

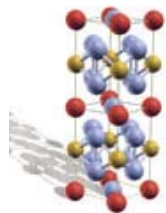
# Understanding Correlated Electron Materials: A Dynamical Mean Field Perspective

**RUTGERS**  
THE STATE UNIVERSITY  
OF NEW JERSEY

Kristjan Haule



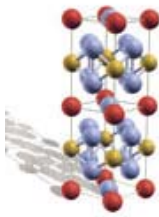
Work in collaboration with Gabi Kotliar



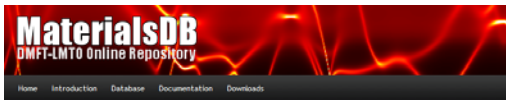
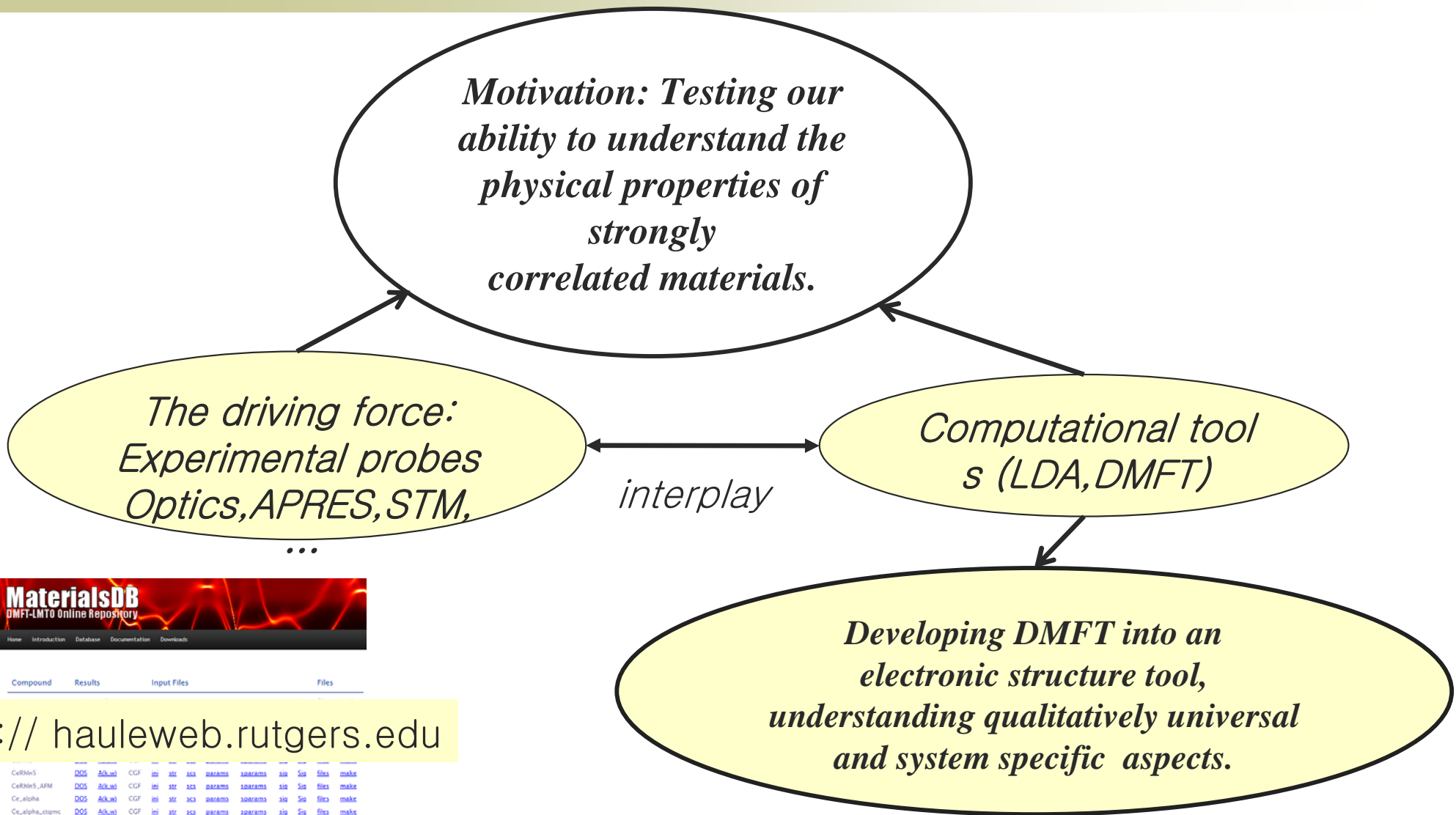
Support:



KITP, Santa Barbara, 2010



# Motivation

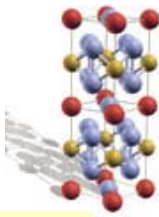


[http:// hauleweb.rutgers.edu](http://hauleweb.rutgers.edu)

Compound	Results	Input Files	Files
CeRhIn5	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
CeRhIn5_AFM	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
Ce_alpha	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
Ce_alpha_ctqmc	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
Ce_gamma	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
Ce_gamma_ctqmc	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
Cm	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
PuAm	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
PuSb	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
PuTe	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	
Pu_delta	DOS <a href="#">Abl_w</a>	CGF <a href="#">ini</a> <a href="#">str</a> <a href="#">scf</a> <a href="#">PARAMS</a> <a href="#">SOBPARAMS</a> <a href="#">sio</a> <a href="#">Sio</a> <a href="#">files</a> <a href="#">make</a>	

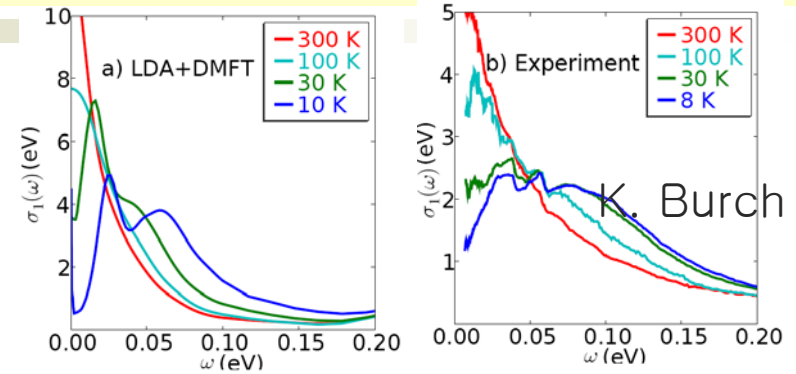
*Developing DMFT into an electronic structure tool, understanding qualitatively universal and system specific aspects.*

Wien2K+DMFT  
 multiorbital CTQMC, full potential basis, charge self-consistent



Protracted screening and multiple

J.H.Shim, KH, G.Kotliar, *Science* 318, 1615 (2007)



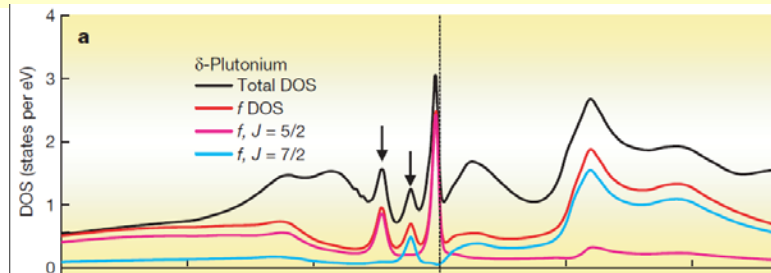
K. Burch et.al.

Tools allow to identify system specific fingerprints which gives us confidence in our understanding of correlated electron material

S

Quasiparticle multiplets in Plutonium

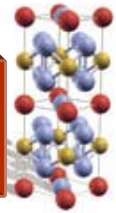
J.H.Shim, KH, G.Kotliar, *Nature* 446, 513 (2007).



Hidden Order in URu2Si2, Kondo effect

KH, G. Kotliar, *Nature Physics* 5, 796 - 799 (2009).

# URu<sub>2</sub>Si<sub>2</sub> - heavy fermion with hidden



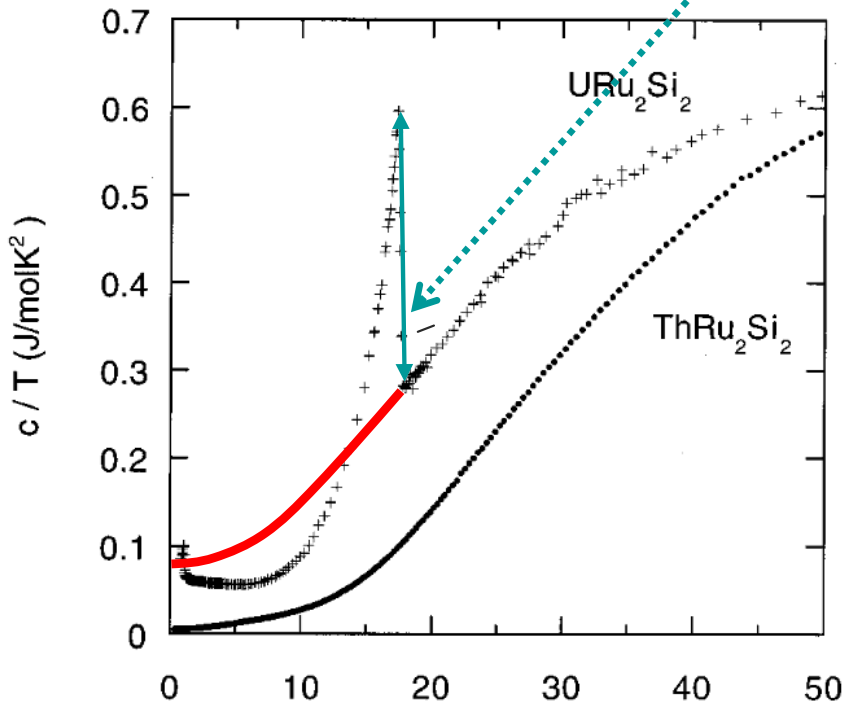
URu<sub>2</sub>Si<sub>2</sub>: T. T. M. Palstra, A. A. Menovsky, J. van den Berg, A. J. Dirkmaat, P. H. Kes,

G. J. Nieuwenhuys and J. A. Mydosh, Physical Review Letters 55, 2727 (1985).  
 Curie-Weiss:  $\mu_z^{\text{eff}} \sim 2.2 \mu_B$  Second order phase transition

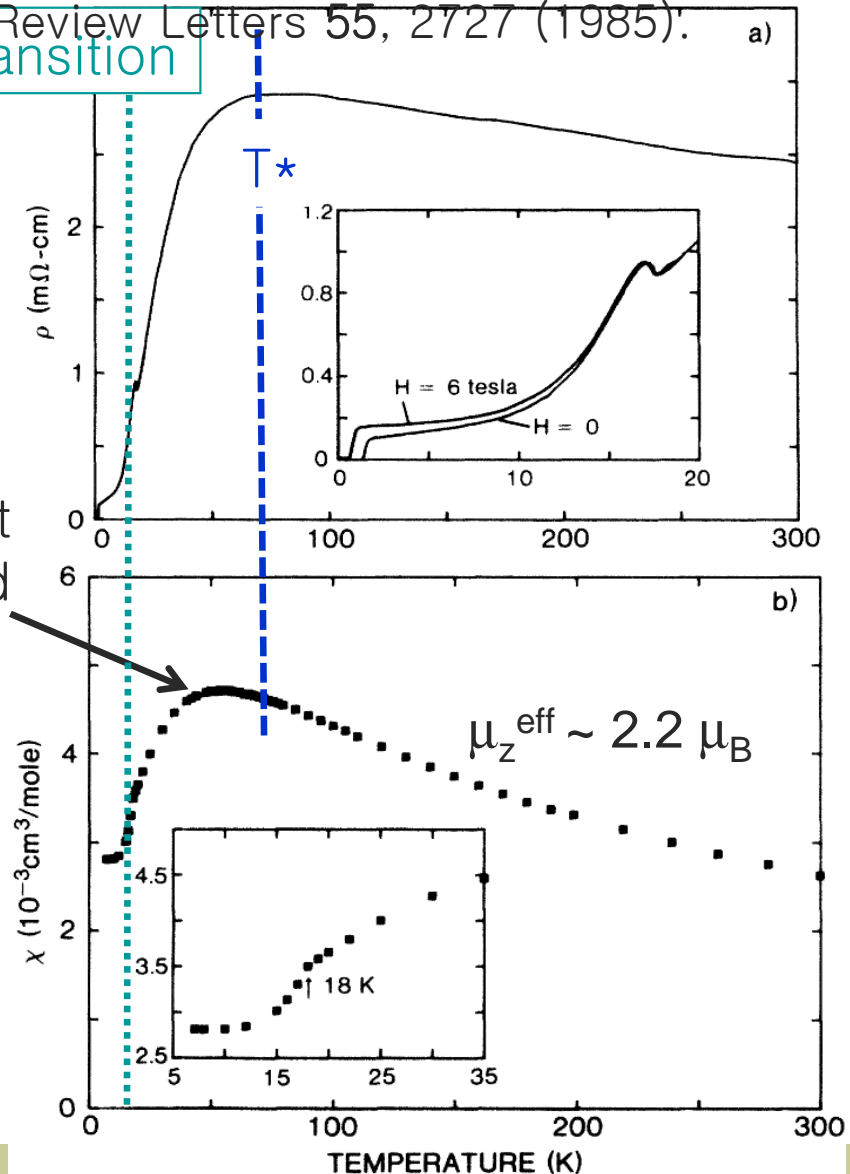
*Elec. cv:*  $\gamma \sim 70 \text{ mJ/mol K}^2$

Coherence:  $T^* \sim 70 \text{ K}$

SOPT:  $T_0 \sim 17.8 \text{ K}$

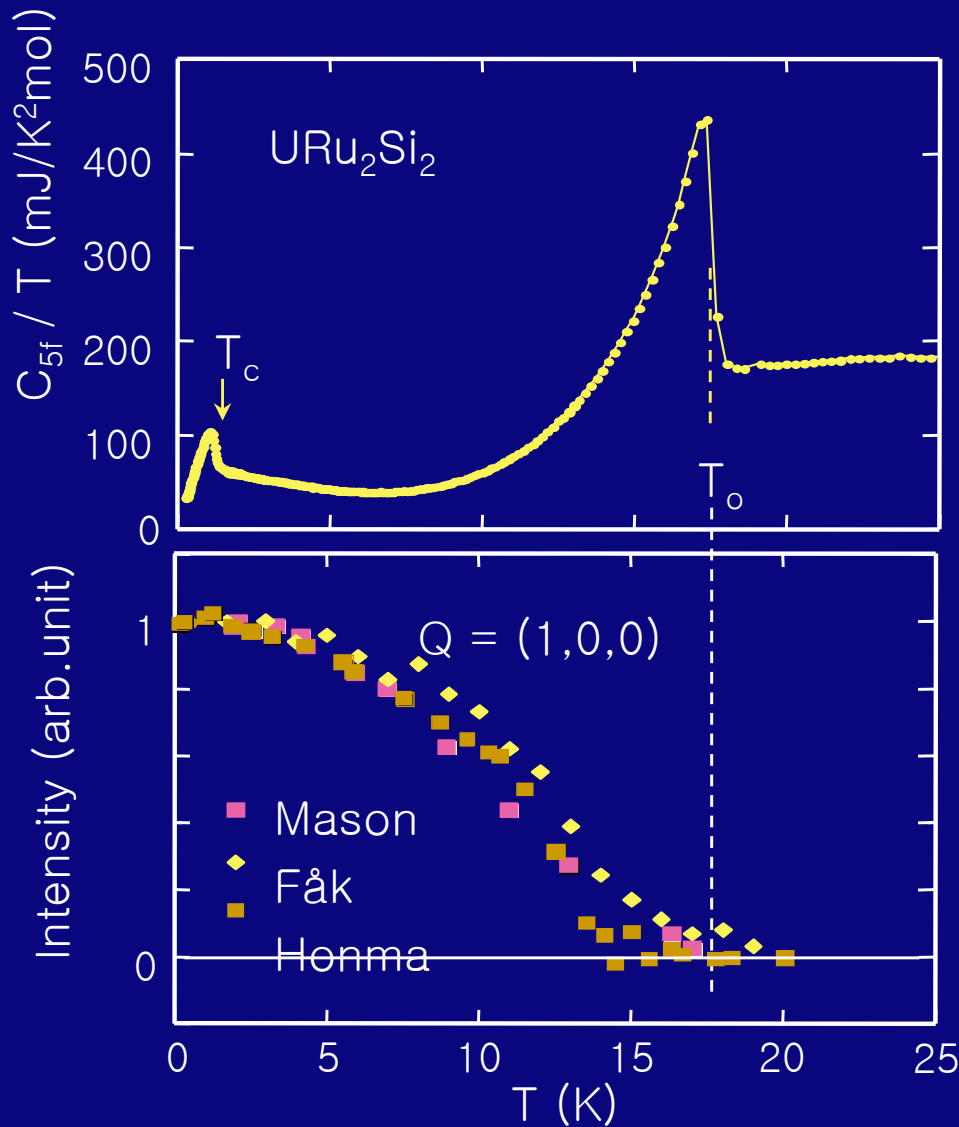
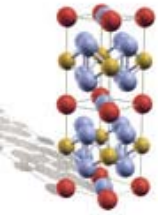


Moment screened



N. H. van Dijk, PRB 56, 14493 (1997).

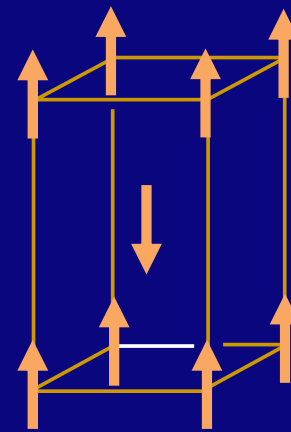
# Neutron Scattering, Specific heat vs. magnetic Bragg-peak intensity. Tc's.



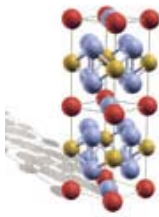
$$S_{\text{mag}} \sim 0.2 R \ln 2$$



$$\mu_{\text{ord}} \sim 0.01 - 0.04 \mu_B$$



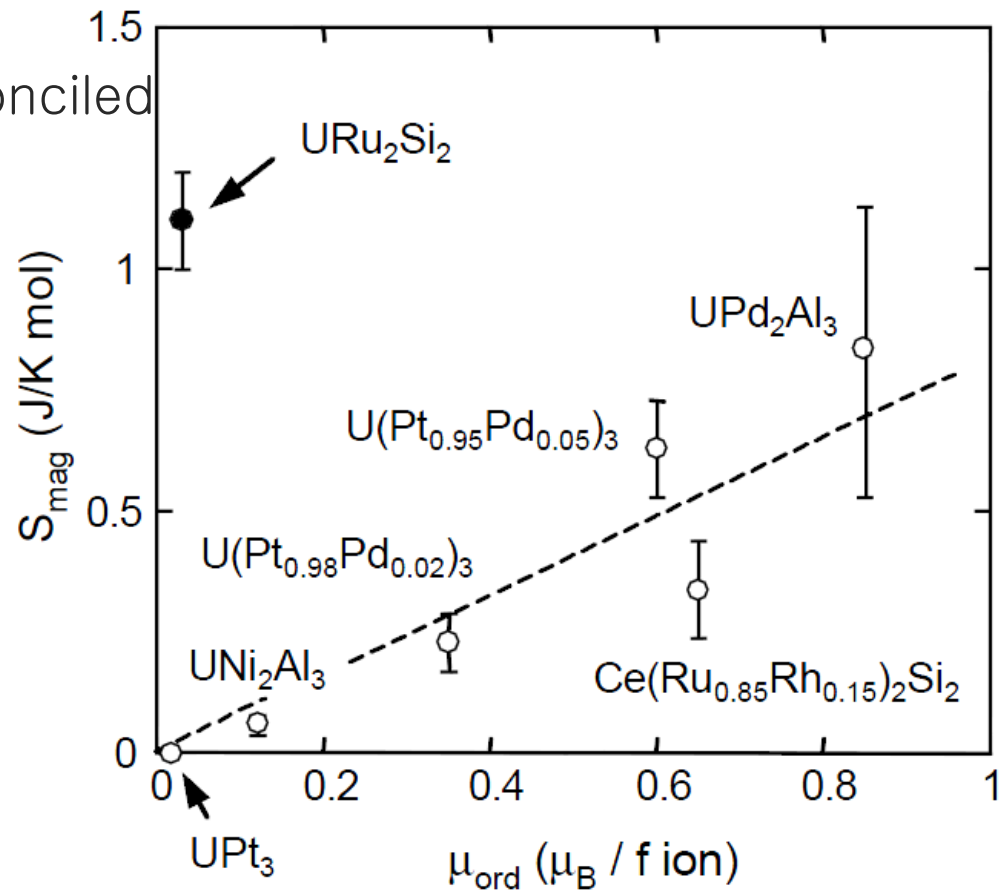
Type-I AF  $\xi_c \sim 100 \text{ \AA}$   
 $\xi_a \sim 300 \text{ \AA}$

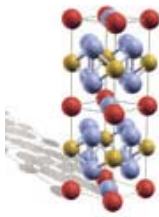


# Hidden Order: The CMT dark matter problem.

- Moment is tiny  
(likely small admixture of AFM phase)
- Large loss of entropy can not be reconciled  
by small moment
- Some other symmetry breaks.  
Hidden order parameter

WHICH?

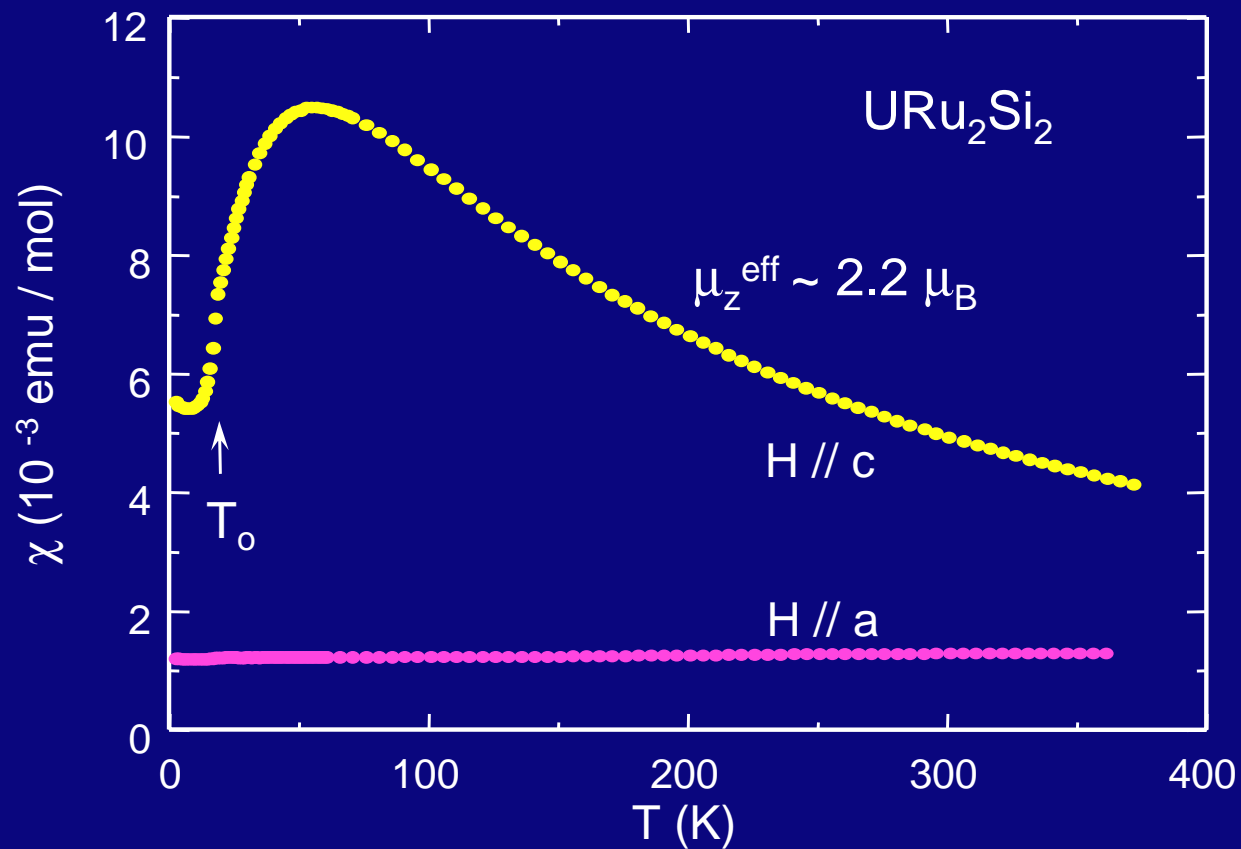




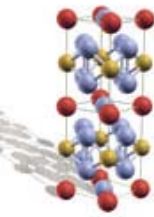
# Magnetic susceptibility

Anisotropy. Moments in z-direction

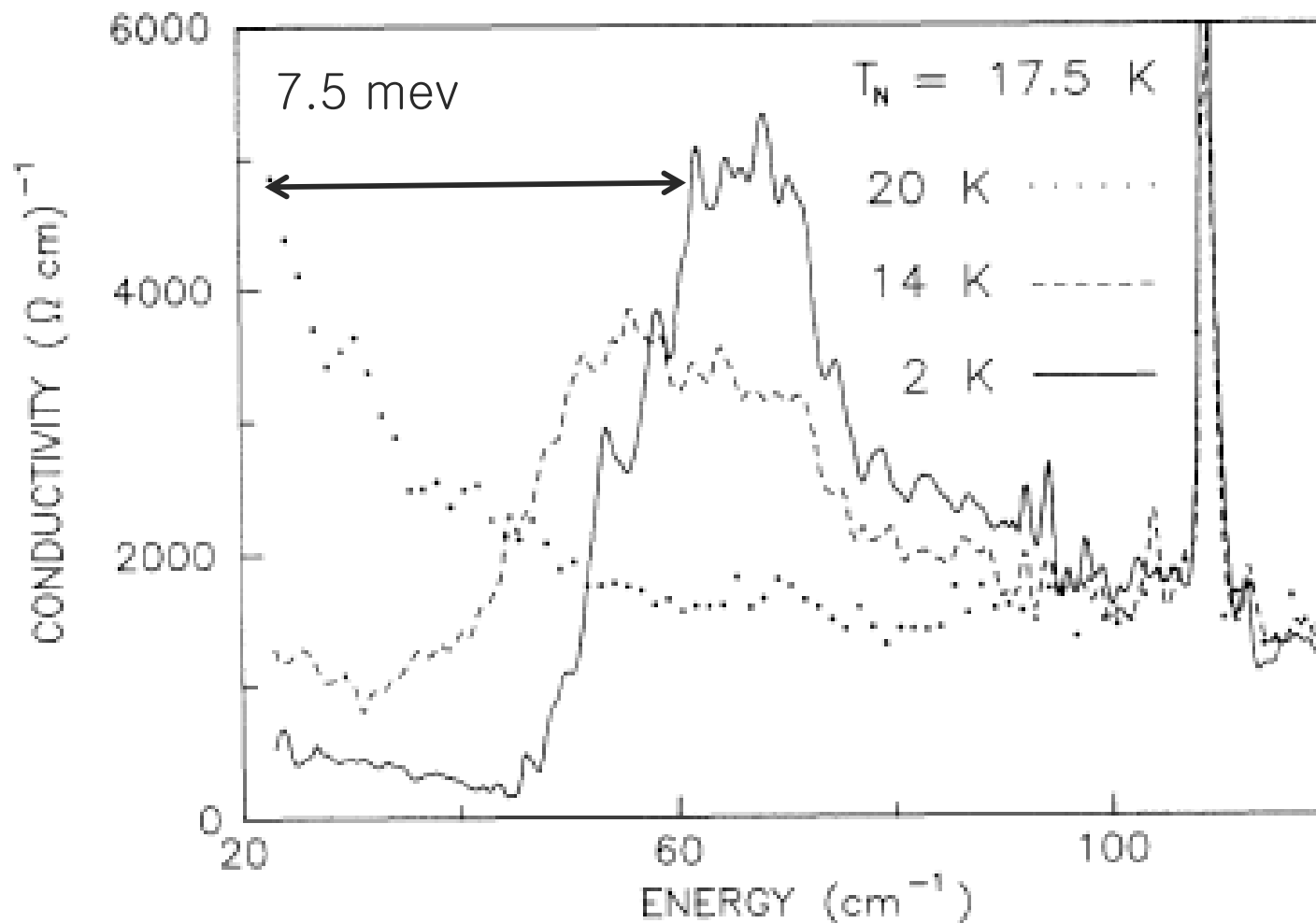
Kondo?  $f^3$   $f^2$  mixed ?



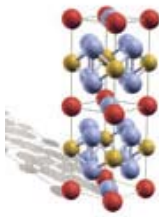
# Optical conductivity



Pseudo-gap opens at  $T_c$ : D. A. Bonn et al. PRL (1988)

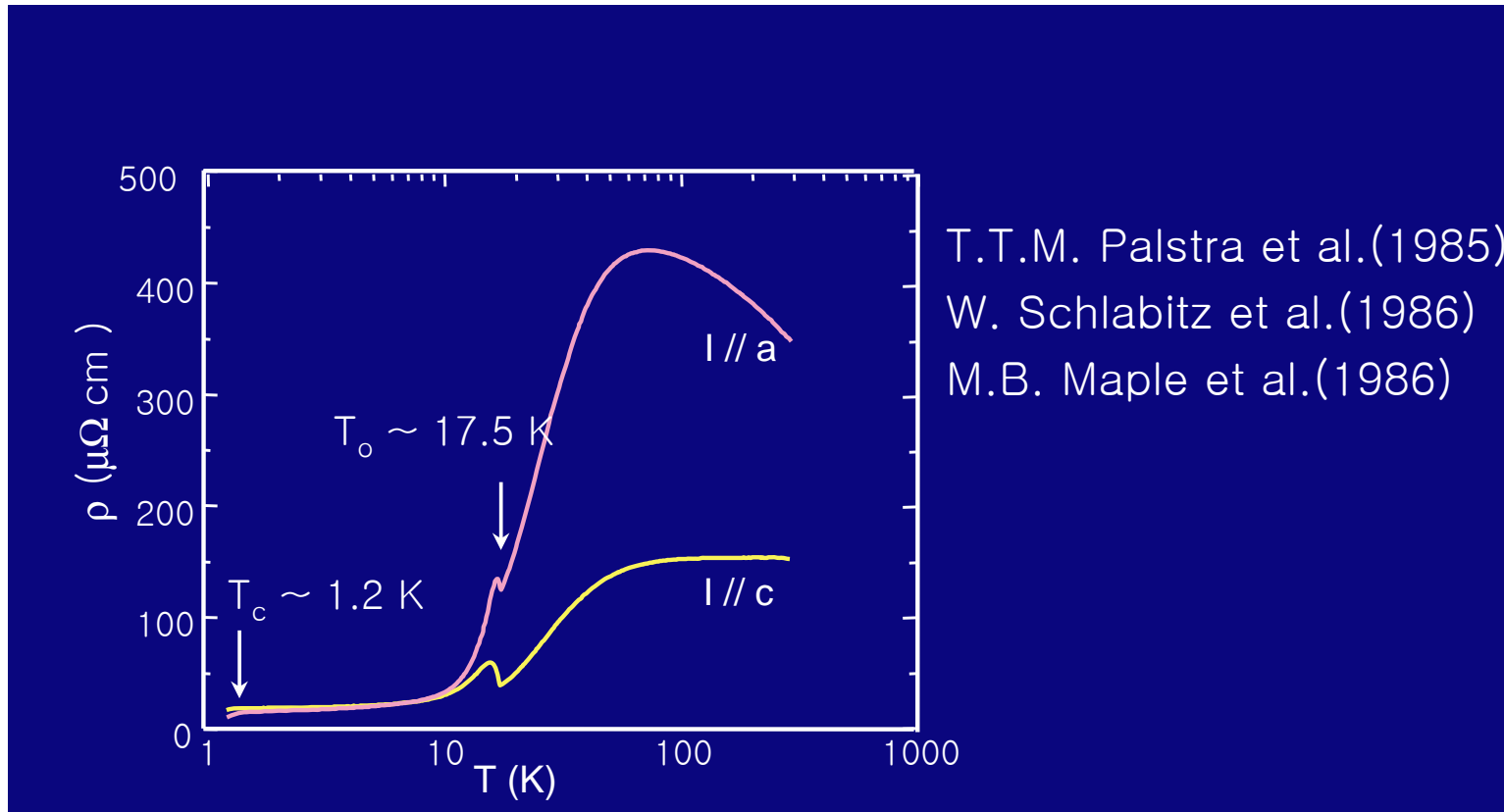






# Resistivity

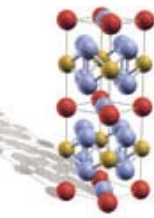
keeps decreasing with decreasing T



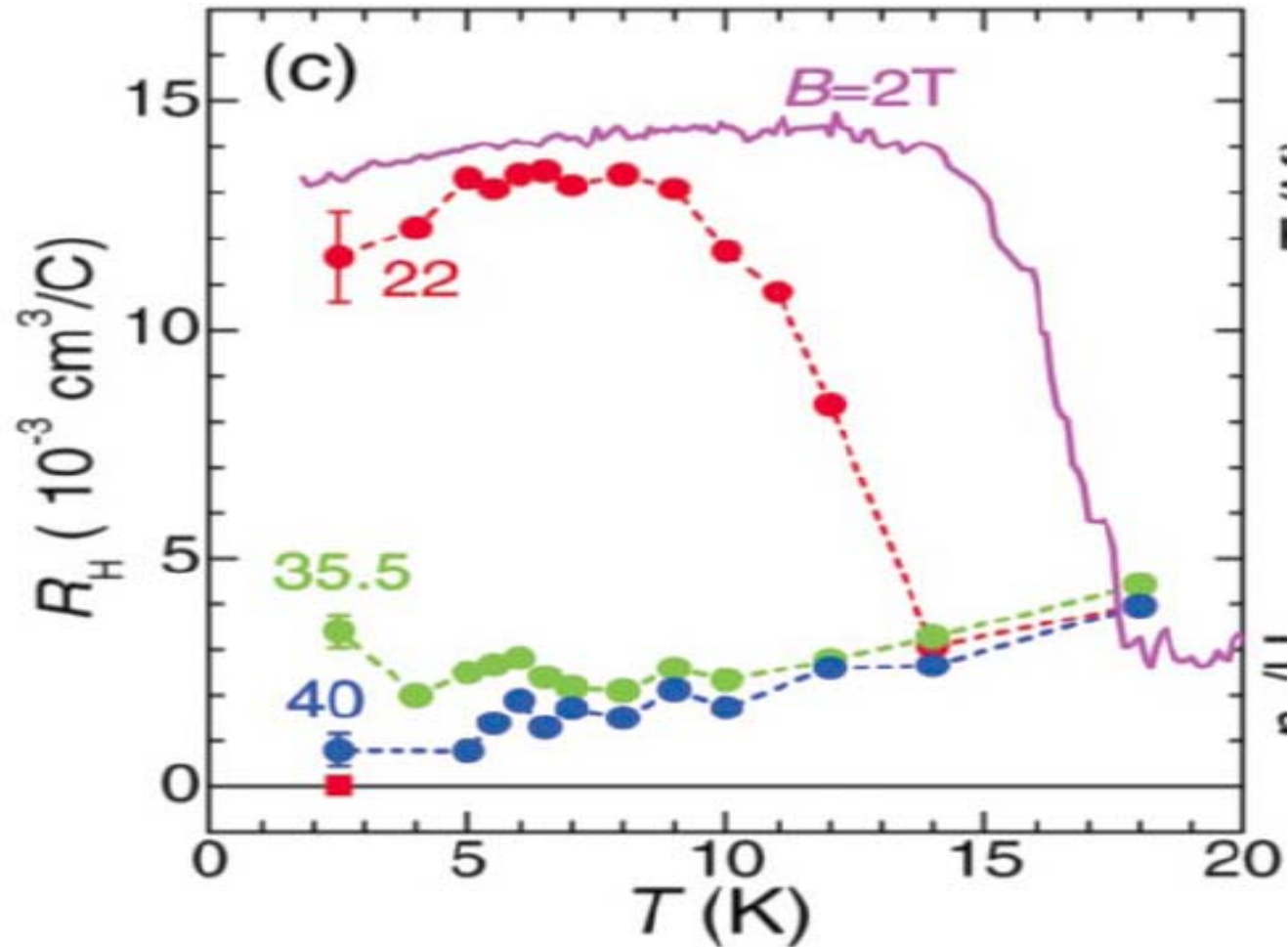
Heavy fermion at high T, low T HO + SC

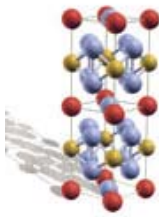
# Hall effect as function of temperature in different external fields,

Y.S. Oh et al. PRL 98, 016401(2007).



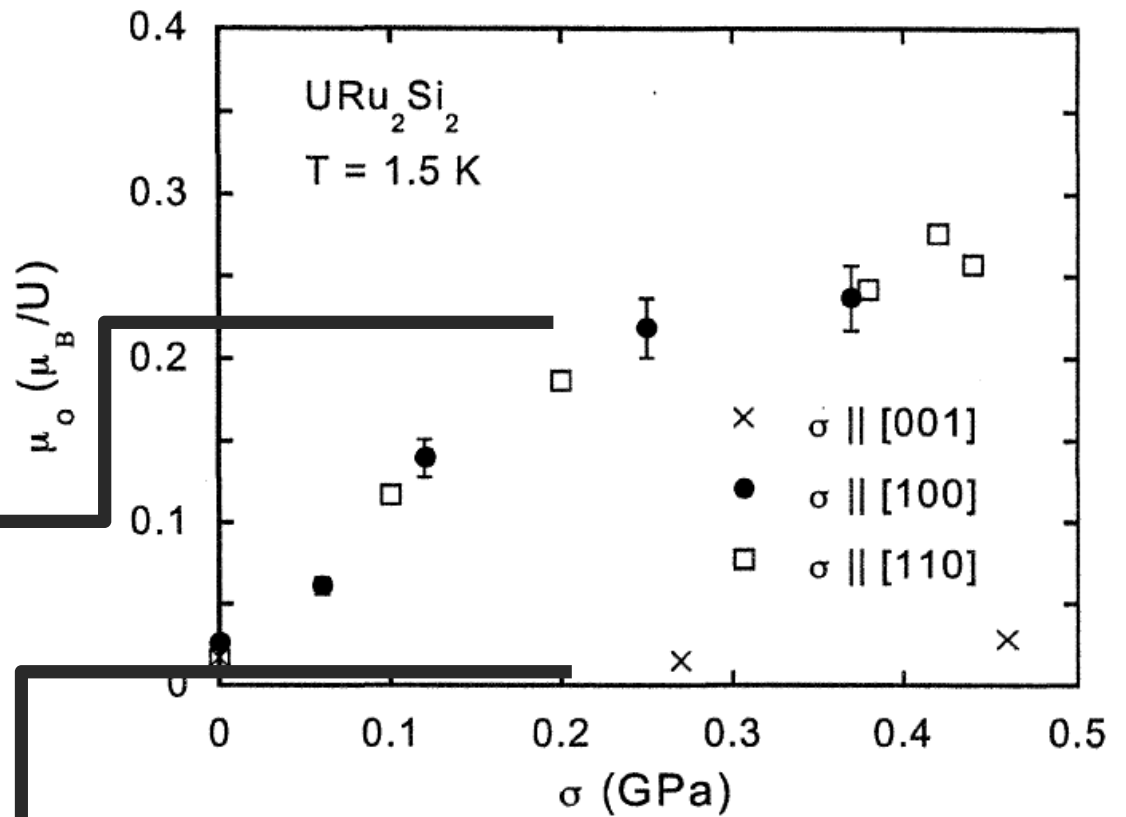
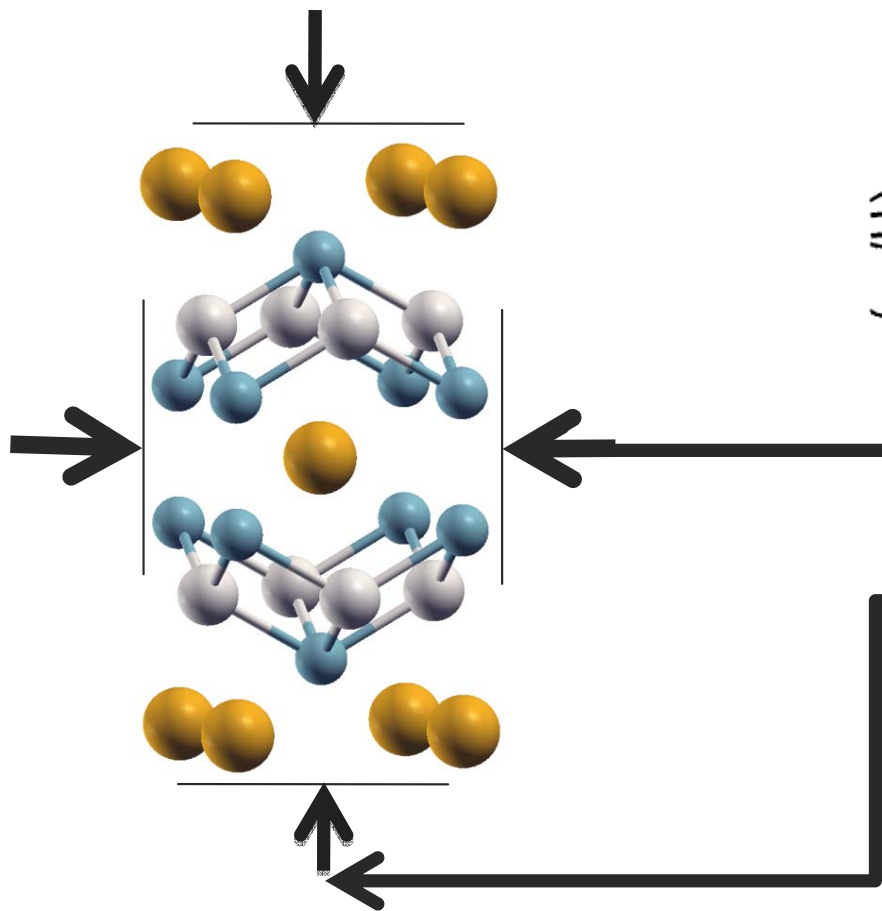
- Fermi surface reconstruction in zero and small field
- Very large fields polarized Fermi liquid.





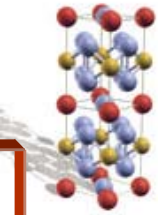
# URu<sub>2</sub>Si<sub>2</sub> Stress in ab plane

Large moment when stress in ab plane  
No moment when stress in c plane



M Yokoyama, JPSJ 71, Supl 264 (2002).

# Adiabatic continuity between HO & AFM phase



PRL 98, 166404 (2007)

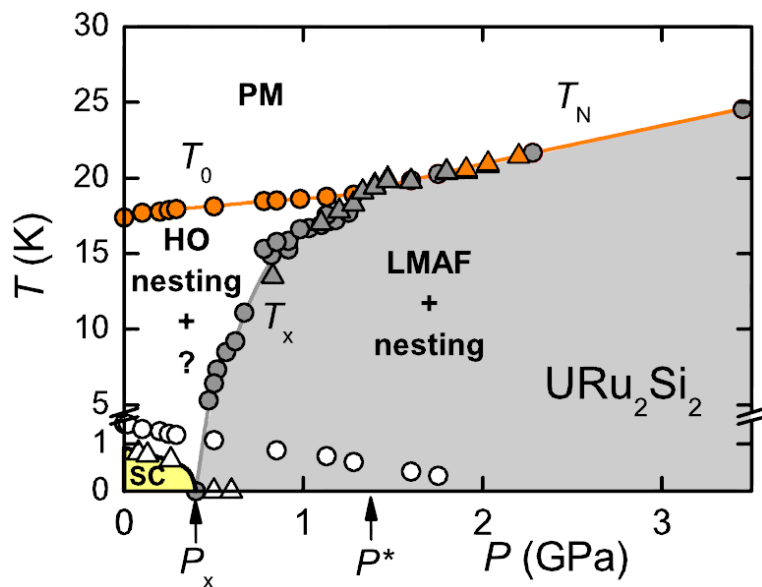
PHYSICAL REVIEW LETTERS

week ending  
20 APRIL 2007

## Field-Induced Fermi Surface Reconstruction and Adiabatic Continuity between Antiferromagnetism and the Hidden-Order State in $\text{URu}_2\text{Si}_2$

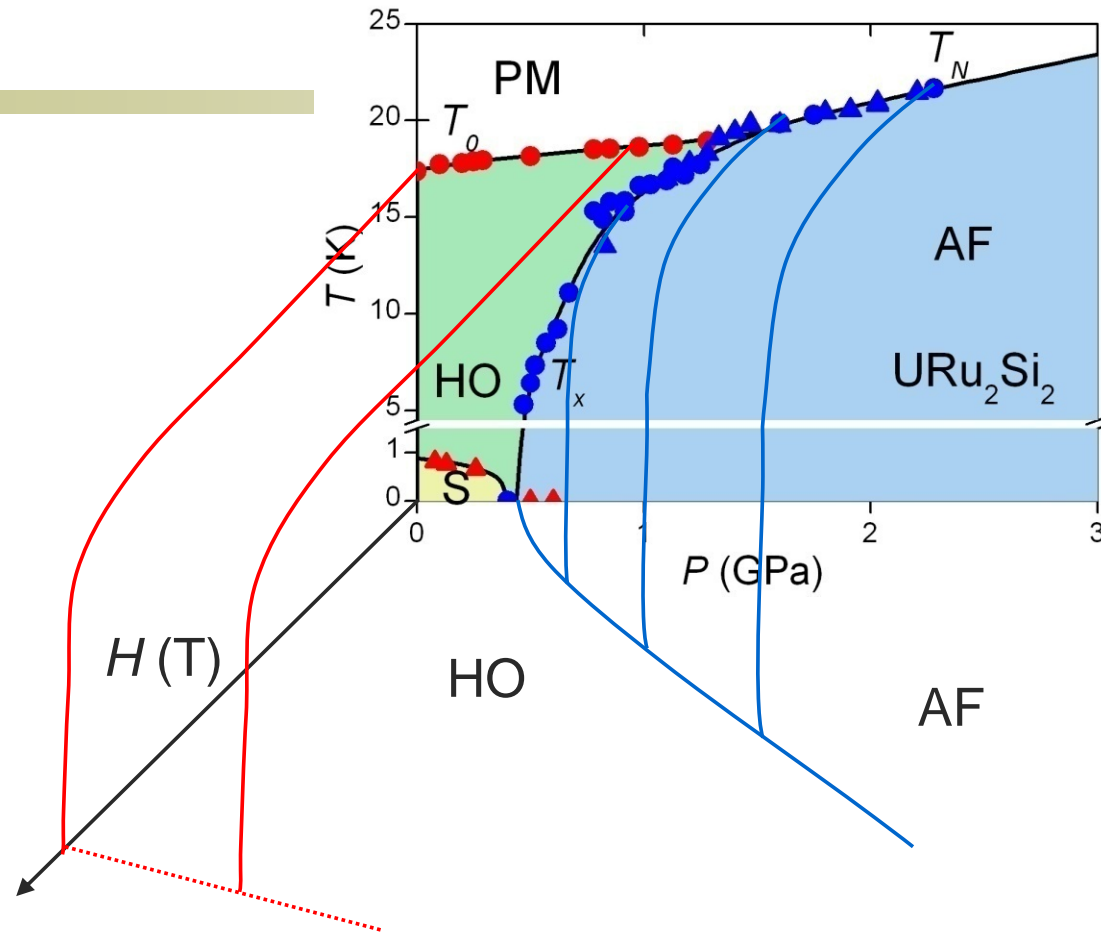
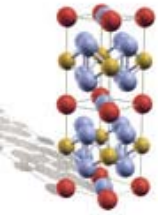
Y. J. Jo,<sup>1</sup> L. Balicas,<sup>1</sup> C. Capan,<sup>2</sup> K. Behnia,<sup>3</sup> P. Lejay,<sup>4</sup> J. Flouquet,<sup>5</sup> J. A. Mydosh,<sup>6</sup> and P. Schlottmann<sup>1</sup>

$H - T$  phase diagram. Instead of phase separation between HO and antiferromagnetism our observations indicate adiabatic continuity between both orderings with field and pressure changing their relative weight.



- HO under pressure converted to AFM phase through 1<sup>st</sup> order transition
- Similar  $T_0$  and  $T_N$
- Almost identical thermodynamic quantities (jump in  $C_v$ ), quantum oscillations

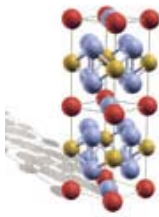
# Pressure magnetic field phase diagram



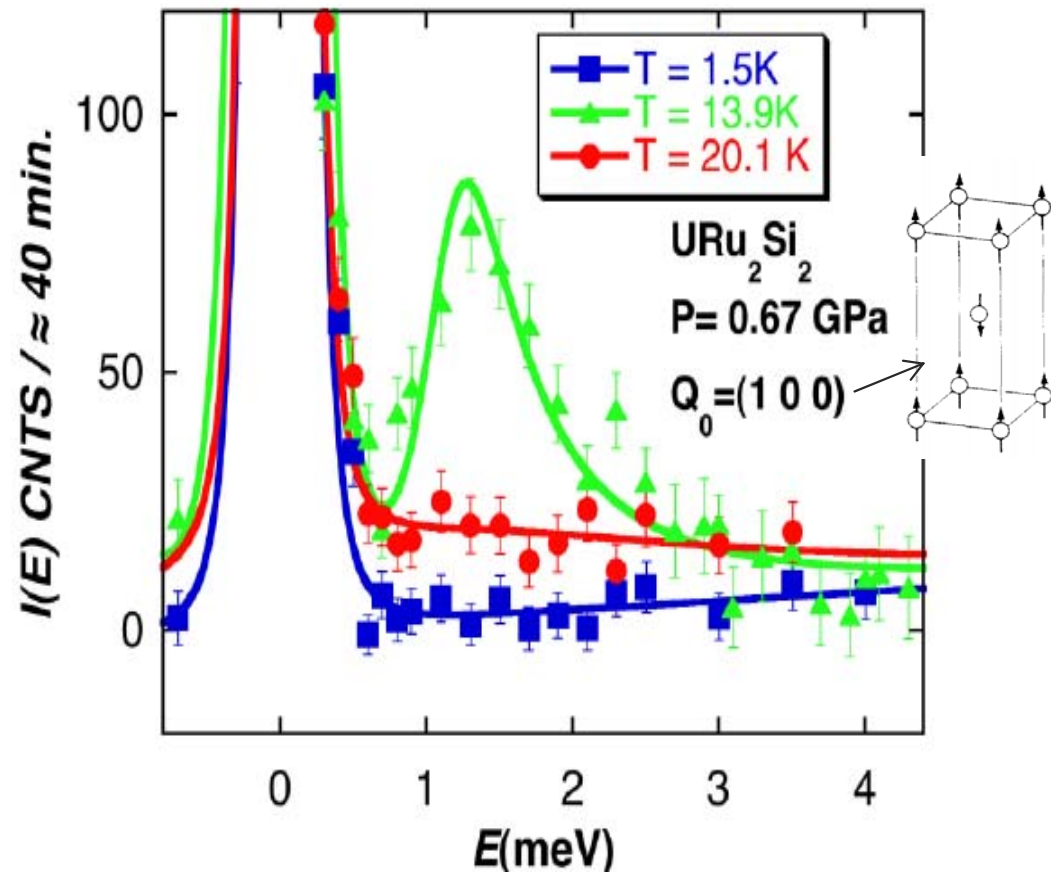
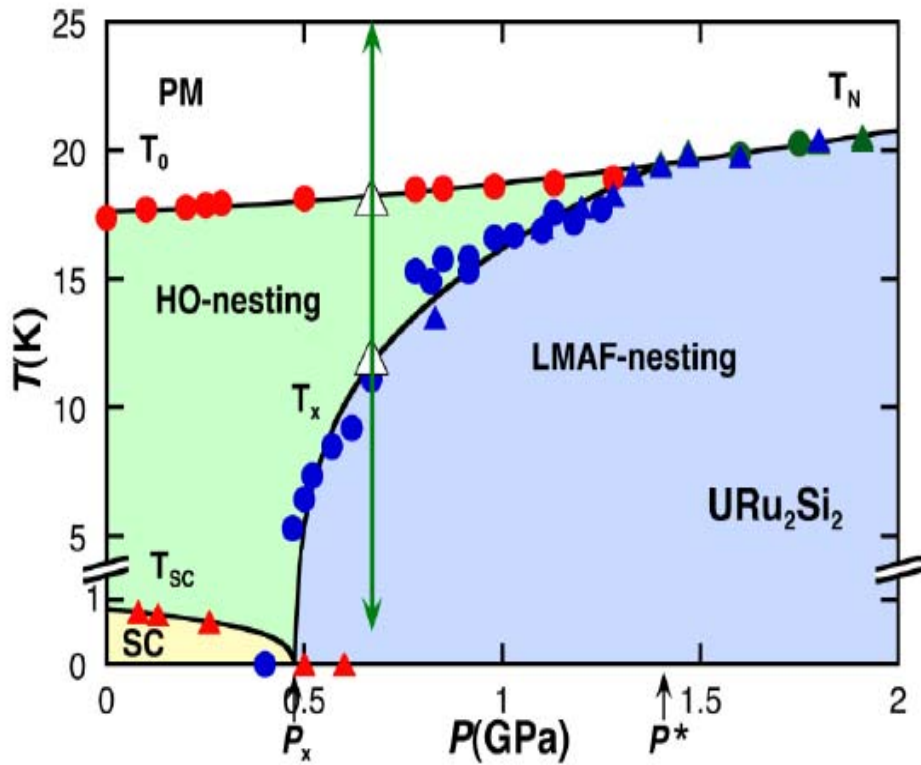
E. Hassinger et.al. PRL 77, 115117 (2008)

Aoki et.al., JPSJ 053701 (2009)

- Pressure induces AF phase, field has opposite effect

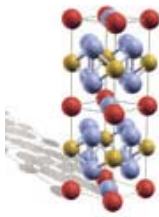


# Key experiment: Neutron scattering

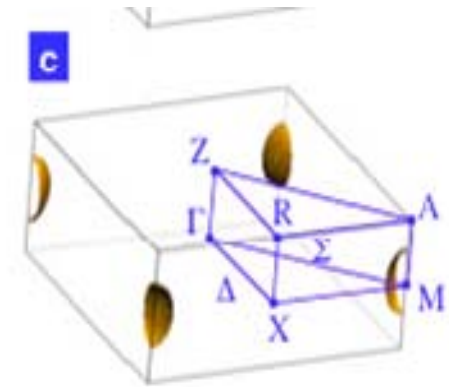
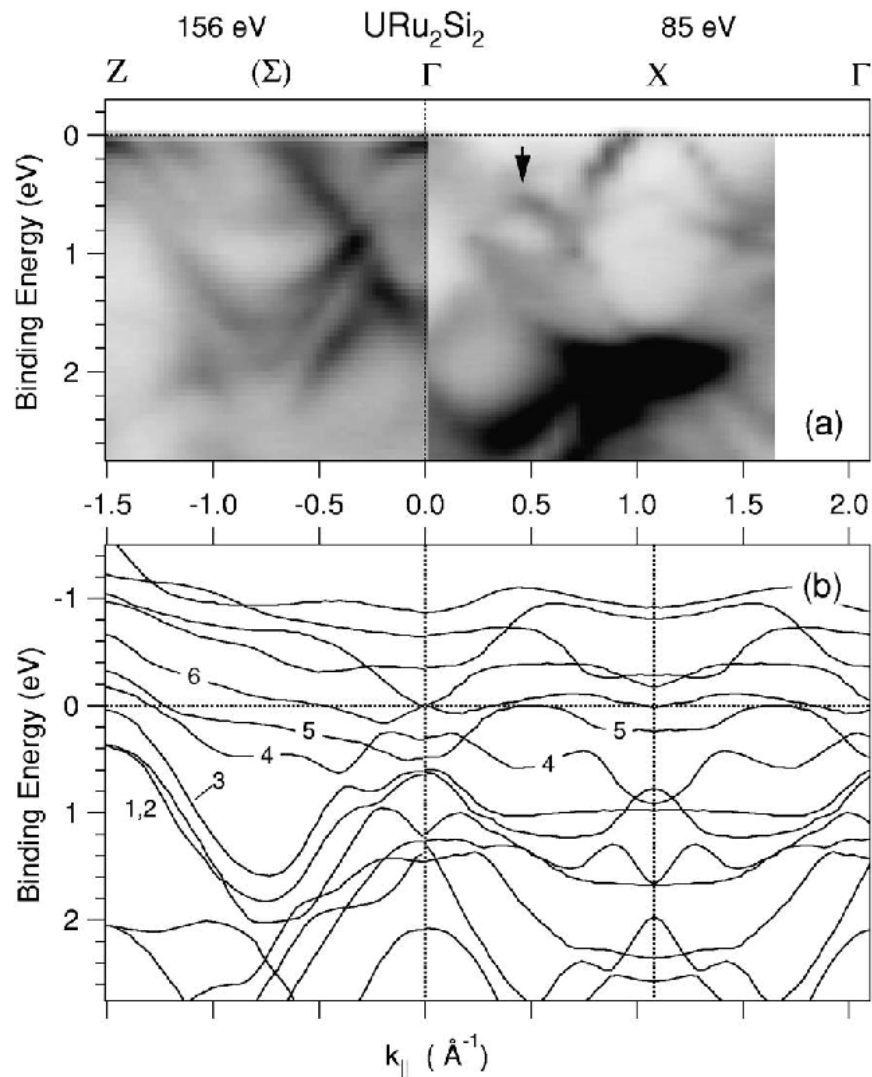


The low energy resonance

A. Villaume, F. Bourdarot, E. Hassinger, S. Raymond, V. Taufour, D. Aoki, and J. Flouquet, PRB 78, 012504 (2008)

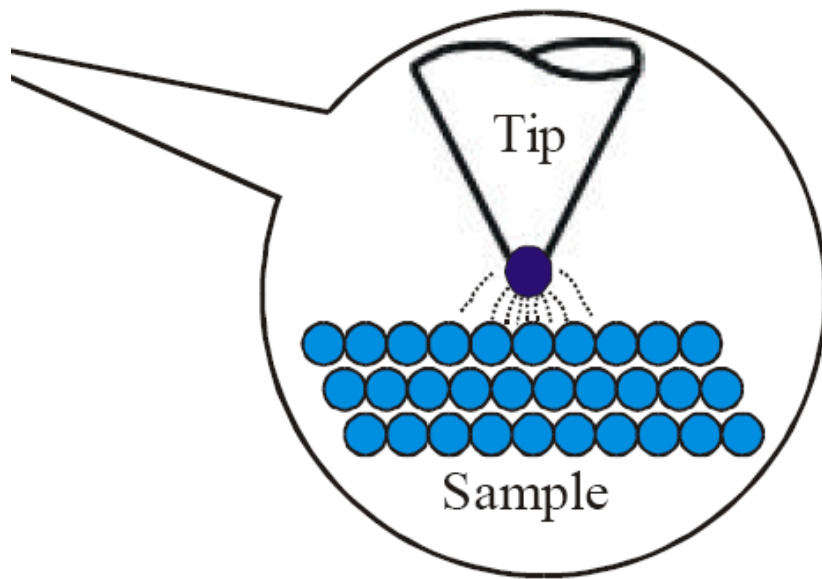
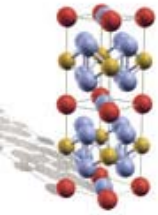


# ARPES does not agree with LDA



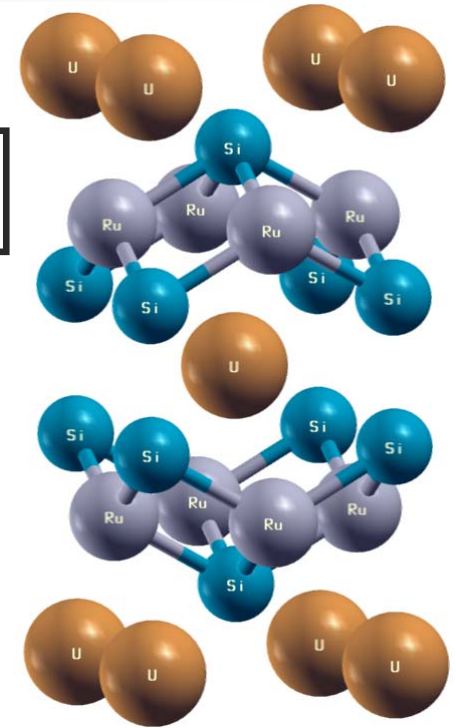
J.D. Denlinger et.al., 2001

# Scanning Tunneling Microscopy

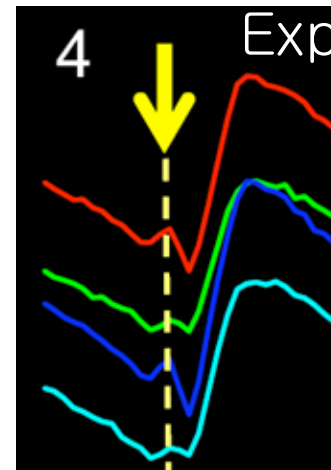


$$dI/dV$$

$$\rho(r, \omega)$$

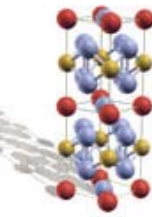


G. Luke, A. Schmidt, P. Wah  
I,  
M. Hamidian, J. C. Davis,  
unpublished

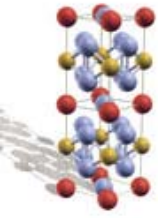




# Comments concerning Hidden Order

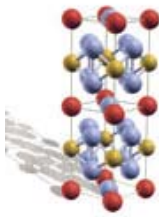


- URu<sub>2</sub>Si<sub>2</sub>, at high temperatures is not too different from a garden variety heavy fermion.
- ARPES does not agree with LDA at 30 K.
- HO converts to LMAF by pressure through a first order line.
- HO can be stabilized by B and destroyed by Rh-doping
- HO and LMAF are remarkably similar  
("Mydosh's adiabatic continuity")
- HO opens some form of a gap in optics.
- HO likely involves an electronic topological transition  
[Hall Effect, also Nernst]
- HO exhibits two INS modes: (1,0,0)@1.5-2.0meV and (1.4,0,0)@5meV of longitudinal fluctuations/excitations.
- HO (but not LAMF) turns into superconductivity at 1.7 K.



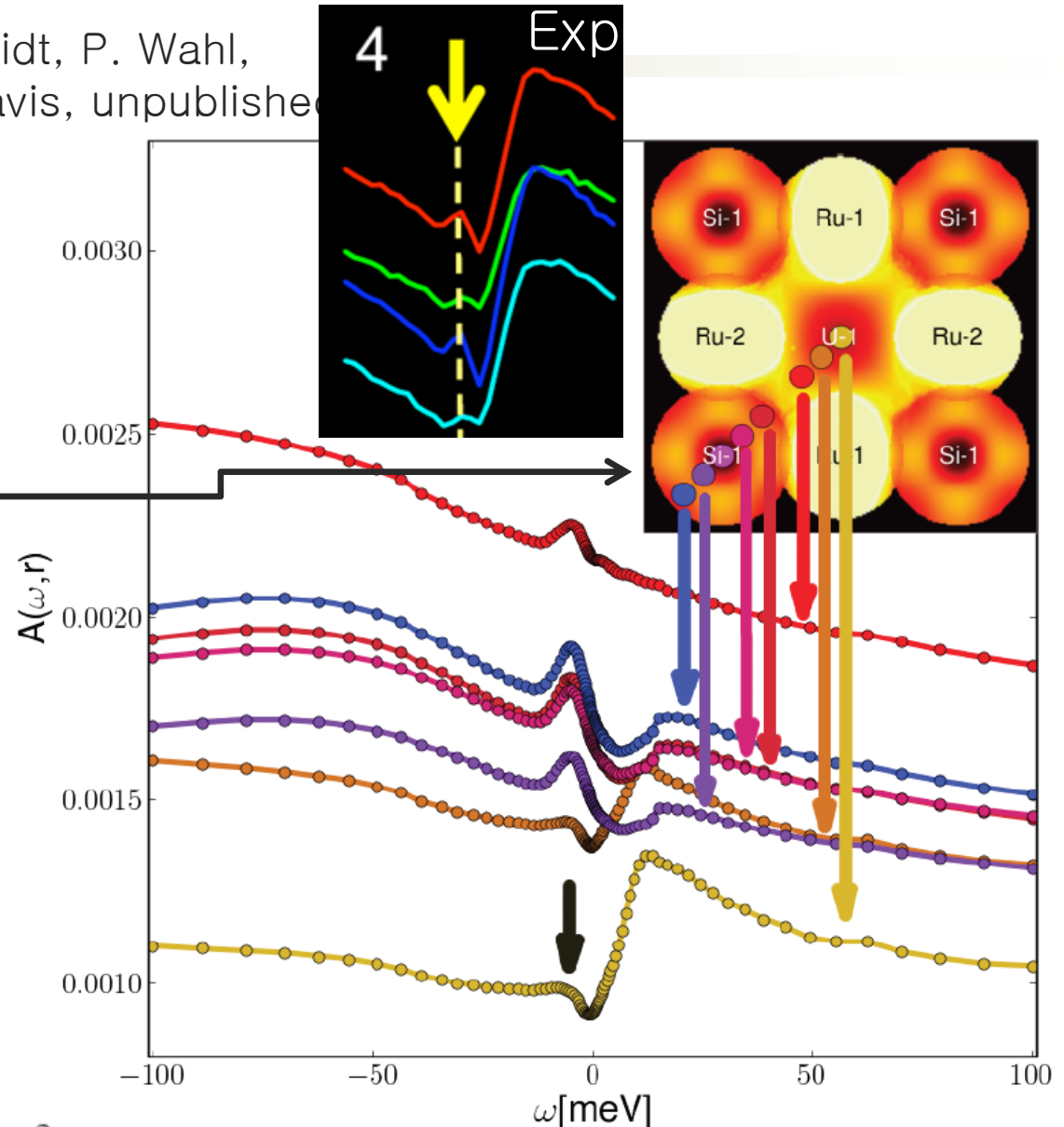
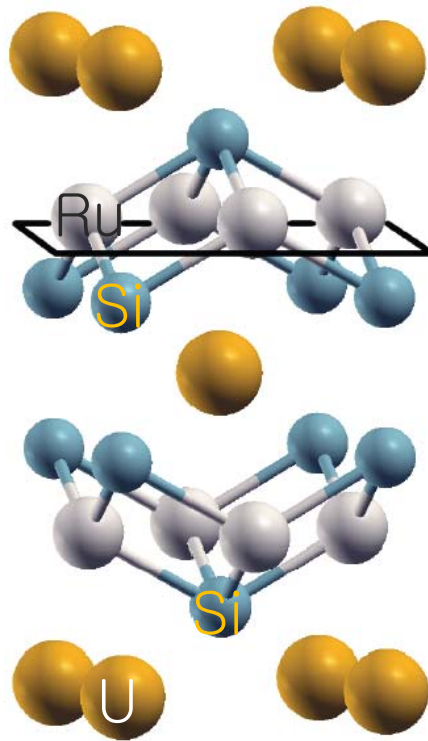
## Some proposals for the hidden order in the literature

- Lev. P. Gorkov: 1996:
  - *Mixed valency, coupling to lattice degrees of freedom.*
- Chandra *et al.*, Nature '02
  - *Incommensurate Orbital Antiferromagnetism (based on "old" NMR)*
- Mineev & Zhitomirsky, PRB '05
  - *SDW (with tiny moment... problem with entropy)*
- Varma & Zhu, PRL '06
  - *Helical Order, Pomeranchuk instability of the Fermi surface ?*
- Elgazaar, & Oppeneer, Nature Materials '08
  - *DFT: antiferromagnetic order parameter, but weak AFM moment (can not explain large entropy loss, stress, adiabatic continuity, moment in z dir....)*
- Santini and Amoretti PRL 04
  - *Quadrupolar ordering.*
- Fazekas and Kiss PRB 07
  - *Octupolar ordering. [ Many Many more , even recently , including us ☺]*



# Dynamical Mean Field calculation

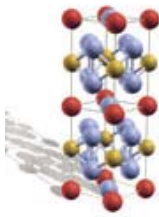
G. Luke, A. Schmidt, P. Wahl,  
M. Hamidian, J. C. Davis, unpublished



Fano lineshape:

$$A(\omega) \propto [(q^2 - 1) + 2q(\omega/\Gamma)] / [(\omega/\Gamma)^2 + 1]$$

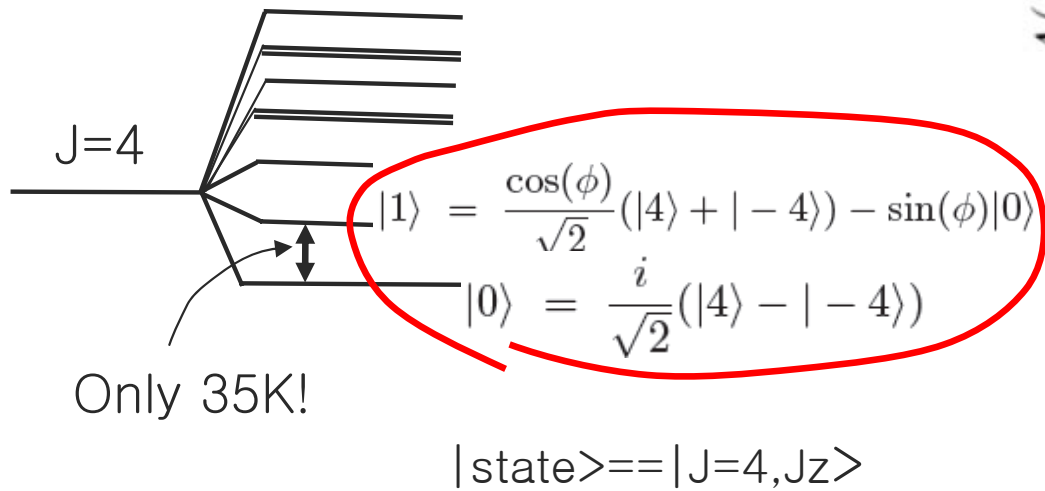
$q \sim 1.24$ ,  $\Gamma \sim 6.8 \text{ meV}$ , very similar to exp



# Origin of gapping?

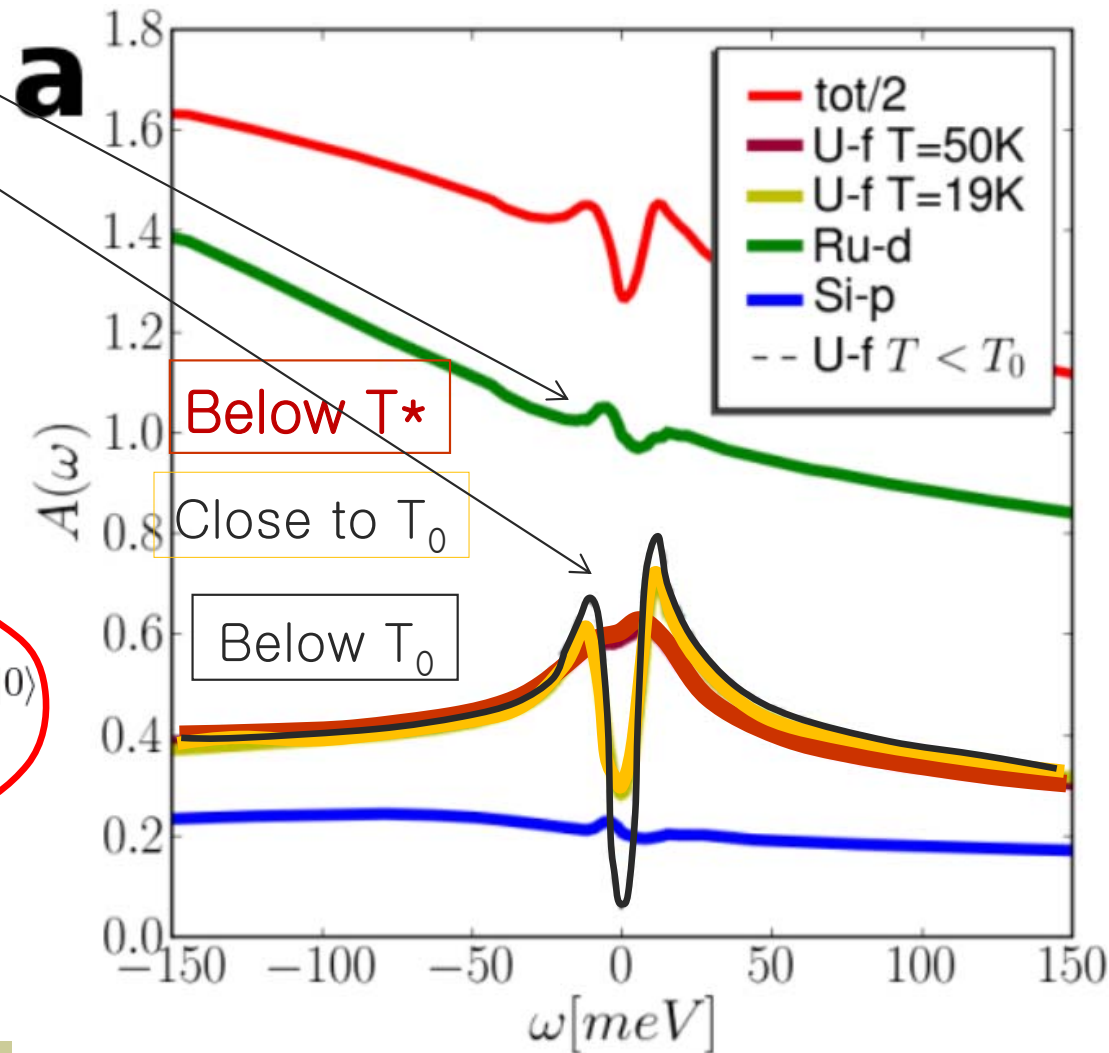
Small effects on spd electrons  
Large effect on U-f's

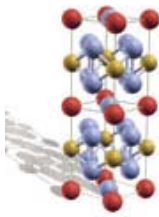
Ground state atomic multiplet of  $f^2$  configuration in tetragonal field



Only 35K!

Partial DOS



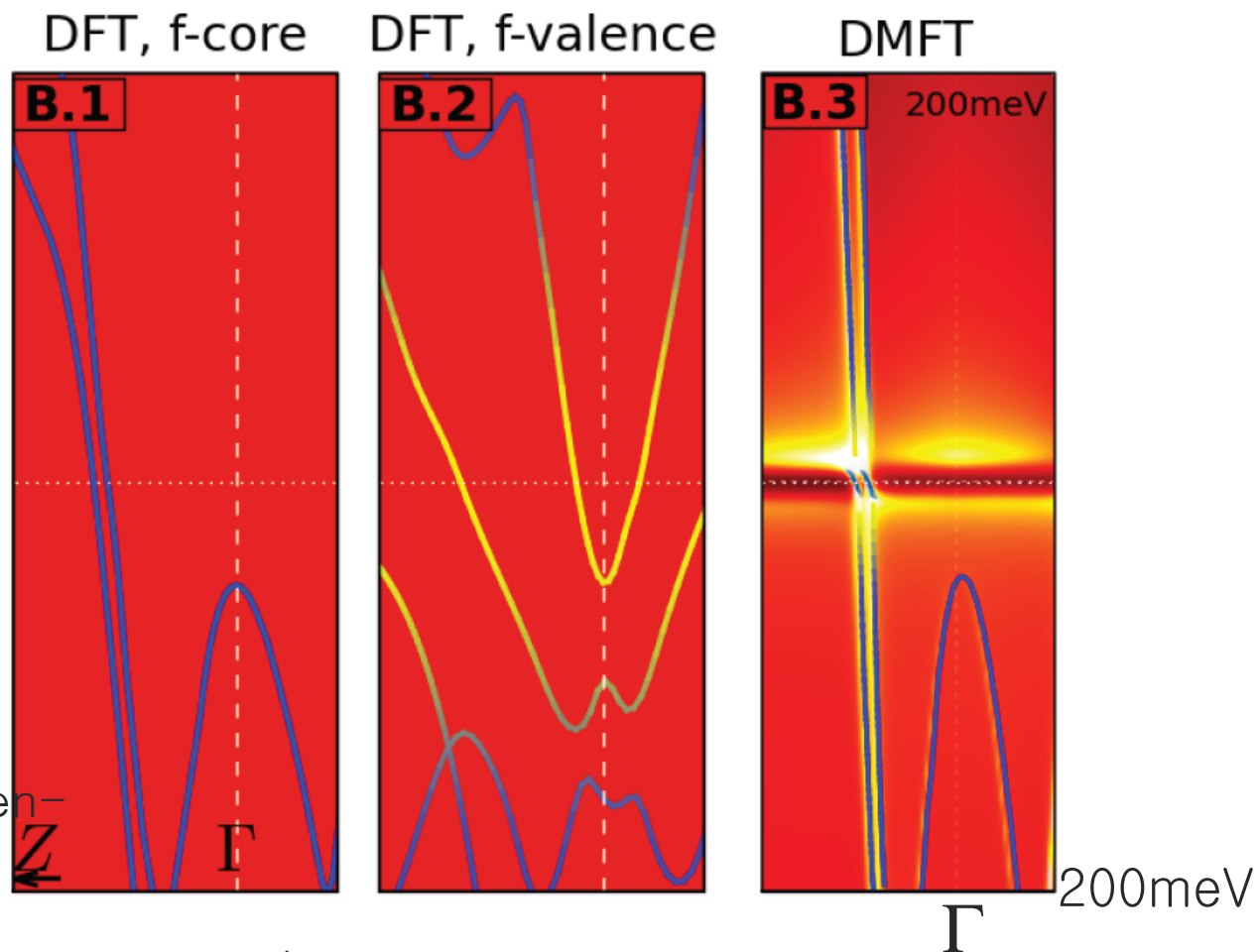


# Partially Arrested Kondo effect

- DFT f-core: good description of bands 30meV away from EF
- DFT f-valence: many f-bands at EF, substantial disagreement with ARPES & DMFT

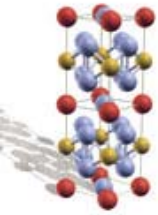
DMFT:  
very narrow region of f-spectral weight  $\pm 10\text{meV}$  around EF appears below  $T^* \sim 70\text{K}$   
Below 35K, partial gap starts to open  
 $\rightarrow$  singlet to singlet Kondo effect

On resonance



At low temperature, two broken symmetry states!

# DMFT allows two broken symmetry states at low T



Density matrix for U 5f state  
the J=5/2 subspace

Large moment phase:

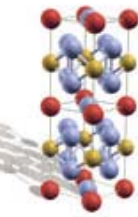
$$\Delta = \begin{pmatrix} J=5/2 & -5/2 & -3/2 & -1/2 & 1/2 & 3/2 & 5/2 \\ -5/2 & \Delta_A & 0 & 0 & 0 & \Delta_\epsilon & 0 \\ -3/2 & 0 & \Delta_B & 0 & 0 & 0 & \Delta'_\epsilon \\ -1/2 & 0 & 0 & \Delta_C & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & \Delta'_C & 0 & 0 \\ 3/2 & \Delta_\epsilon & 0 & 0 & 0 & \Delta'_B & 0 \\ 5/2 & 0 & \Delta'_\epsilon & 0 & 0 & 0 & \Delta'_A \end{pmatrix}$$

Moment free phase:

$$\Delta = \begin{pmatrix} J=5/2 & -5/2 & -3/2 & -1/2 & 1/2 & 3/2 & 5/2 \\ -5/2 & \Delta_A & 0 & 0 & 0 & \Delta_\epsilon + \Delta_\alpha & 0 \\ -3/2 & 0 & \Delta_B & 0 & 0 & 0 & \Delta'_\epsilon + \Delta_\beta \\ -1/2 & 0 & 0 & \Delta_C & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & \Delta_C & 0 & 0 \\ 3/2 & \Delta_\epsilon - \Delta_\alpha & 0 & 0 & 0 & \Delta_B & 0 \\ 5/2 & 0 & \Delta_\epsilon - \Delta_\beta & 0 & 0 & 0 & \Delta_A \end{pmatrix}$$

tetragonal symmetry broken →  
these terms nonzero

# Valence histogram point of view



The DMFT density matrix has most weight in two singlet  $f^2$  configurations

Close to  $f^2$  “Kondo” limit, ( $n_f \sim 2.2$ ),  $J=4$ , two low lying singlets.

Therefore there are two singlets relevant at low energies but they are not Kramer doublets: Conspiracy between cubic crystal field splittings and tetragonal splittings bring these two states close.

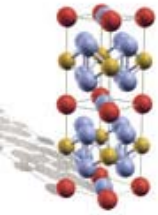
This is why URu<sub>2</sub>Si<sub>2</sub> is sort of unique.

$$|0\rangle = \frac{i}{\sqrt{2}}(|4\rangle - |-4\rangle)$$

$$|1\rangle = \frac{\cos(\phi)}{\sqrt{2}}(|4\rangle + |-4\rangle) - \sin(\phi)|0\rangle$$

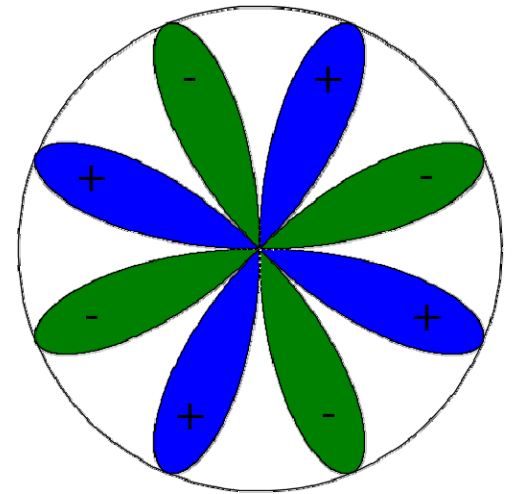
Both states are time-reversal invariant

# DMFT order parameter



$$|0\rangle = \frac{i}{\sqrt{2}}(|4\rangle - |-4\rangle)$$

$$|1\rangle = \frac{\cos(\phi)}{\sqrt{2}}(|4\rangle + |-4\rangle) - \sin(\phi)|0\rangle$$



Order parameter:

$$\psi_i = \langle X_{01}(\mathbf{R}_i) \rangle \begin{cases} \text{Im}\psi \propto \langle J_z \rangle \\ \text{Re}\psi \propto \langle (J_x J_y + J_y J_x)(J_x^2 - J_y^2) \rangle \end{cases}$$

Different orientation gives different phases: adiabatic continuity explained!

Does *not* break the *time reversal*, *nor C4* symmetry. It breaks *inversion* symmetry.

In the atomic limit:

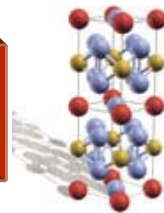
$$|gs\rangle = \cos(\theta)|0\rangle + \sin(\theta)e^{i\varphi}|1\rangle$$

$$\langle gs|\mathbf{J}|gs\rangle = 4 \cos(\phi) \sin(2\theta) \sin(\varphi) * (0, 0, 1)$$

Moment only in z-direction!

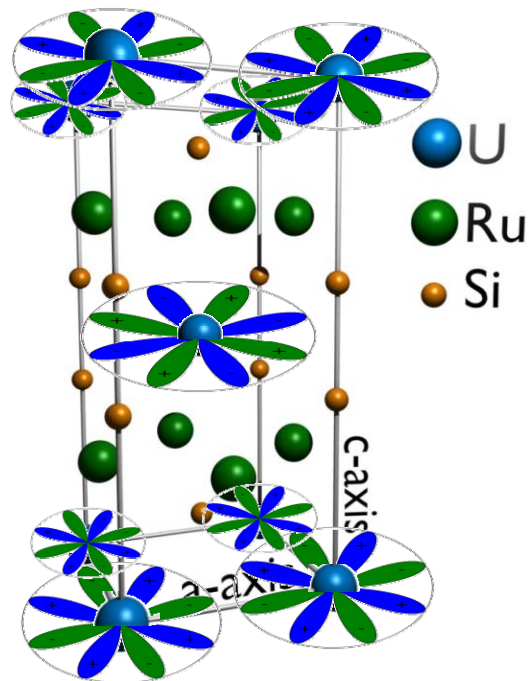
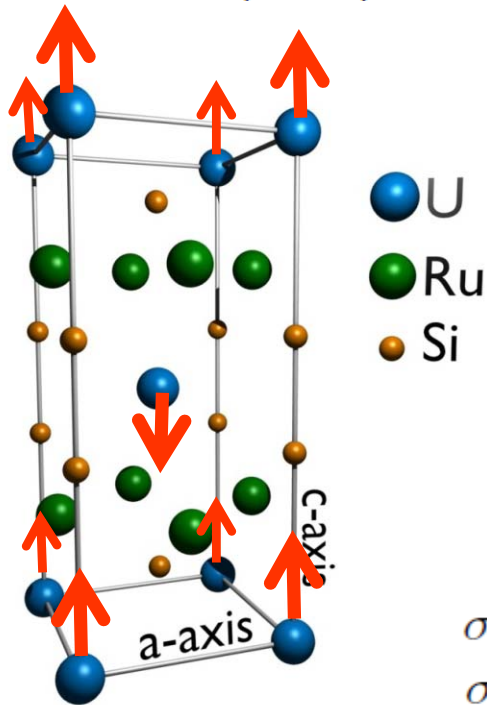


# The two broken symmetry states



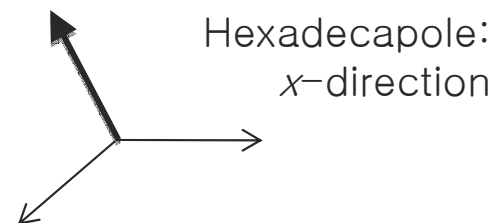
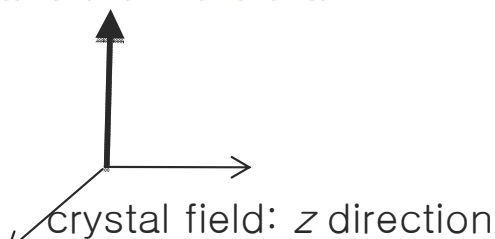
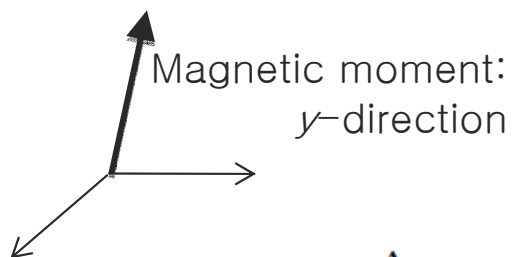
$$\text{Im}\psi \propto \langle J_z \rangle$$

$$\text{Re}\psi \propto \langle (J_x J_y + J_y J_x)(J_x^2 - J_y^2) \rangle$$



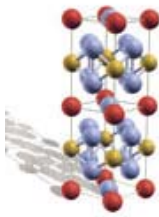
## *A toy model*

$$\begin{aligned} \sigma_3 &= |\emptyset\rangle\langle\emptyset| - |1\rangle\langle 1| \\ \sigma_1 &= |\emptyset\rangle\langle 1| + |1\rangle\langle\emptyset| \\ \sigma_2 &= i(|1\rangle\langle\emptyset| - |\emptyset\rangle\langle 1|) \end{aligned}$$



XY-Ising

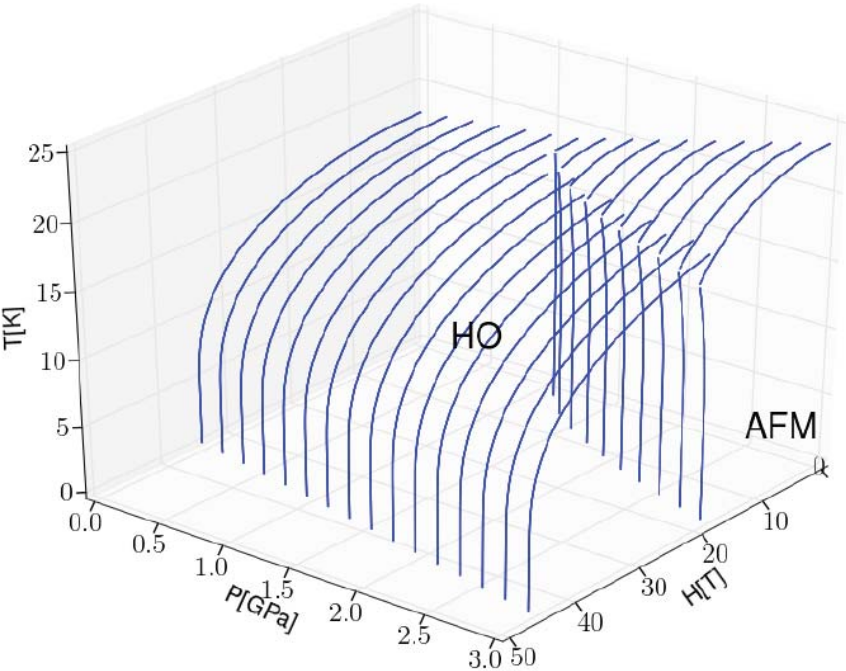
$$H = \sum_i -\frac{\Delta}{2}\sigma_3^i - \frac{1}{2}g\mu_B B |\langle 1|J_z|\emptyset\rangle| \sigma_2^i + \sum_{i,j} \frac{1}{2}(J_{ij}^1 \sigma_1^i \sigma_1^j + J_{ij}^2 \sigma_2^i \sigma_2^j)$$



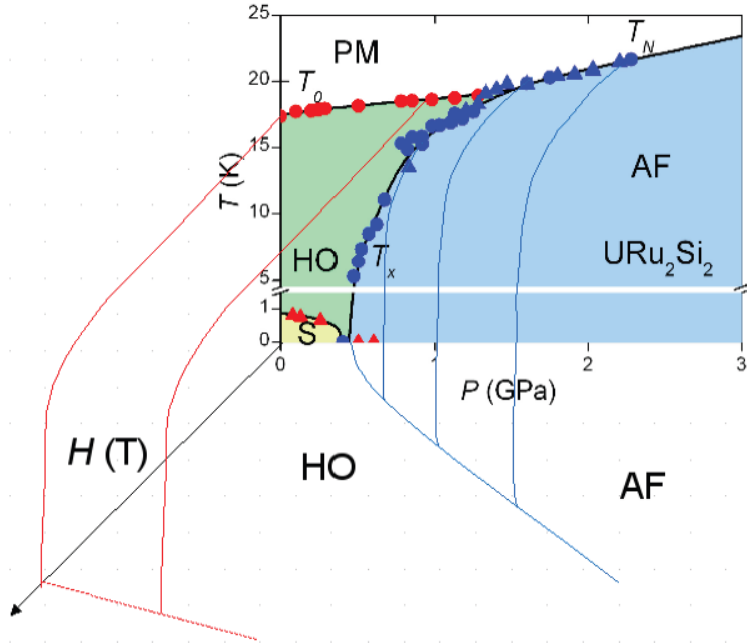
# HO & AFM in magnetic field

KH & G.Kotliar, arxiv: arXiv:0907.3892

$$F[h, \psi] = \frac{1}{2} \sum_{ij, \alpha=(1,2)} J_{ij}^{\alpha} \psi_i^{\alpha} \psi_j^{\alpha} - \sum_{i, \alpha=(1,2)} (h_i^{\alpha} + b^{\alpha}) \psi_i^{\alpha} - T \sum_i \log \left( \cosh \left( \beta \sqrt{(\Delta/2)^2 + (h_i^1)^2 / 2 + (h_i^2)^2 / 2} \right) \right)$$



Mean field



Exp. by E. Hassinger et.al. PRL 77, 115117 (2008)

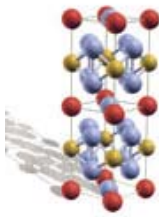
Only two fitting parameters:  $J_{eff}^1$ ,  $J_{eff}^2$

$$J_{eff} = 4|J_1| + 8J_2$$

determined by exp. transition temperature

$$J_{eff}^1 = \frac{\Delta}{\tanh(\Delta/(2T_0))}$$

$$J_{eff}^2 = \frac{\Delta}{\tanh(\Delta/(2T_N))}$$



# HO & AFM under stress

KH & G.Kotliar, arxiv: arXiv:0907.3892

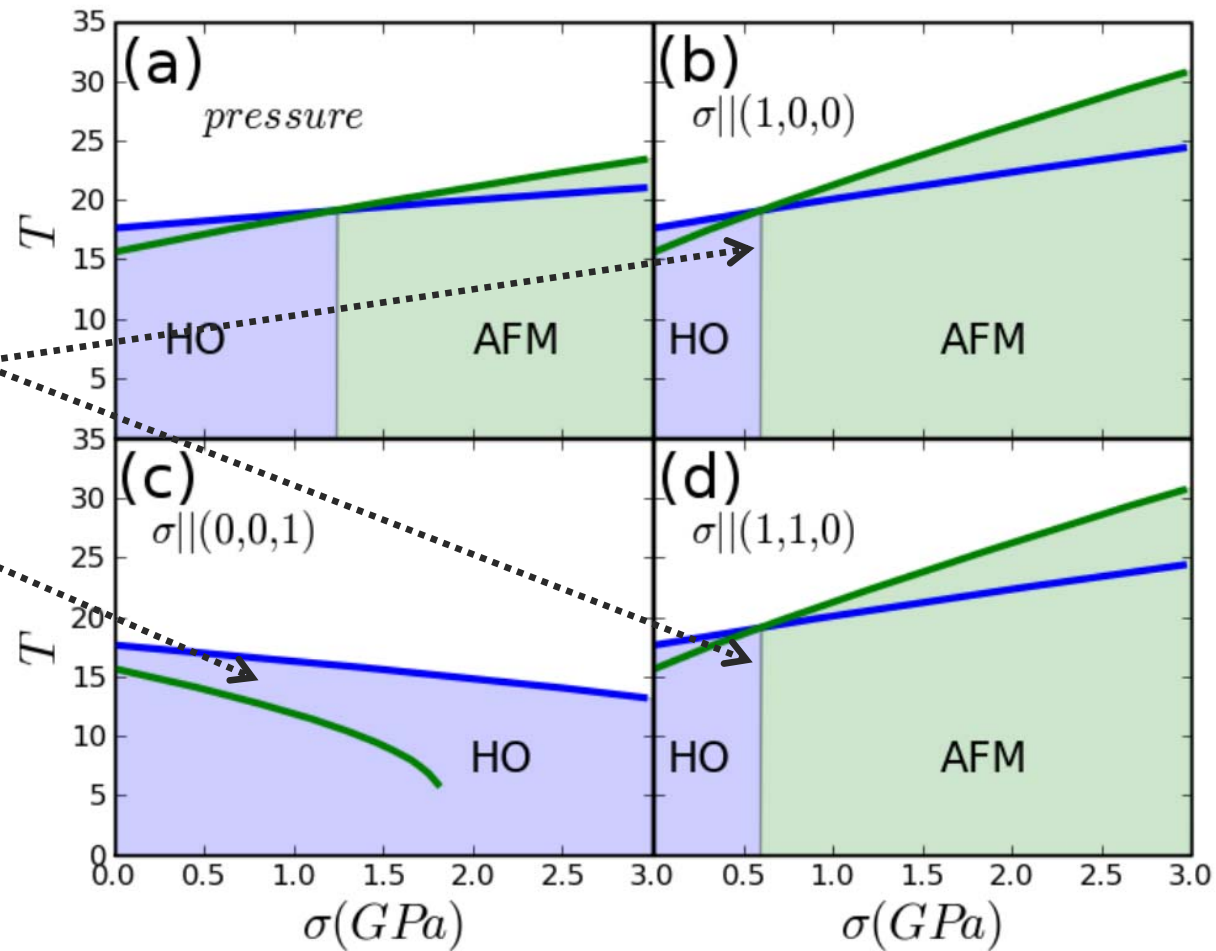
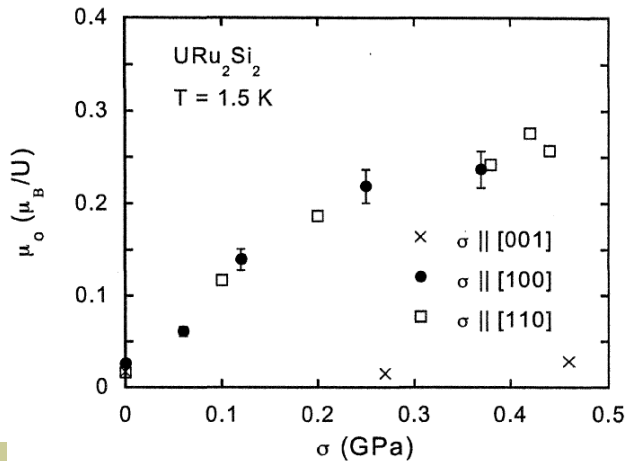
s sensitive to compression (strain), modeled by:  $J_{ij}^\alpha \rightarrow J_{ij}^\alpha (1 + g_\alpha(\epsilon_{xx} + \epsilon_{yy}))$

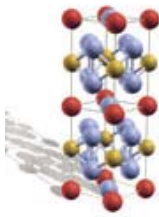
$$\sigma = C\epsilon$$

Very different effect of  
in plane stress  
and uniaxial stress

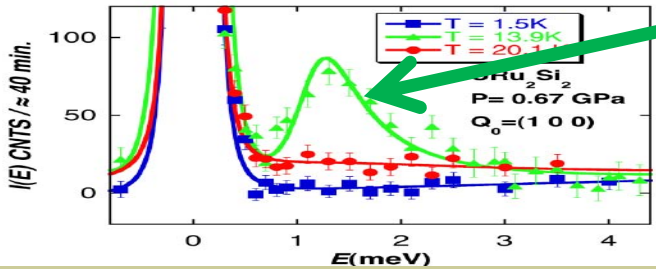
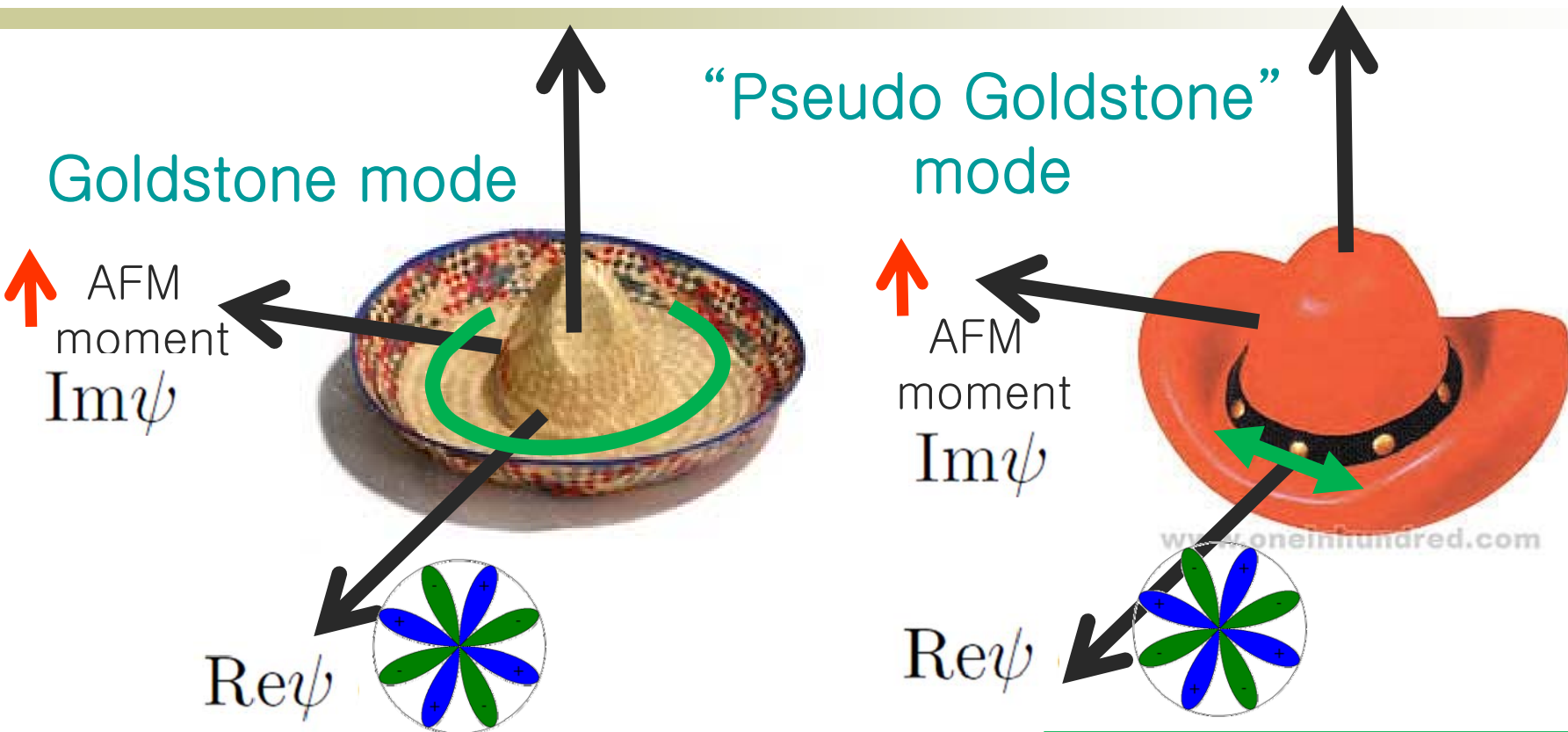
In plane stress favors AFM state

c-axis stress favors HO



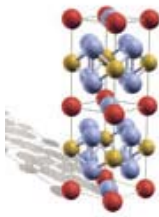


# Neutron scattering experiments



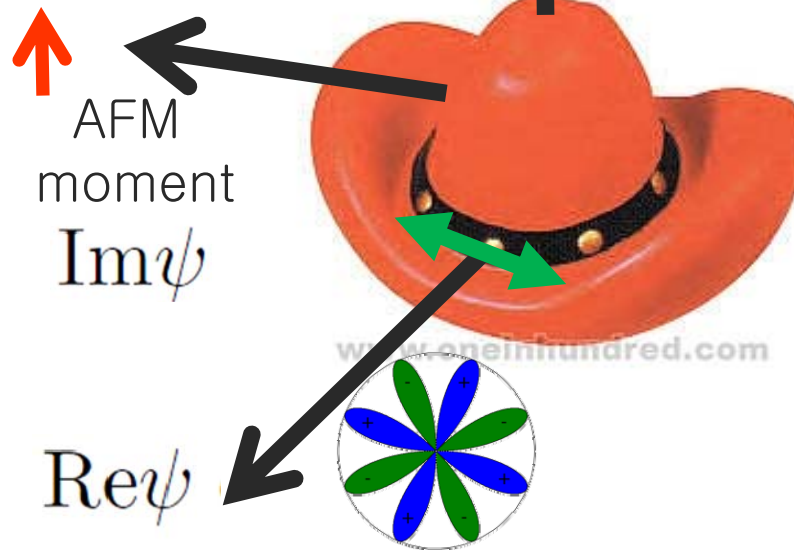
Symmetry is approximate  
“Pseudo-Goldstone” mode  
Fluctuation of  $m$  – finite mass

The exchange constants  $J$   
are slightly different in the two phases ( $\sim 6\%$ )



# Predictions for the mode

“Pseudo Goldstone”  
mode

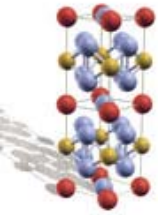


## Predictions:

The mode energy should decrease with pressure  
(since the difference in the exchange constants decreases with increasing pressure)

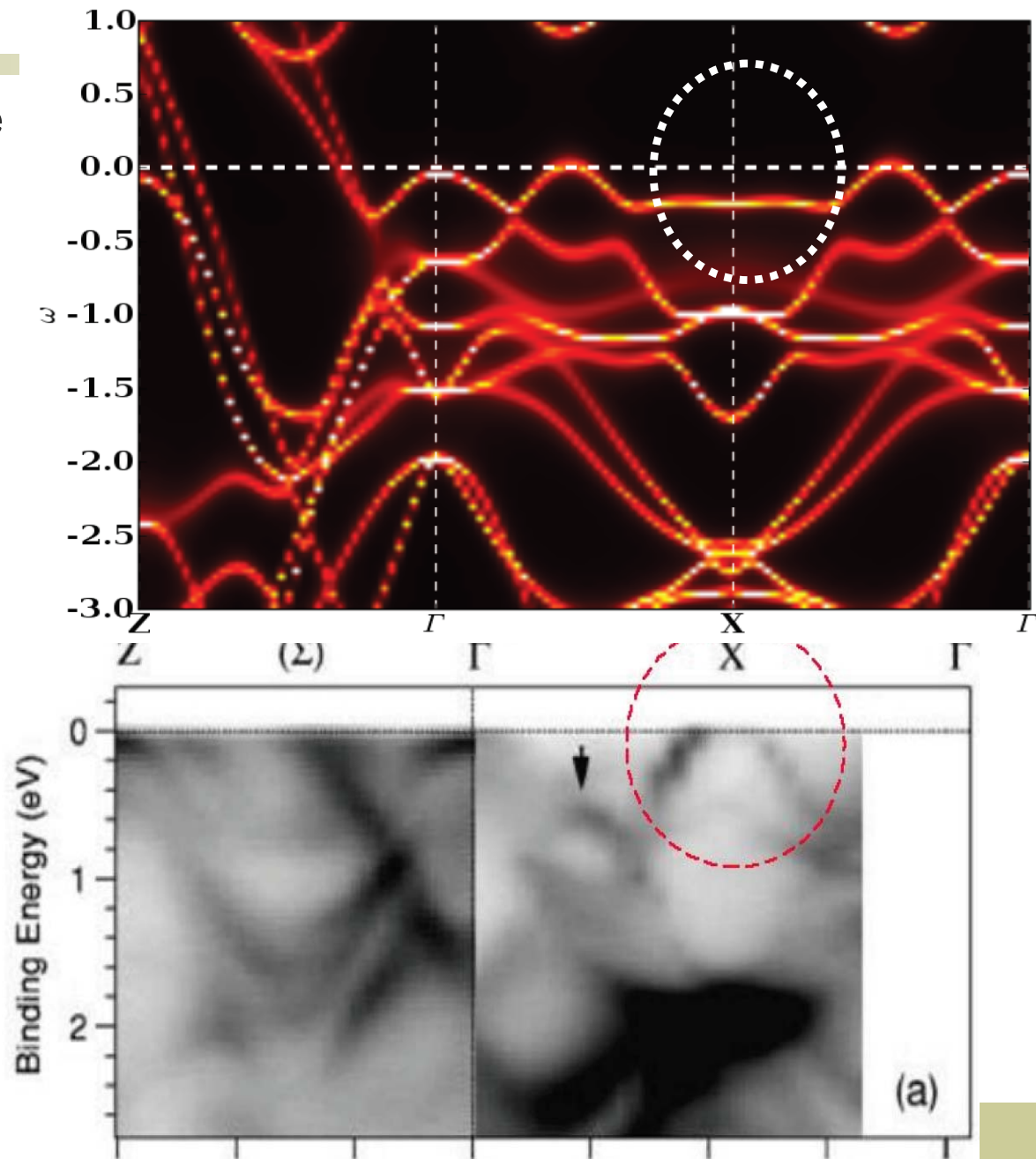
The mode energy should increase with magnetic field  
(since the magnetic field destabilizes the antiferromagnetic phase).

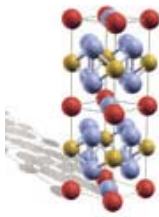
# DMFT $A(k, \omega)$ vs ARPES



Off resonance

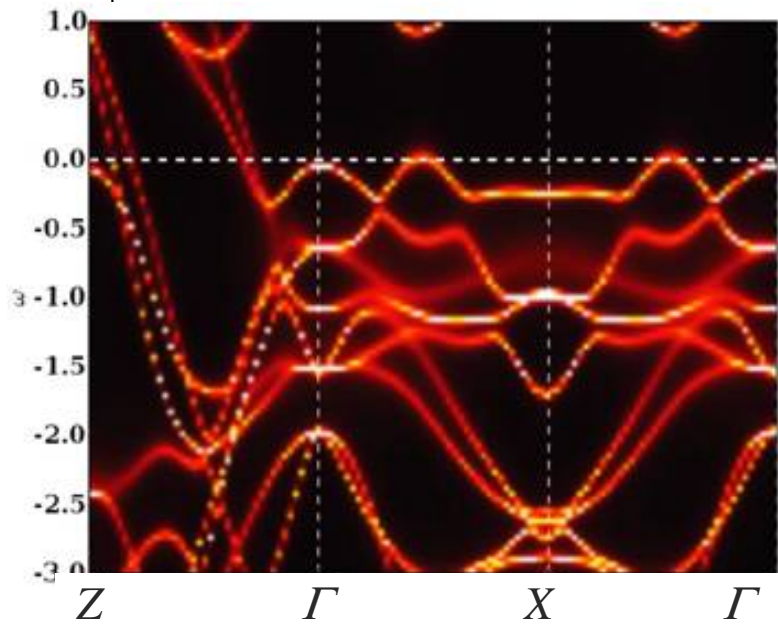
Very good agreement,  
except at X point





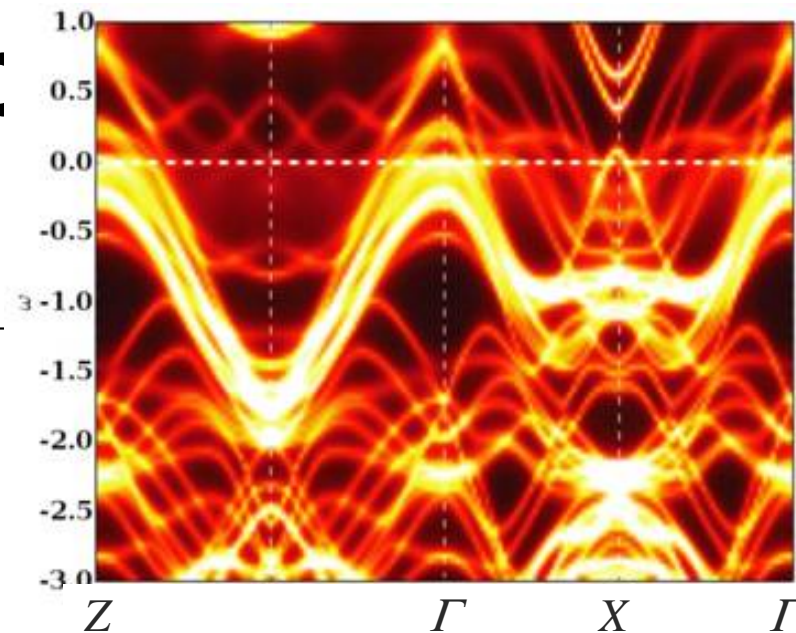
# Surface origin of pocket at X point

*LDA+DMFT - bulk*

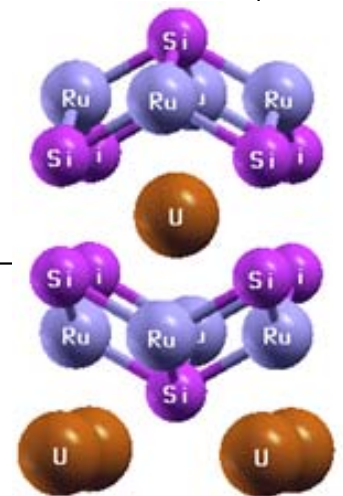


- No hole-pocket at the X-point.*

*LDA+DMFT - Si-terminated surface slab*

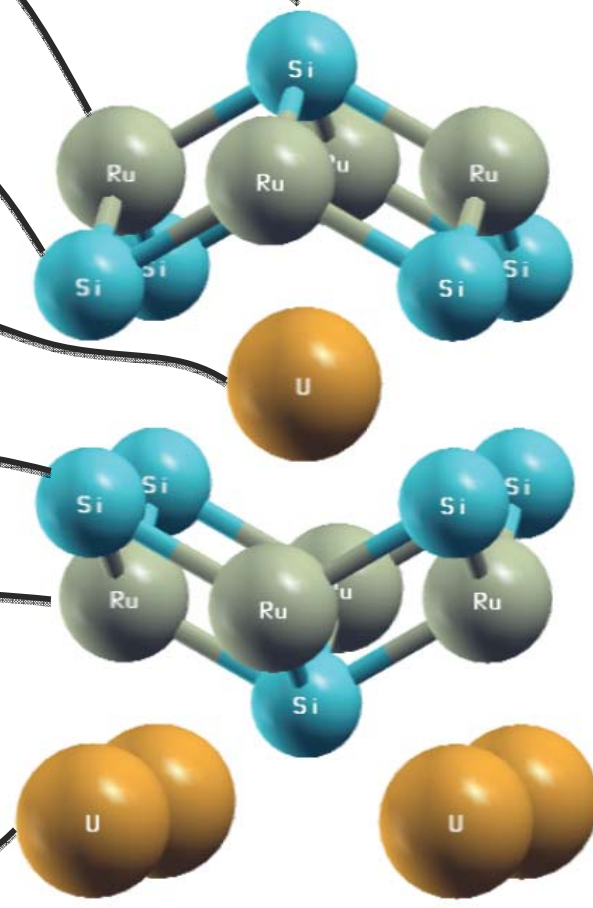
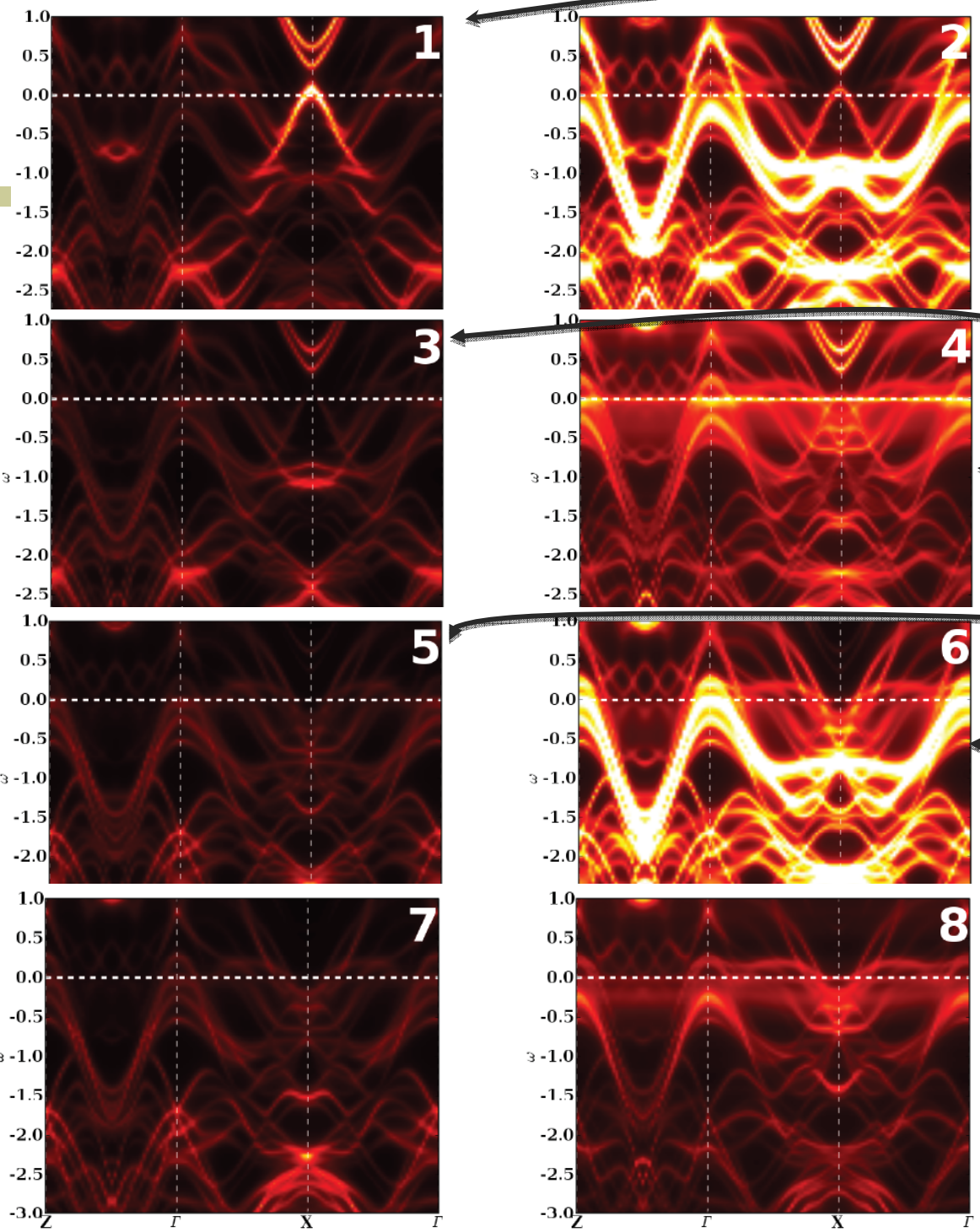


- Hole pocket surface state appears at X-point!*





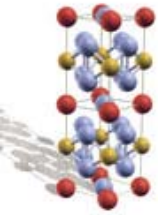
# Layer resolved spectra



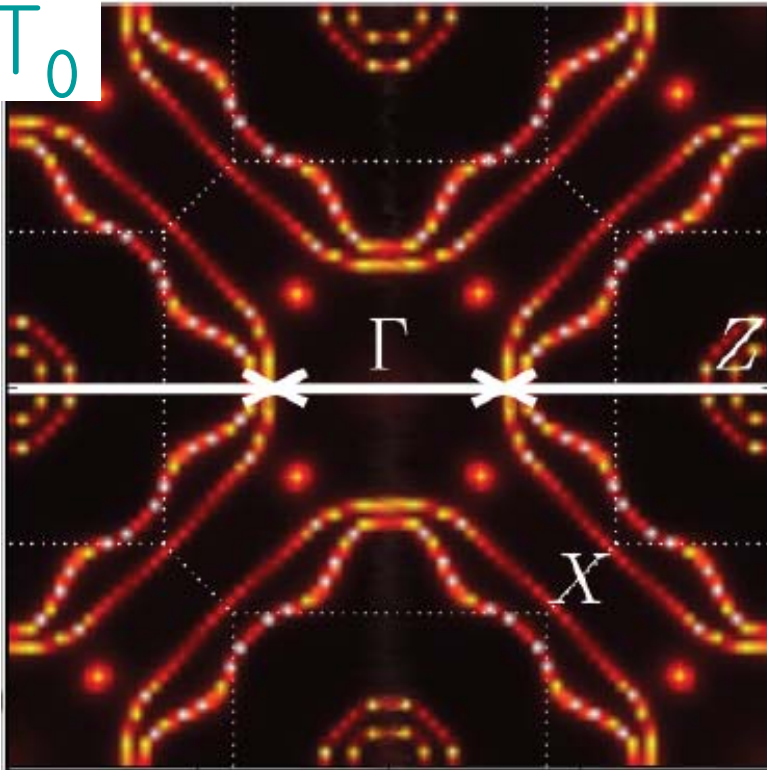
1  
2  
3  
4  
5  
6  
7  
8



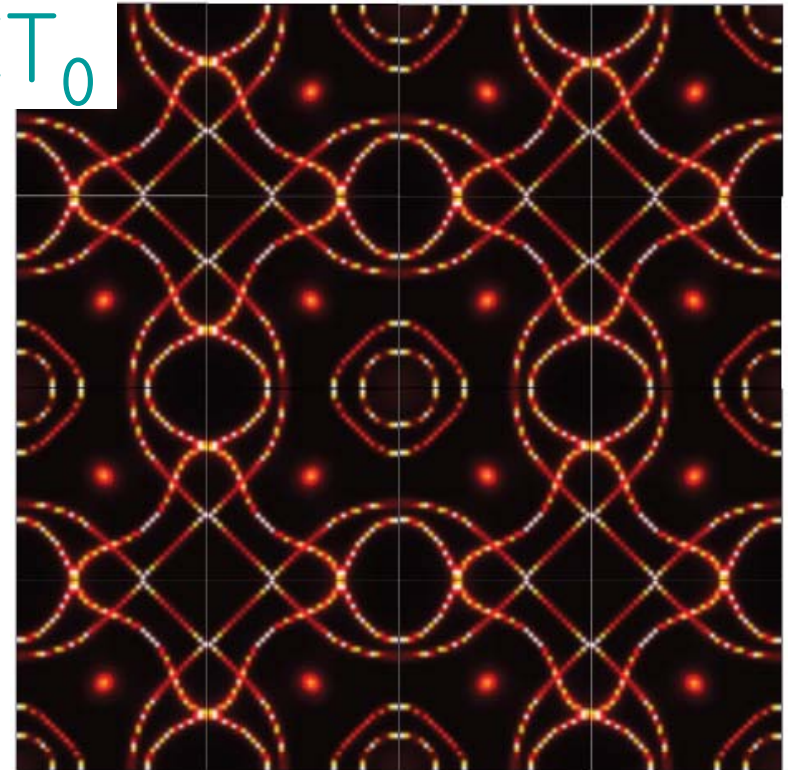
# Fermi surface nesting, reconstruction below $T_c$



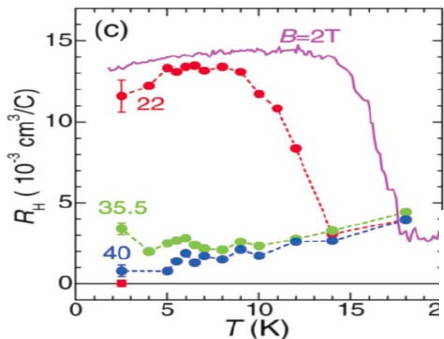
$T > T_0$



$T < T_0$

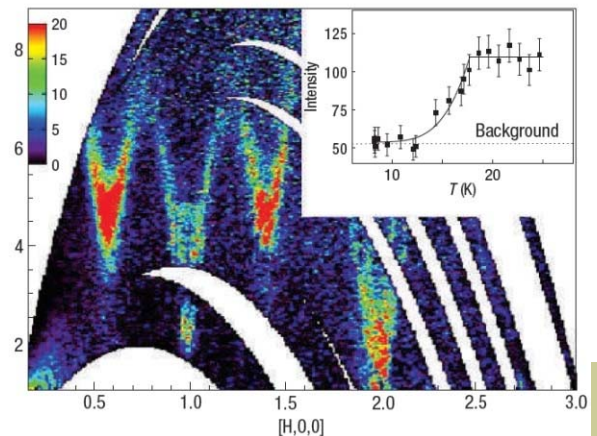


Nesting  $0.6a^*$  and  $1.4a^*$



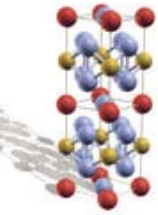
Fermi surface  
reconstruction

2 incommensurate peaks  
 $(0.6, 0, 0)$ ,  $(1.4, 0, 0)$

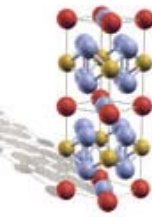


Wiebe et.al. 2008

# Conclusions



- DMFT tools can be used to understand (predict) properties of correlated materials
- Kondo effect in  $\text{URu}_2\text{Si}_2$  is partially arrested below crystal field splitting energy.
  - Gives room to ordered states, either AFM state or orbital order.
  - AFM state and hidden order can be unified by a complex order parameter: “adiabatic continuity”
  - Hidden order has hexadecapole character  
(does not break time reversal symmetry, nor  $C_4$  symmetry)
  - In the hidden order, fluctuations of the magnetic moment as a pseudo-Goldstone mode
  - In AFM state there is a pseudo-Goldstone mode of hexadecapole symmetry



Thank you!