2 (and 3)-Quantum Field Theory

Michael H. Freedman
Microsoft Station Q.
February 23, 2009

- Resides on a higher functional level (name recalls 2-categories, 3-categories, etc…)

- Resides on a higher functional level (name recalls 2-categories, 3-categories, etc…)
- If QFT computes some "fuzziness" around classical trajectories…

- Resides on a higher functional level (name recalls 2-categories, 3-categories, etc…)
- If QFT computes some "fuzziness" around classical trajectories…
- then *n*-QFT should compute another kind of "fuzziness" around the unitary evolution of a QFT.

- Resides on a higher functional level (name recalls 2-categories, 3-categories, etc…)
- If QFT computes some "fuzziness" around classical trajectories…
- then *n*-QFT should compute another kind of "fuzziness" around the unitary evolution of a QFT.
- The idea of passing to a high (functional) level is quite general

- Resides on a higher functional level (name recalls 2-categories, 3-categories, etc…)
- If QFT computes some “fuzziness” around classical trajectories…
- then *n*-QFT should compute another kind of “fuzziness” around the unitary evolution of a QFT.
- The idea of passing to a high (functional) level is quite general
- We will briefly touch on the “easier” and “harder” cases: *n*-QM and *n*-string QFT.

- Continuing the thought, there are: 4-QFTs, 5-QFTs, … , α-QFTs, α any ordinal, but we won't consider all these now.

- Continuing the thought, there are: 4-QFTs, 5-QFTs, … , α-QFTs, α any ordinal, but we won't consider all these now.
- In QFT we may think (loosely) of Fock space as polynomials or functions on fields, or "spanned" by fields ϕ_i, so we write $\psi = \sum a_i \,|\, \phi_i \rangle \in$ Fock as a "wave functional."

- Continuing the thought, there are: 4-QFTs, 5-QFTs, … , α-QFTs, α any ordinal, but we won't consider all these now.
- In QFT we may think (loosely) of Fock space as polynomials or functions on fields, or "spanned" by fields ϕ_i, so we write $\psi = \sum a_i \,|\, \phi_i \rangle \in$ Fock as a "wave functional."
- In 2-QFT operators will act on 2-Fock, the linear space spanned by wave functionals, $\Psi = \sum b_i \,|\, \psi_i \rangle$, i.e. non-linear functionals of wave functionals.

- In spirit, *n*-QFT is the opposite of "constructive field theory."

- In spirit, *n*-QFT is the opposite of "constructive field theory."
- Instead of making some portion of quantum field theory rigorous, I try to catch the overall tune and then hum it other keys.

- In spirit, *n*-QFT is the opposite of "constructive field theory."
- Instead of making some portion of quantum field theory rigorous, I try to catch the overall tune and then hum it other keys.
- *n*-QFT may be:

- In spirit, *n*-QFT is the opposite of "constructive field theory."
- Instead of making some portion of quantum field theory rigorous, I try to catch the overall tune and then hum it other keys.
- *n*-QFT may be:
  - useless (i.e. out of tune)

- In spirit, *n*-QFT is the opposite of "constructive field theory."
- Instead of making some portion of quantum field theory rigorous, I try to catch the overall tune and then hum it other keys.
- *n*-QFT may be:
  - useless (i.e. out of tune)
  - a way of understanding unitarity as emergent

- In spirit, *n*-QFT is the opposite of "constructive field theory."
- Instead of making some portion of quantum field theory rigorous, I try to catch the overall tune and then hum it other keys.
- *n*-QFT may be:
  - useless (i.e. out of tune)
  - a way of understanding unitarity as emergent
  - a way to cook up effective descriptions of hierarchical or strongly interacting systems

- In spirit, *n*-QFT is the opposite of "constructive field theory."
- Instead of making some portion of quantum field theory rigorous, I try to catch the overall tune and then hum it other keys.
- *n*-QFT may be:
  - useless (i.e. out of tune)
  - a way of understanding unitarity as emergent
  - a way to cook up effective descriptions of hierarchical or strongly interacting systems
  - all of the above

- I would like to thank the physicists with whom I have discussed this ides.

- I would like to thank the physicists with whom I have discussed this ides.
- But also, I do not wish to embarrass them or tarnish their reputations.

- I would like to thank the physicists with whom I have discussed this ides.
- But also, I do not wish to embarrass them or tarnish their reputations.
- Let me say that I have enjoyed discussions with Chetan Nayak and Israel Klitch.

- I would like to thank the physicists with whom I have discussed this ides.
- But also, I do not wish to embarrass them or tarnish their reputations.
- Let me say that I have enjoyed discussions with Chetan Nayak and Israel Klitch.
  - But have received endorsements from neither.

In *n*-QFT there will be:

In *n*-QFT there will be:

- *n*-Hilbert space

In *n*-QFT there will be:

- *n*-Hilbert space
- *n*-Fock space

In *n*-QFT there will be:

- *n*-Hilbert space
- *n*-Fock space
- *n*-c^{+}_{H} , *n*-c_{H} *n*-second quantized operators

must be a better name

In *n*-QFT there will be:

- *n*-Hilbert space
- *n*-Fock space
- *n*-c^{+}_{H} , *n*-c_{H} *n*-second quantized operators

  must be a better name
- *n*-*H* *n*-Hamiltonian

In *n*-QFT there will be:

- *n*-Hilbert space
- *n*-Fock space
- *n*-c^{+}_{H} , *n*-c_{H} *n*-second quantized operators

  must be a better name
- *n*-H *n*-Hamiltonian
- *n*-U unitary evolution at level *n*

In *n*-QFT there will be:

- *n*-Hilbert space
- *n*-Fock space
- $n\text{-}c^{+}_{H}$, $n\text{-}c_{H}$ — *n*-second quantized operators

  must be a better name
- *n*-*H* — *n*-Hamiltonian
- *n*-*U* — unitary evolution at level *n*
- *n*-S — *n*-Lagrangian

In *n*-QFT there will be:

- *n*-Hilbert space
- *n*-Fock space
- $n\text{-}c^{+}_{\mathrm{H}}$, $n\text{-}c_{\mathrm{H}}$ — *n*-second quantized operators

  must be a better name
- *n*-*H* — *n*-Hamiltonian
- *n*-*U* — unitary evolution at level *n*
- $n\text{-}\mathcal{S}$ — *n*-Lagrangian
- *n*-*S* — *n*-action

- But there are only ordinary 1-observables.

- But there are only ordinary 1-observables.

- Why?

- But there are only ordinary 1-observables.

  - Why?

- Observables are not really constituents of the quantum theory but a bridge to the classical world.

- But there are only ordinary 1-observables.

- Why?

- Observables are not really constituents of the quantum theory but a bridge to the classical world.
- I stick with the familiar observables: field strength, charge, momentum, etc…, not knowing how to observe any new operators.

- I have not yet said explicitly what a 2-QFT is

- I have not yet said explicitly what a 2-QFT is
- Thank you for your patience.

- I have not yet said explicitly what a 2-QFT is
- Thank you for your patience.
- I first want to introduce the topological notation for function spaces.

- X^Y means {functions: Y W X}

- X^Y means {functions: Y W X}

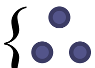
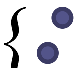
- Pneumonic: 2^3 = {functions: {• • •} W {• •}} = 8

- X^{Y} means {functions: Y W X}

- Pneumonic: $2^3 = \{\text{functions: } \{\bullet\,\bullet\,\bullet\} \text{ W } \{\bullet\,\bullet\}\} = 8$

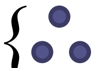
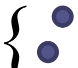
- Convention: $X^{Y^{Z}}$ means $X^{(Y^{Z})}$

- X^Y means {functions: Y W X}

- Pneumonic: 2^3 = {functions: {} W {}} = 8

- Convention: X^{Y^Z} means $X^{(Y^Z)}$
  - (since $(X^Y)^Z \equiv X^{Y \times Z}$ is something much smaller)

- X^Y means {functions: Y W X}

- Pneumonic: 2^3 = {functions: {} W {}} = 8

- Convention: X^{Y^Z} means $X^{(Y^Z)}$
  - (since $(X^Y)^Z \equiv X^{Y \times Z}$ is something much smaller)

  - (another identity: $X^{Y_1 \amalg Y_2} \equiv X^{Y_1} \times X^{Y_2}$)

- Using this very crude notation, let’s describe the “home” of:
  - quantum mechanics
  - field theory
  - quantum field theory
  - string quantum field theory
  - nonlinear sigma models
  - gauge field theory
  - etc…

- QM happens in 0 ? (actually $L^2(\Box)$ or $L^2(\Box^n) = \otimes_n L^2(\Box)$)

- QM happens in $0^{?}$ (actually $L^2(□)$ or $L^2(□^n) = \otimes_n L^2(□)$)

- FT happens in $□^{□^3}$; real field $\phi \in □^{□^3}$

- QM happens in 0 ? (actually $L^2(\square)$ or $L^2(\square^n) = \otimes_n L^2(\square)$)

- FT happens in $\square^{\square^3}$; real field $\phi \in \square^{\square^3}$

- QFT happens in Fock space $\square^{\square^{\square^3}}$;

  wave functional $\psi = \sum a_i \,|\, \phi_i \rangle$, $\sum a_i^2 = 1$

- As you see, my notation ignores all analytic detail:

- As you see, my notation ignores all analytic detail:
- $V \equiv V^*$, “linear combinations of” ≡ “functions on”

- As you see, my notation ignores all analytic detail:
- $V \equiv V^*$, "linear combinations of" ≡ "functions on"
- eigen functions treated "as if" they existed in L^2

- As you see, my notation ignores all analytic detail:

- $V \equiv V^*$, “linear combinations of” ≡ “functions on”

- eigen functions treated “as if” they existed in L^2

- I will confuse: polynomials = power series = functions = distributions

- As you see, my notation ignores all analytic detail:

- $V \equiv V^*$, “linear combinations of” ≡ “functions on”

- eigen functions treated “as if” they existed in L^2

- I will confuse: polynomials = power series = functions = distributions

- In spite of the explosion of literal cardinalities, I will always imagine the work goes on in a nice separable Hilbert space, such as $L^2(?)$.

- A final example: to get string-QFT from QFT, you fiddle around at the "<u>top</u>" of the tower:

$$\square^{\square^{\square^{3}}} \rightsquigarrow \square^{\square^{M^{S^1}}}$$

Fock space of a QFT-Fock space

stringy Fock space of a string QFT

M an 10-manifold, S^1 a circle which sweeps out a world sheet Σ in time.

- Of course, QFTs come in minor variations:

X a manifold is a "nonlinear sigma model"

is a gauge field theory

etc…

- From our perspective, these are minor variations.

- From our perspective, these are minor variations.
- 2-field theory adds a 0 at the bottom of the tower:

wave functional

$$\psi \in \square^{\square^{\square^3}}$$

2-wave functional

$$\Psi \in \square^{\square^{\square^{\square^3}}}$$

- From our perspective, these are minor variations.
- 2-field theory adds a 0 at the bottom of the tower:

wave functional

$$\psi \in \square^{\square^{\square^{3}}}$$

2-wave functional

$$\Psi \in \square^{\square^{\square^{\square^{3}}}}$$

- 3-field theory treats 3-wave functionals $\Psi \in \square^{\square^{\square^{\square^{\square^{3}}}}}$

and so on…

- The usual passage from H to S, the "path integral formulation of QFT," is based on the ability to restrict fields on ?^{4} to ?$^{3} \times t$.

- The usual passage from H to S, the "path integral formulation of QFT," is based on the ability to restrict fields on ?^{4} to ?$^3 \times t$.
- Let's see how this works set theoretically.

- The usual passage from H to S, the "path integral formulation of QFT," is based on the ability to restrict fields on $?^4$ to $?^3 \times t$.
- Let's see how this works set theoretically.
- It is important to be able to restrict fields to time slices. You will notice that the restriction maps exist naturally only for k-fields, k odd.

- The usual passage from *H* to S , the "path integral formulation of QFT," is based on the ability to restrict fields on ?^{4} to ?$^3 \times t$.
- Let's see how this works set theoretically.
- It is important to be able to restrict fields to time slices. You will notice that the restriction maps exist naturally only for *k*-fields, *k* odd.
- For *k* even, pass to the linear duals V↔V* (and we won't worry about it!)

- On adding a functional level, inclusion and restriction alternate.

$$\square^3 \times t \hookrightarrow \square^4 \qquad \text{(inclusion of spaces)}$$

$$\square^{\square^3 \times t} \longleftarrow \square^{\square^4} \qquad \text{(restriction of fields)}$$

$$\square^{\square^{\square^3 \times t}} \hookrightarrow \square^{\square^{\square^4}} \qquad \text{(inclusion of 2-fields)}$$

$$\square^{\square^{\square^{\square^3 \times t}}} \longleftarrow \square^{\square^{\square^{\square^4}}} \qquad \text{(restriction of 3-fields)}$$

etc…

- All books on QFT derive the evolution U from the Hamiltonian H as a "path integral" over fields ϕ weighted by $e^{-iS(\phi)}$, S the action of an ordinary Lagrangian, i.e. a 1-Lagrangian.

- All books on QFT derive the evolution *U* from the Hamiltonian *H* as a "path integral" over fields ϕ weighted by $e^{-iS(\phi)}$, *S* the action of an ordinary Lagrangian, i.e. a 1-Lagrangian.
- Given, say, a 2-Hamiltonian 2-*H*, there will be a 2-Lagrangian, 2-S , constructed as a "path integral" over 2-fields

$\boldsymbol{\phi} \in \square^{\square^{\square^4}}$ weighted by $e^{-i(2\text{-}S(\boldsymbol{\phi}))}$.

- All books on QFT derive the evolution U from the Hamiltonian H as a "path integral" over fields ϕ weighted by $e^{-iS(\phi)}$, S the action of an ordinary Lagrangian, i.e. a 1-Lagrangian.
- Given, say, a 2-Hamiltonian 2-H, there will be a 2-Lagrangian, 2-S , constructed as a "path integral" over 2-fields

  $\boldsymbol{\phi} \in \square^{\square^{\square^4}}$ weighted by $e^{-i(2\text{-}S(\boldsymbol{\phi}))}$.
- Formally, this 2-evolution 2-U is <u>perfectly</u> unitary. The 2-evolution naturally "drags along" an <u>ordinary</u> 1-level linear evolution but this is <u>not</u> unitary and only becomes unitary in the <u>squeezed</u> limit.

- By fiddling with the top of the tower we can produce 2-string FT:

$$\int_{\text{all 2-D field theories}} \longrightarrow \int_{\text{all 2-D 2-field theories}}$$

$$(\text{a string action}) \in \square^{M^{\Sigma}} \qquad (\text{a 2-string action}) \in \square^{\square^{M^{\Sigma}}}$$

- Or simplifying the top of the tower we produce 2-quantum mechanics

$$\psi \in O = \Box^{\Box^{pt.}} \qquad \Psi \in 2\text{-}O = \Box^{\Box^{\Box}}$$

wave function

2-wave function

- Or simplifying the top of the tower we produce 2-quantum mechanics

$$\psi \in \mathrm{O} = \square^{\square^{pt.}} \qquad\qquad \Psi \in 2\text{-}\mathrm{O} = \square^{\square^{\square}}$$

↗ wave function ↗ 2-wave function

- To get a picture of how 2-QM might work, consider as a model for part of 2-O consisting of Ψ µ 2-O made from just two Dirac functions,

$$\Psi = \frac{\sqrt{2}}{2}|\psi_1\rangle + \frac{\sqrt{2}}{2}|\psi_2\rangle$$

where we think of ψ_i = amplitude for particle i in position x.

- A 2-Hamiltonian for such a ψ might be chosen analogous to an ordinary Hamiltonian for a "molecule" moving in a potential:

$$2\text{-}H = \tfrac{1}{2} p_{x_1}^2 + \tfrac{1}{2} p_{x_2}^2 + V(x_1 - x_2) + \tfrac{x_1^2}{2} + \tfrac{x_2^2}{2} + \tfrac{\lambda}{4!} x_1^4 + \tfrac{\lambda}{4!} x_2^4$$

where $p_{x_i} = i\partial_{x_i}$, acting inside kets
and, for example:

$$p_{x_1 + x_2} \psi = \tfrac{\sqrt{2}}{2} \left| i\partial_{x_1} \psi_1 \right\rangle + \tfrac{\sqrt{2}}{2} \left| i\partial_{x_2} \psi_2 \right\rangle$$

- Passing to a center of mass coordinate $\frac{x_1+x_2}{2}$:

- Passing to a center of mass coordinate $\frac{x_1+x_2}{2}$:
- If $\lambda = 0$:

$$2\text{-}H = \tfrac{1}{2} p^2_{x_1+x_2} + \left(\tfrac{x_1+x_2}{2}\right)^2$$

$$+ \tfrac{1}{2} p^2_{x_1-x_2} + \left(\tfrac{x_1-x_2}{2}\right)^2 + V(x_1 - x_2),$$

So the center of mass is still SHO.

- Passing to a center of mass coordinate $\frac{x_1+x_2}{2}$:
- If $\lambda = 0$:

$$2\text{-}H = \tfrac{1}{2}p^2_{x_1+x_2} + \left(\tfrac{x_1+x_2}{2}\right)^2 + \tfrac{1}{2}p^2_{x_1-x_2} + \left(\tfrac{x_1-x_2}{2}\right)^2 + V(x_1 - x_2),$$

So the center of mass is still SHO.

- If λ û 0, the center of mass wave function

$$\psi(c) = \int dx_1 \left(\phi_1(x_1) + \phi_2(2c - x_1)\right) / norm^2$$

does not evolve unitarily.

- Passing to a center of mass coordinate $\frac{x_1+x_2}{2}$:
- If $\lambda = 0$:

$$2\text{-}H = \tfrac{1}{2} p^2_{x_1+x_2} + \left(\tfrac{x_1+x_2}{2}\right)^2$$
$$+ \tfrac{1}{2} p^2_{x_1-x_2} + \left(\tfrac{x_1-x_2}{2}\right)^2 + V(x_1 - x_2),$$

So the center of mass is still SHO.

- If λ û 0, the center of mass wave function

$$\psi(c) = \int dx_1 \left(\phi_1(x_1) + \phi_2(2c - x_1)\right) / norm^2$$

does not evolve unitarily.

- Solution at the 1-level is induced by "ket erasure" defined below.

- I see two potential uses for *n*-QFT, *n*-QM, *n*-string QFT

- I see two potential uses for *n*-QFT, *n*-QM, *n*-string QFT
  - 1) If the sad day arrives when we are confronted with evidence of non-unitarity, then we will have a canonical way to look for corrections.

- I see two potential uses for *n*-QFT, *n*-QM, *n*-string QFT
  - 1) If the sad day arrives when we are confronted with evidence of non-unitarity, then we will have a canonical way to look for corrections.
  - This could happen because:

- I see two potential uses for *n*-QFT, *n*-QM, *n*-string QFT
  - 1) If the sad day arrives when we are confronted with evidence of non-unitarity, then we will have a canonical way to look for corrections.
  - This could happen because:
    - a) gravity refuses to harmonize with unitarity

- I see two potential uses for *n*-QFT, *n*-QM, *n*-string QFT
  - 1) If the sad day arrives when we are confronted with evidence of non-unitarity, then we will have a canonical way to look for corrections.
  - This could happen because:
    - a) gravity refuses to harmonize with unitarity
    - b) quantum computers don't work properly

- I see two potential uses for *n*-QFT, *n*-QM, *n*-string QFT
    - 1) If the sad day arrives when we are confronted with evidence of non-unitarity, then we will have a canonical way to look for corrections.
    - This could happen because:
        - a) gravity refuses to harmonize with unitarity
        - b) quantum computers don't work properly
        - c) philosophers convince us that the classical world cannot be partial trace applied to unitary evolution.

- 2) Formal manipulations in 2-QFT, e.g. of (perturbed) Gaussian integrals, at higher (functional) levels:

- 2) Formal manipulations in 2-QFT, e.g. of (perturbed) Gaussian integrals, at higher (functional) levels
- will produce:

- 2) Formal manipulations in 2-QFT, e.g. of (perturbed) Gaussian integrals, at higher (functional) levels
- will produce:
  - 2-ghosts, 2-Hubbard Stratonovich, 2-perturbative expansions, etc…

- 2) Formal manipulations in 2-QFT, e.g. of (perturbed) Gaussian integrals, at higher (functional) levels
- will produce:
  - 2-ghosts, 2-Hubbard Stratonovich, 2-perturbative expansions, etc…
  - may suggest new effective descriptions of hierarchical and/or highly interacting systems

- 2) Formal manipulations in 2-QFT, e.g. of (perturbed) Gaussian integrals, at higher (functional) levels
- will produce:
  - 2-ghosts, 2-Hubbard Stratonovich, 2-perturbative expansions, etc…
  - may suggest new effective descriptions of hierarchical and/or highly interacting systems
  - such as FQHE states

field	wave functional	Fock(H) = F	$\mathbb{S}^{\lambda}(\phi)$ and S^{λ}
$\phi\in\square^{\square^4}$	$\psi(\phi)\in\square^{\square^{\square^4}}$ $\psi=\Sigma a_i\,\lvert\phi_i\rangle,\ \ \Sigma\lvert a_i\rvert^2=1$	$=\square^{\square^{\square^3}}\}\,H$ $=e^H$ $=\square\oplus H\oplus(H\otimes_S H)\oplus\cdots$	$\int dx^4\left((\nabla\phi)^2-\frac{m^2}{2}\phi^2-\frac{\lambda}{4!}\phi^4\right)$ for $\lambda=0$, solve in k-space $\lvert 0\rangle$ & $c_k^\dagger$ generate F
2-field	2-wave functional	2-Fock(H) = 2-F	2-$\mathbb{S}^{0,\lambda}(\boldsymbol{\phi})$ and 2-$S^{0,\lambda}$
$\boldsymbol{\phi}\in\square^{\square^{\square^4}}$	$\boldsymbol{\psi}(\boldsymbol{\phi})\in\square^{\square^{\square^{\square^4}}}$ $\boldsymbol{\psi}=\Sigma a_i\,\lvert\boldsymbol{\phi}_i\rangle,\ \ \Sigma\lvert a_i\rvert^2=1$	= Fock(Fock(H)) $=\square^{\square^{\square^{\square^3}}}$ $=\square\oplus F\oplus(F\otimes_S F)\oplus\cdots$	$\int<\phi\left((\boldsymbol{\nabla}\boldsymbol{\phi})^2_\phi-\frac{m}{2}\lvert\boldsymbol{\phi}\rvert^2-\frac{\lambda}{4!}\lvert\boldsymbol{\phi}\rvert^4\right)$ using translations in $\square^{\square^4}$, $\mathrm{H}\in(\square^*)^{\square^4}$ for $\lambda=0$ solve in H-space $\lVert 0\rangle\rangle$ & $2\text{-}c^\dagger_{\mathrm{H}}$ generate 2-F, $[c^\dagger_{\mathrm{H}_i}, c_{\mathrm{H}_j}]_\xi=\delta_{ij}$
3-field	3-wave functional	3-Fock(H) = 3-F	3-$\mathbb{S}^{0,\lambda}$
$\boldsymbol{\phi}\in\square^{\square^{\square^{\square^4}}}$	$\boldsymbol{\psi}(\boldsymbol{\phi})\in\square^{\square^{\square^{\square^{\square^4}}}}$ $\boldsymbol{\psi}=\Sigma a_i\,\lvert\boldsymbol{\phi}_i\rangle,\ \ \Sigma\lvert a_i\rvert^2=1$	=Fock^3(H) $=\square^{\square^{\square^{\square^{\square^3}}}}$ $=\square\oplus 2\text{-}F\oplus(2\text{-}F\otimes_S 2\text{-}F)\oplus\cdots$	$\int<\boldsymbol{\phi}\left((\boldsymbol{\nabla}\boldsymbol{\phi})^2_{\boldsymbol{\phi}}-\frac{m}{2}\lvert\boldsymbol{\phi}\rvert^2-\frac{\lambda}{4!}\lvert\boldsymbol{\phi}\rvert^4\right)$ $\lvert\lVert 0\rangle\rangle\rangle$ & $3\text{-}c^\dagger_{\lvert\mathrm{H}}$ generates 3-F, $[c^\dagger_{\lvert\mathrm{H}_i}, c_{\lvert\mathrm{H}_j}]_\xi=\delta_{ij}$ etc…

- Notations:

$$\nabla_{\phi'} \Phi\big|_{\phi} = \left(\Phi(\phi - \Delta\phi') - \Phi(\phi)\right) \Big/ \Delta \|\phi'\|_{L^2}$$

$$\nabla \Phi\big|_{\phi} = \left(\int_{\|\phi'\|_{L^2}=1} d\phi' \left(\nabla_{\phi'} \Phi\big|_{\phi}\right)^2 \right)^{\frac{1}{2}}$$

parallel formulae give $\nabla \Phi\big|_{\Phi}$, etc…

- Introduce the "small gradient" $\boldsymbol{\nabla}_x$ based on $x \in \square^4$ (not $\square^4$) translation:

Define $\phi_{\Delta x}(x) := \phi(x - \Delta x)$

then define $\boldsymbol{\nabla}_x \boldsymbol{\phi}\big|_{\phi} = \left(\boldsymbol{\phi}(\phi_{\Delta x}) - \boldsymbol{\phi}(\phi)\right) \Big/ \Delta x$

x µ ?^{3} or x µ ?^{4} depending on context.

- Define a family of 2-actions:

$$2\text{-}L^{c,\lambda} = |\nabla \boldsymbol{\phi}|_{\phi}|^2 - \frac{m^2}{2}|\boldsymbol{\phi}|^2 - \frac{\lambda}{4!}|\boldsymbol{\phi}|^4 - c\,\underbrace{\text{variance}(\boldsymbol{\phi})}, \; c \ddagger 0$$

$$\langle|\boldsymbol{\phi}|^2\rangle - \langle\boldsymbol{\phi}\rangle^2$$

$$= \int_{<\phi} |\boldsymbol{\phi}(\phi)|^2 - |\int_{<\phi} \boldsymbol{\phi}(\phi)|^2$$

- Define a family of 2-actions:

$$2\text{-}L^{c,\lambda} = |\nabla \boldsymbol{\phi}|_{\phi}|^2 - \frac{m^2}{2}|\boldsymbol{\phi}|^2 - \frac{\lambda}{4!}|\boldsymbol{\phi}|^4 - c\,\underbrace{\text{variance}(\boldsymbol{\phi})}, \; c \ddagger 0$$

$$\langle|\boldsymbol{\phi}|^2\rangle - \langle\boldsymbol{\phi}\rangle^2$$

$$= \int_< \phi\,|\,\boldsymbol{\phi}(\phi)\,|^2 - |\int_< \phi\boldsymbol{\phi}(\phi)\,|^2$$

- As $c \rightarrow ƒ$, the 2-physics of 2-$L^{c,\lambda}$ is expected to concentrate on 2-fields, "rules," $\boldsymbol{\phi}$ which are nearly Dirac: $\boldsymbol{\phi}$ … $\delta\phi$, some ϕ.

- In the $c \rightarrow ƒ$ limit, only x µ ?⁴ translations have bounded energy among general variations so ∇ is expected to reduce to ∇_x

- In the $c \to ƒ$ limit, only x µ $?^4$ translations have bounded energy among general variations so ∇ is expected to reduce to ∇_x
- This effectively deletes the “0” with the arrow next to it in the table shown earlier.

- In the $c \to$ ƒ limit, only x µ ?^{4} translations have bounded energy among general variations so ∇ is expected to reduce to ∇_x

- This effectively deletes the "0" with the arrow next to it in the table shown earlier.

- Thus, $c \to$ ƒ "squeezes" 2-QFT back to ordinary QFT with $1/c$ the small parameter.

<u>field</u> $\phi \in □^{□^4}$	<u>wave functional</u> $\psi(\phi) \in □^{□^{□^4}}$ $\psi = \Sigma a_i \mid \phi_i \rangle, \quad \Sigma \mid a_i \mid^2 = 1$	<u>Fock(H) = F</u> $= □^{□^{□^3}\}} H$ $= e^H$ $= □ \oplus H \oplus (H \otimes_S H) \oplus \cdots$	<u>$S^{\lambda}(\phi)$ and S^{λ}</u> $\int dx^4 \left((\nabla \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \right)$ for $\lambda = 0$, solve in k-space $\mid 0 \rangle$ & $c_k^{\dagger}$ generate F
<u>2-field</u> $\boldsymbol{\phi} \in □^{□^{□^4}}$	<u>2-wave functional</u> $\boldsymbol{\psi}(\boldsymbol{\phi}) \in □^{□^{□^{□^4}}}$ $\boldsymbol{\psi} = \Sigma a_i \mid \boldsymbol{\phi}_i \rangle, \quad \Sigma \mid a_i \mid^2 = 1$	<u>2-Fock(H) = 2-F</u> = Fock(Fock(H)) $= □^{□^{□^{□^3}}}$ $= □ \oplus F \oplus (F \otimes_S F) \oplus \cdots$	<u>2-$S^{0,\lambda}(\boldsymbol{\phi})$ and 2-$S^{0,\lambda}$</u> $\int < \phi \left((\boldsymbol{\nabla \phi})_{\phi}^2 - \frac{m}{2} \mid \boldsymbol{\phi} \mid^2 - \frac{\lambda}{4!} \mid \boldsymbol{\phi} \mid^4 \right)$ using translations in $□^{□^4}$, $\mathrm{H} \in (□^*)^{□^4}$ for $\lambda = 0$ solve in H-space $\parallel 0 \rangle\rangle$ & $2\text{-}c_{\mathrm{H}}^{\dagger}$ generate 2-F, $[c_{\mathrm{H}_i}^{\dagger}, c_{\mathrm{H}_j}]_{\xi} = \delta_{ij}$
<u>3-field</u> $\boldsymbol{\phi} \in □^{□^{□^{□^4}}}$	<u>3-wave functional</u> $\boldsymbol{\psi}(\boldsymbol{\phi}) \in □^{□^{□^{□^{□^4}}}}$ $\boldsymbol{\psi} = \Sigma a_i \mid \boldsymbol{\phi}_i \rangle, \quad \Sigma \mid a_i \mid^2 = 1$	<u>3-Fock(H) = 3-F</u> =Fock^3(H) $= □^{□^{□^{□^{□^3}}}}$ $= □ \oplus 2\text{-}F \oplus (2\text{-}F \otimes_S 2\text{-}F) \oplus \cdots$	<u>3-$S^{0,\lambda}$</u> $\int < \boldsymbol{\phi} \left((\boldsymbol{\nabla \phi})_{\boldsymbol{\phi}}^2 - \frac{m}{2} \mid \boldsymbol{\phi} \mid^2 - \frac{\lambda}{4!} \mid \boldsymbol{\phi} \mid^4 \right)$ $\parallel\mid 0 \rangle\rangle\rangle$ & $3\text{-}c_{\mid\mathrm{H}}^{\dagger}$ generates 3-F, $[c_{\mid\mathrm{H}_i}^{\dagger}, c_{\mid\mathrm{H}_j}]_{\xi} = \delta_{ij}$ etc…

- Similarly, let us define a 3-action:

$$3\text{-}L^{c,\lambda} = |\nabla \phi|_{\phi}|^2 - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4!}|\phi|^4 - \text{squeezing term}$$

where squeezing term conceptually is:

$$const \cdot \min_{\phi_0} \underbrace{\int <\phi| \phi(\phi) - \phi(\phi_0)|^2}_{f(\phi)}$$

- Similarly, let us define a 3-action:

$$3\text{-}L^{c,\lambda} = |\nabla \phi|_{\phi}|^2 - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4!}|\phi|^4 - \text{squeezing term}$$

where squeezing term conceptually is:

$$const \cdot \min_{\phi_0} \underbrace{\int < \phi | \phi(\phi) - \phi(\phi_0)|^2}_{f(\phi)}$$

Setwise, evaluation includes $\{\text{fields}\} \subset \square^{\{\text{fields}\}}$ by

$$\phi_{\phi}(\phi) := \phi(\phi)$$

- Similarly, let us define a 3-action:

$$3\text{-}L^{c,\lambda} = |\nabla \boldsymbol{\phi}|_{\boldsymbol{\phi}}|^2 - \frac{m^2}{2}|\boldsymbol{\phi}|^2 - \frac{\lambda}{4!}|\boldsymbol{\phi}|^4 - \text{squeezing term}$$

where squeezing term conceptually is:

$$const \cdot \min_{\phi_0} \underbrace{\int_{<} \boldsymbol{\phi} | \boldsymbol{\phi}(\boldsymbol{\phi}) - \boldsymbol{\phi}(\phi_0)|^2}_{f(\phi)}$$

Setwise, evaluation includes $\{\text{fields}\} \subset \square^{\square^{\{\text{fields}\}}}$ by

$$\boldsymbol{\phi}_{\phi}(\boldsymbol{\phi}) := \boldsymbol{\phi}(\phi)$$

An analytically more convenient squeeze term:

$$\beta' \int_{<} \phi e^{-\beta f(\phi)}, \ \ \beta' >> 0, \beta >> 0$$

- As with 2-fields, we now expect as $\beta \to ƒ$ that the "physics" of 3-fields will squeeze down to evaluation of 3-fields of the form $\boldsymbol{\phi}_{\phi}(\boldsymbol{\phi}) = \boldsymbol{\phi}(\phi)$, i.e. a 1-field ϕ.

- As with 2-fields, we now expect as $\beta \to f$ that the "physics" of 3-fields will squeeze down to evaluation of 3-fields of the form $\boldsymbol{\phi}_{\phi}(\boldsymbol{\phi}) = \boldsymbol{\phi}(\phi)$, i.e. a 1-field ϕ.

- Also expected: $\int < \boldsymbol{\phi} \,|\, \boldsymbol{\nabla\phi}|_{\boldsymbol{\phi}}|^2 \rightsquigarrow \int dx^4 \,|\, \nabla \phi \,|^2$

- As with 2-fields, we now expect as $\beta \rightarrow f$ that the "physics" of 3-fields will squeeze down to evaluation of 3-fields of the form $\boldsymbol{\phi}_{\phi}(\boldsymbol{\phi}) = \boldsymbol{\phi}(\phi)$, i.e. a 1-field ϕ.

- Also expected: $\int < \boldsymbol{\phi} | \boldsymbol{\nabla} \boldsymbol{\phi} |_{\boldsymbol{\phi}} |^2 \rightsquigarrow \int dx^4 \, | \nabla \phi |^2$

- Similarly for the mass and interaction terms

- Since 3 is odd, 3-fields naturally restrict to “time slices.”

$$\square^{\square\square\square^{3}\times t} \xleftarrow{\text{restriction}} \square^{\square\square\square^{4}}$$

- Since 3 is odd, 3-fields naturally restrict to “time slices.”

$$\square^{\square^{\square^{\square^{3}\times t}}} \xleftarrow{\text{restriction}} \square^{\square^{\square^{\square^{4}}}}$$

- The path integral allows the formal derivation of a unitary evolution 3-*U* starting from a Hermitian 3-Hamiltonian 3-*H*.

- Since 3 is odd, 3-fields naturally <u>restrict</u> to "time slices."

$$\square^{\square^{\square^{\square^{3}\times t}}} \xleftarrow{\text{restriction}} \square^{\square^{\square^{\square^{4}}}}$$

- The path integral allows the formal derivation of a unitary evolution 3-*U* starting from a Hermitian 3-Hamiltonian 3-*H*.
- This can also be done replacing "3" with "2" (i.e. at the 2-level) by passing to the linear duals:

$$\left(\square^{\square^{\square^{3}\times t}}\right)^{*} \xleftarrow{\text{restriction}} \left(\square^{\square^{\square^{4}}}\right)^{*}$$

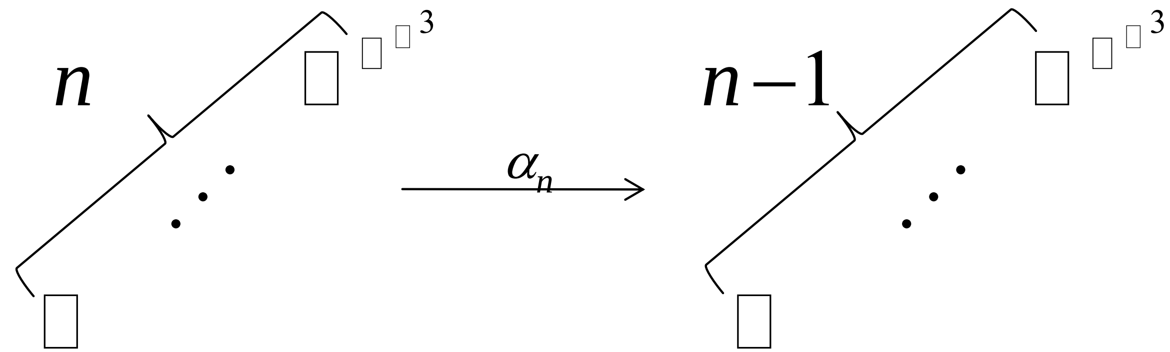
- Two final points should be explained:

- Two final points should be explained:
  - How the evolution at level n drags along a linear but not-quite unitary evolution at all levels $m < n$.

- Two final points should be explained:
  - How the evolution at level n drags along a linear but not-quite unitary evolution at all levels $m < n$.
  - Observables.

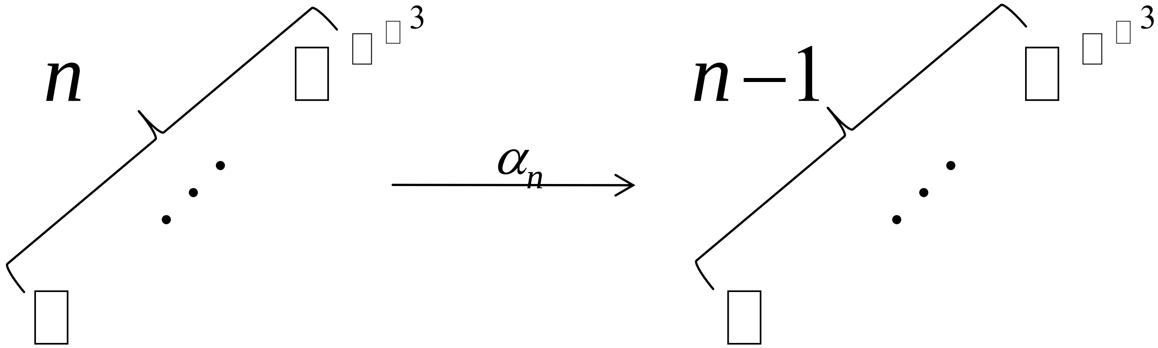
- Two final points should be explained:
  - How the evolution at level n drags along a linear but not-quite unitary evolution at all levels $m < n$.
  - Observables.
- For both of these we must define the "ket erasure" maps α_n.

- "erase kets and extend linearly" defines a linear map:



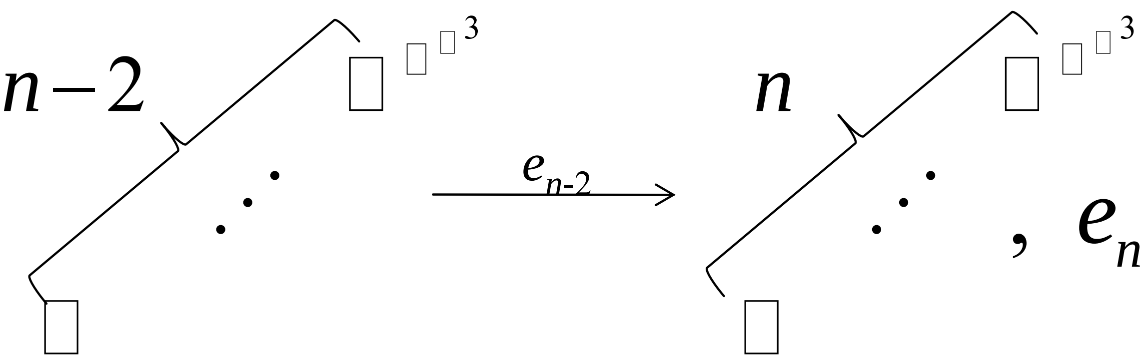
$$\sum a_i \,|\, \phi_i^n\rangle \xrightarrow{\alpha_n} \sum \tilde{a}_i \phi_i^n \quad \text{where } \tilde{a}_i = \begin{cases} \overline{a}_i \text{ for } n \text{ odd} \\ a_i \text{ for } n \text{ even} \end{cases}$$

- “erase kets and extend linearly” defines a linear map:



$$\sum a_i \,|\, \phi_i^n \rangle \xrightarrow{\alpha_n} \sum \tilde{a}_i \phi_i^n \quad \text{where } \tilde{a}_i = \begin{cases} \bar{a}_i \text{ for } n \text{ odd} \\ a_i \text{ for } n \text{ even} \end{cases}$$

- There is also our familiar evaluation map $e_{n\text{-}2}$



$$,\ e_{n-2}\phi_0^{n-2}(\phi^{n-1}) = \phi^{n-1}(\phi_0^{n-2})$$

- Formally

$$\alpha_{n-1} \circ \alpha_n \circ e_{n-2} = id_{n-2} \text{ (up to infinite constant)}$$

- Formally

$$\alpha_{n-1} \circ \alpha_n \circ e_{n-2} = id_{n-2} \text{ (up to infinite constant)}$$

- Proof: If $\boldsymbol{\phi}(\boldsymbol{\phi}) = \boldsymbol{\phi}(\phi_0)$ then $\boldsymbol{\phi} = \sum_i \boldsymbol{\phi}_i(\phi_0)\,|\boldsymbol{\phi}_i\rangle$

$$\alpha_2 \boldsymbol{\phi} = \sum_i \boldsymbol{\phi}_i(\phi_0)\boldsymbol{\phi}_i = \sum_i b_{i0}\boldsymbol{\phi}_i \text{ where we have written:}$$

$$\boldsymbol{\phi}_i = \sum_j b_{ij}\,|\,\phi_j\rangle$$

$$\alpha_1\alpha_2\boldsymbol{\phi} = \sum_{i,j} b_{i0}\bar{b}_{ij}\phi_j = \sum_i b_{i0}\bar{b}_{i0}\phi_0 + \sum_{i,j\neq 0} b_{i0}\underbrace{\bar{b}_{ij}}\phi_j$$

$$= \infty(\phi_0) + \sum_{j\neq 0} 0\phi_j$$

assumes all values with
j fixed and *i* varying:
symmetry » cancellation

- Measurement will merely be by a Hermitian operator V on ordinary Fock space $F = \square^{\square\,\square^{3}}$. The protocol is "reduce then observe":

- Measurement will merely be by a Hermitian operator V on ordinary Fock space $F = \square^{\square^{3}}$. The protocol is "reduce then observe":

- $\psi^n \xrightarrow{\alpha_n} \psi^{n-1} \rightarrow \cdots \rightarrow \psi^1$ observe λ_i of V with $prob\,|a_i|^2$, $\psi^1 = \sum_i a_i \psi_i^1$

where $\{\psi_i^1\}$ is an eigen-basis for V.

- Measurement will merely be by a Hermitian operator V on ordinary Fock space $F = \square^{\square \square^3}$. The protocol is "reduce then observe":

- $\psi^n \xrightarrow{\alpha_n} \psi^{n-1} \rightarrow \cdots \rightarrow \psi^1$ observe λ_i of V with $prob\,|a_i|^2$, $\psi^1 = \sum_i a_i \psi_i^1$

where $\{\psi_i^1\}$ is an eigen-basis for V.

- presume n = odd. Then successive evaluation maps promote ψ^1 back to level n where n-U evolves it until the next measurement by some V' also acting on ordinary Fock space $F = \square^{\square \square^3}$.

- Measurement will merely be by a Hermitian operator V on ordinary Fock space $F = \square^{\square\square 3}$. The protocol is "reduce then observe":

- $\psi^n \xrightarrow{\alpha_n} \psi^{n-1} \rightarrow \cdots \rightarrow \psi^1$ observe λ_i of V with $prob\,|a_i|^2$, $\psi^1 = \sum_i a_i \psi_i^1$

where $\{\psi_i^1\}$ is an eigen-basis for V.

- presume n = odd. Then successive evaluation maps promote ψ^1 back to level n where n-U evolves it until the next measurement by some V' also acting on ordinary Fock space $F = \square^{\square\square 3}$.

- If the level n-evolution is sufficiently squeezed then n-U evolves very nearly within evaluation subspace F ð n-F and exact unitarity on n-F implies that a nearly exact unitarity will be observed on F.

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue:

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue: Unitarity is only emergent.

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue: Unitarity is only emergent.
- Applications:

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue: Unitarity is only emergent.
- Applications: none.

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue: Unitarity is only emergent.
- Applications: none.
- Target application:

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue: Unitarity is only emergent.
- Applications: none.
- Target application: strings/gravity, FQHE

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue: Unitarity is only emergent.
- Applications: none.
- Target application: strings/gravity, FQHE (whichever proves easier)

Summary/Outlook

- Formally: quantum mechanics, QFT, and string QFT can all be promoted to higher functional levels: non-linear functionals of wave functionals ad infinitum.
- The familiar foundations all extend at a purely formal level.
- Vice/virtue: Unitarity is only emergent.
- Applications: none.
- Target application: strings/gravity, FQHE (whichever proves easier)

- Thank you for your attention.

Appendix

- Side note on higher level creation operators:
  - 2-c_H creates a set of states of varying particle numbers, e.g. the set may contain a scalar, a singleton of momentum k, linear combinations of pairs (k' ... $_s$ k''), etc….
  - In other words, 2-c_H creates an arbitrary element of Fock space.
  - 3-c_H creates sets of sets of states, i.e. an arbitrary element of 2-Fock space.
  - and so on…

Appendix

- Note on unitarity of U derived from S (derived from H):

  - $S =$ ûS reverses sign with reversal of orientation of slab: ? $\times [0,1]$

  - $$U_{ij} = \int_0^1 e^{iS} = \overline{\int_1^0 e^{iS}} = \overline{U}_{ji}^{-1}$$

  - Level n is formally identical to level one.