

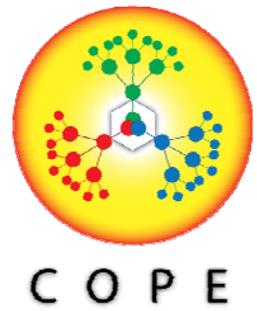


A unified explanation of the Kadowaki-Woods ratio

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Discoveries of ratios whose values are constant within broad classes of materials have led to many deep physical insights. The Kadowaki-Woods ratio (KWR)^{1,2} compares the temperature dependence of a metal's resistivity to that of its heat capacity; thereby probing the relationship between the electron-electron scattering rate and the renormalisation of the electron mass. However, the KWR takes very different values in different materials^{3,4}. Here we introduce a ratio⁵, closely related to the KWR, that includes the effects of carrier density and spatial dimensionality and takes the same (predicted) value in organic charge transfer salts, transition metal oxides, heavy fermions and transition metals - despite the numerator and denominator varying by ten orders of magnitude. Hence, in these materials, the same emergent physics is responsible for the mass enhancement and the quadratic temperature dependence of the resistivity and no exotic explanations of their KWRs are required.

Introduction

- In a Fermi liquid the electronic contribution to the heat capacity increases linearly with temperature: $C_{el}(\eta) = \gamma T$.
- When electron-electron scattering dominates over electron-phonon scattering the low temperature resistivity is given by $\rho(\eta) = \rho_0 + AT^2$, where the magnitude of A is determined by the strength of the electron-electron scattering
- Rice¹ found that in a range of transition metals $A/\gamma^2 = a_{TM} = 0.4 \mu\Omega \text{ cm mol}^{-2} \text{ K}^2 \text{ J}^{-2}$.
- Kadowaki and Woods² found that in the heavy fermion materials $A/\gamma^2 = a_{HF} = 10 \mu\Omega \text{ cm mol}^{-2} \text{ K}^2 \text{ J}^{-2}$.

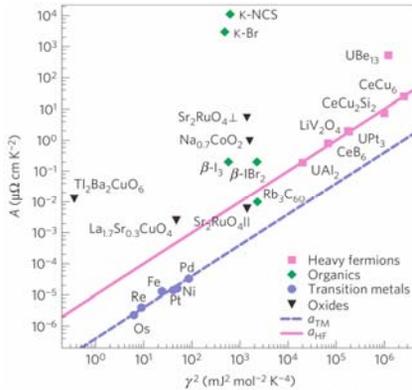


Fig. 1: The standard Kadowaki-Woods plot. It can be seen that the data for the transition metals and heavy fermions (other than UBe_{13}) fall onto two separate lines. However, a wide range of other strongly correlated metals do not fall on either line or between the two lines. In labelling the data points we use the following abbreviations: κ -Br is κ -(BEDT-TTF)₂Cu(CN)₂Br; κ -NCS is κ -(BEDT-TTF)₂Cu(NCS)₂; β -I₃ is β -(BEDT-TTF)₂I₃; and β -Br₂ is β -(BEDT-TTF)₂Br₂. For Sr_2RuO_4 we show data for A measured with the current both perpendicular and parallel to the basal plane. Further details of the data are reported in Ref. 5.

Key questions

- Why is the KWR constant within (a) the heavy fermions and (b) the transition metals - despite the large variation in their effective masses?
- Why is the KWR larger in the heavy fermions than in the transition metals?
- Why are such large and varied KWRs observed in layered materials (e.g., oxides and organics)?

The self energy (Σ)

- Describes the interactions between electrons
- The real part, Σ' , describes the shift in the energy due to electron-electron interactions ($m^*/m = 1 - \partial\Sigma'/\partial\omega$)
- The imaginary part, Σ'' , gives the quasiparticle lifetime ($\Sigma'' = \hbar/\tau$)

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This research was supported under the Australian Research Council's (ARC) Discovery Projects funding scheme (project DP0878523). B.J.P. is the recipient of an ARC Queen Elizabeth II Fellowship (DP0878523).

Many-body theory

Following Miyake, Matsuura & Varma⁶ we take a phenomenological momentum independent form for the imaginary part of self energy:

$$\Sigma''(\omega, T) = -\frac{\hbar}{2\tau_0} - s \frac{\omega^2 + (\pi k_B T)^2}{\omega^2} \quad \text{for } |\omega^2 + (\pi k_B T)^2| < \omega^{*2}$$

At higher energies we smoothly cut off Σ'' . In the unitary scattering limit one finds that

$$A = \frac{16nk_B^2}{\pi\hbar e^2 \langle v_{0x}^2 \rangle D_0^2 \omega^{*2}}$$

where D_0 is the bare density of states at the Fermi energy, n is the electron density and v_{0x} is the bare Fermi velocity in the x direction. Here all of the many-body effects are encapsulated by ω^* . We calculate the real part of the self energy via the Kramers-Kronig relationship. In the limit $m^* \gg m$ we find that

$$\gamma = \gamma_0 \left(1 - \frac{\partial \Sigma'}{\partial \omega}\right) = \gamma_0 \left(1 + \frac{4s_0 \xi}{\pi \omega^*}\right)$$

where γ_0 is the bare γ and ξ is a factor of order unity. Again, all of the many-body effects are encapsulated by ω^* . Thus we find that the Kadowaki-Woods ratio is

$$\frac{A}{\gamma^2} = \frac{81}{4\pi\hbar k_B^2 e^2 \xi^2 n D_0^2 \langle v_{0x}^2 \rangle}$$

Notice that the ratio is independent of ω^* . Thus the ratio is *unrenormalised*. But, while the first factor is universal, the second factor dependent on the bare properties of the material. This suggests that a more fundamental ratio is

$$\frac{A f_{dx}(n)}{\gamma^2} = \frac{81}{4\pi\hbar k_B^2 e^2} \quad (1)$$

where $f_{dx}(n) = n D_0 \langle v_{0x}^2 \rangle \xi^2$ encapsulates the unrenormalised material dependent properties of the system

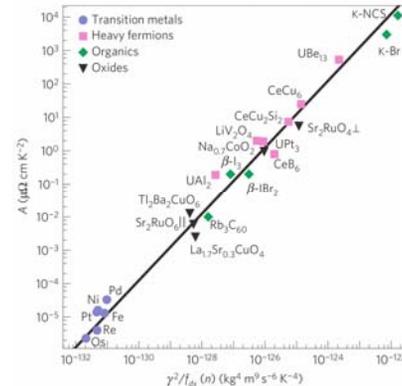


Fig. 2: Comparison of the ratio defined in Eq. (1) with experimental data. It can be seen that, in all of the materials studied, the data are in excellent agreement with our prediction (line). The abbreviations in the data point labels are the same as in Fig. 1. $f_{dx}(n)$ is calculated from a 3D or quasi-2D free electron model as appropriate, see Ref. 5 for details.

Conclusions

- The absolute value of the KWR does not contain any information about the 'strength' of the electronic correlations
- The same correlations determine A and γ in all of the materials studied
- The correlation effects in the KWR ratio cancel because of the Kramers-Kronig relation for the self energy (fluctuation-dissipation theorem)
- We can now answer the three questions we posed earlier
 - The KWR is constant within (a) the heavy fermions and (b) the transition metals because of the Kramers-Kronig relation, but also because the bare properties $[f_{dx}(n)]$ are basically constant within each class of material
 - The KWR is larger in the heavy fermions than in the transition metals because $f_{dx}(n)$ is smaller in the transition metals
 - There are large and varied KWRs in the layered materials (oxides and organics) because $f_{dx}(n)$ is small and varied in these materials