One-loop techniques for many legs

J. Ph. Guillet, G. Heinrich, N. Kauer, N. Mahmoudi, C. Schubert

Thomas Binoth

University of Würzburg

LoopFest III
1–3 April, 2004

Content

- Motivation
- Automatic computation of one-loop amplitudes:
  The GOLEM project
- Formalism for Hexagon calculations
- Examples: $\phi\phi \to \phi\phi\phi\phi,\ \gamma\gamma \to \phi\phi\phi\phi,\ gg \to \gamma\gamma g$
- Summary/Outlook

Jet rates at the LHC

Number of jets:	3	4	5	6	7	8
σ/nb	91.4	6.54	0.46	0.032	0.002	0.0002

$p_T(jet) > 60$ GeV, $\theta_{ij} > 30^0$, $|\eta_j| < 3$ [Draggiotis,Kleiss,Papadopoulos, hep-ph/0202201]

Multi–particles/jet production plays a very important role !!!

Problems with leading order predictions:

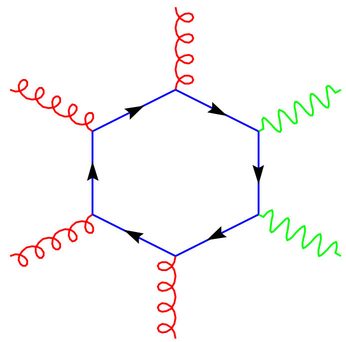
- Scale dependence: N–jet cross sections $\sim \alpha_s(\mu)^N \Rightarrow$ predictions rely on NLO corrections
- Peripheral phase space regions: degenerate partonic configurations at LO are sensitive to extra parton emission
- Jet structure: the more partons the better

Backgrounds for Higgs boson search

To make a reliable estimate of backgrounds for the relevant signal signatures

- $PP \to H$ by Gluon Fusion (NNLO), $H \to \gamma\gamma$, $H \to WW^{(*)}$
- $PP \to Hj$ (NLO)
- $PP \to Hjj$ by Weak Boson Fusion (NLO), $H \to \tau^+\tau^-$, $H \to WW^{(*)}$, $H \to \gamma\gamma$

the following NLO computations are needed:



- $PP \to \gamma\gamma + 0, 1, 2\,\text{jets}$
- $PP \to WW^* + 0, 1, 2\,\text{jets}$
- $PP \to t\bar{t} + 0, 1, 2\,\text{jets}$
- $PP \to \tau^+\tau^- + 0, 1, 2\,\text{jets}$

“industrial” production of NLO amlpitudes needed !!!

Automated computation of multi-parton processes at NLO

- $2 \to 2$ easy, $2 \to 3$ hard, $2 \to 4$ examples unknown
- Standard Packages: FF, FeynArts, FeynCalc, FormCalc, Loop-Tools,...,

$2 \to N$ at NLO needs:

- Real $2 \to N+1$ corrections: $d\sigma^R_{NLO}[2 \to N+1]$
- Virtual $2 \to N$ corrections: $d\sigma^V_{NLO}[2 \to N]$
- explicit cancellations of IR divergencies

  [for an alternative see: D. Soper]

> It seems feasible to construct an automatic program which produces finite, numerically stable NLO amplitudes for up to $2 \to 4$ partonic processes in terms of FORTRAN, C,... functions which can be used to build NLO Monte Carlo programs.

[Z. Nagy, D.E. Soper, JHEP 0309, 055 (2003), S. Dittmaier, hep-ph/0308246 , W.T. Giele, E.W.N. Glover, hep-ph/0402152.]

Our aim is the construction of a . . .

G eneral
O ne
L oop
E valuator for
M atrix Elements

. . . for QCD $\oplus$ EW processes !

Tree $2 \rightarrow 5$ matrix elements for SM processes...

- Helicity amplitudes sorted in irreducible colour structures!

...do not seem to be a problem...

- Several packages exist:
  MadGraph, GRACE, CompHep, Omega, Amegic++, Alpgen,...

...up to the usual trouble with unstable particles.

One-loop $2 \to N$, $N \leq 4$ matrix elements for SM processes

- Sort amplitudes by helicity, colour and analyticity properties!

- Generate graphs for a given amplitude (e.g. QGRAPH, FeynArts, ...)

- Make tensor decomposition of scattering amplitude
  $\Rightarrow$ To be done once and forever for $6V$,$4V + f\bar{f}$,$2V + 2S + f\bar{f}$,...

- Project onto tensor coefficients
  $\Rightarrow$ Lorentz indices saturated,
  tensor reduction of 5, 6 point functions not problematic ($k \cdot p_j$ reducible)
  $\Rightarrow$ no high negative powers of Gram determinants, good for numerical stability

- Algebraic n-dim tensor reduction of 1- to 4-point functions
  (or use numeric approach as outlined below)

- Sort into basis set of two, three, four point scalar integrals
  $\Rightarrow$ Basis set for each amplitude can be determined automatically
  $\Rightarrow$ IR, UV divergent part can be isolated explicitly!

- Simplify coefficients analytically using algebraic programs

Explicit cancellations of IR divergences

- Use Dipole formalism for massive partons

[S. Catani, S. Dittmaier, M.H. Seymour, Z. Trocsányi, hep–ph/0201036]

$$\sigma_{NLO} = \sigma_{NLO}[2 \to N+1] + \sigma_{NLO}[2 \to N]$$

$$= \int_{N+1} \left[(d\sigma_R)_{\epsilon=0} - \left(\sum_{\text{Dipoles}} d\sigma_B \otimes dV_{\text{Dipole}} \right)_{\epsilon=0} \right] + \int_N \left[d\sigma_V + d\sigma_B \otimes \mathbf{I}(\epsilon) \right]_{\epsilon=0}$$

- Analytical work done!

Representation of amplitudes by tensor integrals:

$$\Gamma^{c,\lambda}(p_j, m_j) = \sum_{\{c_i\},\alpha} f^{\{c_i\}} \mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{(q_1^2 - m_1^2)\ldots(q_N^2 - m_N^2)} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\ldots,\mu_R} \, I_N^{\mu_1\ldots\mu_R}(p_j, m_j)$$

$$I_N^{\mu_1\ldots\mu_R}(p_j, m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k_1^\mu \ldots k_R^\mu}{(q_1^2 - m_1^2)\ldots(q_N^2 - m_N^2)} \quad , \quad q_j = k - r_j = k - p_1 \ldots - p_j$$

Separate Lorentz structure from integrals:

$$I_N^{\mu_1\ldots\mu_R} = \sum_{m=0}^{[R/2]} \left(-\frac{1}{2}\right)^m \sum_{j_1,\ldots,j_{R-2m}=1}^{N-1} \left[g^{\cdot\cdot}_{(m)} r^{\cdot}_{j_1} \ldots r^{\cdot}_{j_{R-2m}} \right]^{\{\mu_1\ldots\mu_R\}} I_N^{n+2m}(j_1, \ldots, j_{R-2m})$$

$$I_N^n(j_1, \ldots, j_R) = (-1)^N \Gamma(N - n/2) \int_0^\infty d^N z \, \delta(1 - \sum_{l=1}^N z_l) \frac{z_{j_1} \ldots z_{j_R}}{(z \cdot S \cdot z)^{N-n/2}}$$

Numerical approach possible after extracting IR/UV divergences.

Reduction of scalar integrals

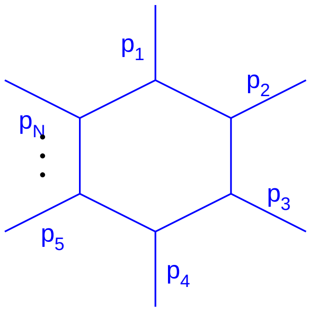
[Z. Bern, L. Dixon, D. Kosower, Nucl.Phys.B412 (1994); T.B., J.P. Guillet, G. Heinrich, Nucl.Phys.B572 (2000).]

$$I_N^n = (-1)^N \Gamma(N - \frac{n}{2}) \int_0^1 d^N z \frac{\delta(1 - \sum_{j=1}^N z_j)}{\left(\sum_{i,j=1}^N S_{ij}\, x_i\, x_j\right)^{N-\frac{n}{2}}}$$

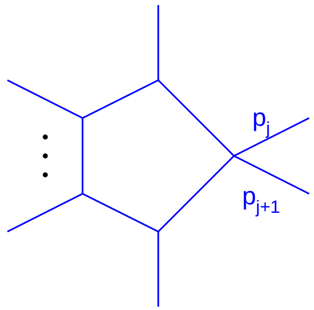
$$S_{ij} = G_{ij} - G_{ii}/2 - G_{jj}/2 + m_i^2 + m_j^2$$

$$G_{ij} = 2\, r_i \cdot r_j \quad , \quad r_j = p_1 + \ldots + p_j \quad , \quad n = 4 - 2\,\epsilon$$

$I_N^n =$



$= \sum_{j=1}^{N} B_j$



$$+ \begin{cases} -(1+2\epsilon)\frac{\det(G)}{\det(S)} I_N^{n+2} & , N = 4 \\ \mathcal{O}(\epsilon) & , N = 5 \\ 0 & , N \geq 6 \end{cases}$$

$$\sum_{j=1}^{N} S_{ij} B_j = -1 \quad \Leftrightarrow \quad B_i = -\sum_{j=1}^{N} S_{ij}^{-1}$$

Any N point integral can be represented by n–dimensional triangle functions and (n+2) dimensional box functions. The latter are infrared finite.

Five and six point functions

- Hexagons/pentagons are expressed by the functions I_3^n and I_4^{n+2}
- The isolation of infrared singularities is straightforward!!!

$$
\begin{aligned}
I_5^n \;=\; & (B_1B_{12}+B_2B_{21})I^n_{3,12}+(B_1B_{13}+B_3B_{31})I^n_{3,13}+B_1(B_{12}+B_{13}+B_{14}+B_{15})I^{n+2}_{4,1}\\
&+(B_2B_{23}+B_3B_{32})I^n_{3,23}+(B_2B_{24}+B_4B_{42})I^n_{3,24}+B_2(B_{21}+B_{23}+B_{24}+B_{25})I^{n+2}_{4,2}\\
&+(B_3B_{34}+B_4B_{43})I^n_{3,34}+(B_3B_{35}+B_5B_{53})I^n_{3,35}+B_3(B_{31}+B_{32}+B_{34}+B_{35})I^{n+2}_{4,3}\\
&+(B_4B_{45}+B_5B_{54})I^n_{3,45}+(B_4B_{41}+B_1B_{14})I^n_{3,14}+B_4(B_{41}+B_{42}+B_{43}+B_{45})I^{n+2}_{4,4}\\
&+(B_5B_{51}+B_1B_{15})I^n_{3,15}+(B_5B_{52}+B_2B_{25})I^n_{3,25}+B_5(B_{51}+B_{52}+B_{53}+B_{54})I^{n+2}_{4,5}
\end{aligned}
$$

$$
\begin{aligned}
I_6^n = &\\
&\{[B_1(B_{12}B_{123}+B_{13}B_{132})+B_2(B_{21}B_{123}+B_{23}B_{231})+B_3(B_{31}B_{132}+B_{32}B_{231})]\,I^n_{3,123}+5\text{ c.p.}\}\\
&+\{[B_1(B_{12}B_{124}+B_{14}B_{142})+B_2(B_{21}B_{214}+B_{24}B_{241})+B_4(B_{41}B_{412}+B_{42}B_{421})]\,I^n_{3,124}+5\text{ c.p.}\}\\
&+\{[B_1(B_{13}B_{134}+B_{14}B_{143})+B_3(B_{31}B_{314}+B_{34}B_{341})+B_4(B_{41}B_{413}+B_{43}B_{431})]\,I^n_{3,134}+5\text{ c.p.}\}\\
&+\{[B_1(B_{13}B_{135}+B_{15}B_{153})+B_3(B_{31}B_{315}+B_{35}B_{351})+B_5(B_{51}B_{513}+B_{53}B_{531})]\,I^n_{3,135}+1\text{ c.p.}\}\\
&+\{(B_1B_{12}+B_2B_{21})(B_{123}+B_{124}+B_{125}+B_{126})I^{n+2}_{4,12}+5\text{ c.p.}\}\\
&+\{(B_1B_{13}+B_3B_{31})(B_{132}+B_{134}+B_{135}+B_{136})I^{n+2}_{4,13}+5\text{ c.p.}\}\\
&+\{(B_1B_{14}+B_4B_{41})(B_{142}+B_{143}+B_{145}+B_{146})I^{n+2}_{4,14}+2\text{ c.p.}\}
\end{aligned}
$$

Reduction of tensor integrals

$$I_N^n(l_1,\ldots,l_R) = (-1)^N \Gamma(N-\frac{n}{2}) \int_0^1 d^N z\, \delta(1-\sum_{j=1}^N z_j) \frac{z_{l_1}\ldots z_{l_R}}{\left(\sum_{i,j=1}^N S_{ij}\, x_i\, x_j\right)^{N-\frac{n}{2}}}$$

$$I_N^n(l_0,\ldots,l_R) = \sum_{k=1}^R S^{-1}_{l_0 l_k} I_N^{n+2}(l_1,\ldots,l_{k-1},l_{k+1},\ldots,l_R)$$

$$+B_{l_0}(N-n-R-1) I_N^{n+2}(l_1,\ldots,l_p) - \sum_{j=1}^N S^{-1}_{j l_0} I^n_{N-1,j}(l_1,\ldots,l_R)$$

Each N–point integral with a non–trivial numerator can be represented by scalar integrals with shifted dimensions.

- The integrales I_N^{n+2m} are representable by I_N^{n+2m-2} and I_{N-1}^{n+2m-2} integrals

Each N–point integral with non–trivial numerator can be represented by scalar integrals $I_1^n, I_2^n, I_3^{n+2m}, I_4^{n+2}$. $m>0$ only for the infra-red divergent case where closed formulas are known.

Subtraction formalism for tensor integrals

For numeric approach sufficient to isolate IR part from tensor integrals:

$$I_N^{\mu_1,\dots\mu_R} \sim \sum \tau_{l_1,\dots l_R}^{\mu_1,\dots\mu_R} \; I_N^n(l_1,\dots,l_R)$$
$$I_N^n(l_1,\dots,l_R) \sim I_N^{n+2}(l_1,\dots,l_{R-1}) \oplus I_{N-1}^n(l_1,\dots,l_{R-1})$$

- Integrales $\sim I_N^{n+2}$ are IR finite $\Rightarrow$ numerical evaluation!
- Iteration for integrals $\sim I_{N-1}^n$ till analytical evaluation possible.

From the numerical point of view:

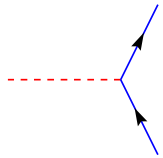
$$I_N^{n+2}(l_1,\dots,l_R) \sim I_N^{n+2}$$

Numerical methods for scalar graphs already developed!

Six scalar scattering in the Yukawa model

[T.B., J.P. Guillet, G. Heinrich, C. Schubert, Nucl. Phys. B615 (2001)]

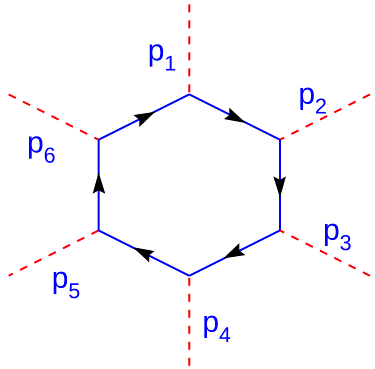
$$\mathcal{L}_{\mathrm{Yukawa}} = \bar{\psi}\,[\,i\,\partial\!\!\!/ - \lambda\,\phi\,]\,\psi + \mathcal{L}_{\phi} \quad , \quad \mathcal{L}_{\phi\psi\bar{\psi}} = i\lambda$$



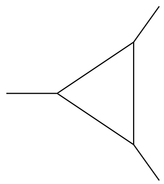
$$\mathcal{M}(\phi\phi \to \phi\phi\phi\phi) = -\frac{\lambda^6}{(4\pi)^2\,6}\sum_{\pi\in\mathcal{S}_6} G(\pi_1,\ldots,\pi_6)$$

$$G(1,2,3,4,5,6) = \int \frac{d^nk}{i\pi^{n/2}}\frac{tr(\,q\!\!\!/_1 \ldots\ q\!\!\!/_6)}{q_1^2\ldots q_6^2} =$$

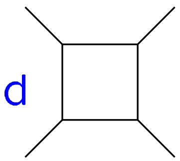
$$q_j = k - p_1 - \ldots - p_j$$



- Tensor structure can be simplified
- each diagram is IR,UV finite $\Rightarrow$ expressible by



and



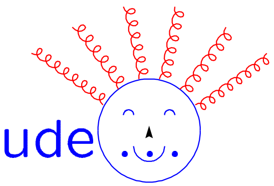
$|_{n+2}$

Result for $\mathcal{M}(\phi\phi \to \phi\phi\phi\phi)$:

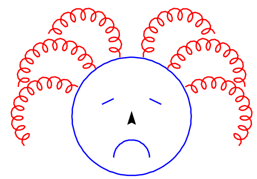
$$\mathcal{M}(\phi\phi \to \phi\phi\phi\phi) = -\frac{\lambda^6}{(4\pi)^2} \sum_{\pi \in \mathcal{S}_6} \mathcal{A}(p_{\pi_1}, p_{\pi_2}, p_{\pi_3}, p_{\pi_4}, p_{\pi_5}, p_{\pi_6})$$

$$\mathcal{A}(p_1, p_2, p_3, p_4, p_5, p_6) = \frac{2}{3} I_3^n(p_{12}, p_{34}, p_{56})$$
$$+ \left\{ \frac{B_1 \left[\mathrm{tr}(6123) - 2 s_{61}(s_{123} - s_{12})\right]}{s_{123} s_{345} - s_{12} s_{45}} + \frac{B_2 \left[\mathrm{tr}(1234) - 2 s_{34}(s_{123} - s_{23})\right]}{s_{234} s_{123} - s_{23} s_{56}} \right\}$$
$$\times \log\left(\frac{s_{12}}{s_{123}}\right) \log\left(\frac{s_{23}}{s_{123}}\right)$$
$$- \left\{ B_1 - \frac{B_2 \left[\mathrm{tr}(1234) - 2 s_{34}(s_{123} - s_{23})\right]}{2[s_{234} s_{123} - s_{23} s_{56}]} - \frac{B_6 \left[\mathrm{tr}(5612) - 2 s_{56}(s_{345} - s_{61})\right]}{2[s_{345} s_{234} - s_{61} s_{34}]} \right\}$$
$$\times \left[\log\left(\frac{s_{12}}{s_{234}}\right) \log\left(\frac{s_{56}}{s_{234}}\right) + \log\left(\frac{s_{34}}{s_{234}}\right) \log\left(\frac{s_{12}}{s_{56}}\right) \right]$$

Example for a non–trivial non–supersymmetric hexagon amplitude



. . . probably the most simple one



Example: $\Gamma(\gamma\gamma \to \phi\phi\phi\phi)$ in the gauged Yukawa Model

Tensor decomposition of amplitude:

$$\Gamma^{\lambda_1\lambda_2}(\gamma\gamma \to \phi\phi\phi\phi) = \epsilon_1^{\mu,\lambda_1}\epsilon_2^{\nu,\lambda_2}\mathcal{M}_{\mu\nu} \quad , \quad \mathcal{M}_{\mu\nu} = A\, g_{\mu\nu} + \sum_{l,j=1}^{5} B_{lj}\, r_l^\mu r_j^\nu$$

Projection on tensor coefficients:

$$\mathcal{P}^{\mu\nu} = \frac{1}{n-5}[g^{\mu\nu} - 2\sum_{l,j=1}^{5} H_{lj} r_l^\mu r_j^\nu] \quad , \quad \mathcal{R}_l^\mu = 2\sum_{j=1}^{5} H_{lj}\, r_j^\mu \, , \qquad H = G^{-1} \, , \, G_{ij} = 2 r_i \cdot r_j$$

$\Rightarrow$

$$\tilde{A}(\mathcal{M}) = \mathcal{P}^{\mu\nu}\mathcal{M}_{\mu\nu} = A \quad , \quad \tilde{B}_{ij}(\mathcal{M}) = \left[\mathcal{R}_i^\mu \mathcal{R}_j^\nu - 2H_{ij}\mathcal{P}^{\mu\nu}\right]\mathcal{M}_{\mu\nu} = B_{ij}$$

Transversality: $\epsilon_1 \cdot p_1 = \epsilon_2 \cdot p_2 = 0$

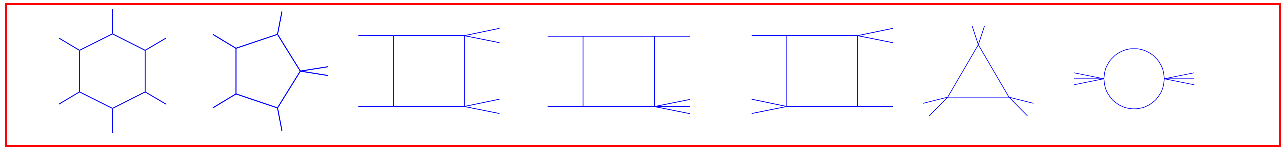
Gauge invariance: solve Ward identities $\mathcal{M}^{p_1\epsilon_2} = \mathcal{M}^{\epsilon_1 p_2} = 0$
or work in helicity basis $\epsilon_1 \cdot p_1 = \epsilon_1 \cdot p_2 = \epsilon_2 \cdot p_1 = \epsilon_2 \cdot p_2 = 0$

$\Rightarrow$ Only 9 out of 26 tensor coefficients needed!

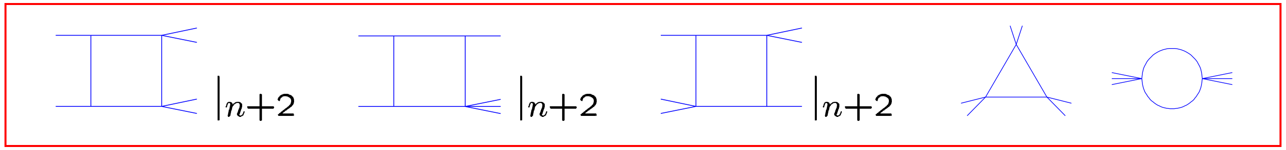
$\Gamma(\gamma\gamma \to \phi\phi\phi\phi)$ in the gauged Yukawa Model

$A, B_{ij} \sim$ linear combinations of Feynman parameter integrals with numerators.

Apply "tensor" reduction:



Apply scalar reduction:



UV/IR finite $\Rightarrow$ 3-point functions $\sim 1/\epsilon, 1/\epsilon^2$, poles of 2-point functions cancel!

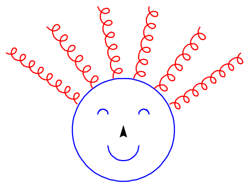
Helicity amplitudes $\leftrightarrow$ field strength tensors: $\mathcal{F}_j^{\mu\nu} = p_j^\mu \epsilon_j^\nu - p_j^\nu \epsilon_j^\mu$:

$$\begin{aligned} \Gamma^{\lambda_1\lambda_2} &= A\, \epsilon_1^{\lambda_1} \cdot \epsilon_2^{\lambda_2} + \sum_{j,l=3..5} B_{jl}\, p_j \cdot \epsilon_1^{\lambda_1}\, p_l \cdot \epsilon_2^{\lambda_2} \\ &= -\frac{A}{s_{12}}\, tr(\mathcal{F}_1^{\lambda_1} \mathcal{F}_2^{\lambda_2}) + 4 \sum_{i,j=3}^{5} \frac{B_{jl}}{s_{12}^2}\, p_j \cdot \mathcal{F}_1^{\lambda_1} \cdot p_2\, p_i \cdot \mathcal{F}_2^{\lambda_2} \cdot p_1 \end{aligned}$$

Result for $\mathcal{M}(\gamma\gamma \to \phi\phi\phi\phi)$:

$$\mathcal{M}^{++}(\gamma\gamma \to \phi\phi\phi\phi) = -\frac{\lambda^4 g^2}{(4\pi)^2}\frac{\varepsilon_1^+ \cdot \varepsilon_2^+}{s_{12}} \sum_{\pi \in S_4} \{C(1,\pi_3,2,\pi_4,\pi_5,\pi_6)$$
$$\times[F_1(s_{\pi_4\pi_5}, s_{\pi_5\pi_6}, s_{12\pi_3}) + F_1(s_{1\pi_3}, s_{2\pi_3}, s_{12\pi_3}) - 2\,F_1(s_{1\pi_3}, s_{3\pi_4}, s_{1\pi_3\pi_4})$$
$$+2\,F_2(s_{12\pi_3}, s_{2\pi_4\pi_5}, s_{1\pi_3}, s_{\pi_4\pi_5}) - F_2(s_{1\pi_3\pi_4}, s_{2\pi_4\pi_5}, s_{1\pi_3}, s_{2\pi_5}) - F_2(s_{1\pi_3\pi_4}, s_{2\pi_3\pi_4}, s_{\pi_3\pi_4}, s_{\pi_5\pi_6})]$$
$$+2\,F_2(s_{1\pi_3\pi_4}, s_{2\pi_3\pi_4}, s_{\pi_3\pi_4}, s_{\pi_5\pi_6}) + (1 \leftrightarrow 2)\}$$

$$C(1,2,3,4,5,6) = 2 + \frac{\mathrm{tr}^+(3456)B_5 + \mathrm{tr}^+(1234)B_1}{s_{34}} + \frac{\mathrm{tr}^+(6123)B_2 + \mathrm{tr}^+(4561)B_4}{s_{16}}$$
$$F_1(s_1,s_2,s_3) = -\log\left(\frac{s_1}{s_3}\right)\log\left(\frac{s_2}{s_3}\right)$$
$$F_2(s_1,s_2,s_3,s_4) = -\frac{1}{2}\log^2\left(\frac{s_3}{s_4}\right) - \mathrm{Li}_2\left(1 - \frac{s_1 s_2}{s_3 s_4}\right)$$
$$\mathrm{tr}^+(jl\ldots) = \mathrm{tr}((1+\gamma_5)\,\not{p}_j \not{p}_l \ldots)/2$$



Non–linear constraintes between Mandelstam variables are represented linearly through reduction coefficients B_j !!!

$\mathcal{M}^{+-}$ less compact but still numerically stable.

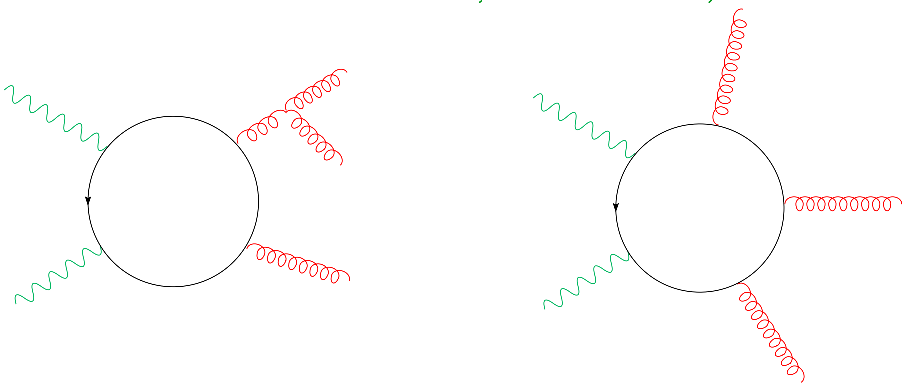
The $\gamma\gamma \to ggg$ amplitude

- Relevant for $\gamma\gamma +$jet background for Higgs+jet production
- Amplitude indirectly known from $gg \to ggg$ [Z. Bern, L. Dixon, D. Kosower, Phys.Rev.Lett.70 (1993)]

Independent helicity structures: $\Gamma^{+++++}, \Gamma^{++++-}, \Gamma^{+++--}, \Gamma^{+-+++}, \Gamma^{+-++-}, \Gamma^{--+++}$

Exercise: calculate helicity amplitudes in a modular way

- Box, pentagon topologies
- One colour structure: $\sim f^{abc}$
- Sort by analytical structure: $1, I_2^n, I_{3,1\text{mass}}^n, I_{3,2\text{mass}}^n, I_{4,1\text{mass}}^{n+2}$



The amplitude $\Gamma^{+++++}[\gamma\gamma ggg \to 0]$

With the reference momenta p_5, p_1, p_2, p_3, p_4 products of $\epsilon_j \cdot p_k$ can be treated uniformly for all graphs $2\mathrm{tr}^-(p_j,\ldots) = \mathrm{tr}(p_j) - \mathrm{tr}(\gamma_5, p_j, \ldots)$:

$$\prod_{j=1}^{5} \epsilon_j \cdot q_j = \frac{\mathrm{tr}^-(p_1, q_1, \ldots, p_5, q_5)}{4\sqrt{2} < 51 >< 12 >< 23 >< 34 >< 45 >}$$

$$\Gamma^{+++++} = \frac{1}{4\sqrt{2} < 51 >< 12 >< 23 >< 34 >< 45 >} \mathrm{tr}^+(1,2,3,5) \frac{s_{12}}{s_{35}}$$

$$\mathcal{F}_j^{\mu\nu} = p_j^\mu \epsilon_j^\nu - p_j^\nu \epsilon_j^\mu$$

$$\Gamma^{+++++} = f^{abc} \frac{\mathrm{tr}(\mathcal{F}_1^+ \mathcal{F}_2^+)\mathrm{tr}(\mathcal{F}_3^+ \mathcal{F}_4^+ \mathcal{F}_5^+)}{2 s_{34} s_{45} s_{35}}$$

Spinor helicity formalism:

$$\epsilon_\mu^+(p) = \frac{< n^- | \gamma_\mu | p^- >}{\sqrt{2} < n^- | p^+ >}, \; p|p^\pm >= 0, \; |j^\pm > := |p_j^\pm >, \; < jl > := < j^- | l^+ >, \; [jl] := < j^+ | l^- >$$

The amplitude $\Gamma^{++++-}[\gamma\gamma \to ggg]$

$$\Gamma^{++++-} = f^{abc} \frac{A + \epsilon(p_1,p_2,p_3,p_4)\, B}{4\sqrt{2} < 41 >< 32 >< 13 >< 14 > [51] < 24 > [45]}$$

$$\Gamma^{++++-} = f^{abc} \frac{\mathrm{Tr}(\mathcal{F}_1^+ \mathcal{F}_2^+)\mathrm{Tr}(\mathcal{F}_3^+ \mathcal{F}_4^+)}{s_{12}^2 s_{34}^2} [C^{++++-}\, p_1 \cdot \mathcal{F}_5^- \cdot p_3 - (3 \leftrightarrow 4)]$$

$$C^{++++-} = -\frac{s_{45}s_{13}s_{14}}{s_{35}s_{15}s_{24}} - \frac{s_{13}s_{45}}{s_{15}s_{35}} + \frac{s_{45}^2}{s_{15}s_{24}} - \frac{s_{12}^2 + s_{45}^2 - s_{12}s_{45}}{s_{35}s_{15}} + \frac{s_{13}s_{15}}{s_{23}s_{45}} + \frac{s_{13} - s_{34}}{s_{23}}$$
$$-\frac{s_{34}s_{45}}{s_{23}s_{15}} + \frac{s_{15} - s_{25}}{s_{45}} - \frac{s_{23} + s_{35}}{s_{13}} - \frac{s_{23}s_{25}}{s_{13}s_{45}} + \frac{s_{34} + s_{12}}{s_{15}}$$

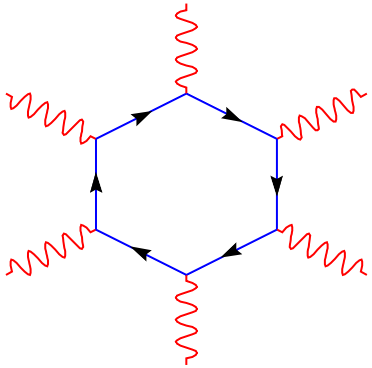
- All helicity amplitudes have structure $tr(FF)tr(FFF)$ or $tr(FF)tr(FF)p \cdot F \cdot p$
- Ward identities manifestly fulfilled, Bose symmetry $1 \leftrightarrow 2$, $3 \leftrightarrow 4$ strong check
- Automated simplification procedures used

Useful identities for field strength tensors:

Permutation and flipping rules:

$$\begin{aligned}
\text{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\, p_i\cdot\mathcal{F}_3^+\cdot p_j &= \frac{\text{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)}{s_{13}^2 s_{23}}\{[s_{23}(s_{1j}s_{3i}-s_{1i}s_{3j})+s_{13}(s_{2i}s_{3j}-s_{2j}s_{3i})]p_3\cdot\mathcal{F}_2^+\cdot p_1 \\
&\quad + s_{12}s_{13}(s_{3i}\, p_3\cdot\mathcal{F}_2^+\cdot p_j - s_{3j}\, p_3\cdot\mathcal{F}_2^+\cdot p_i)\} \\
\text{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\text{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+) &= \text{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)\text{Tr}(\mathcal{F}_2^+\mathcal{F}_4^+)\left(\frac{\text{tr}^+(1243)}{s_{13}s_{24}}\right)^2 \\
&= \frac{\text{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)}{s_{13}^2}\frac{\text{Tr}(\mathcal{F}_2^+\mathcal{F}_4^+)}{s_{24}^2}[\text{tr}(1243)\text{tr}^+(1243) - s_{12}s_{13}s_{34}s_{24}] \\
\text{Tr}(\mathcal{F}_1^+\mathcal{F}_2^-) &= 0
\end{aligned}$$

Under construction: $\Gamma(\gamma\gamma \to \gamma\gamma\gamma\gamma)$:



- automatic FORM code generation
- modular programming for general 6–point kinematics
- n-dimensional reduction successful

Procedures are starting point of GOLEM

Numeric approach for 6–point processes

- Representation of an amplitude by a basis set of scalar or tensor integrals:

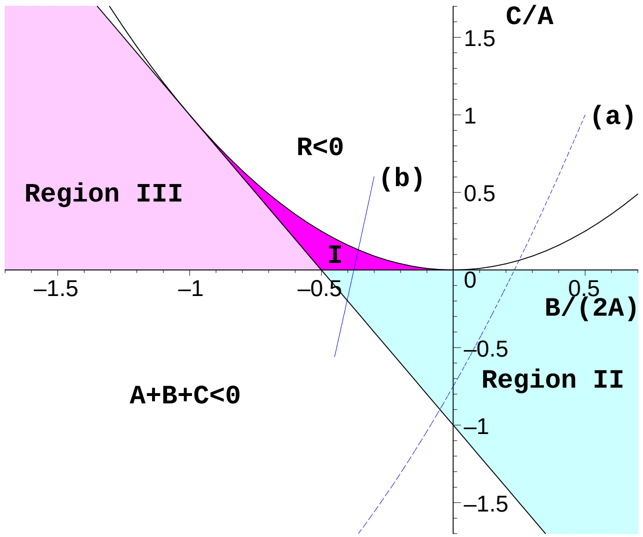
  What is an adequate set? $I_4^{n+2}(l_1, l_2, \ldots)$, $I_3^n(l_1, l_2, \ldots)$, I_2^n

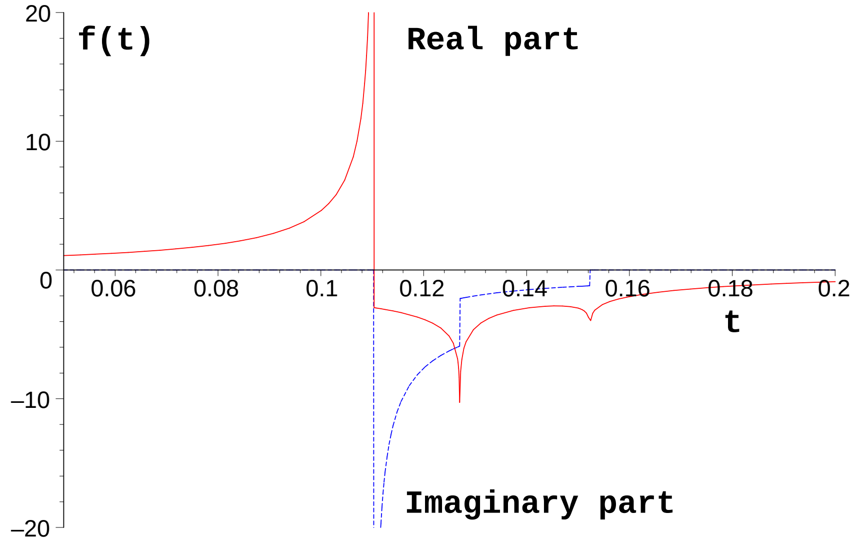
- Numerical evaluation of the basis functions after isolation of infrared divergences.
- Exercise: calculate the most difficult basis function numerically: the hexagon diagram.

  [T.B., G. Heinrich, N. Kauer, hep–ph/0210023 (2002)]

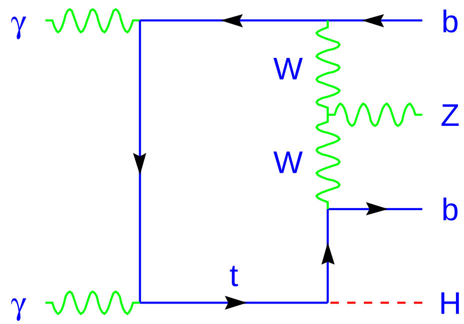
Numerical evaluation of the massive Hexagon function

- Problem: integrable singularities because of internal thresholds
- Reduction to 1- and 2-dimensional integrations
- Structure of singularities analysed
  $\Rightarrow$ constructive proof that singularities are of type $1/\sqrt{x}$ or $\log x$
- Iterative numerical methods developed

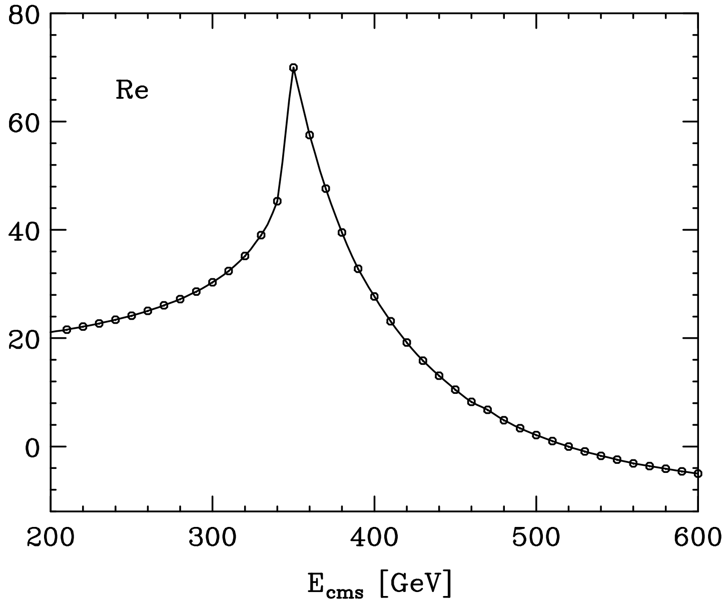


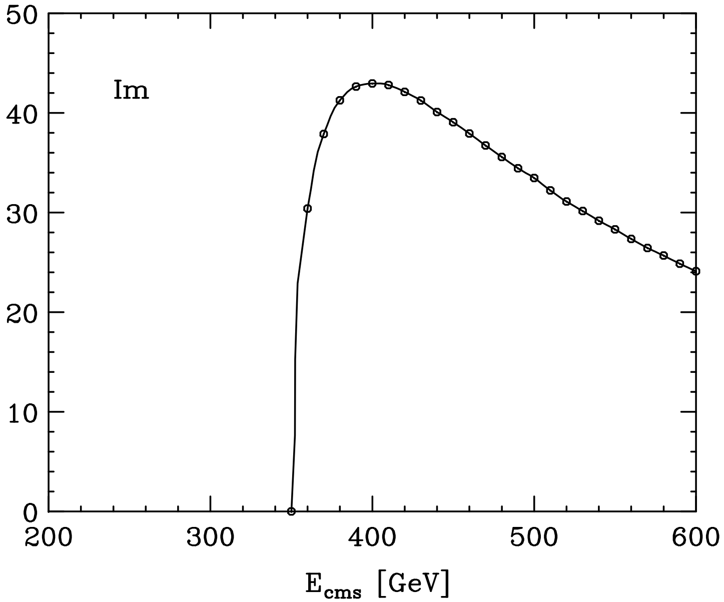


Numerical result for the hexagon:



- Stable result in the threshold region
- Real and imaginary parts are split explicitly ($\rightarrow$ no "$i\delta$" termes)
- the result for the box and the pentagon was compared with "LoopTools" T. Hahn



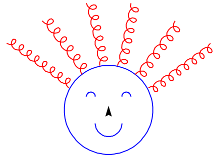


Summary:

- Multi–particle/jet production very important for TeV colliders
  $\Rightarrow$ new methods to calculate such processes at NLO
  $\Rightarrow$ Encouraging results for the Yukawa model
  $\Rightarrow$ Objectif: $PP \rightarrow$ 4 jets at NLO
  $\Rightarrow$ Numerical methods for the electro–weak sector at NLO

- Higgs search needs background calculations for $2 \rightarrow 3, 4$ processes
  $\Rightarrow$ industrial production of NLO calculations needed
  $\Rightarrow$ GOLEM project

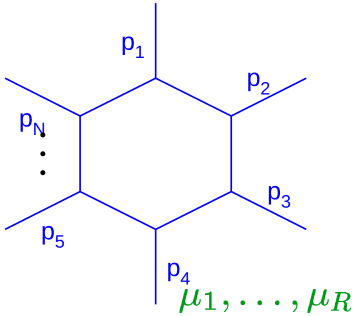
Only a profound knowledge of the Standard Model will allow us to understand new and unexpected phenomena !!!



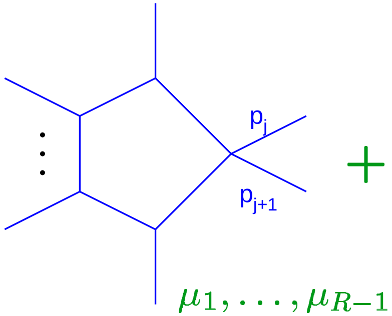
Subtraction formalism for tensor integrals

$$I_N^{\mu_1\cdots\mu_R} = \int \frac{d^n k}{i\pi^{n/2}} \frac{q_1^{\mu_1}\cdots q_R^{\mu_R}}{q_1^2\cdots q_N^2} \quad , \quad q_j = k - r_j \qquad , \quad r_j = p_1 + \ldots + p_j$$

$I_N^{\mu_1\ldots\mu_R} =$



$= \sum_{j=1}^{N} C_{jR}^{\mu_R}$



$$+ \begin{cases} \sim \sum I_N^{n+2m}(l_1,\ldots) & , N < 6 \\ 0 & , N \geq 6 \end{cases}$$

$$\sum_{j=1}^{N} S_{ij} \; C_{jR}^{\mu_R} = (r_i - r_R)^{\mu_R}$$

- Integrales $\sim I_N^{n+2m}(l_1,\ldots)$ are IR finite, no $1/\det(G)$ $\Rightarrow$ stable numerical evaluation!
- Iteration for integrals $\sim I_{N-1}^{\mu_1\ldots\mu_{R-1}}$ till analytical evaluation possible.