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Finite-size scaling of MBL phase transitions

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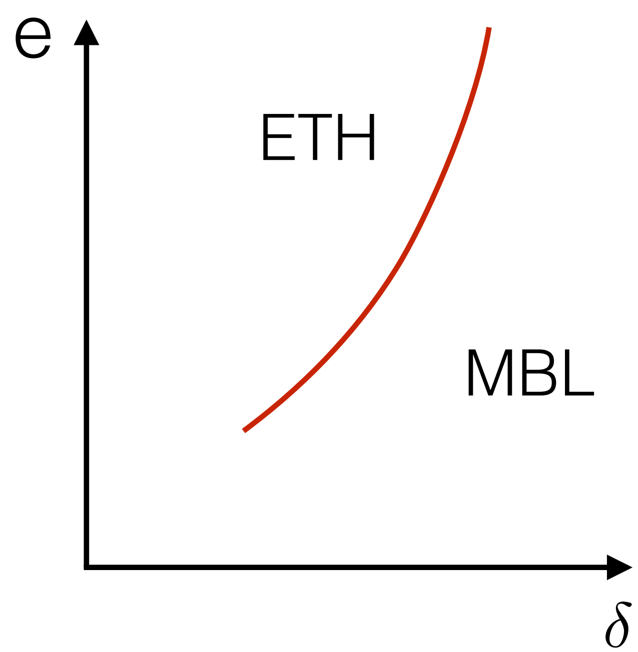
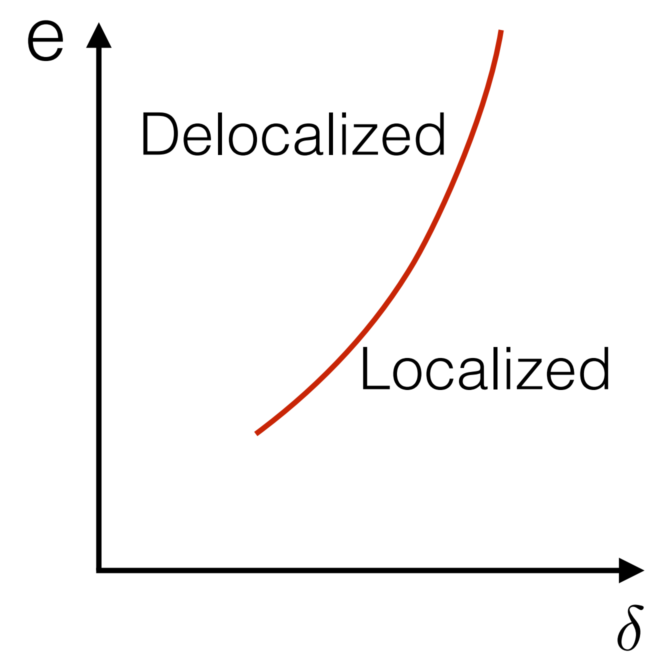
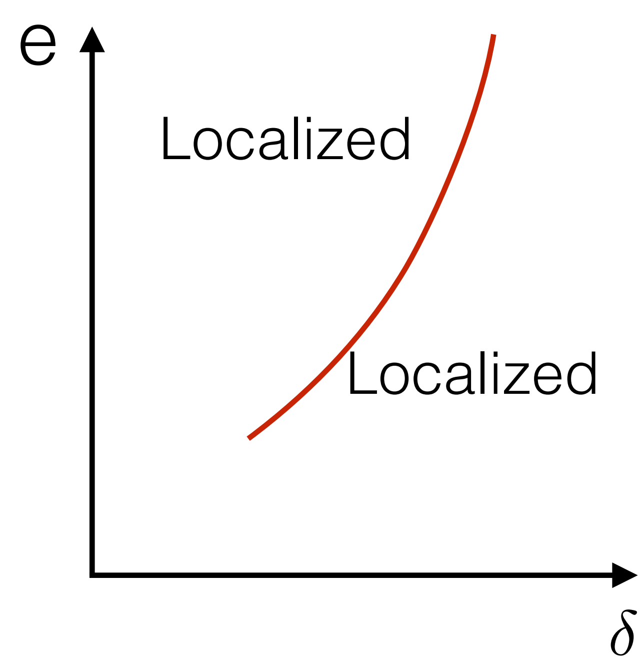
arXiv:1509.04285

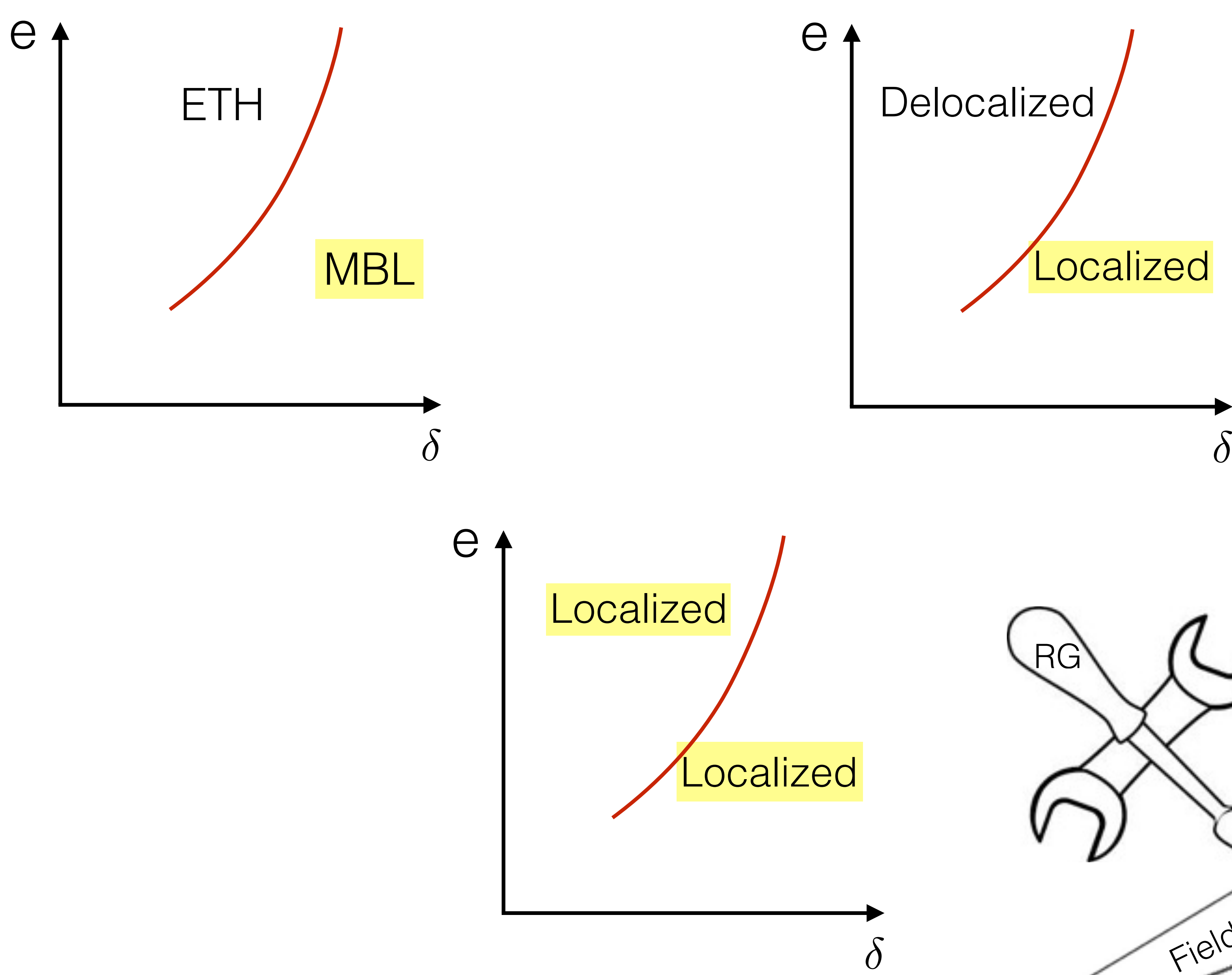
Eigenstate phase transitions

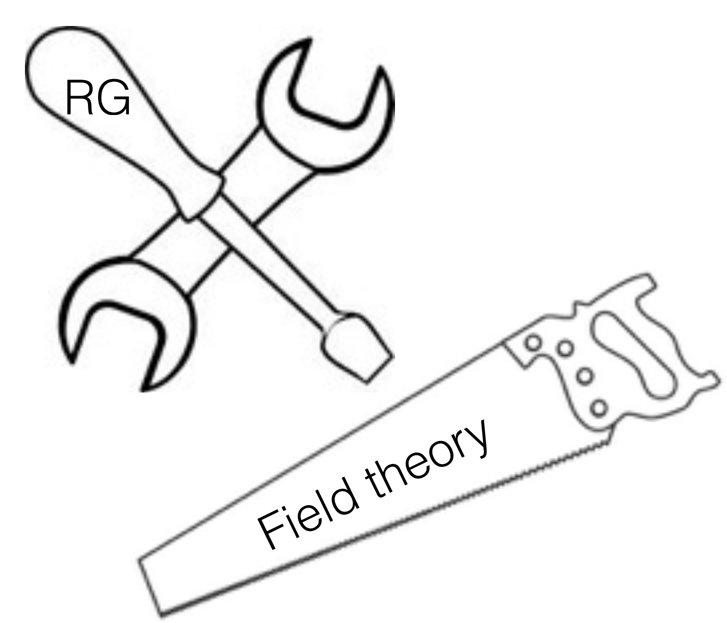




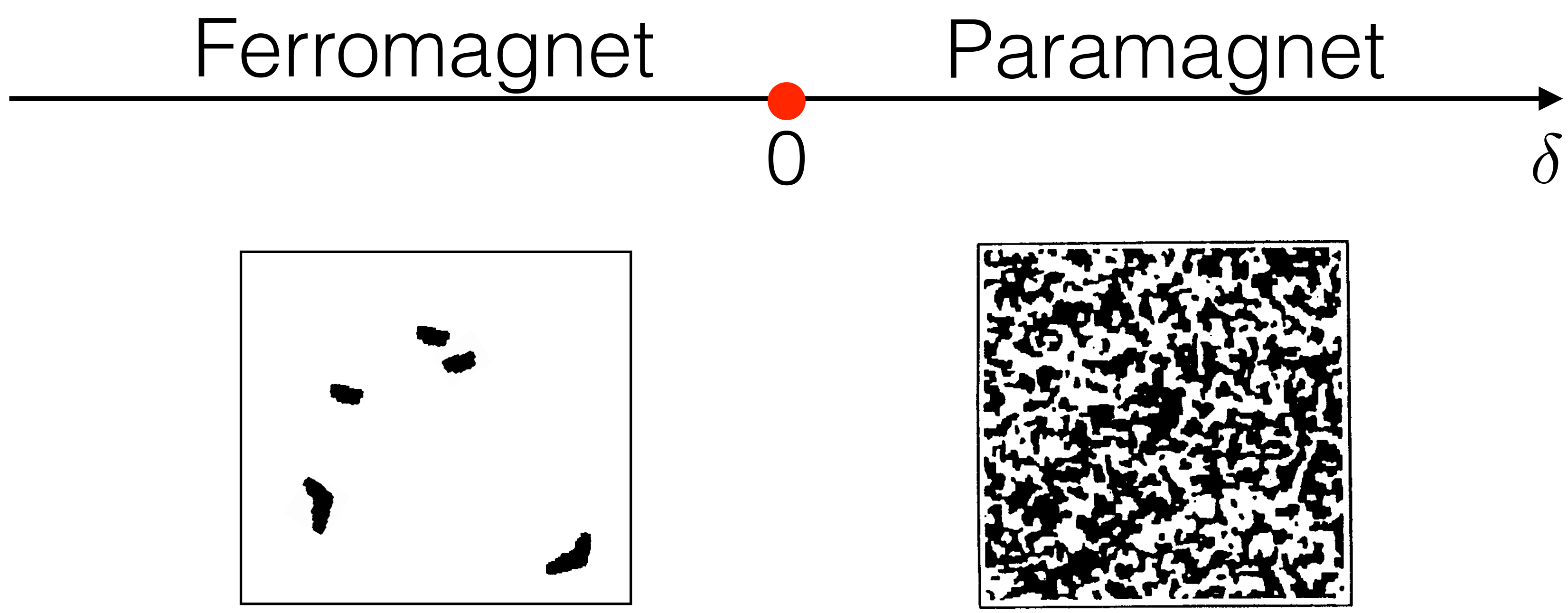


Eigenstate phase transitions





Harris criterion

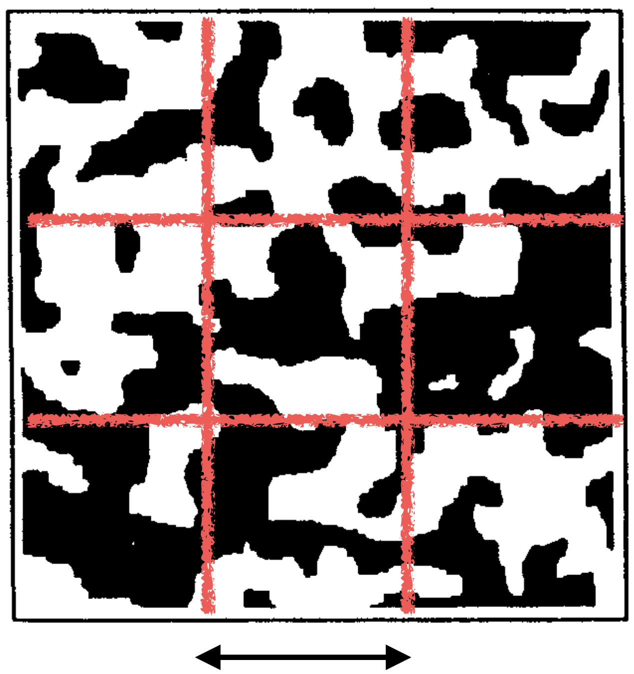


Characterized by $\xi \sim \delta^{-\nu}$

A. B. Harris, J. Phys. C 7, 1671 (1974)

Harris criterion

Add disorder in δ



$\xi \sim \bar{\delta}^{-\nu}$

δ in a box: $\bar{\delta} \pm C\sqrt{\frac{1}{\xi^d}}$

RMS Fluctuation > Mean

⇒ clean fixed point unstable

$\nu \geq 2/d$ for stability

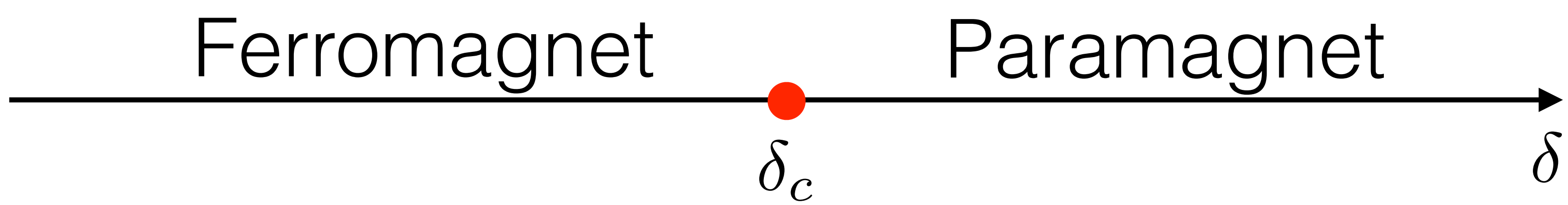
A. B. Harris, J. Phys. C 7, 1671 (1974)

Generalization by CCFS

- What if no reference clean transition?
- Chayes, Chayes, Fisher, Spencer (CCFS)
  - Probability distribution of order parameter
  - Finite-size scaling

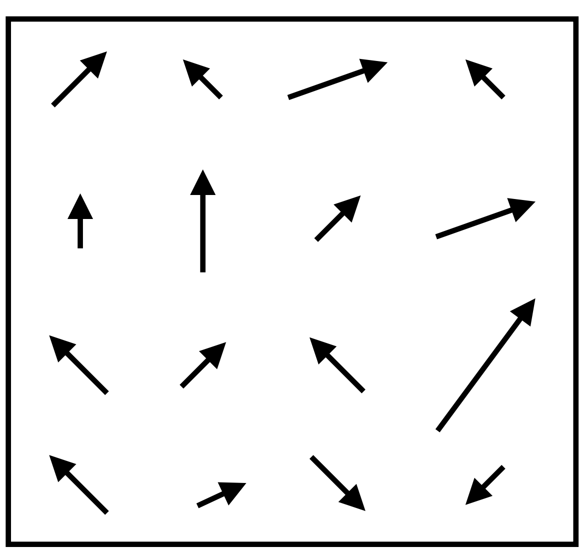
Chayes, Chayes, Fisher, Spencer, PRL 57, 2999 (1986)

Probability distribution of order parameter

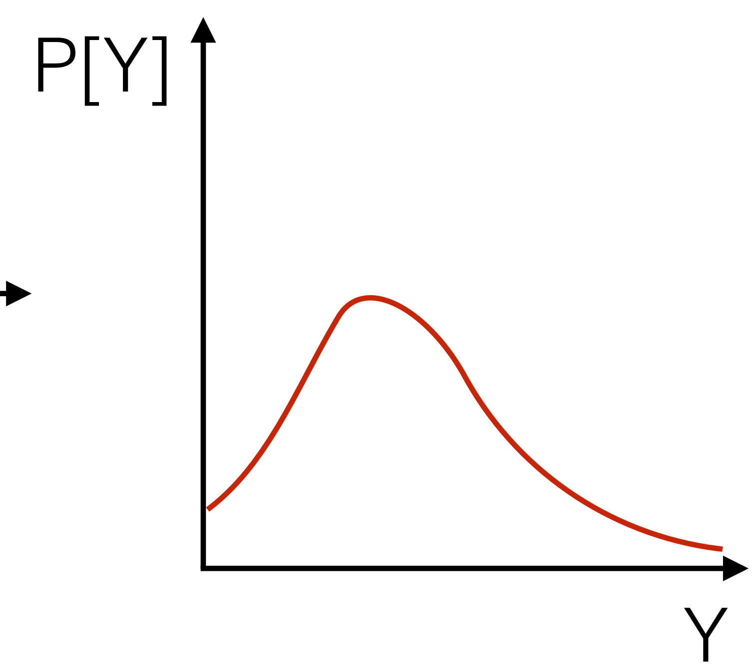


Eg: Variance of disorder

At fixed δ, L

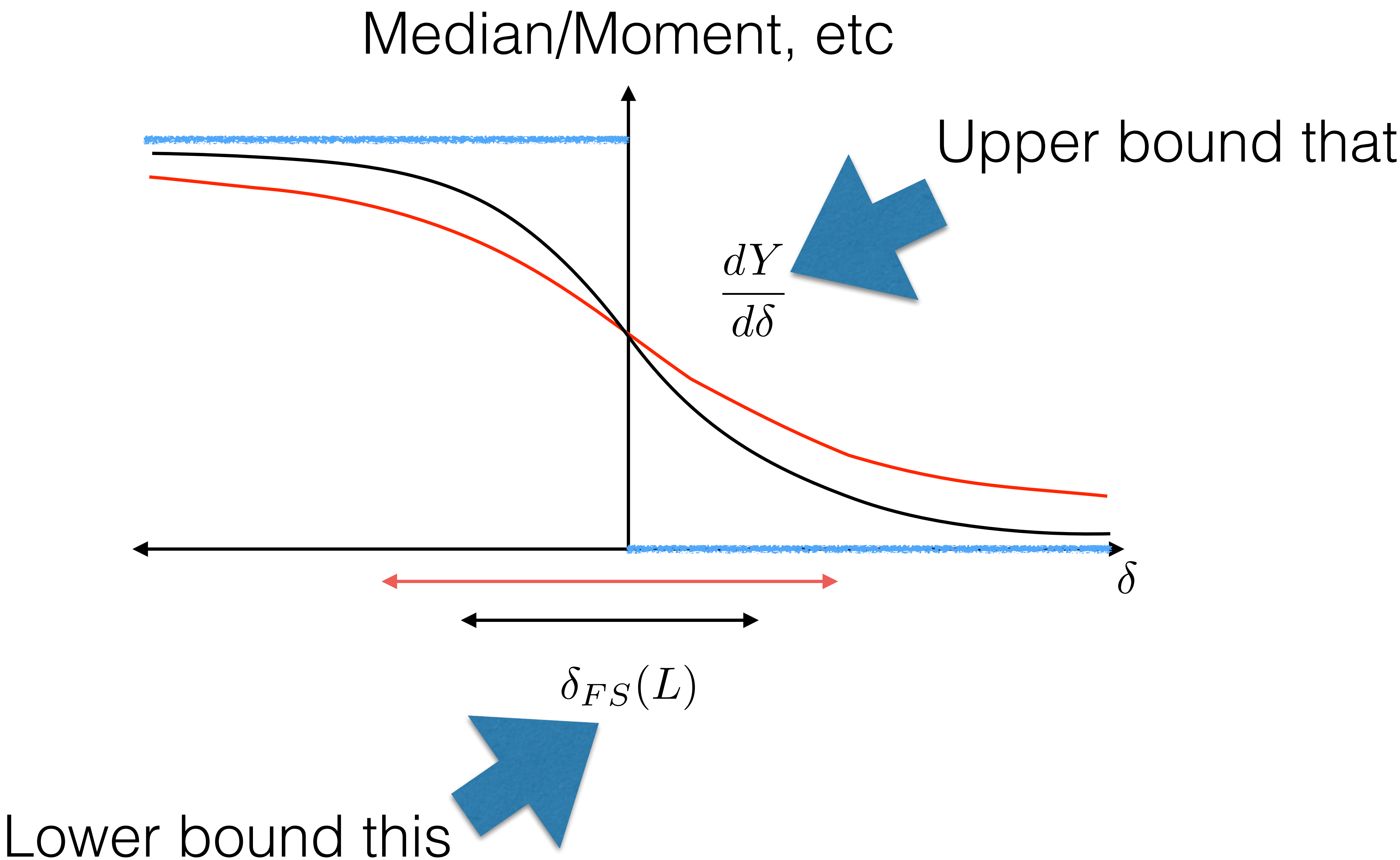


→ Measure order parameter Y for each sample →



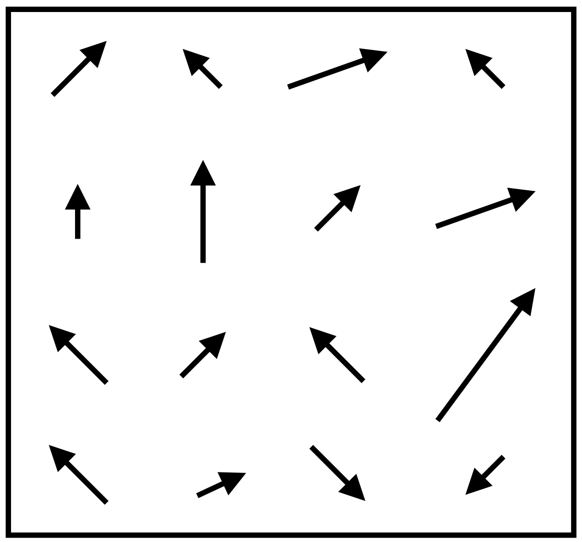
Chayes, Chayes, Fisher, Spencer, PRL 57, 2999 (1986)

Finite-size scaling



Chayes, Chayes, Fisher, Spencer, PRL 57, 2999 (1986)

An elementary upper bound



Disorder distribution

$$p(\text{Disorder}) \propto e^{-E}$$

Disorder susceptibility

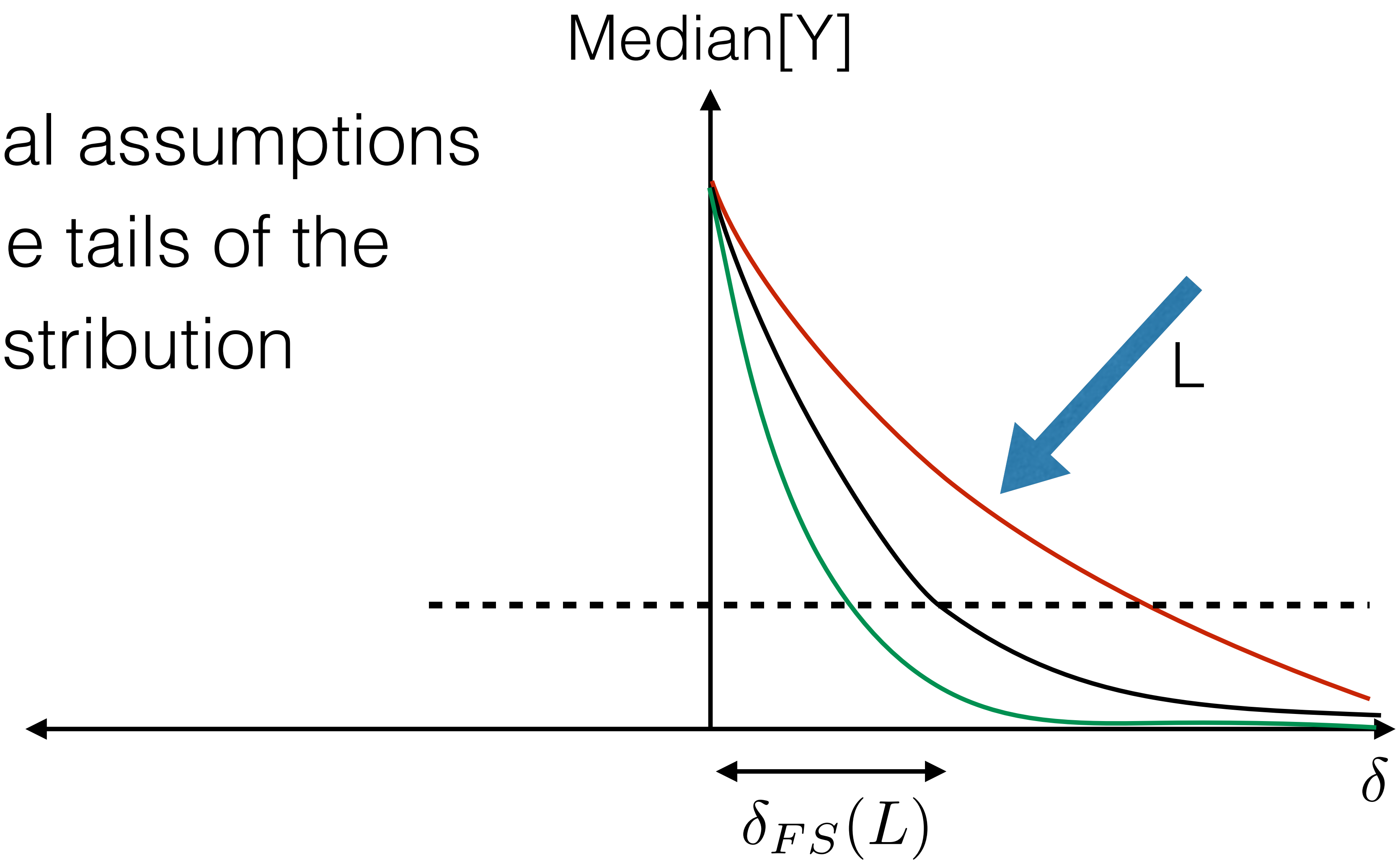
$$\left|\frac{d[X]}{d\delta}\right| \leq \sqrt{[(\partial_\delta E)^2]_c}$$

Disorder local ⇒ Extensive susceptibility

$$\left|\frac{d[X]}{d\delta}\right| \leq \alpha L^{d/2}$$

Tail theorem

Technical assumptions on the tails of the distribution



$$\delta_{FS} \sim L^{-1/\nu_{FS}}$$

$$\nu_{FS} \geq 2/d$$

Chayes, Chayes, Fisher, Spencer, PRL 57, 2999 (1986)

Mean theorem

- Order parameter Y is a bounded random variable
- Finite-size scaling ansatz

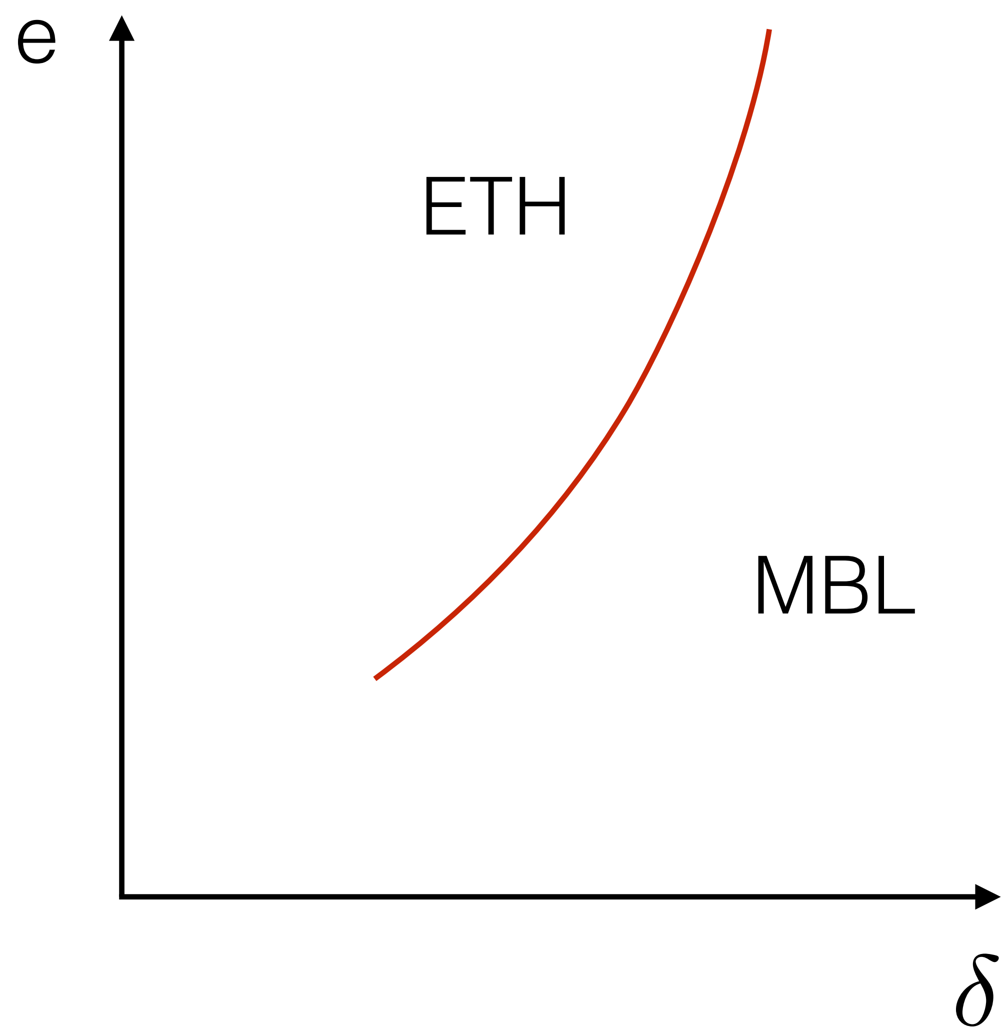
$$[Y](L,\delta) \sim \frac{1}{L^a}\tilde{Y}(L^{1/\nu}\delta)$$

For any short-range correlated quenched disorder

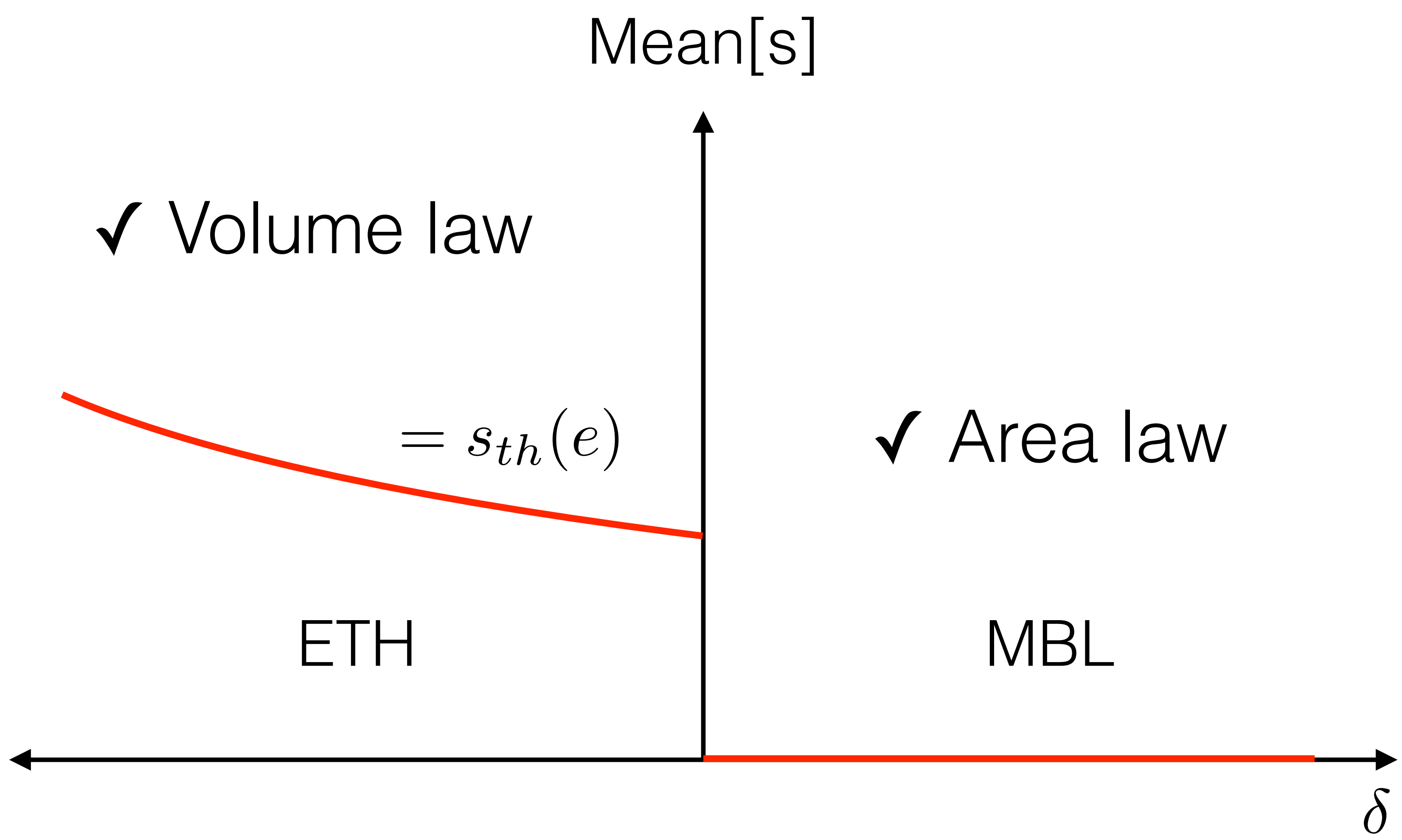
$$\nu \geq \frac{2}{d+2a}$$

Chandran, Laumann, Oganesyan, arXiv:1509.04285 (2015)

Application: MBL-ETH transition



Entanglement entropy density



Pal and Huse, PRB 82, 174411 (2010)

Bauer and Nayak, J Stat Mech P09005 (2013)

Tarun Grover, arXiv:1405.1471 (2014)

Entanglement entropy density

- Finite size scaling ansatz

$$[s](L, L_A, \delta) \sim \frac{1}{L^a}\tilde{s}(L^{1/\nu}\delta, L_A/L)$$

- [s] jumps at transition ⇒ a=0

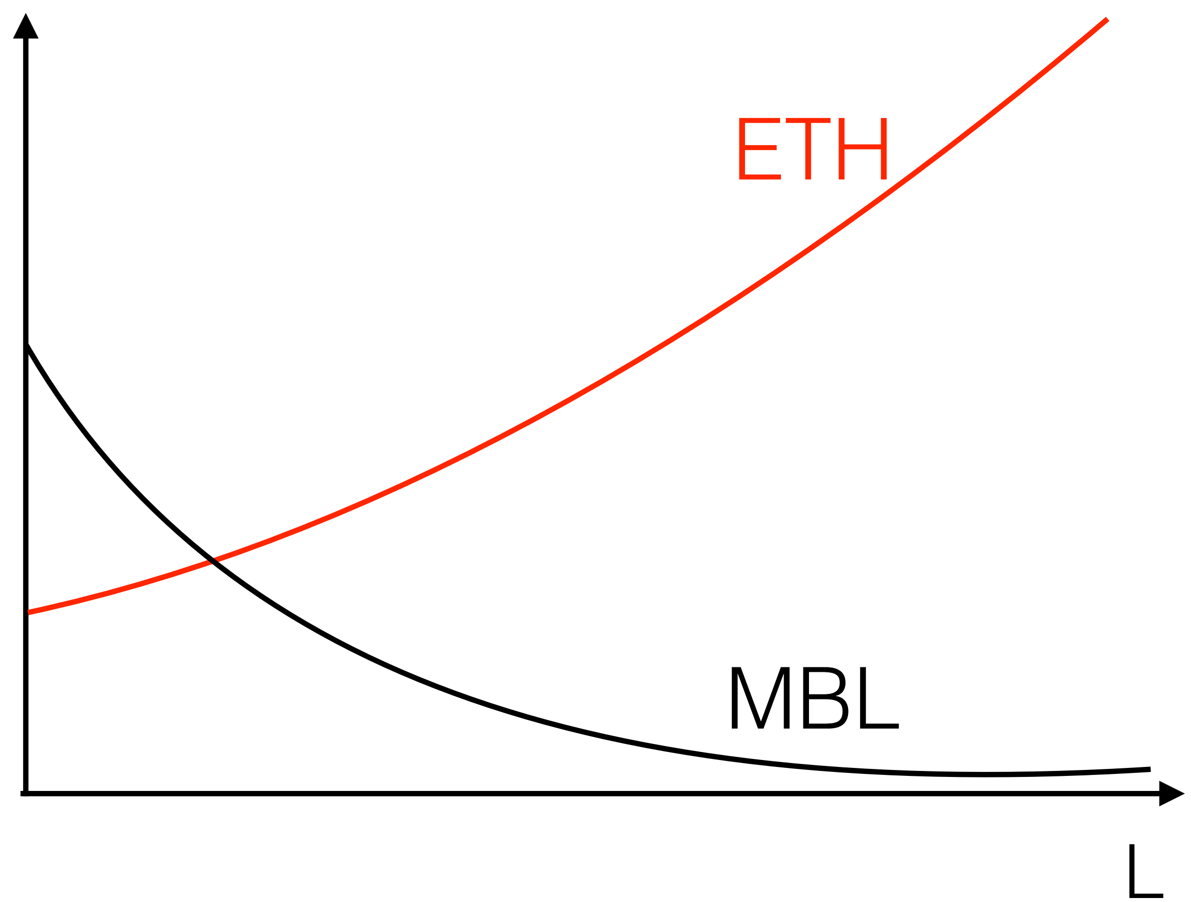
- Mean theorem ⇒ $\nu \geq 2/d$

- If CCFS assumptions apply, $\nu_{FS} \geq 2/d$

Matrix element/Level spacing

Local perturbation $\hat{V}$ effectiveness in hybridizing eigenstates

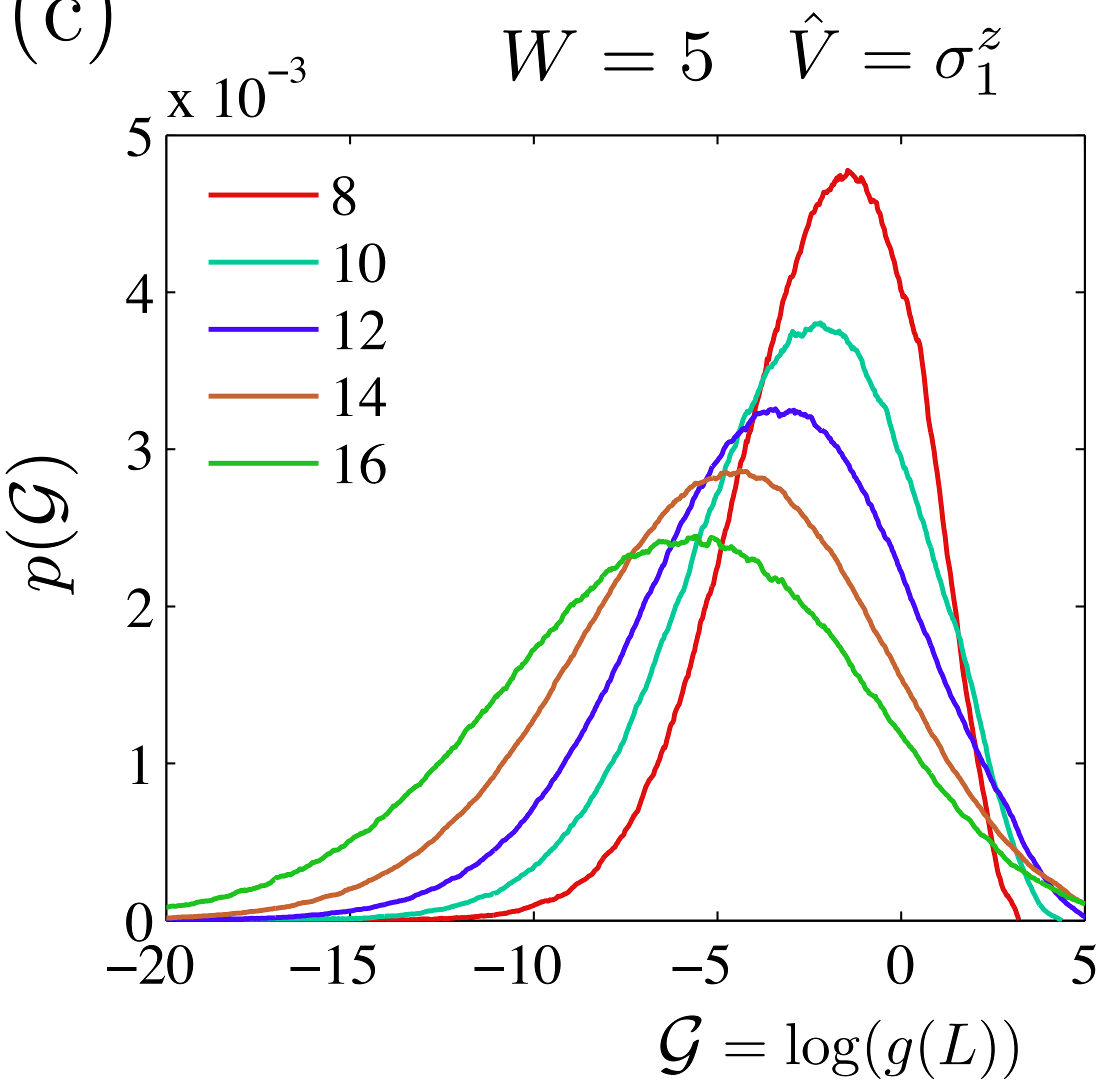
$$g(L) = \frac{|\langle E_{n+1}|\hat{V}|E_n\rangle|}{E_{n+1} - E_n}$$



Serbyn, Papic, and Abanin, arXiv:1507.01635 (2015)
Vosk, Huse, and Altman, PRX 5, 031032 (2015)
Potter, Vasseur, and Parameswaran, PRX 5, 031033 (2015)

Matrix element/Level spacing

(c)



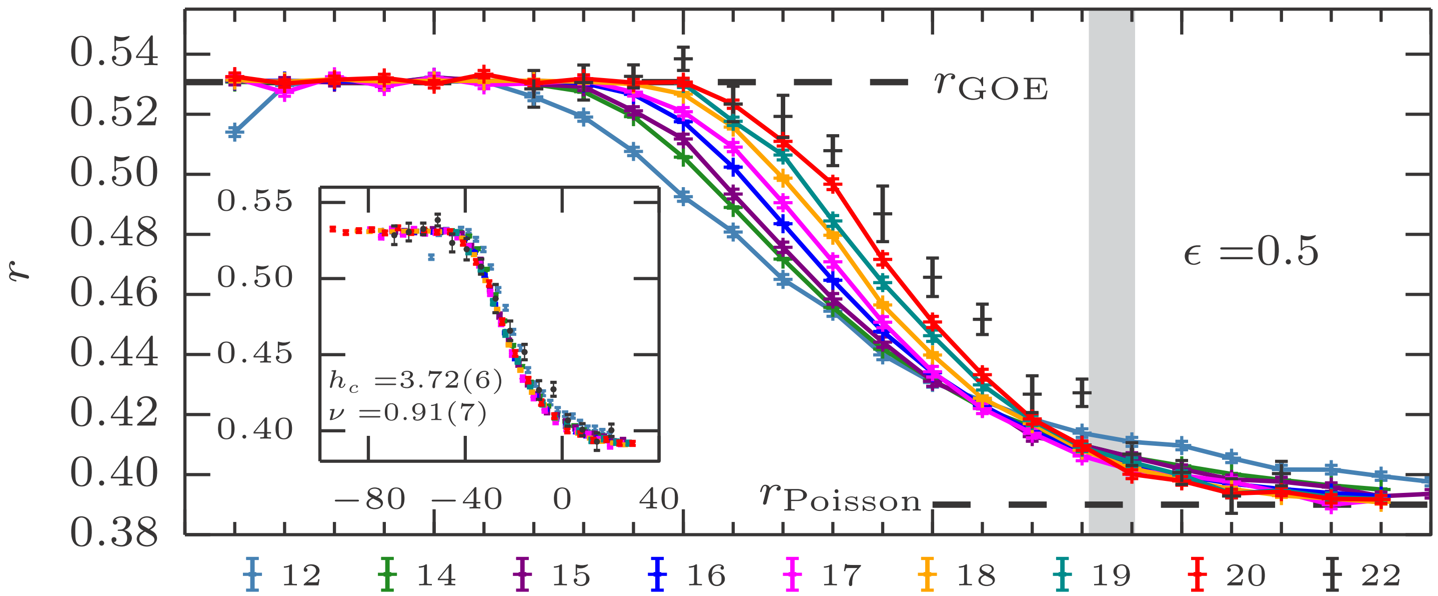
- Fat tails
- Mean and tail theorems don't apply
- No Harris bound on scaling window

Serbyn, Papic, and Abanin, arXiv:1507.01635 (2015)

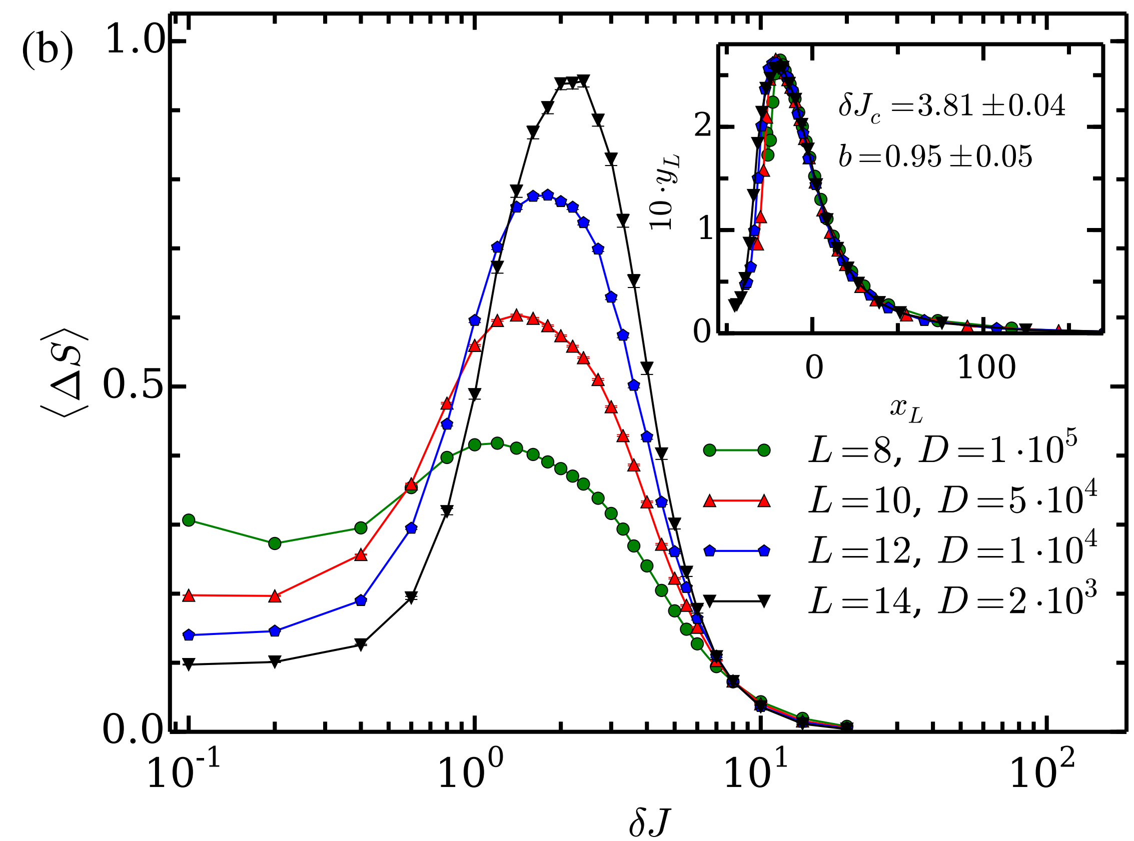
Vosk, Huse, and Altman, PRX 5, 031032 (2015)

Potter, Vasseur, and Parameswaran, PRX 5, 031033 (2015)

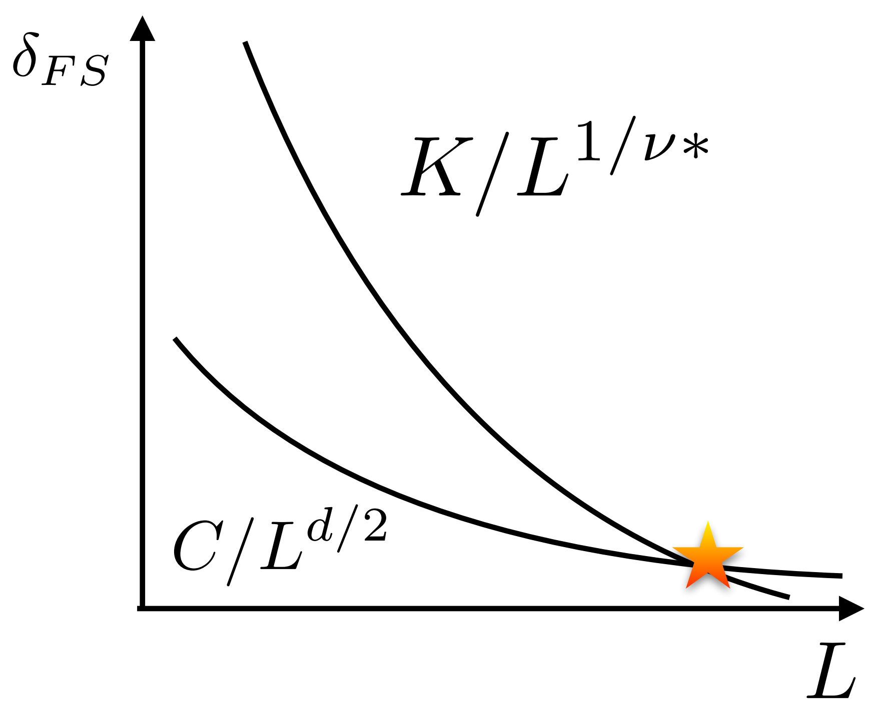
Nightmare on numerics street



Luitz, Laflorencie, Alet, PRB 91, 081103 (2015)



Kjall, Bardarson, Pollmann, PRL 113, 107204 (2014)



Conservative estimate

of asymptotic system sizes

L = 500 - 5000

There's no time but..

- Applies to MBL-delocalized, MBL-MBL transitions
- Can generalize to correlated disorder
- Applies to multiple diverging length scales
- Applies to first order transitions

Take-away messages

- $\nu \geq 2/d$ at MBL-ETH transition
  - Mean entanglement entropy density ([s])
  - Mean level spacing parameter ([r])
  - …
- Going forward
  - Gaussian distributed disorder
  - Medians, entire distribution
  - Caution: collapsing tails can lead to smaller apparent ν