Unconventional localization transitions in high dimensions

Victor Gurarie

Work with S. Syzranov and L. Radzihovsky
M. Gärttner, A.-M. Rey

KITP Nov 2015

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- What changes in higher dimensions

- Localization transitions in higher dimensions

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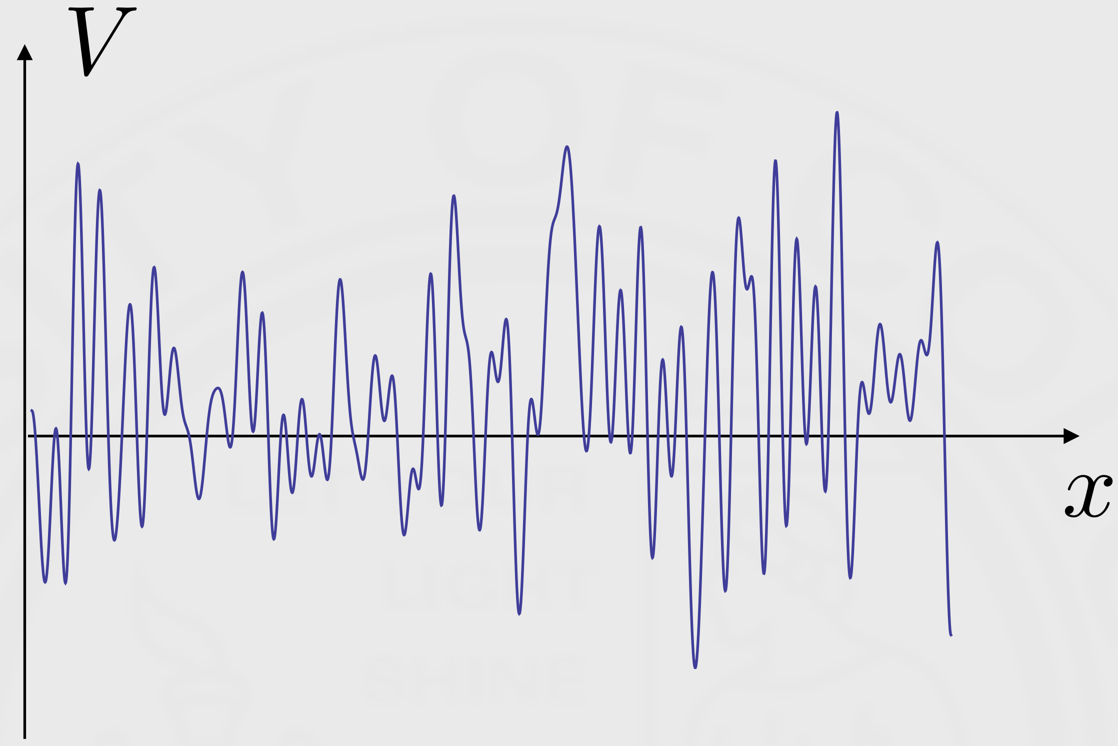
S. V. Syzranov, VG, L. Radzihovsky, PRB 91, 035133 (2015)

S. V. Syzranov, L. Radzihovsky, VG, PRL 114, 166601 (2015)

M. Gärttner, S. V. Syzranov, A. M. Rey, VG, L. Radzihovsky, PRB 92, 041406 (R) (2015)

S. V. Syzranov, P. M. Ostrovsky, VG, L. Radzihovsky, arXiv:1512xxxxx

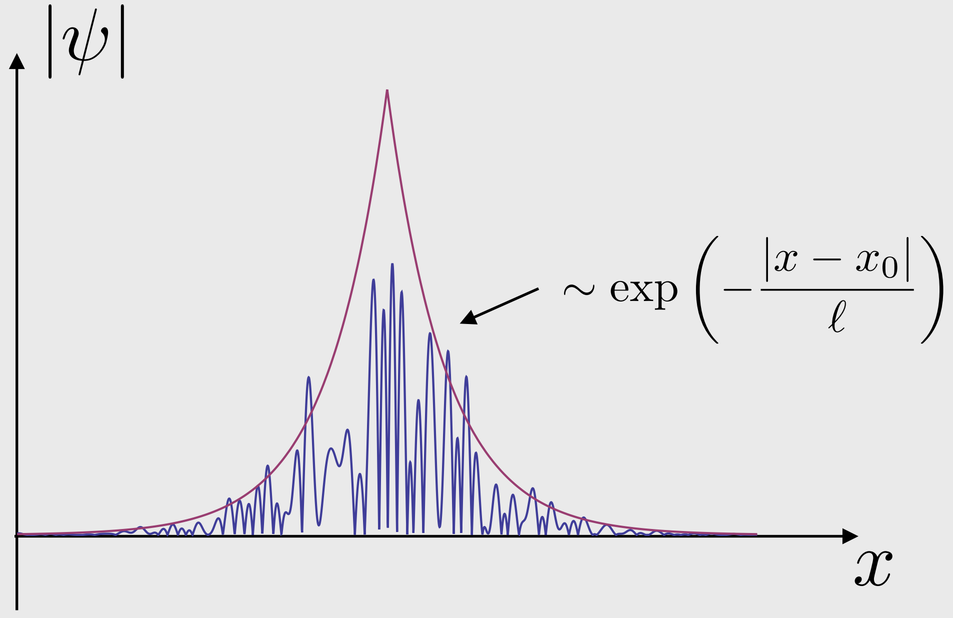
Anderson localization



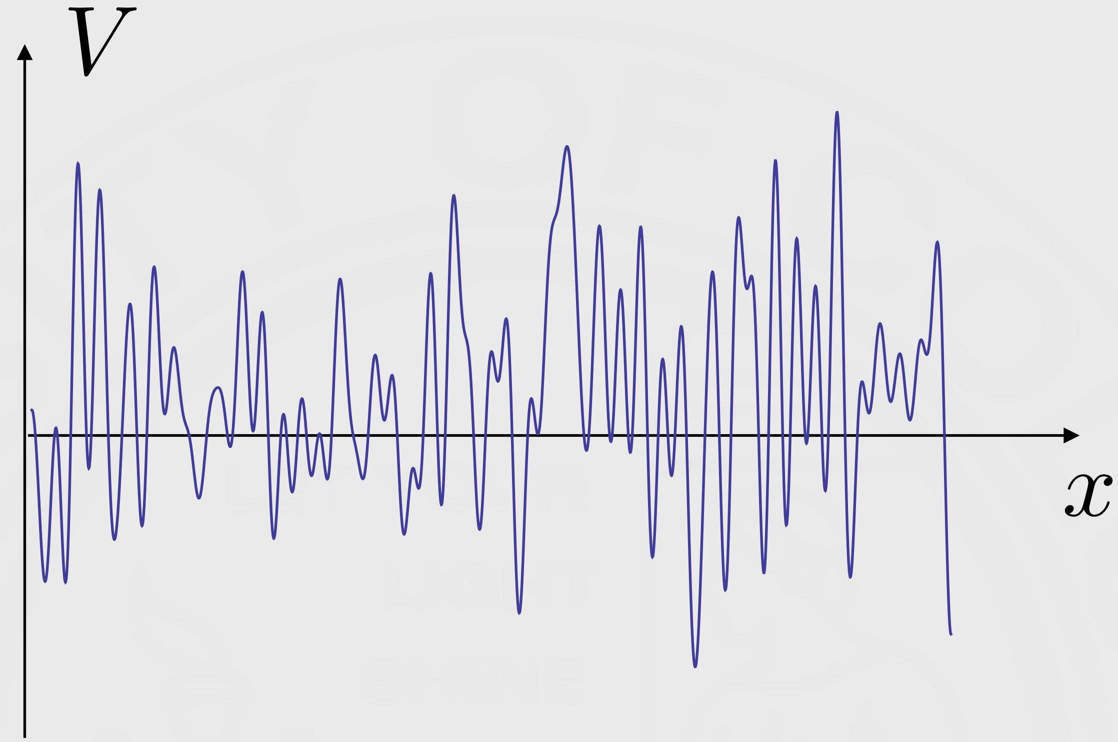
Quantum motion in a random potential

$$-\frac{1}{2m}\Delta\psi + V(\mathbf{r})\psi = E\psi$$

Wave functions at sufficiently low energy (above 2D) are localized



Anderson localization



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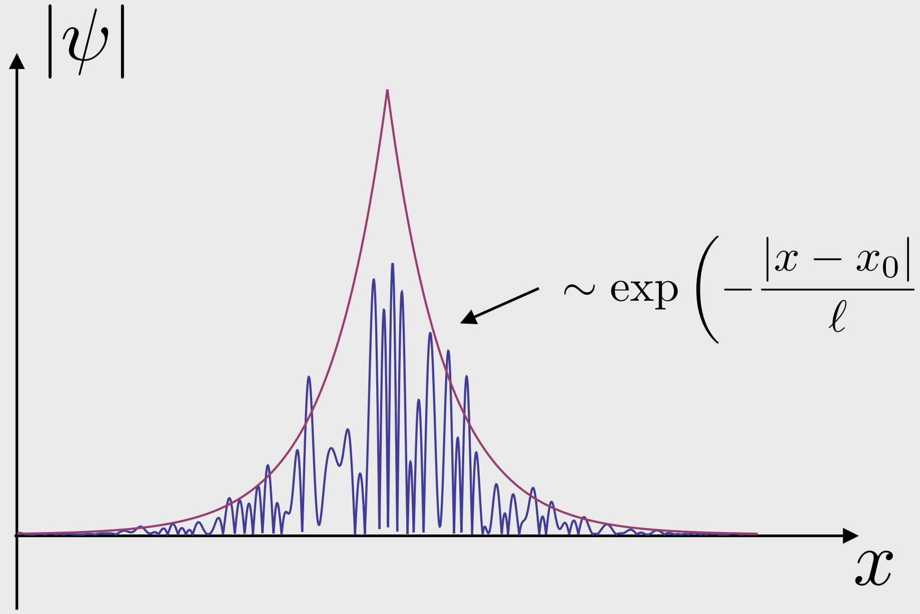

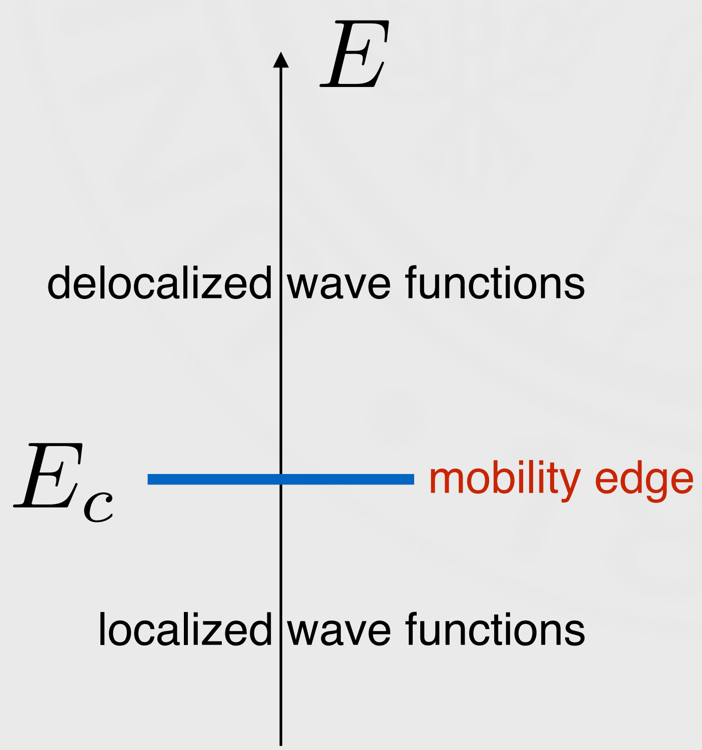

$$\ell \sim \frac{1}{(E_c - E)^{\nu_A}}$$

Anderson localization

Quantum motion in a random potential

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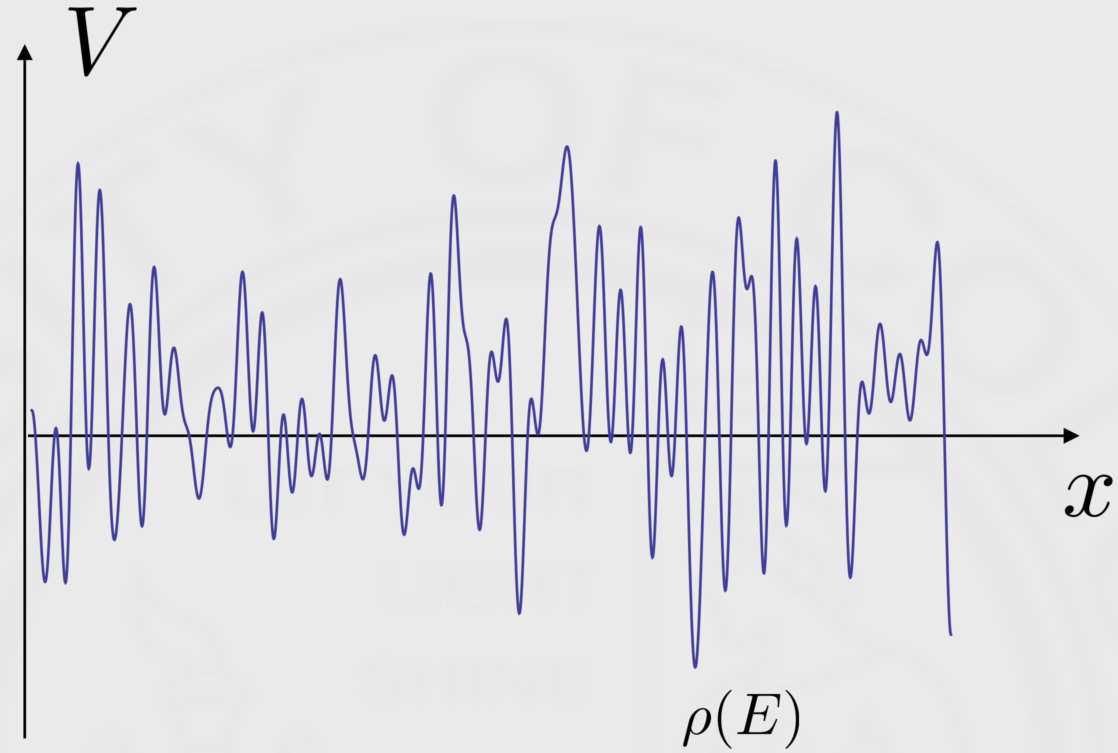

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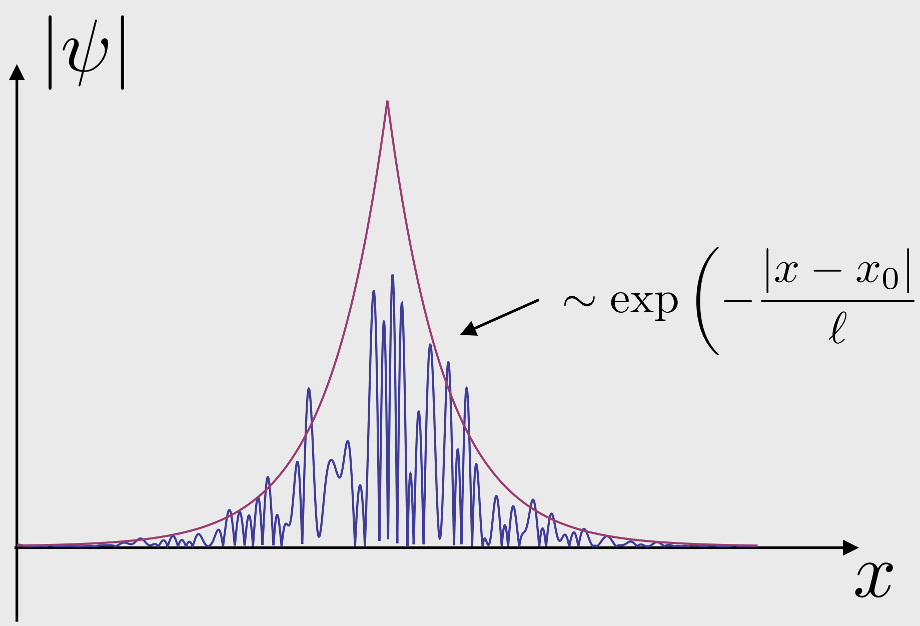

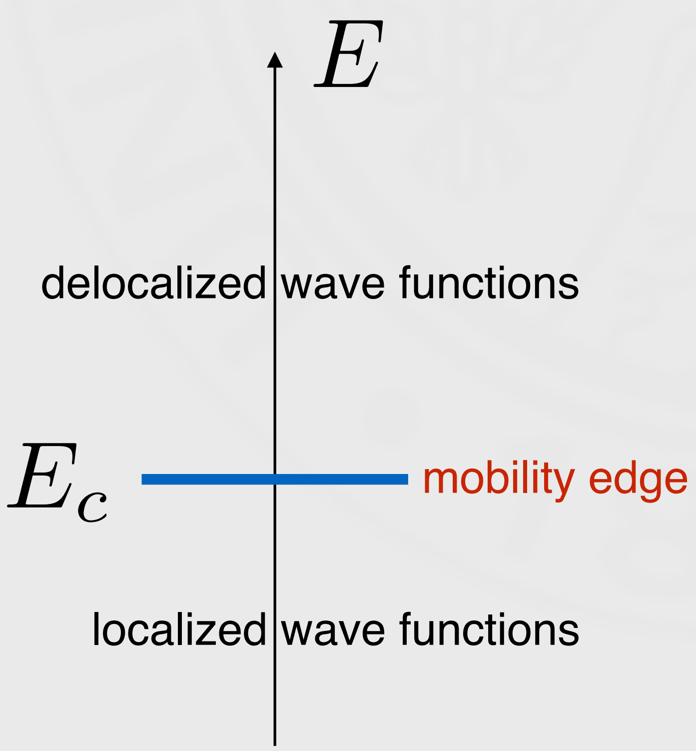

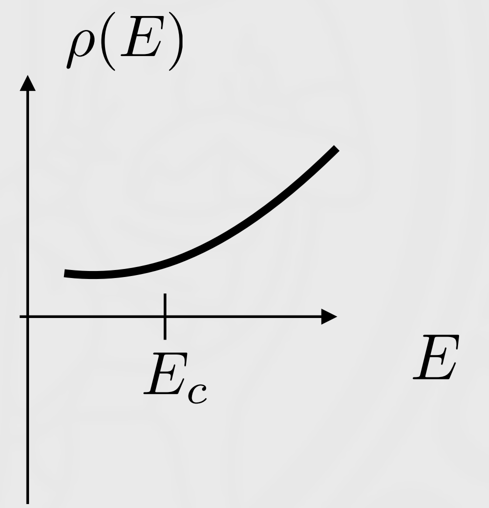

density of states is smooth, "ignores" E_c

$$\ell \sim \frac{1}{(E_c - E)^{\nu_A}}$$

Anderson localization

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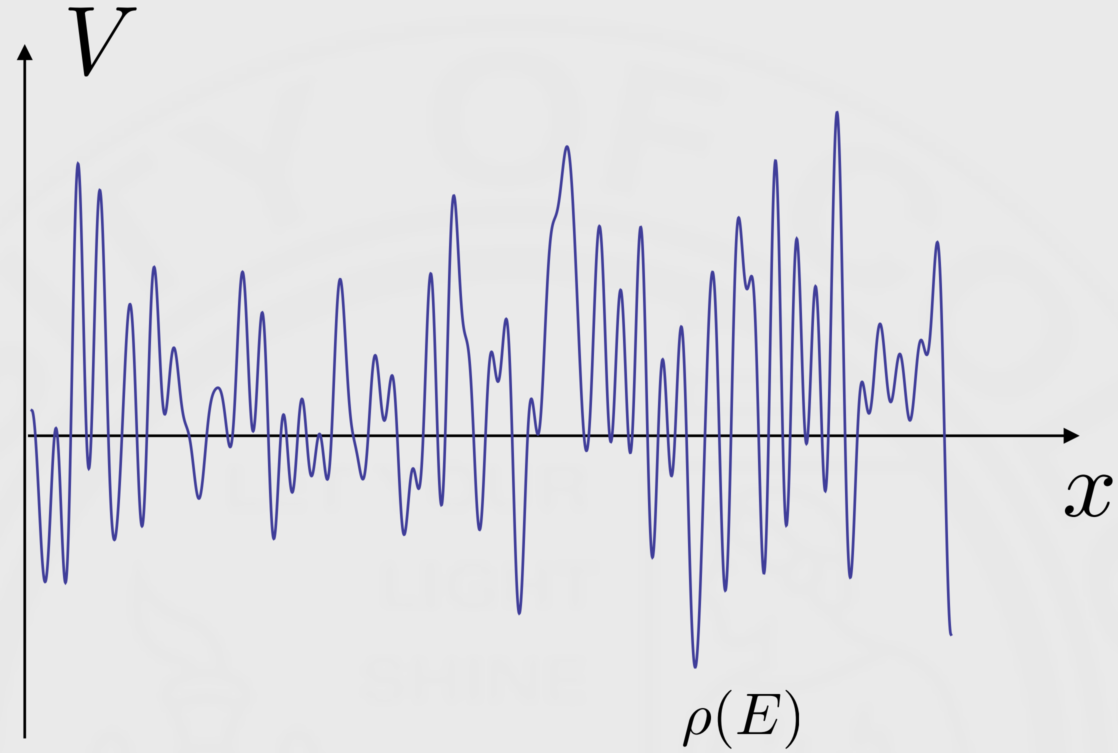

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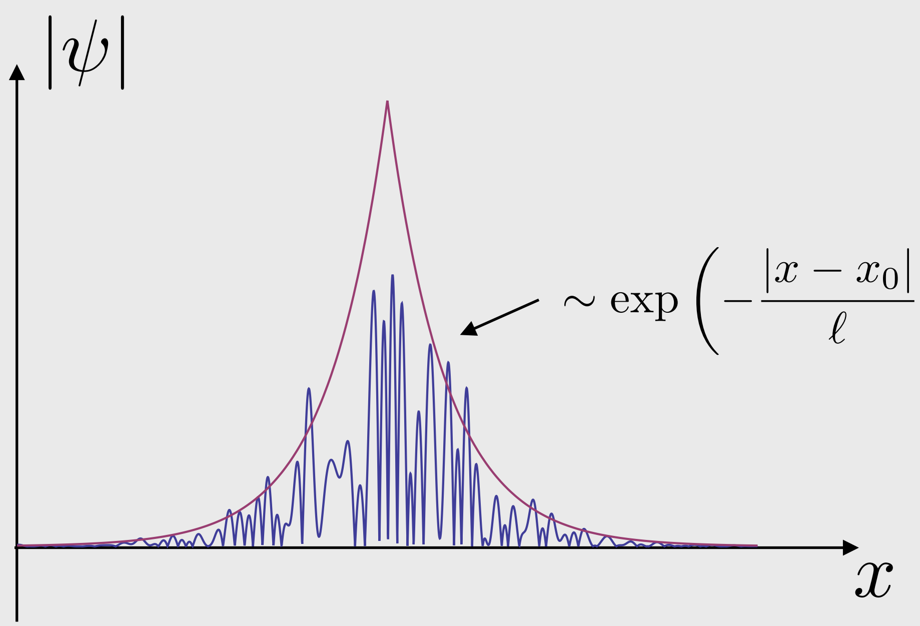

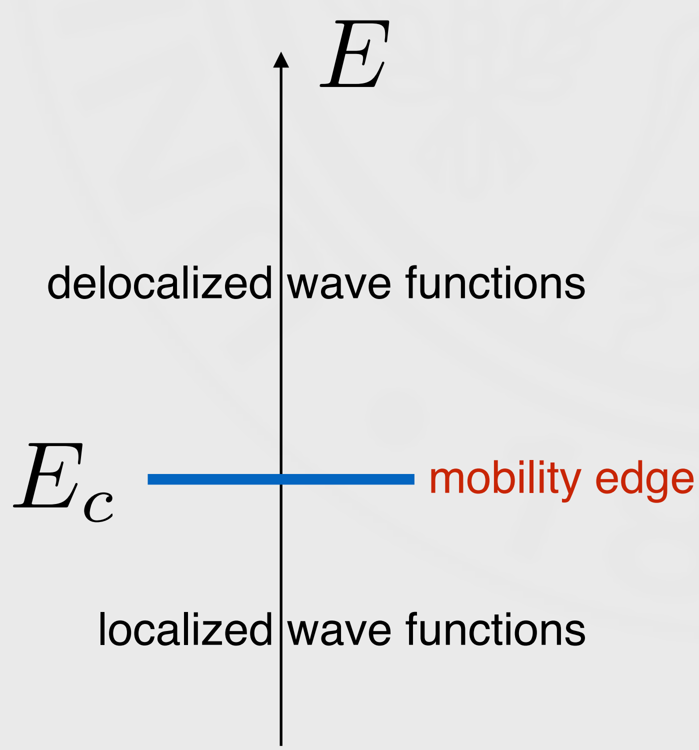

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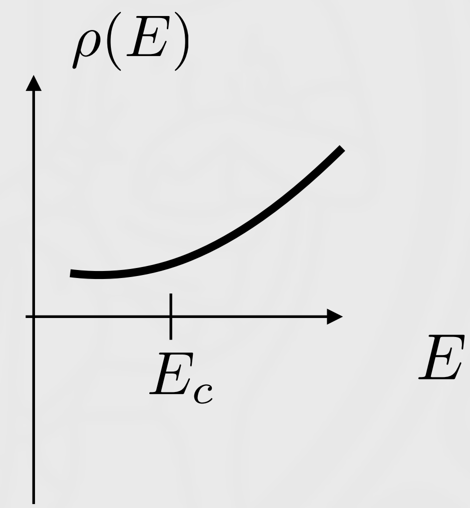

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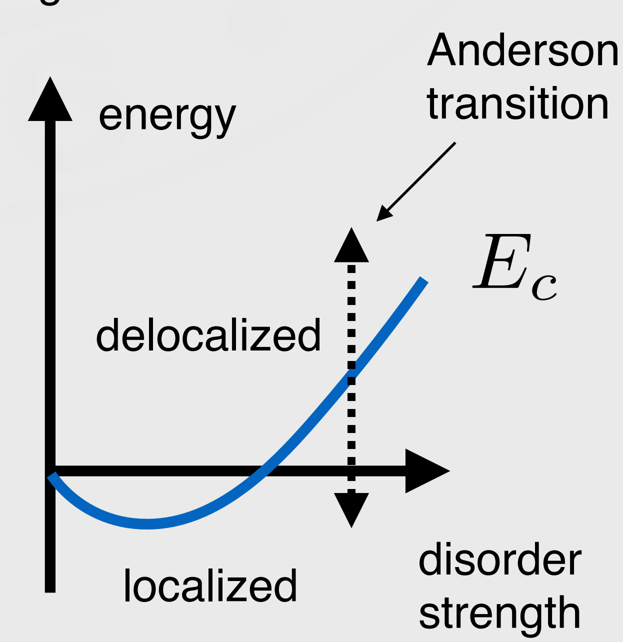


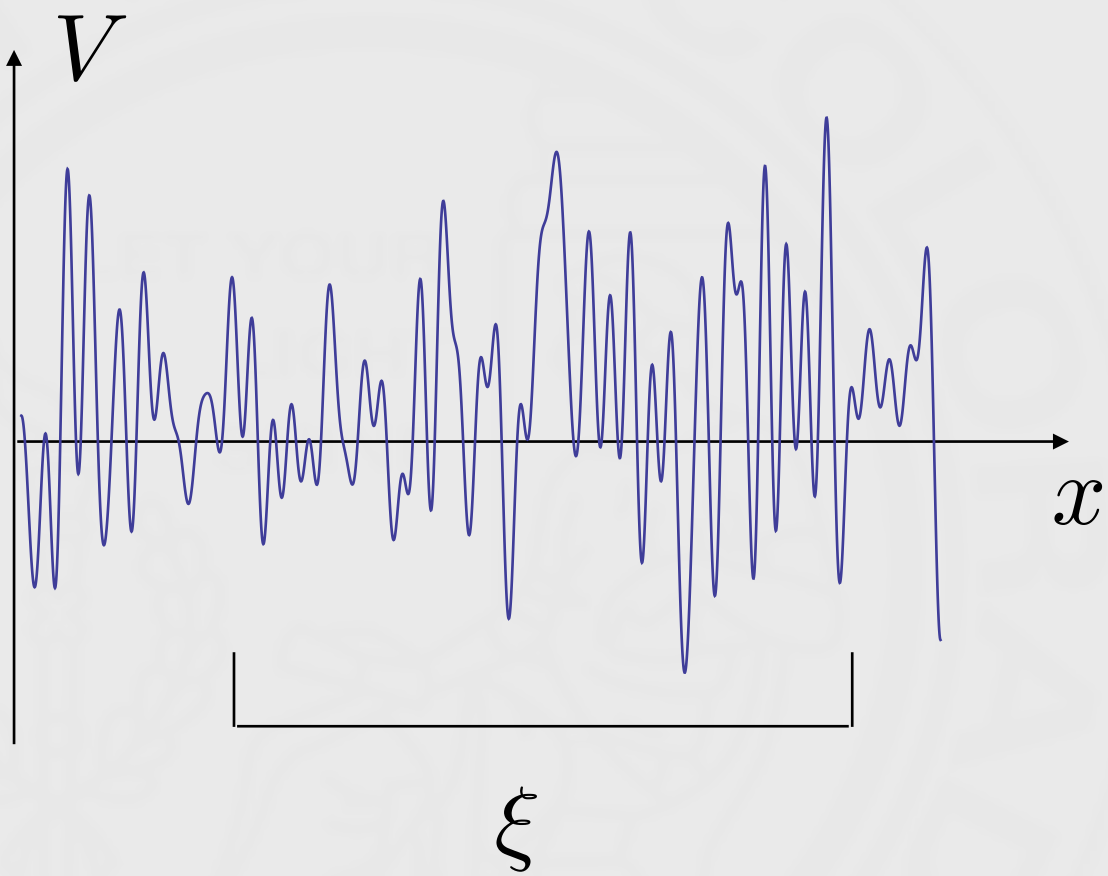
Disorder is “irrelevant” in high-enough Dim

$$H = \sum_{\mathbf{k}} |k|^{\alpha} \, \hat{a}^{\dagger}_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \sum_{\mathbf{r}} V_{\mathbf{r}} \hat{a}^{\dagger}_{\mathbf{r}} \hat{a}_{\mathbf{r}}$$

random potential

$\alpha = 2$ Schrödinger spectrum

$\alpha = 1$ Dirac spectrum



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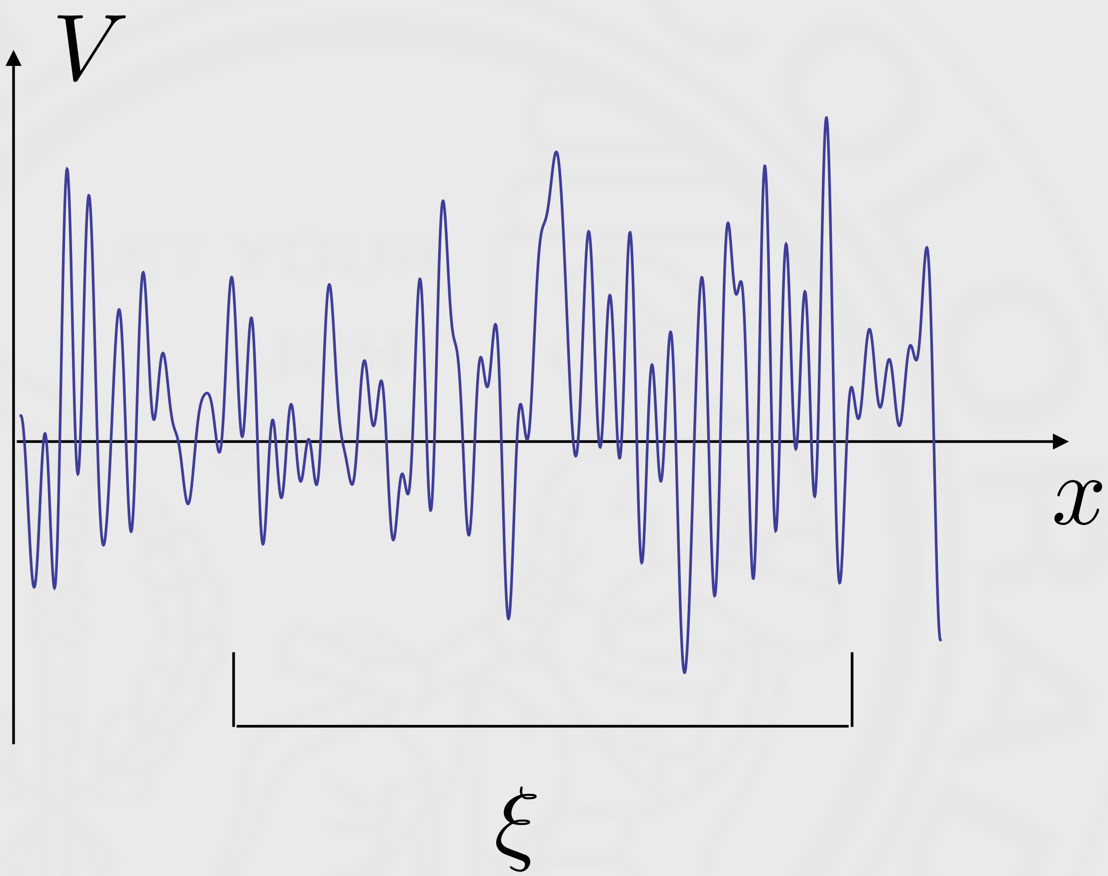

$$K \sim (1/\xi)^{\alpha}$$

$$V \sim \frac{\xi^{\frac{d}{2}}}{\xi^{d}} = \frac{1}{\xi^{\frac{d}{2}}}$$

$d < 2\alpha$ Potential energy dominates at large ξ

$d > 2\alpha$ Kinetic energy dominates at large ξ

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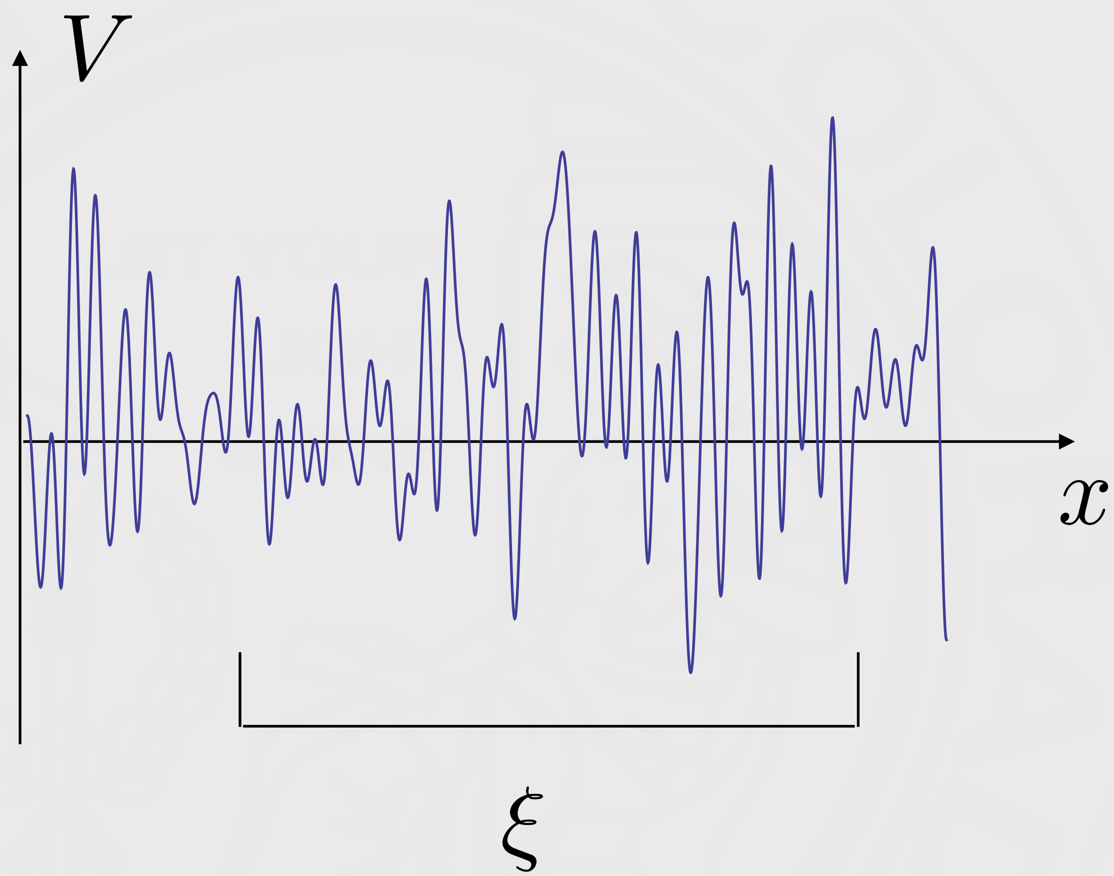

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Gaussian short-ranged correlated potential

$$P(V) \sim \exp\left(-\frac{c}{\gamma}\int d^d r\, V^2(\mathbf{r})\right)$$

Can be encoded in this renormalization group equation for disorder strength

$$\frac{\partial \gamma}{\partial \ell} = (2\alpha - d)\gamma + \ldots$$

random potential strength

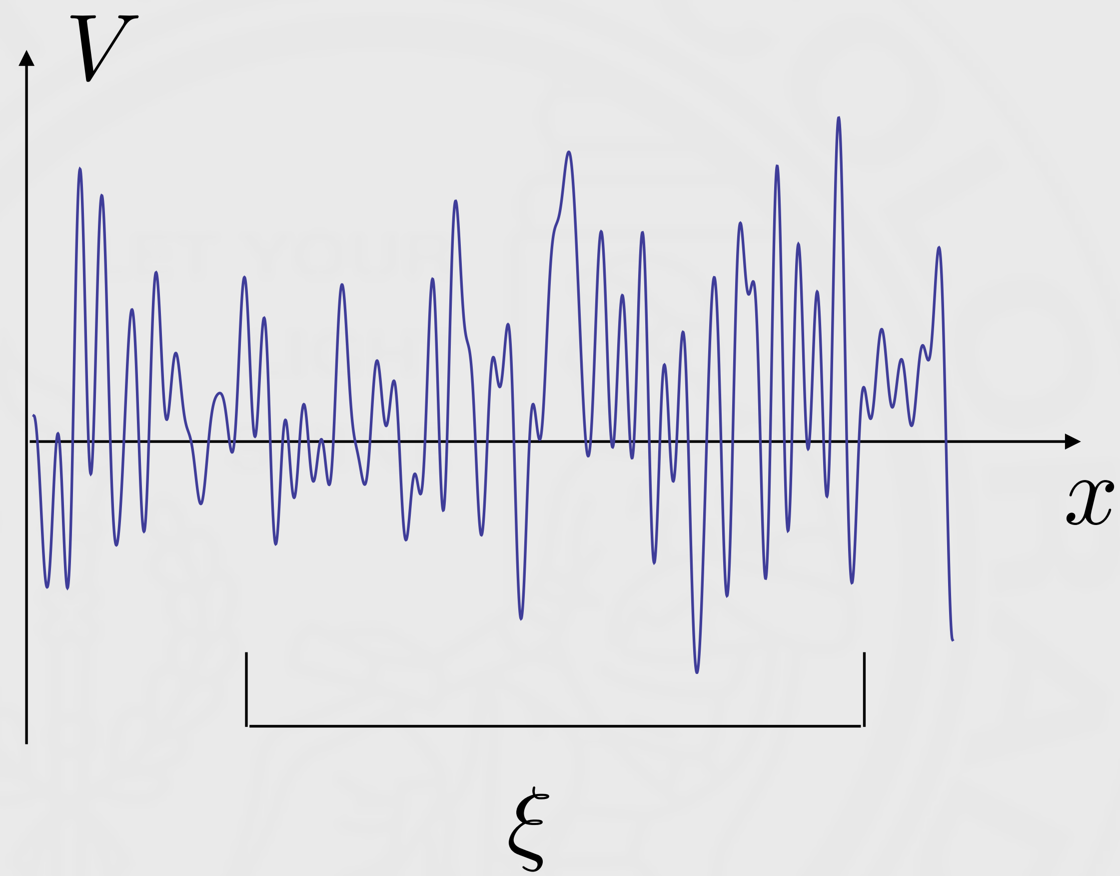
Disorder is “irrelevant” in high-enough Dim

$$H = \sum_{\mathbf{k}} |k|^{\alpha} \, \hat{a}^{\dagger}_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \sum_{\mathbf{r}} V_{\mathbf{r}} \hat{a}^{\dagger}_{\mathbf{r}} \hat{a}_{\mathbf{r}}$$

random potential

$\alpha = 2$ Schrödinger spectrum

$\alpha = 1$ Dirac spectrum



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$d < 2\alpha$ Potential energy dominates at large ξ

$d > 2\alpha$ Kinetic energy dominates at large ξ

$d_c = 4$ Schrödinger spectrum

$d_c = 2$ Dirac spectrum

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$$P(V) \sim \exp\left(-\frac{c}{\gamma} \int d^d r \, V^2(\mathbf{r})\right)$$

Can be encoded in this renormalization group equation for disorder strength

$$\frac{\partial \gamma}{\partial \ell} = (2\alpha - d)\gamma + \ldots$$

random potential strength

RG beyond the lowest order

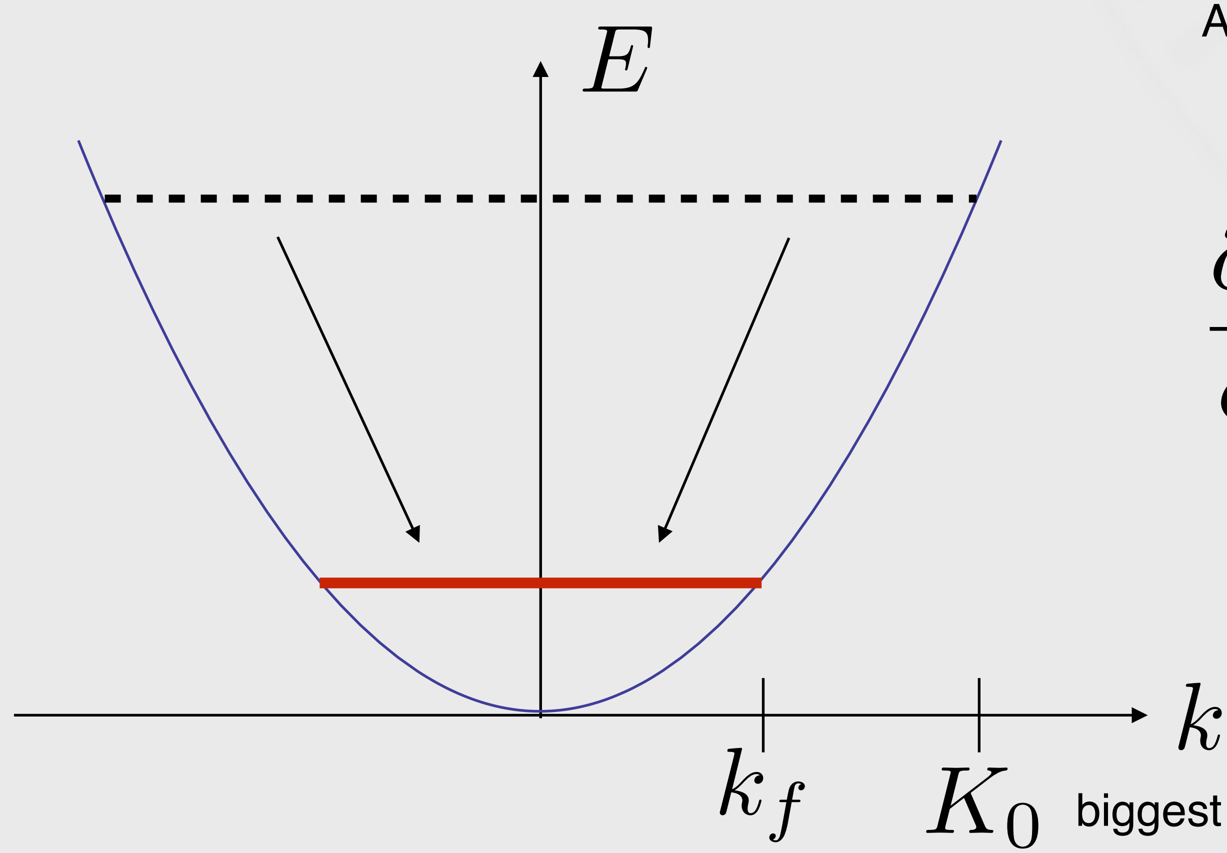
Generating functional for the Green's functions, even a single one (in this example, α=2)

$$Z = \int \mathcal{D}\psi\, e^{i\int d^d\mathbf{r}\, \bar{\psi}_a (E+i0-|\partial_{\mathbf{r}}|^{\alpha}-V(\mathbf{r}))\psi_a}$$

Averaging over disorder

$$Z = \int \mathcal{D}\psi\, e^{i\int d^d\mathbf{r}\left[\bar{\psi}_a (E+i0-|\partial_{\mathbf{r}}|^{\alpha})\psi_a + i\gamma\left(\bar{\psi}_a\psi_a\right)^2\right]}$$

Apply RG to this problem

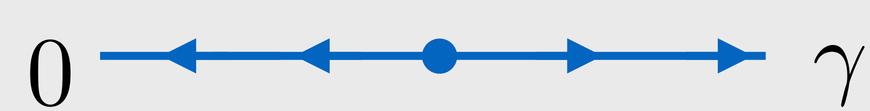


biggest momentum available to the system

$C > 0$

$$\frac{\partial\gamma}{\partial\ell} = (2\alpha - d)\gamma + C\gamma^2 + \dots$$

critical point at $d > 2\alpha$



Origin of the critical point

Motion in imaginary random potential = self avoiding random walk = O(N) model of statistical mechanics in the limit $N \to 0$

De Gennes 1972

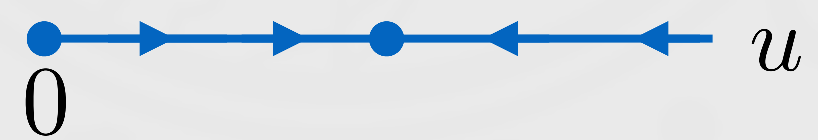
Recall from the studies of the Ising model near 4D

$$\frac{\partial u}{\partial \ell} = (2\alpha - d)\, u - C(N)\, u^2 + \ldots$$

$$C(N) > 0$$

fixed point at $d < 2\alpha$

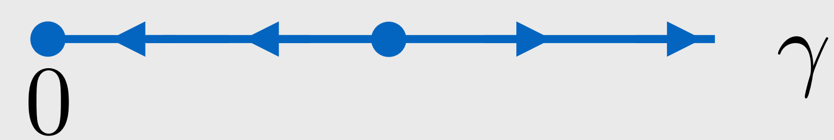
(Wilson-Fisher critical point)



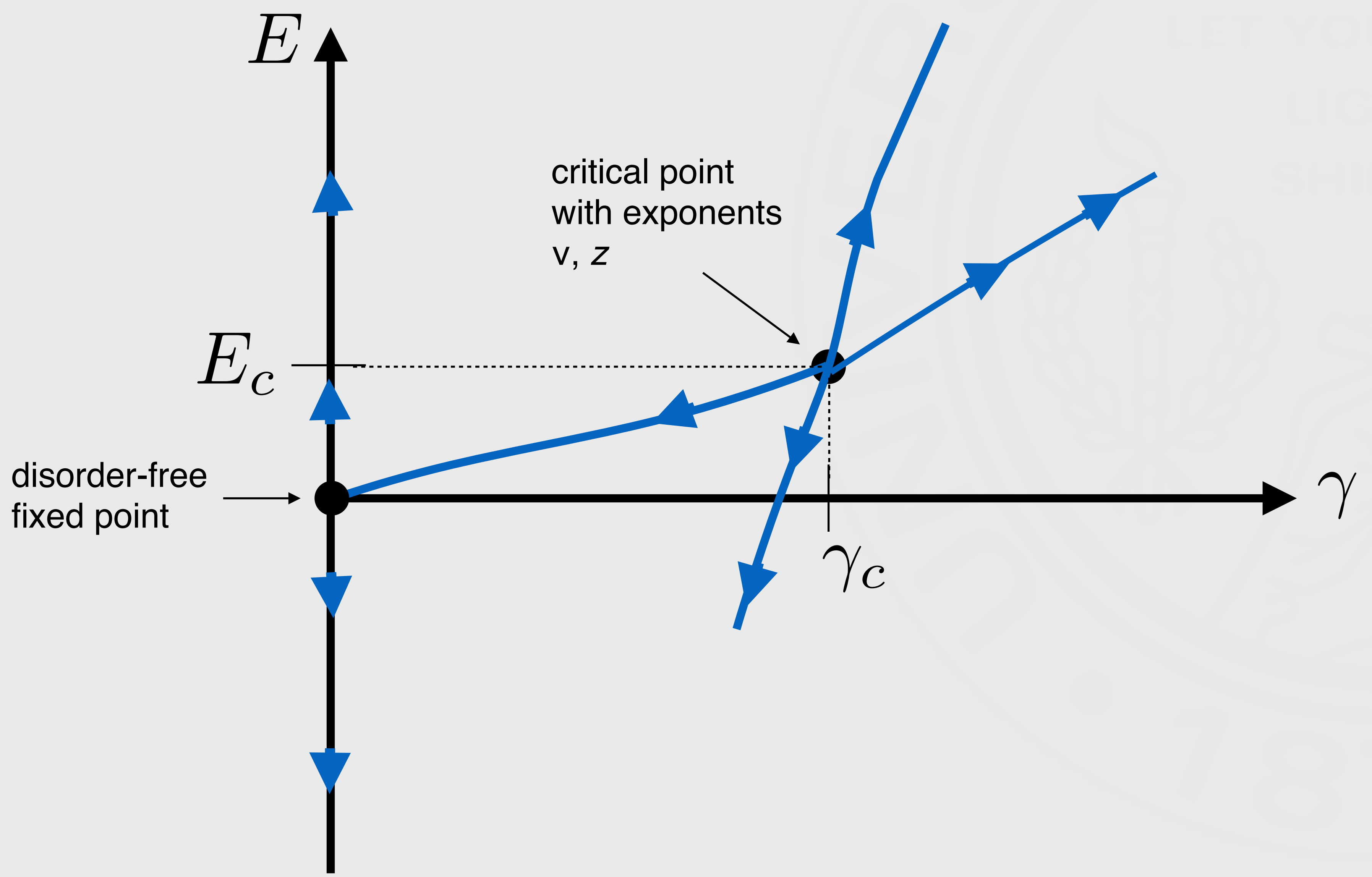
$$u \to -\gamma$$

$$\frac{\partial \gamma}{\partial \ell} = (2\alpha - d)\gamma + C\gamma^2 + \ldots$$

critical point at $d > 2\alpha$



Full RG flow diagram, *d*>2*a*



$$Z = \int \mathcal{D}\psi \, e^{i\int d^d\mathbf{r}\left[\bar{\psi}_a(E+i0-|\partial_\mathbf{r}|^\alpha)\psi_a + i\gamma\left(\bar{\psi}_a\psi_a\right)^2\right]}$$

Consequence: critical DoS

$$Z = \int \mathcal{D}\psi \, e^{i \int d^d\mathbf{r} \left[\bar{\psi}_a (E + i0 - |\partial_\mathbf{r}|^\alpha) \psi_a + i\gamma \left(\bar{\psi}_a \psi_a \right)^2 \right]}$$

Correlation function within this theory produces disorder-averaged Green's functions, in particular gives the density of states

$$\epsilon = d - 2\alpha \ll 1$$

Within ε-expansion

$$\nu \approx \frac{1}{d - 2\alpha}$$

$$z \approx \alpha + \frac{d - 2\alpha}{4}$$

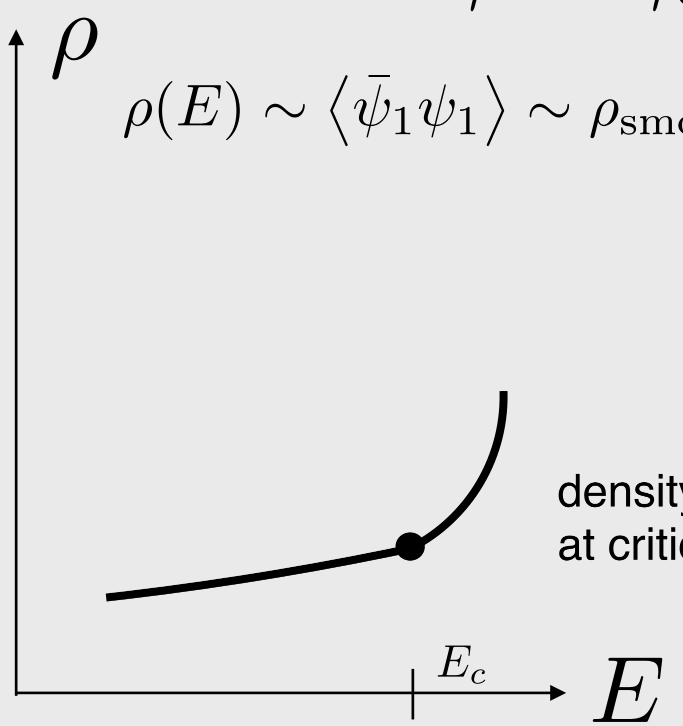
Assumption: RG flow in γ results in a critical point γ_c.

E acquires dimension *z*
γ acquires dimension *ν*

$$z \approx \frac{d}{2} - \frac{d - 2\alpha}{4}$$

$$\gamma = \gamma_c$$

$$\rho(E) \sim \langle \bar{\psi}_1 \psi_1 \rangle \sim \rho_{\mathrm{smooth}}(E) + |E - E_c|^{\frac{d-z}{z}}$$



density of states is singular at critical disorder

More generally

$$\rho(E) \sim \rho_{\mathrm{smooth}}(E) + |E - E_c|^{\frac{d}{z} - 1} \, \Phi \left(\frac{\gamma - \gamma_c}{|E - E_c|^{\frac{1}{z\nu}}} \right)$$

Universal scaling function

Consequence: critical DoS

$$Z = \int \mathcal{D}\psi \, e^{i\int d^d\mathbf{r}\left[\bar\psi_a(E+i0-|\partial_\mathbf{r}|^\alpha)\psi_a + i\gamma\left(\bar\psi_a\psi_a\right)^2\right]}$$

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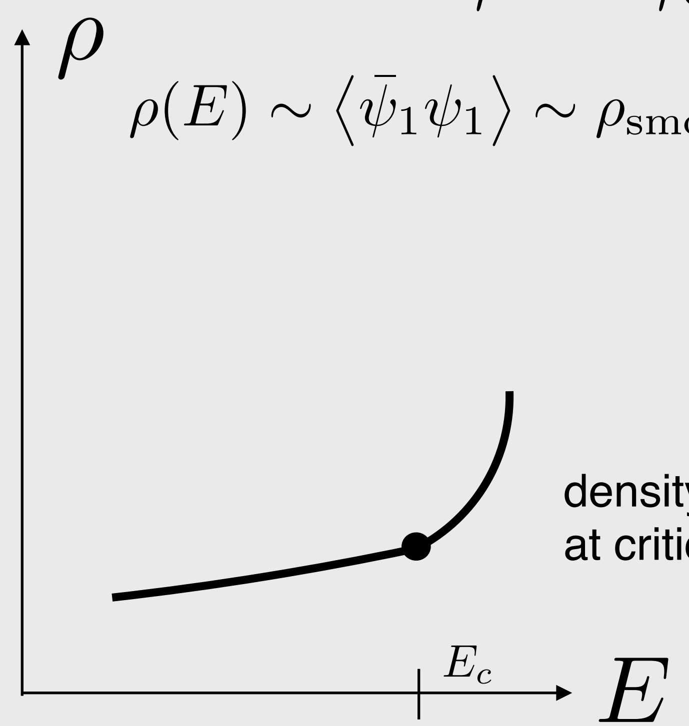
E acquires dimension z
γ acquires dimension ν

$$z \approx \frac{d}{2} - \frac{d-2\alpha}{4}$$

$$\gamma = \gamma_c$$

slightly larger than 1

$$\rho(E) \sim \langle \bar\psi_1 \psi_1 \rangle \sim \rho_{\text{smooth}}(E) + |E-E_c|^{\frac{d-z}{z}}$$



density of states is singular
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More generally

$$\rho(E) \sim \rho_{\text{smooth}}(E) + |E-E_c|^{\frac{d}{z}-1}\,\Phi\left(\frac{\gamma-\gamma_c}{|E-E_c|^{\frac{1}{z\nu}}}\right)$$

Universal scaling function

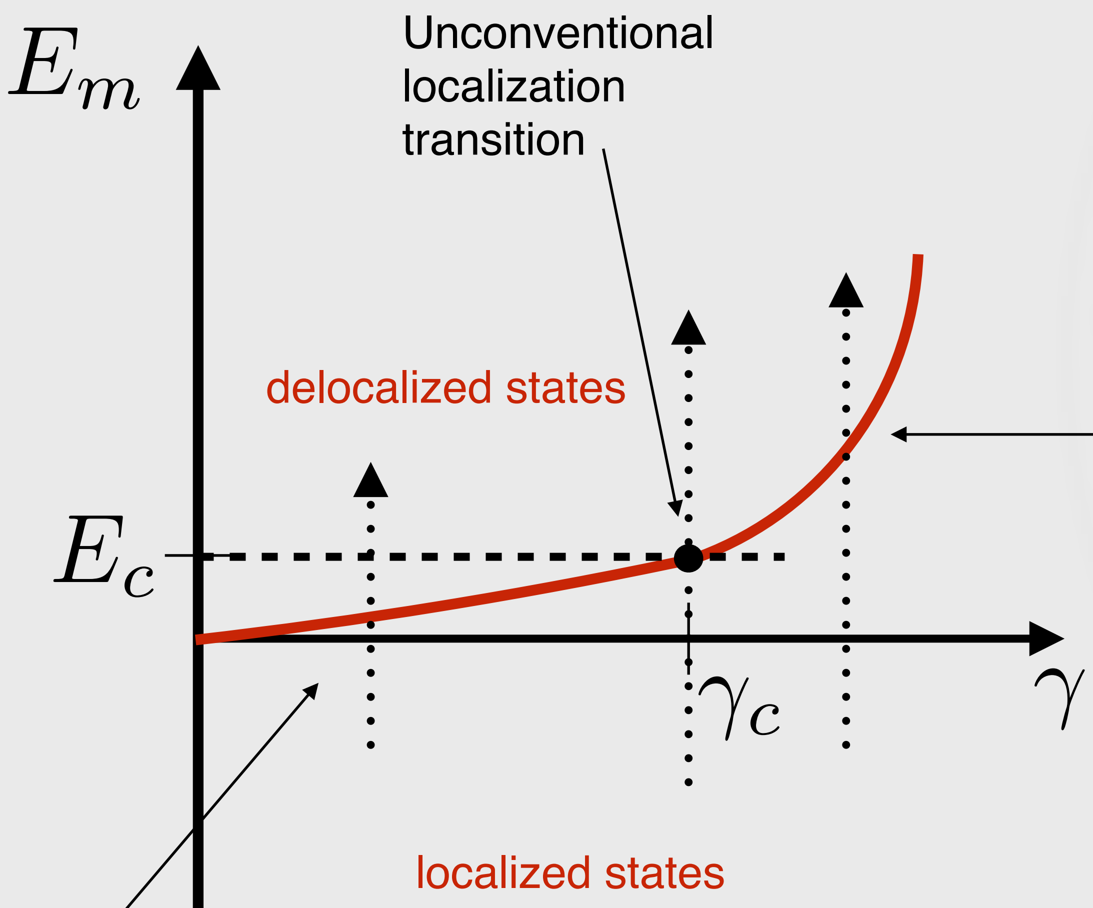
New transitions in higher dimensions

$$\alpha = 2$$

$$d > 4$$

$$\ell \sim \frac{1}{(E_c - E)^{1/z}}$$

ℓ localization length

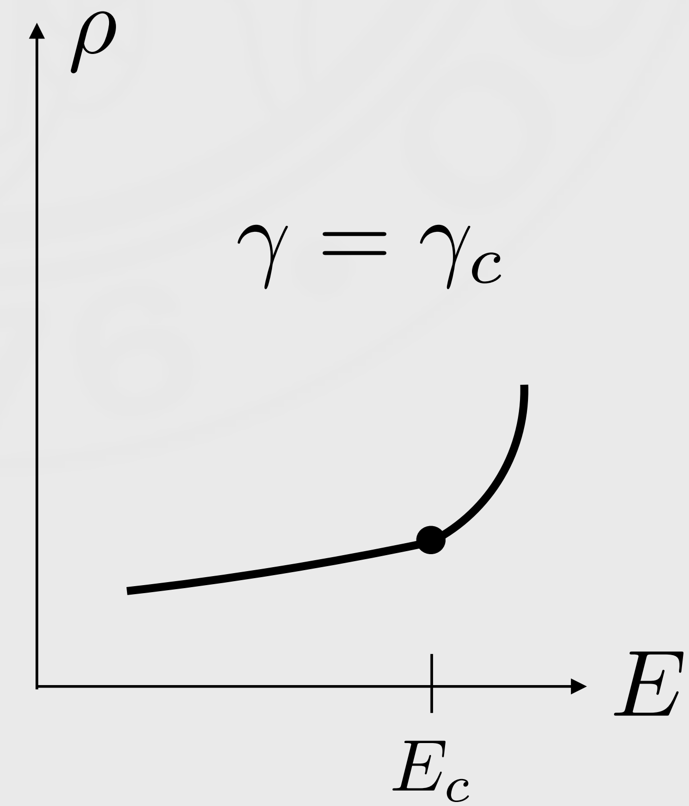


Conventional Anderson transition

$$\ell \sim \frac{1}{(E_c - E)^{\nu_A}}$$

transition between Lifshitz tail states and states unaffected by disorder (not really understood)

$$E_m(\gamma) \sim |\gamma - \gamma_c|^{z\nu}$$

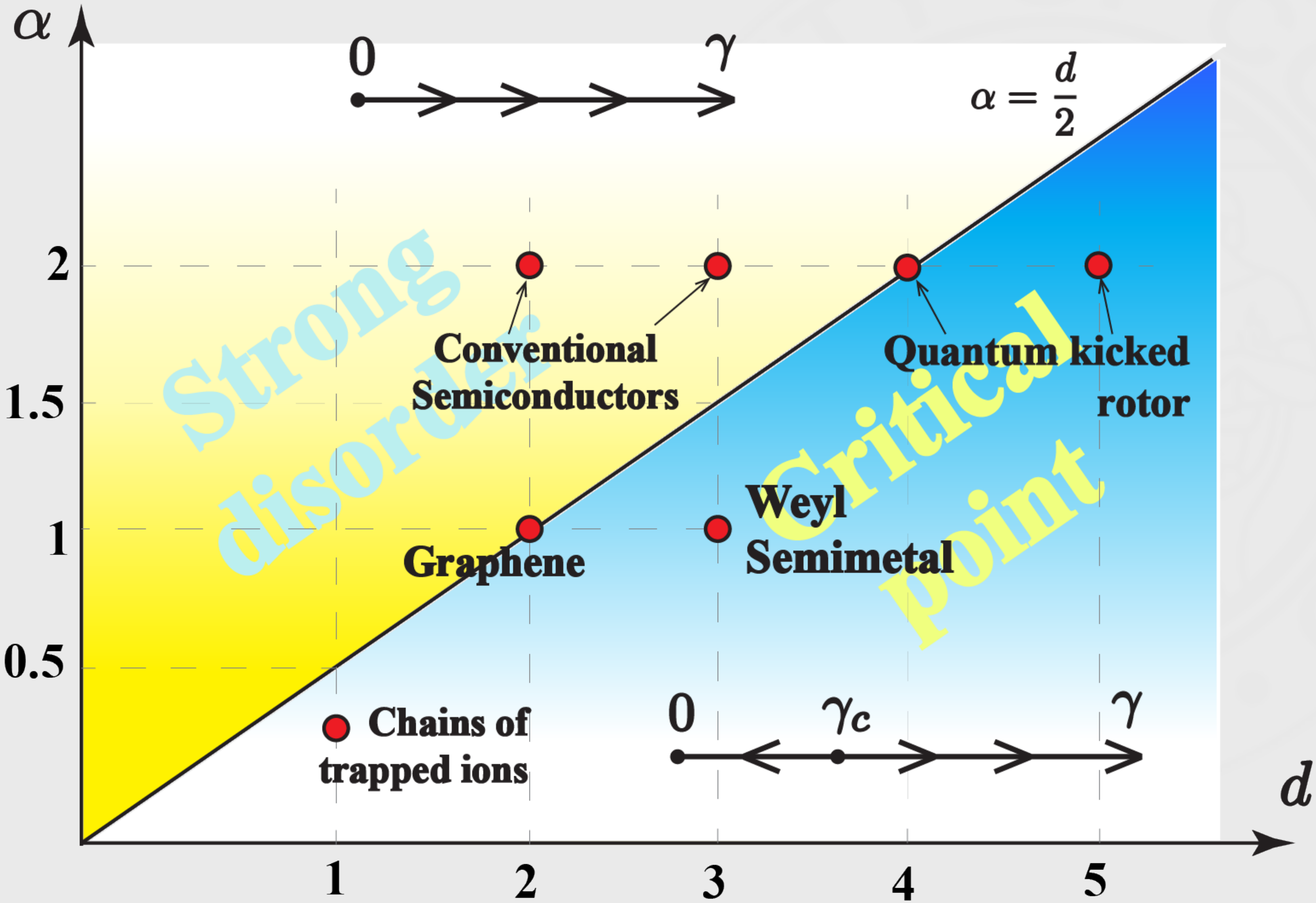


$$\rho \approx \rho_0(E) + C\,|E - E_c|^{\frac{d}{z}-1}$$

density of states is singular at critical disorder

S. Syzranov, L. Radzihovsky, VG (2015)

“Phase diagram” of available setups



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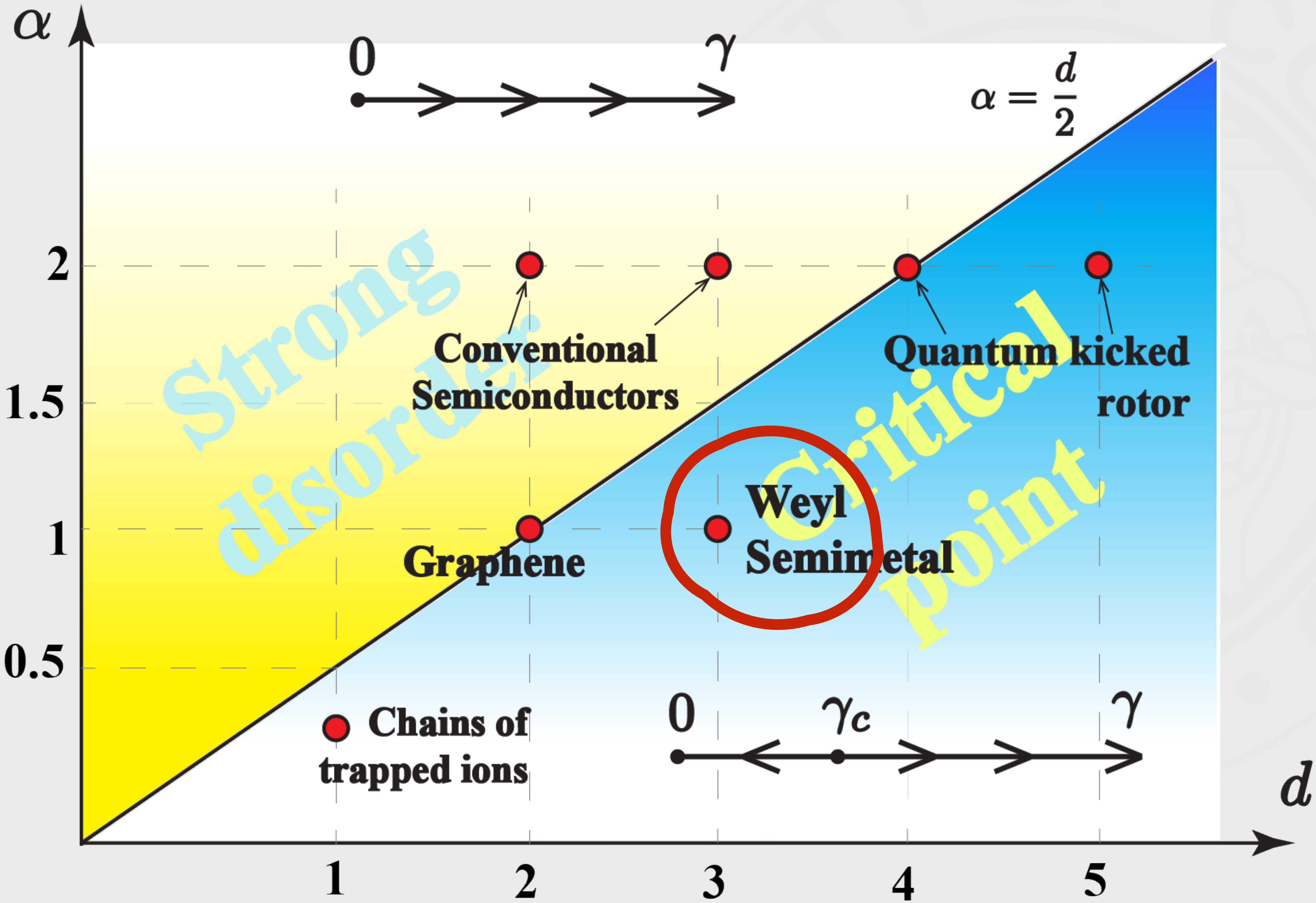


Disordered Dirac (Weyl) Hamiltonian

$$H = v_F \sum_{i=1}^{3} \sigma_i k_i + V(\mathbf{r}) \qquad \alpha = 1 \qquad 3 = d > 2\alpha$$

"Soft" disorder which does not scatter from one Dirac cone to another.

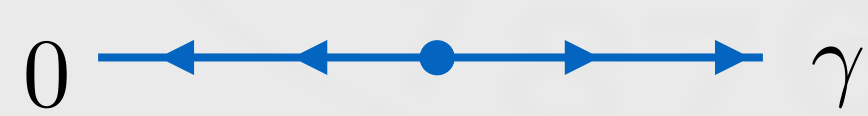
(more complicated disorder proportional to Pauli matrices possible; won't lead to a critical point - so restrict to the potential disorder)

<u>Absence of localization</u> in a disordered single Dirac cone

Goes back to the work on the absence of localization
at the boundary of topological insulators (2007 and earlier)

Study of the DoS goes back to the original work of E. Fradkin (1986)

That pioneering work recognized the existence
of the critical point at d=3, α=1, for the first time.



Recent interest starting from the work of Goswami and Chakravarty, 2011

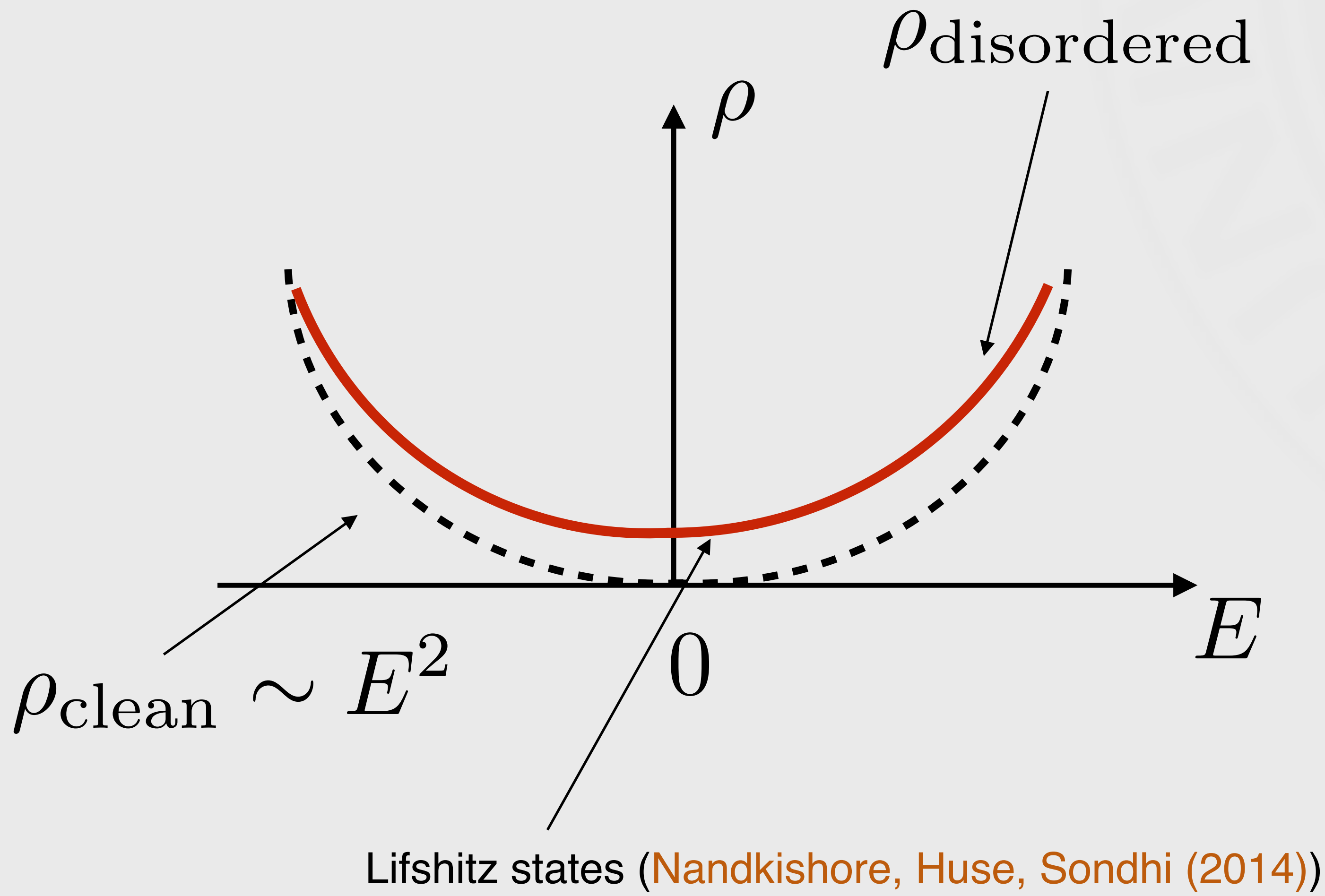
Density of states

$$H = v_F \sum_{i=1}^{3} \sigma_i k_i + V(\mathbf{r})$$

$$\langle V(\mathbf{r}_1) V(\mathbf{r}_2) \rangle = \gamma \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$E_c = 0$$

$$\gamma < \gamma_c$$

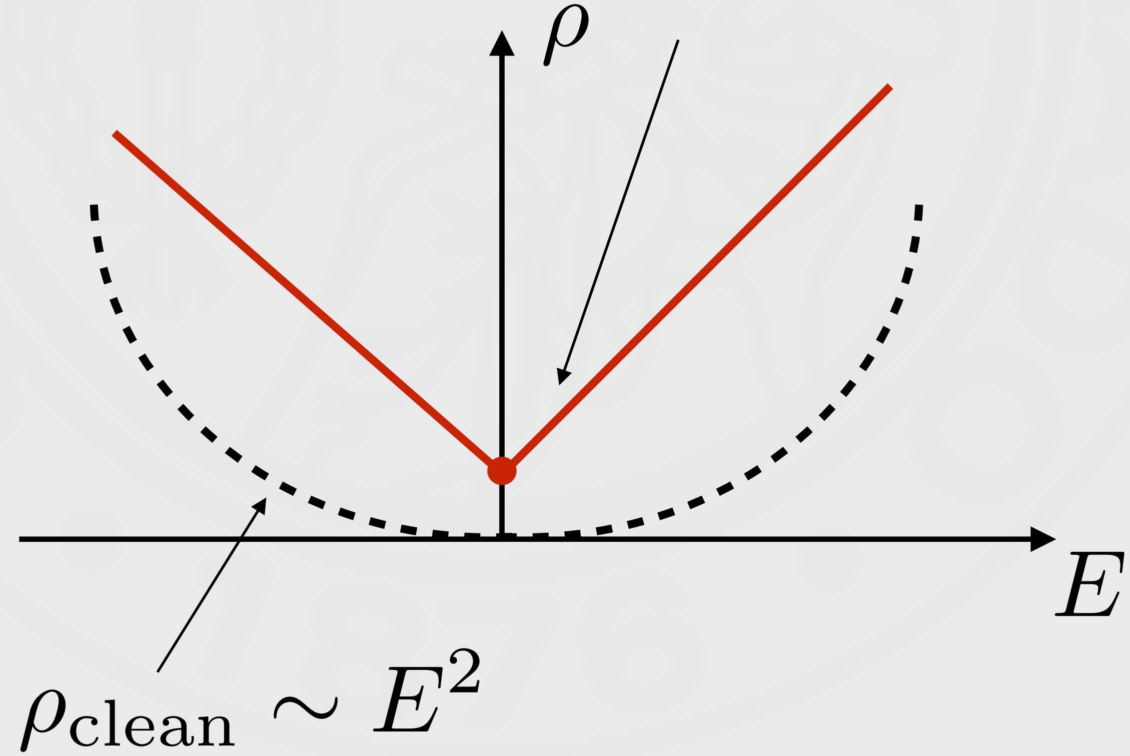


z<d/2: P.M. Ostrovsky, S.V. Syzranov, L. Radzihovsky, VG, in preparation

$$\gamma = \gamma_c \qquad z \approx \frac{d}{2}$$

$$\rho_{\text{disordered}} \sim |E|^{\frac{d}{z}-1}$$

$$d = 3$$



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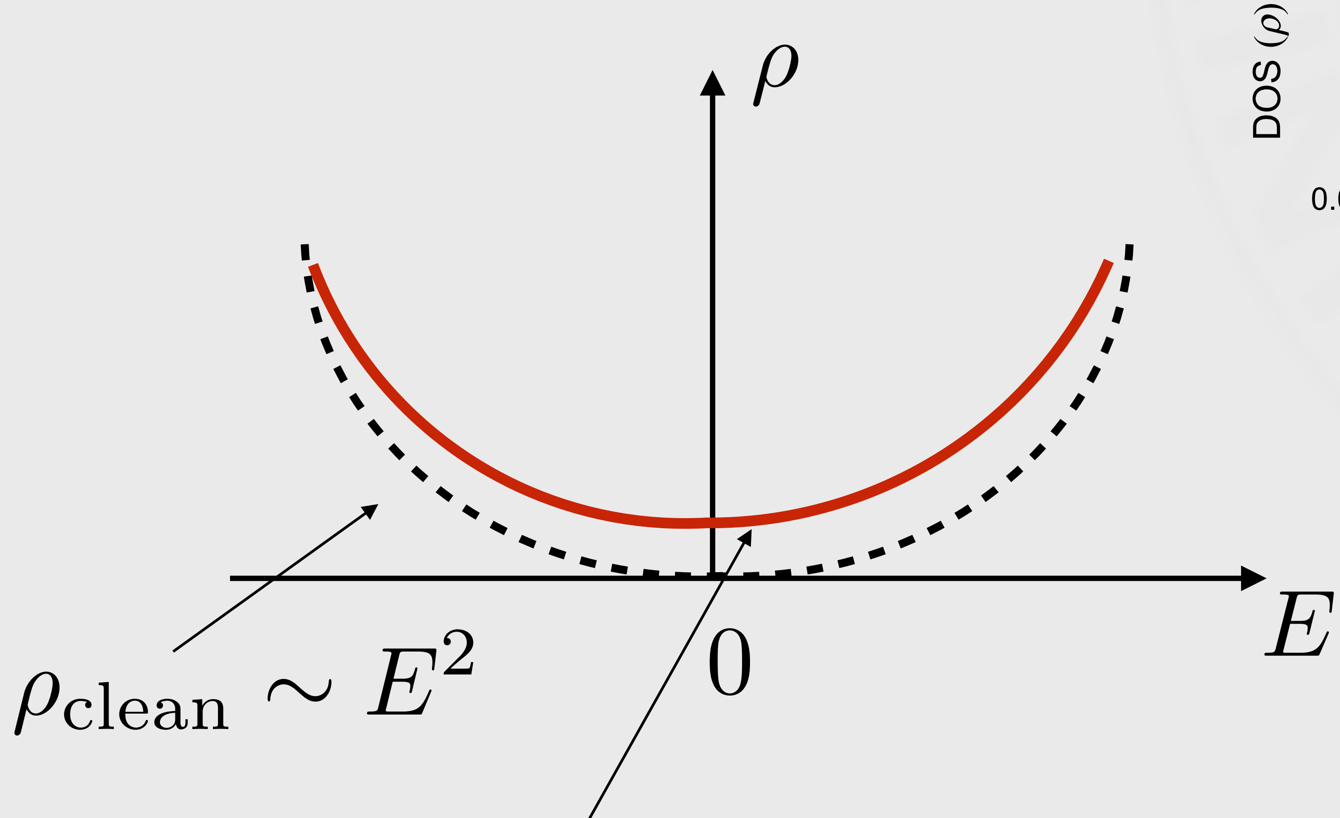
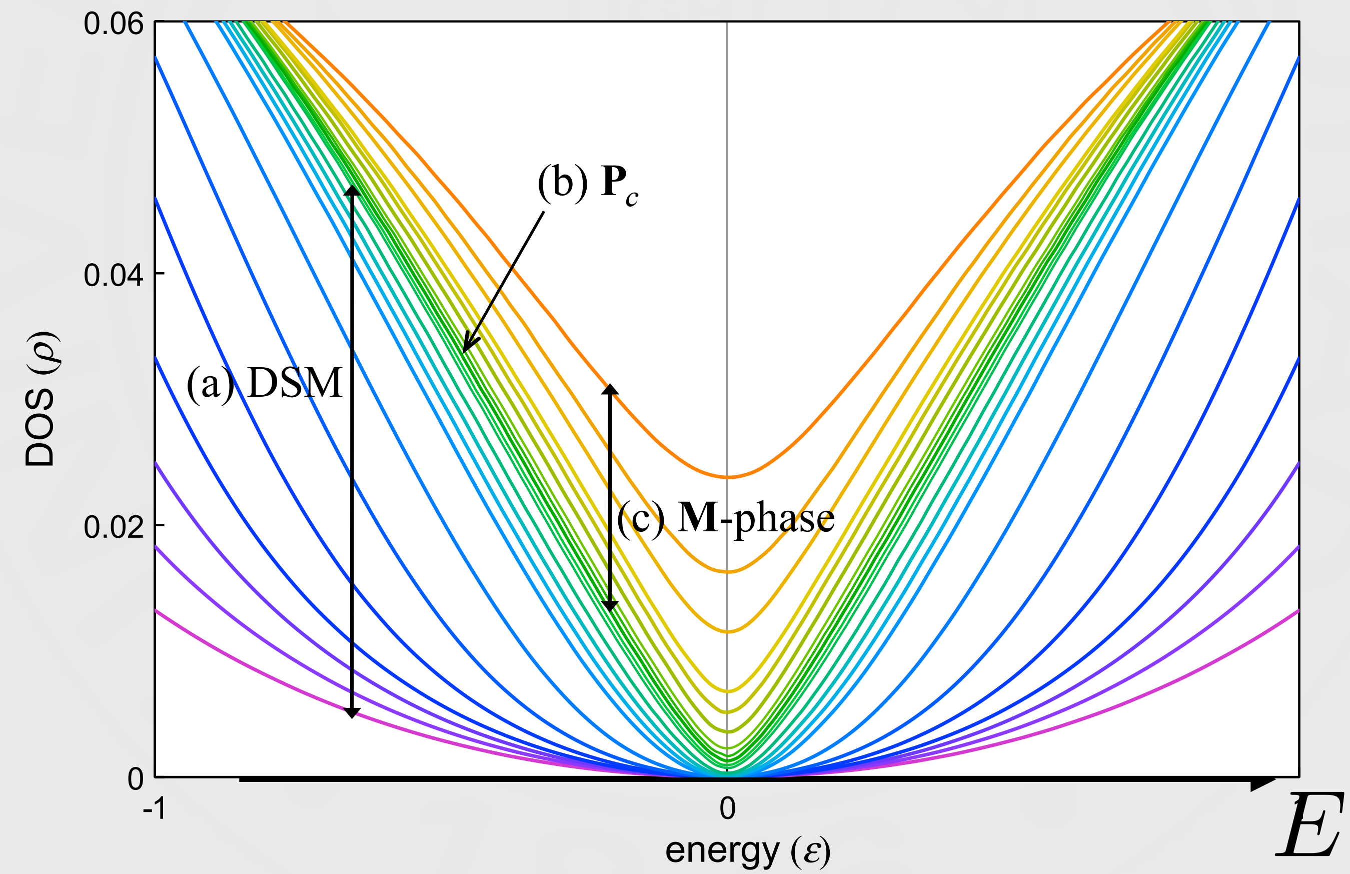
z<d/2: P.M. Ostrovsky, S.V. Syzranov, L. Radzihovsky, VG, in preparation



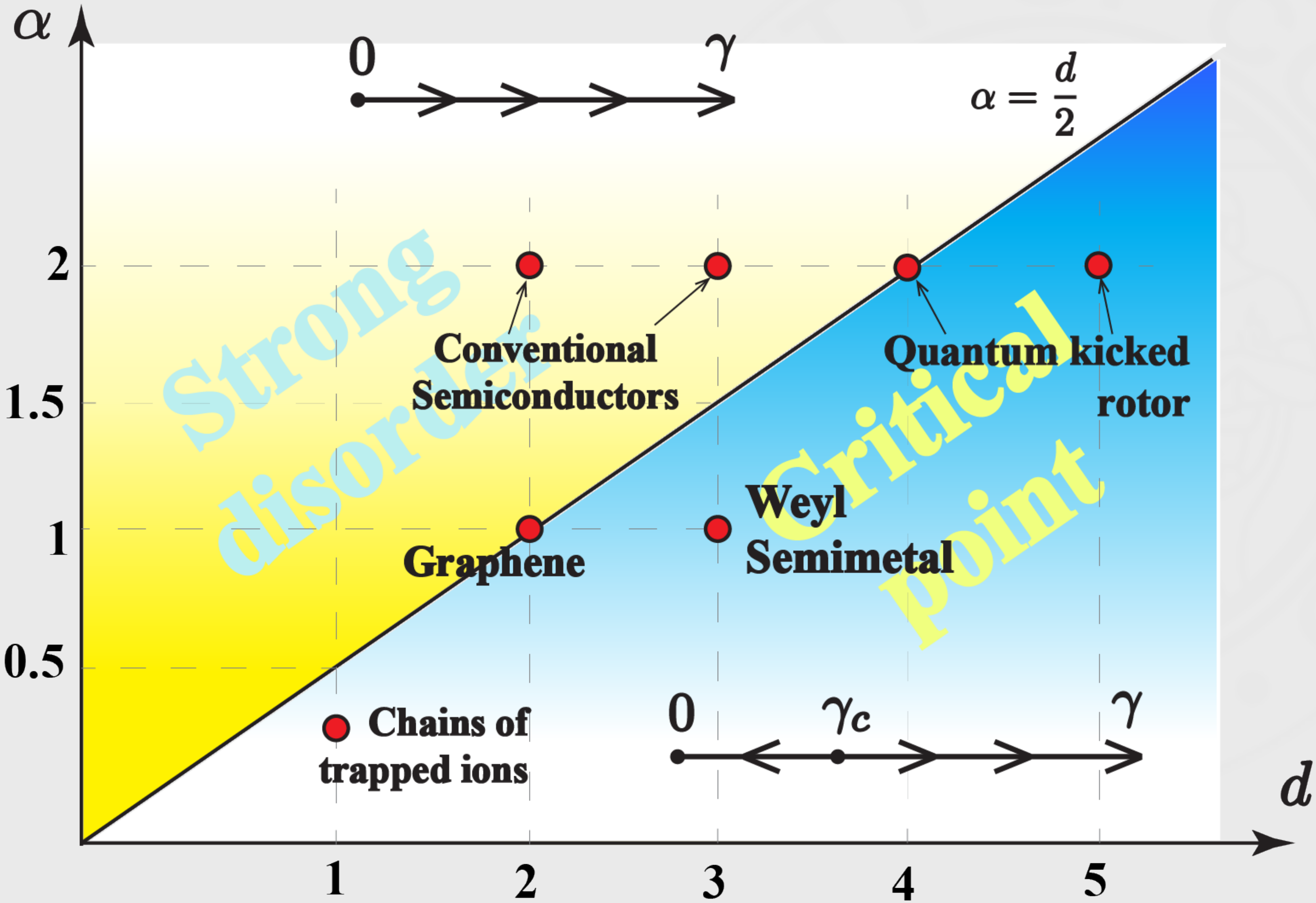
Lifshitz states (Nandkishore, Huse, Sondhi (2014))



Density of states vs energy
From K. Kobatashi, T. Ohtsuki, K.-I. Imura and I. Herbut (2014)

these authors were also the first to write down the scaling ansatz for the DoS

"Phase diagram" of available setups



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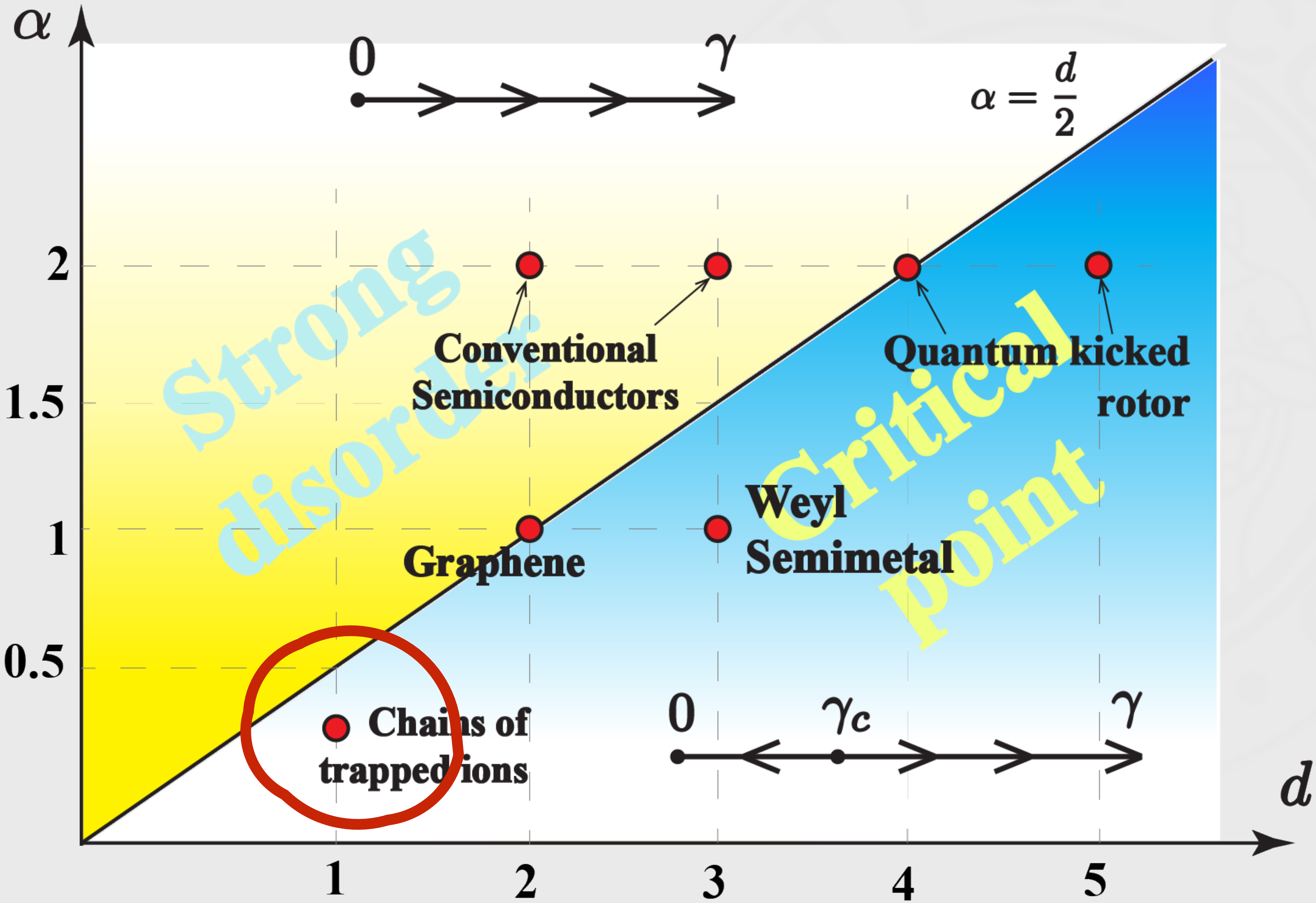


1D model with long-range hopping

Critical density of states if $\alpha < \frac{1}{2}$

(1) $$H = \sum_{nm} \frac{1}{|n-m|^{1+\alpha}} a_n^\dagger a_m + \sum_n V_n a_n^\dagger a_n$$

$$H_0 \sim |k|^\alpha$$

Like disordered Schrödinger equation

(2) $$H = \sum_{nm} \frac{i\,\mathrm{sign}\,(n-m)}{|n-m|^{1+\alpha}} a_n^\dagger a_m + \sum_n V_n a_n^\dagger a_n$$

$$H_0 = |k|^\alpha \mathrm{sign}\, k$$

Like disordered Weyl equation

Long range interacting spin systems

PRL 103, 120502 (2009) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2009

Entanglement and Tunable Spin-Spin Couplings between Trapped Ions Using Multiple Transverse Modes

K. Kim,[1] M.-S. Chang,[1] R. Islam,[1] S. Korenblit,[1] L.-M. Duan,[2] and C. Monroe[1]

[1]*Joint Quantum Institute: Department of Physics, University of Maryland, and National Institute of Standards and Technology, College Park, Maryland 20742, USA*

[2]*FOCUS Center and MCTP, Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*

(Received 2 May 2009; published 16 September 2009)

We demonstrate tunable spin-spin couplings between trapped atomic ions, mediated by laser forces on multiple transverse collective modes of motion. A $\sigma_x\sigma_x$-type Ising interaction is realized between quantum bits stored in the ground hyperfine clock states of $^{171}Yb^+$ ions. We demonstrate entangling gates and tailor the spin-spin couplings with two and three trapped ions. The use of closely spaced transverse modes provides a new class of interactions relevant to quantum computing and simulation with large collections of ions in a single crystal.

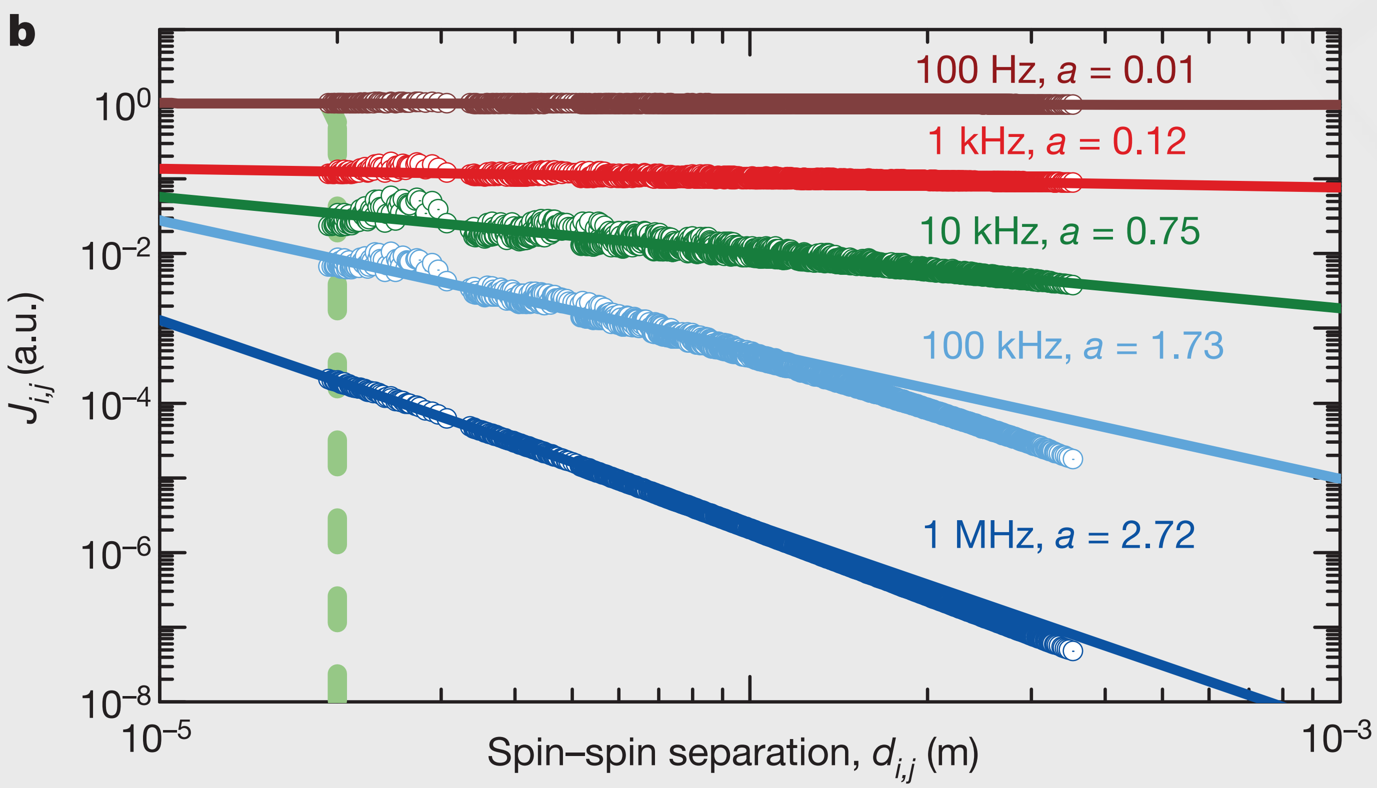
DOI: 10.1103/PhysRevLett.103.120502 PACS numbers: 03.67.Pp, 03.67.Ac, 03.67.Lx, 37.10.Ty

LETTER

doi:10.1038/nature10981

Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins

Joseph W. Britton[1], Brian C. Sawyer[1], Adam C. Keith[2,3], C.-C. Joseph Wang[2], James K. Freericks[2], Hermann Uys[4], Michael J. Biercuk[5] & John J. Bollinger[1]



$$H = \sum_{ij} J_{ij}\sigma_i^x\sigma_j^x + B\sum_i \sigma_i^z$$

$$J_{ij} \sim \frac{1}{|i-j|^a}$$

Long range interaction + disorder

$$H = \sum_{ij} J_{ij}\sigma_i^x\sigma_j^x + \sum_i V_i\sigma_i^z$$

random spin chain

random uncorrelated potential

$$\langle V_i V_j \rangle = \gamma\delta_{ij}$$

Short range J - Real space renormalization group

Dasgupta, Ma, Hu; 1979
D. Fisher, 1994

$$J_{ij} \sim \frac{1}{|i-j|^a}$$

???

Subject of studies on many-body localization

Friday schedule:

11:00AM

Main Seminar Room

- 11:00AM - Rajiv Singh (UC Davis) - *Early Breakdown of area-law entanglement at the many-body delocalization transition*
- 11:30AM - Chris Monroe (UMD) - *Observation of MBL states in trapped ion spin chains*

12:30PM LUNCH BREAK

Long range interaction + disorder

$$H = \sum_{ij} J_{ij} \sigma_i^x \sigma_j^x + \sum_i V_i \sigma_i^z$$

random spin chain

random uncorrelated potential

$$\langle V_i V_j \rangle = \gamma \delta_{ij}$$

Short range *J* - Real space renormalization group	Dasgupta, Ma, Hu; 1979 D. Fisher, 1994
$J_{ij} \sim \frac{1}{\lvert i-j \rvert^a}$???	

Subject of studies on many-body localization

Here: single particle sector: one spin down, the rest of the spins up

$$H = \sum_{ij} J_{ij} a_i^\dagger a_j + \sum_i V_i a_i^\dagger a_i$$

Numerical study of the 1D chain

$$H = \sum_{nm} \frac{i\,\mathrm{sign}\,(n-m)}{|n-m|^{1+\alpha}} a_n^\dagger a_m + \sum_n V_n a_n^\dagger a_n$$

random uncorrelated potential

$$H_0 = |k|^\alpha \mathrm{sign}\, k$$

$$\rho_0(E) = \int \frac{dk}{2\pi}\, \delta\left(E - |k|^\alpha \mathrm{sign}\, k\right) \sim |E|^{\frac{1}{\alpha}-1}$$

$$\alpha < \frac{d}{2} = \frac{1}{2} \qquad d = 1$$

M. Gärttner, S. Syzranov,
A.-M. Rey, VG, L. Radzihovsky
(2015)

$$E_c = 0$$

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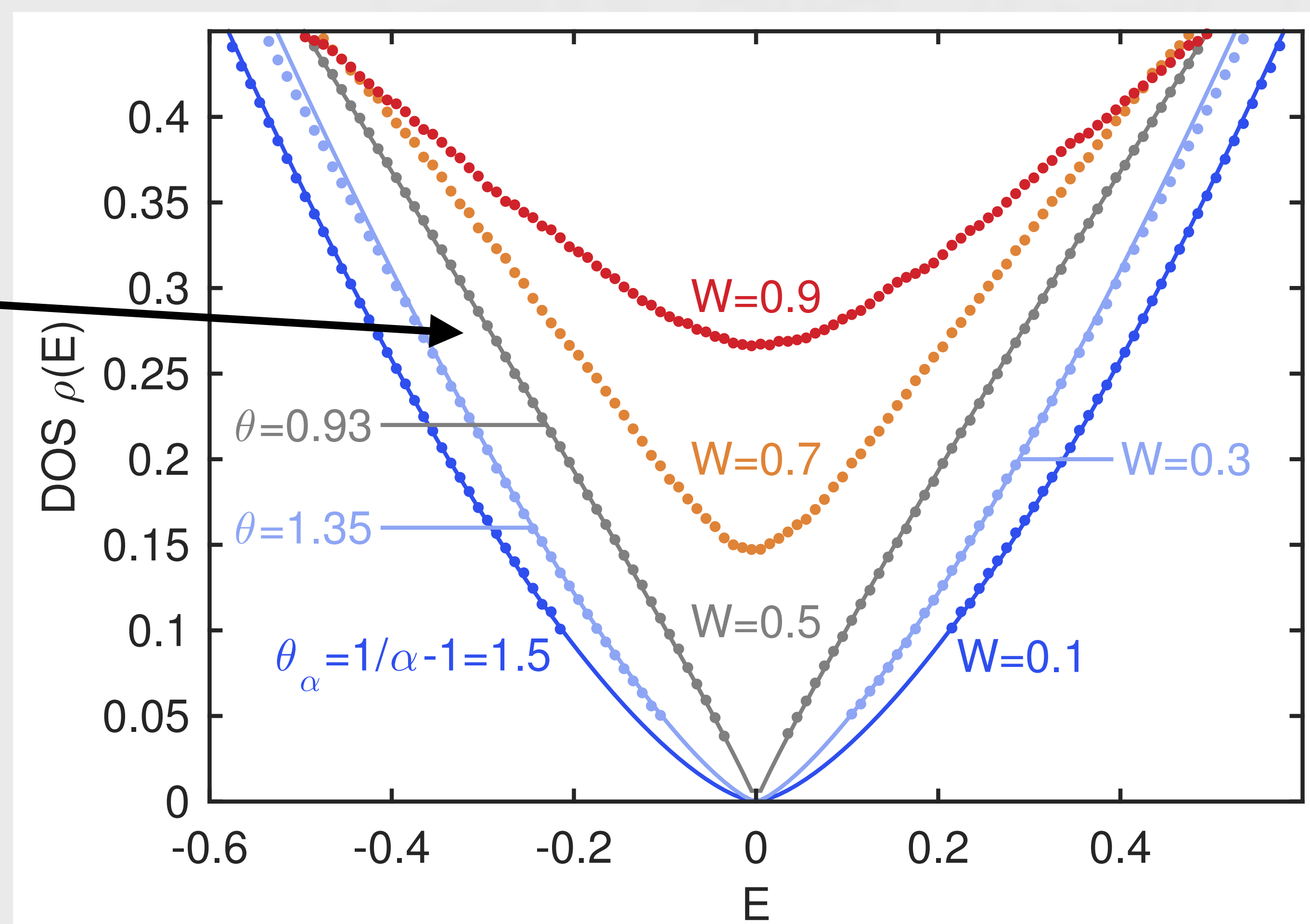
$$\alpha < \frac{d}{2} = \frac{1}{2} \qquad d = 1$$

$$\rho \sim E^{\frac{d}{z}-1}$$

$$z \approx \frac{d}{2}$$



$$\alpha = \frac{2}{5}$$

$$\rho(E) \sim E^\theta$$

$$E_c = 0$$

M. Gärttner, S. Syzranov, A.-M. Rey, VG, L. Radzihovsky (2015)

Potential rounding off of the DoS

A possibility was raised that the singularity in the DoS is actually rounded off.

This could be the effect of non-zero DoS.

The analysis was based on vanishing DoS

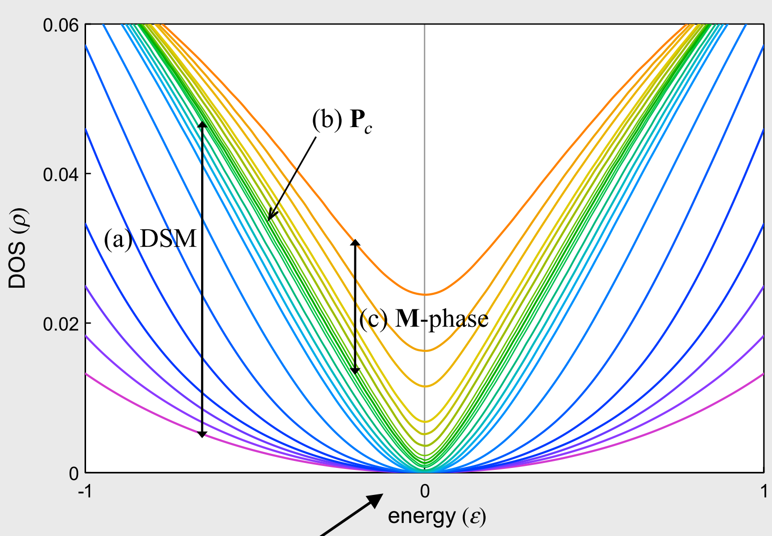
$$\rho_{\text{clean}}(E) \sim E^{\frac{d}{\alpha}-1} \qquad \frac{d}{\alpha} - 1 > 1$$

However, exact DoS is not vanishing as $E \to 0$

Another argument is based on the mapping between α=2, d>4 problem and self-attracting random walks.
Self-attractive random walks are known not to have any transitions above 4D.
Oono (1976); Brydges, Slade (1995)

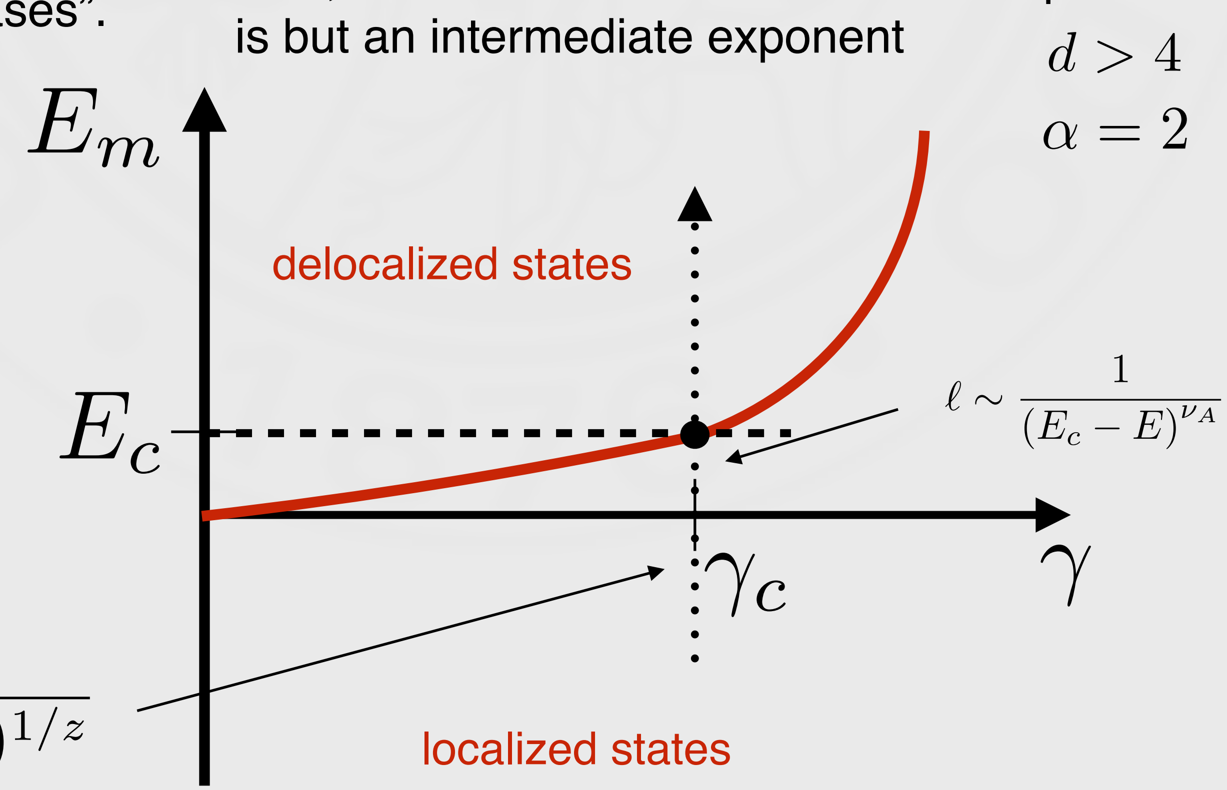
Yet another argument points out the absence of any order parameter to distinguish the weak disorder and strong disorder "phases".

If so, the new localization transition exponent is but an intermediate exponent



For example, DoS here may be rounded off

$$\ell \sim \frac{1}{(E_c - E)^{1/z}}$$



Whether this is so remains an open question

Conclusions and outlook

- New types of Anderson localization and critical behavior in higher dimensions

- Potential rounding off of the singularities and crossovers instead of transitions?

- Numerical test(s)?

- Experiments? Kicked rotor, Dirac materials, long range interacting ions arranged in 1D chains?

- Interactions?