredients for the factorization theorem

$$\sigma(\tau_N < \tau_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots$$

- Expand this formula to $O(\alpha_S^2)$, and turn off all resummation, to get the NNLO cross section below the cut. Need each of these separate functions to NNLO.
- The beam and jet functions depend only on the flavor of the parton (quark, gluon); the soft function depends only on the parton flavors and the external hard directions; the hard function is the only process-dependent piece.

  - *H*@NNLO: for W/H+j, Gehrmann, Tancredi (2011); Gehrmann, Jaquier, Glover, Koukoutsakis (2011) (see also Becher, Bell, Lorentzen, Marti (2013))
  - *B*@NNLO: Gaunt, Stahlhofen, Tackmann (2014)
  - *S*@NNLO: Boughezal, Liu, FP (2015)
  - *J*@NNLO: Becher, Neubert (2006); Becher, Bell (2011)

Within the past two years all ingredients have become available to apply this idea to 2→2 scattering at the LHC!

Recipe for the NNLO calculation

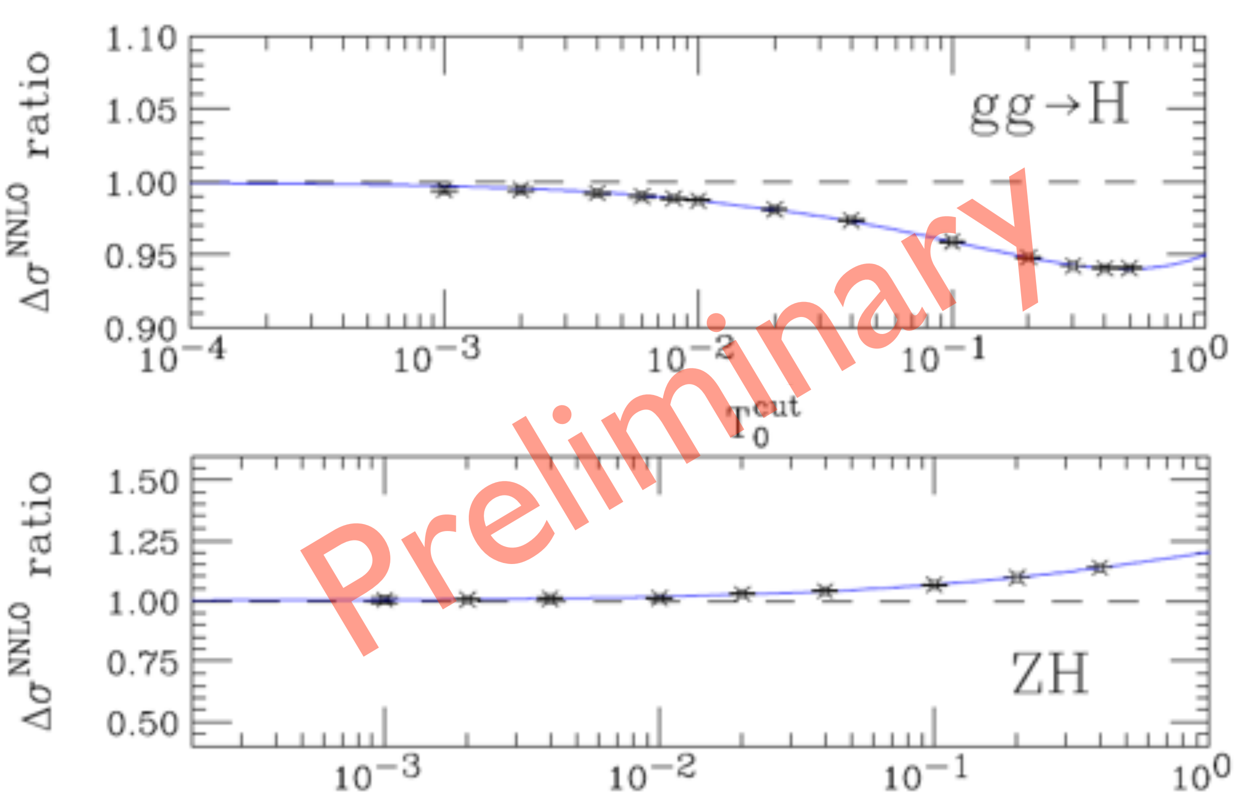
- Generate an event for the N+1 jet process, which may contain either 1 or 2 additional partons beyond Born level. Determine the reference vectors for the N-jettiness measurement by pre-clustering the radiation using any jet algorithm.

- Calculate τ_N for this event. If $\tau_N > \tau_N^{cut}$, keep the event. Such events form the NLO N+1 jet cross section needed for $\sigma(\tau_N > \tau_N^{cut})$. This accounts for the resolved and single-unresolved components of the RR and RV.

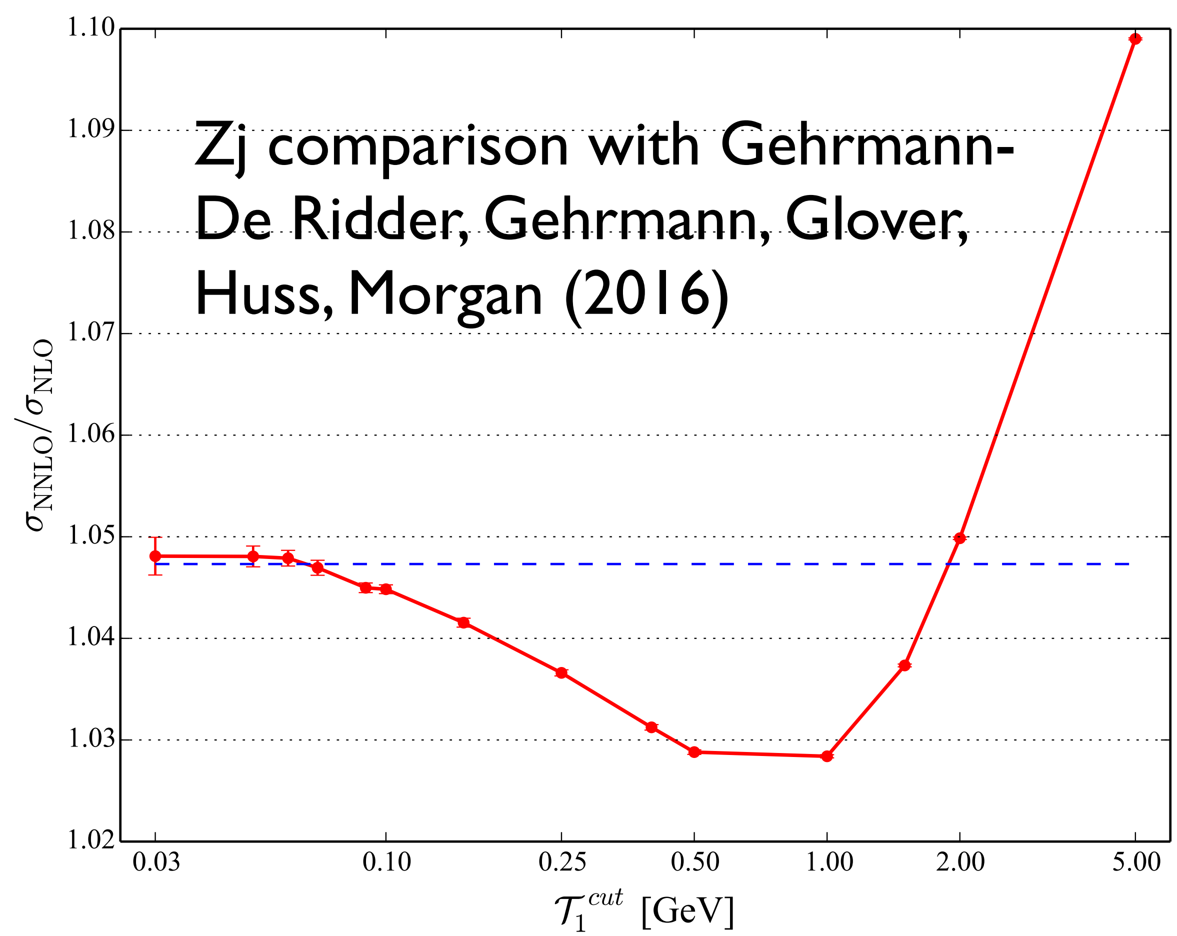
- Obtain the cross section $\sigma(\tau_N < \tau_N^{cut})$ by expanding the resummation formula to $O(\alpha_S^2)$. This accounts for the VV and double-unresolved components of the RR and RV.

Results from N-jettiness subtraction

- Obtained new results for W/H/Z+jet (Boughezal, Focke, Liu, FP; Boughezal, Focke, Giele, Liu, FP; Boughezal, Campbell, Ellis, Focke, Giele, Liu, FP (2015))
- Confirmed existing results for color-singlet processes: H/W/Z (Gaunt, Stahlhofen, Tackmann, Walsh 2015); VH (Campbell, Ellis, Williams 2016)
- Corrected existing result for γγ obtained using q_T-subtraction (Campbell, Ellis, Li, Williams 2016)

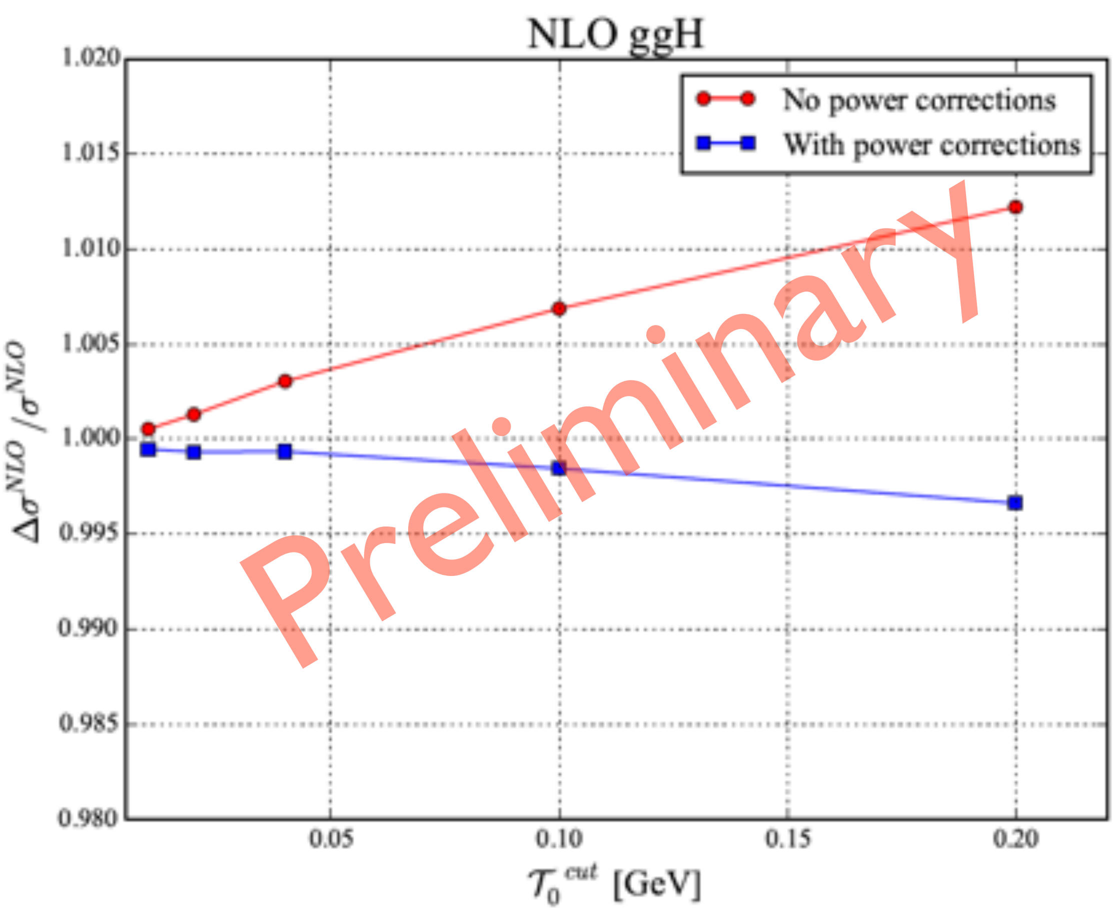


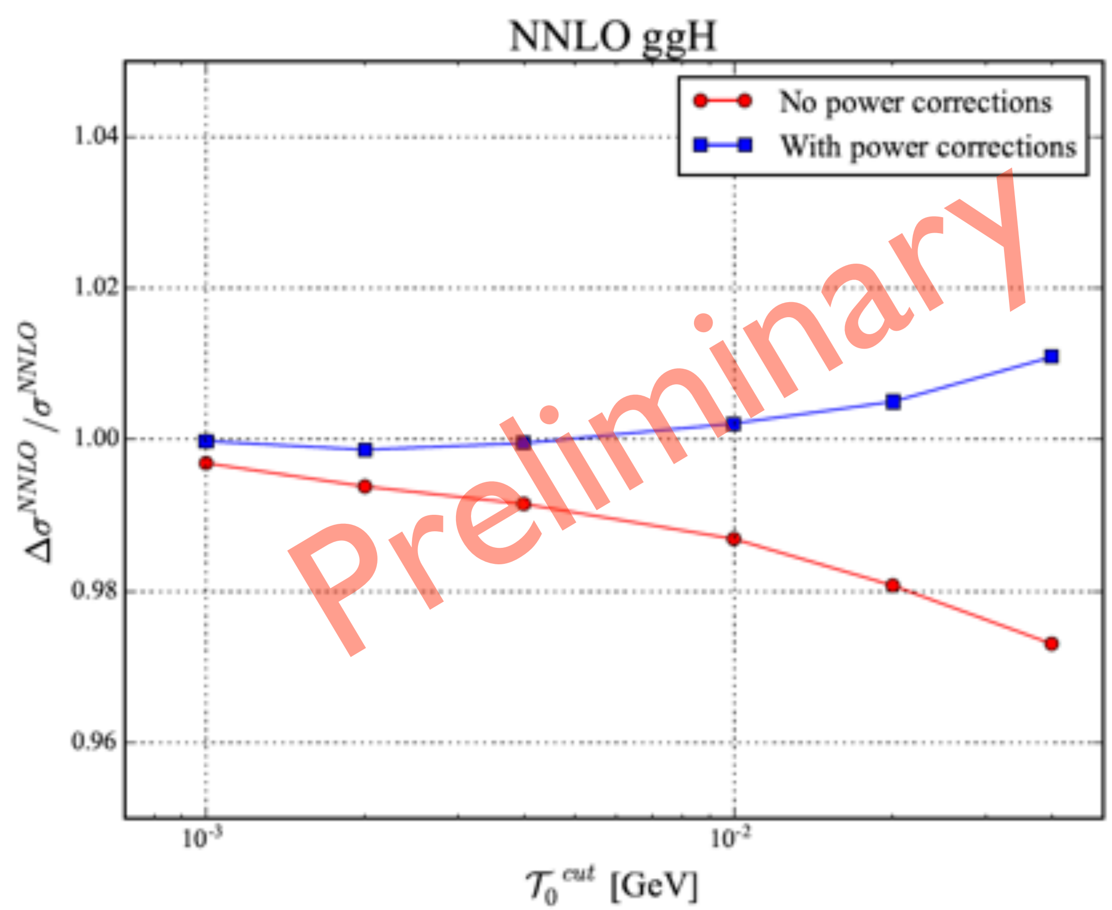
MCFM 8.0: Boughezal, Campbell, Ellis, Focke, Giele, Liu, FP, Williams



Future improvements

- N-jettiness subtraction can be systematically improved by incorporating the power corrections to the factorization formula.
- These corrections are universal and calculable; this allows higher τ_N^{cut} and significantly improves the method.
- Leading NNLO power corrections known (Boughezal, Liu, FP 2016; in progress)





Conclusions

- We are entering the era of NNLO phenomenology in time for LHC Run II
- Numerous NNLO calculations for 2→2 processes that make tests of the SM more `stressful'
- The N-jettiness subtraction scheme allows for precision calculations of jet processes at the LHC
- More results to come!