

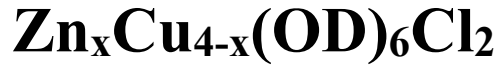
# Toward Microscopic Understanding of Two Frustrated Antiferromagnets

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University of Virginia

## Outline

- **Motivation: Quantum spin liquid states**
- **Quantum spin states in  $\text{Zn}_x\text{Cu}_{4-x}(\text{OD})_6\text{Cl}_2$**
- **Frustrated minority spins in  $\text{GeNi}_2\text{O}_4$**
- **Summary**

## Collaborators



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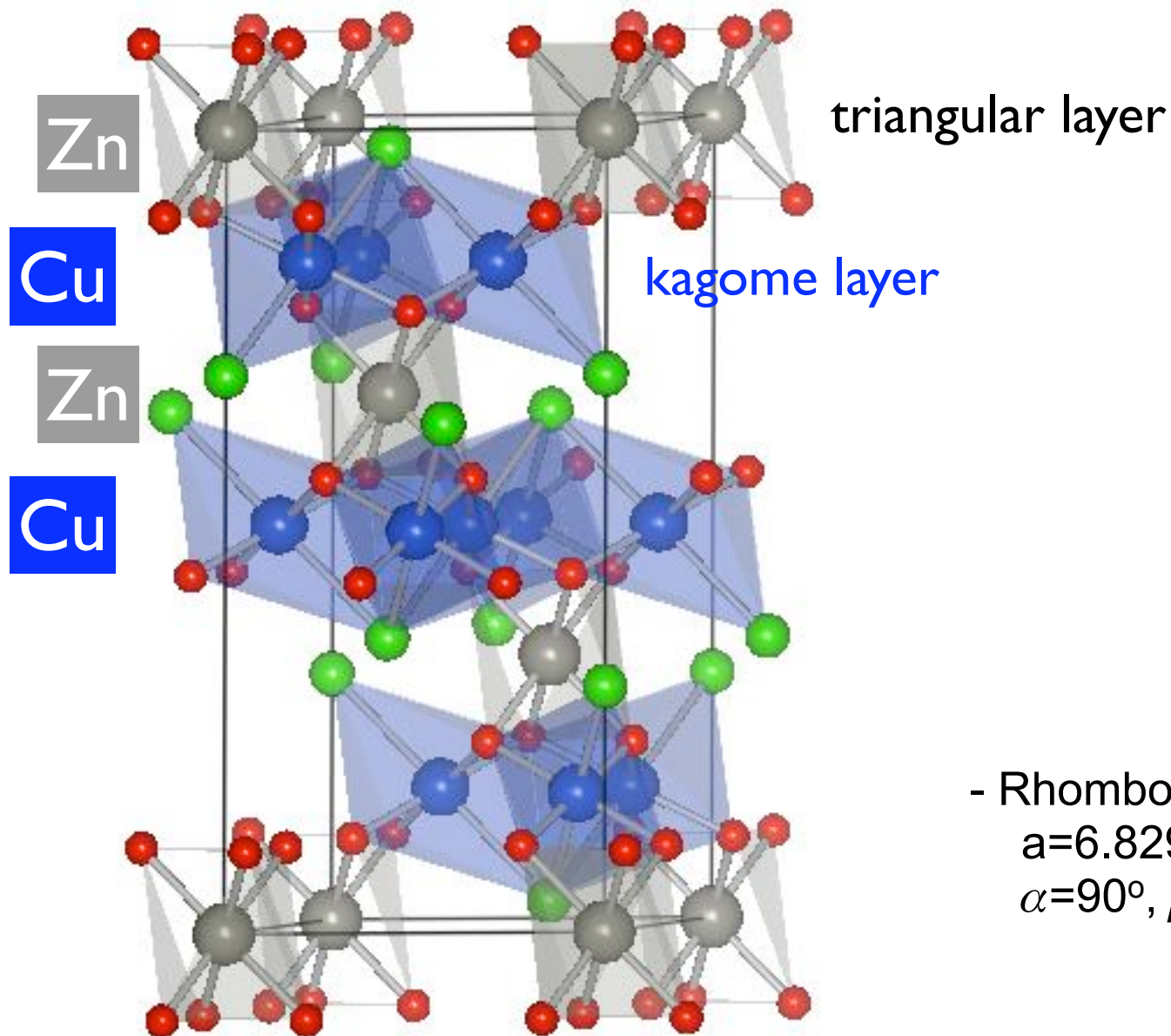
*C. L. Henley*, Cornell U

# **Quantum kagome antiferromagnet**



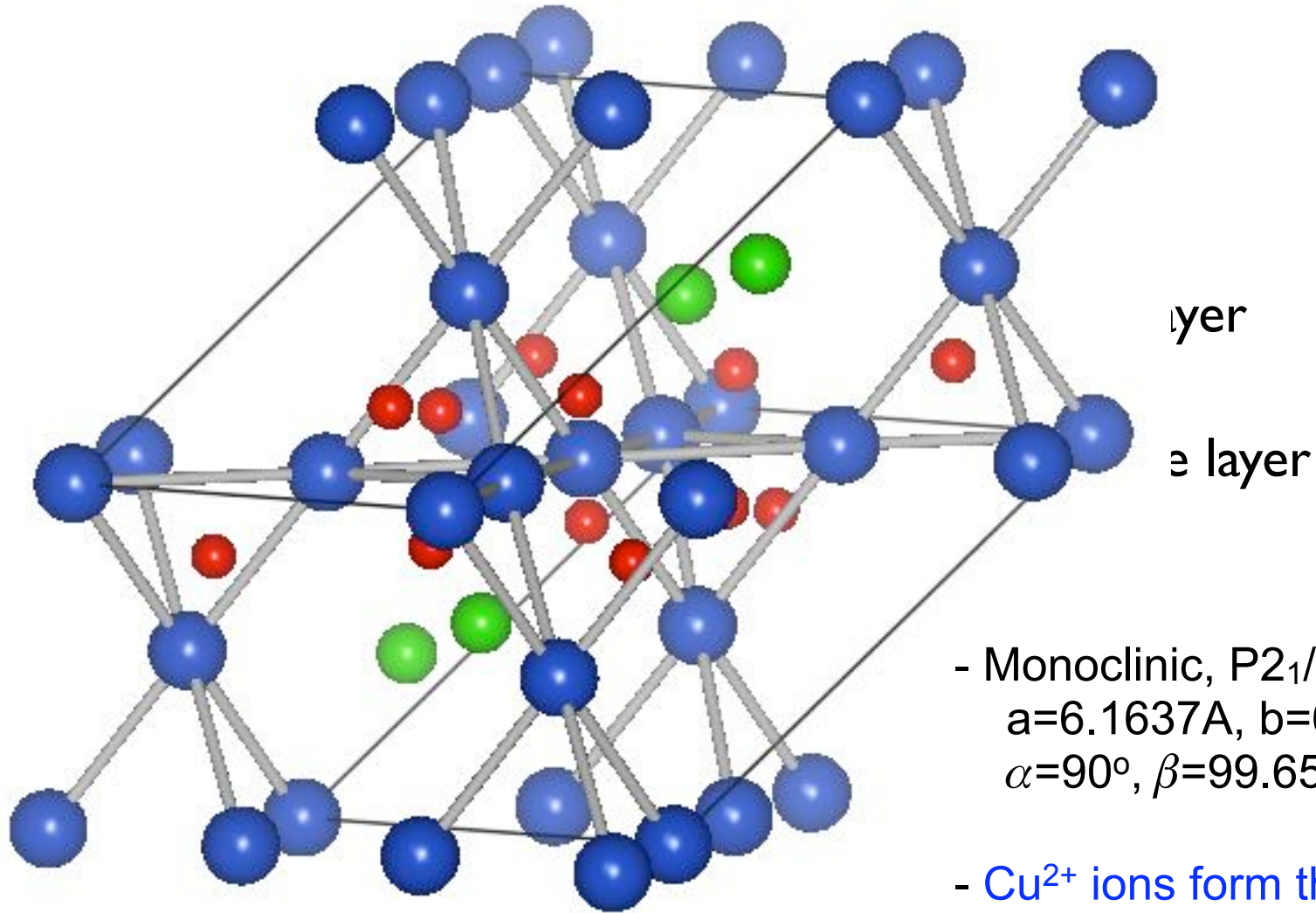
# ZnCu<sub>3</sub>(OD)<sub>6</sub>Cl<sub>2</sub>: A perfect kagome system???

M.P. Shores/G. Nocera *et al.*, JACS (2005)



- Rhombohedral, R-3m (#166),  
a=6.8293Å, b=6.8293, c=14.024Å  
 $\alpha=90^\circ$ ,  $\beta=90^\circ$ ,  $\gamma=120^\circ$

# $\text{Cu}_4(\text{OD})_6\text{Cl}_2$ : a pyrochlore system?

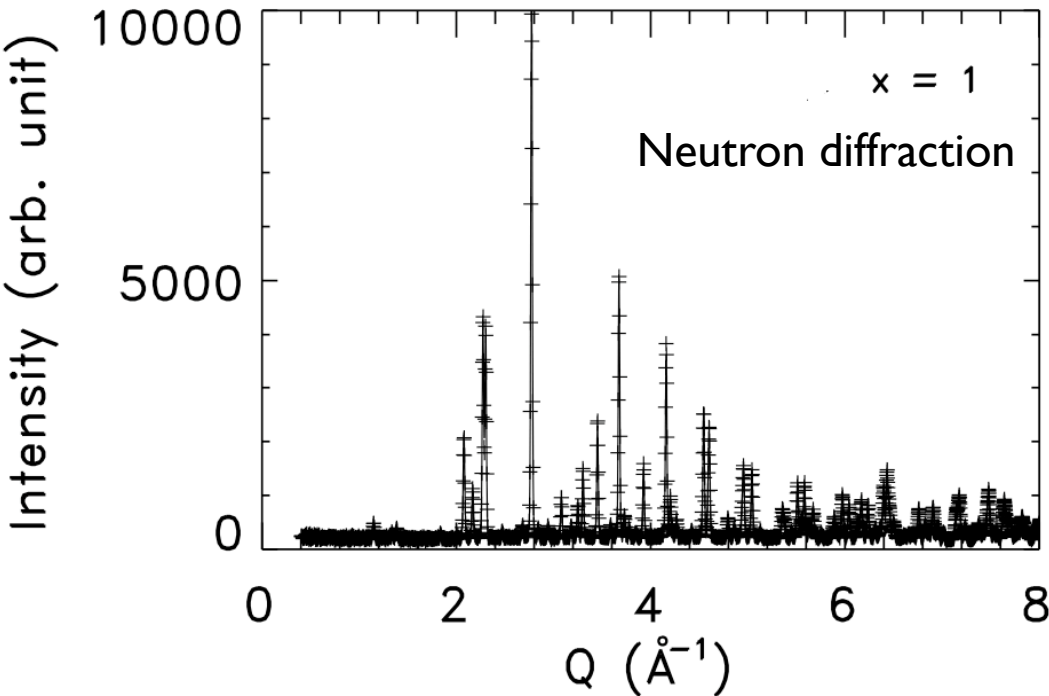


- Monoclinic,  $P2_1/n$  (#14),  
 $a=6.1637\text{\AA}$ ,  $b=6.8166$ ,  $c=9.114\text{\AA}$   
 $\alpha=90^\circ$ ,  $\beta=99.6515^\circ$ ,  $\gamma=90^\circ$

- $\text{Cu}^{2+}$  ions form the pyrochlore lattice.

# ZnCu<sub>4</sub>(OD)<sub>6</sub>Cl<sub>2</sub>: Crystal Structure

S.-H. Lee *et al.*, Nature Materials (2007)  
supplementary information

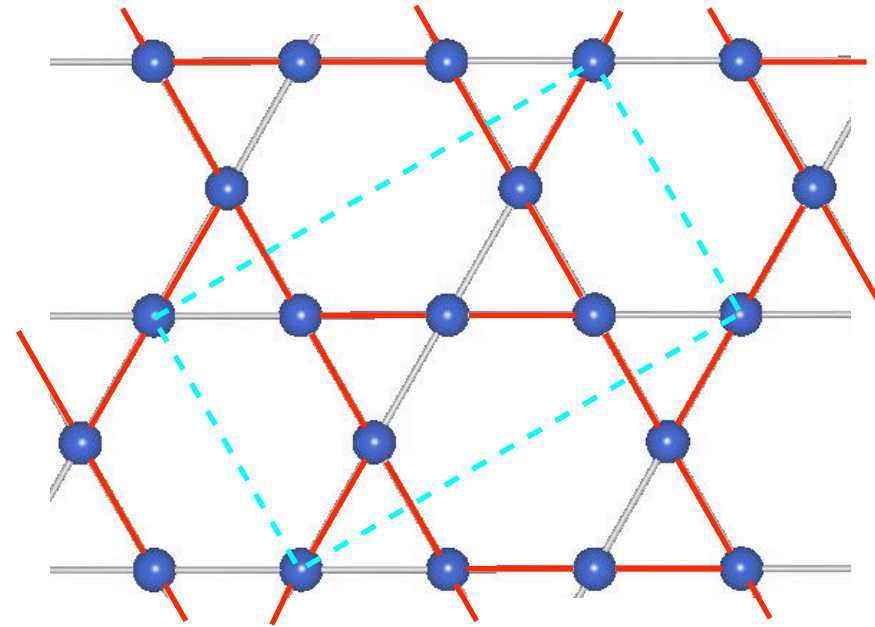
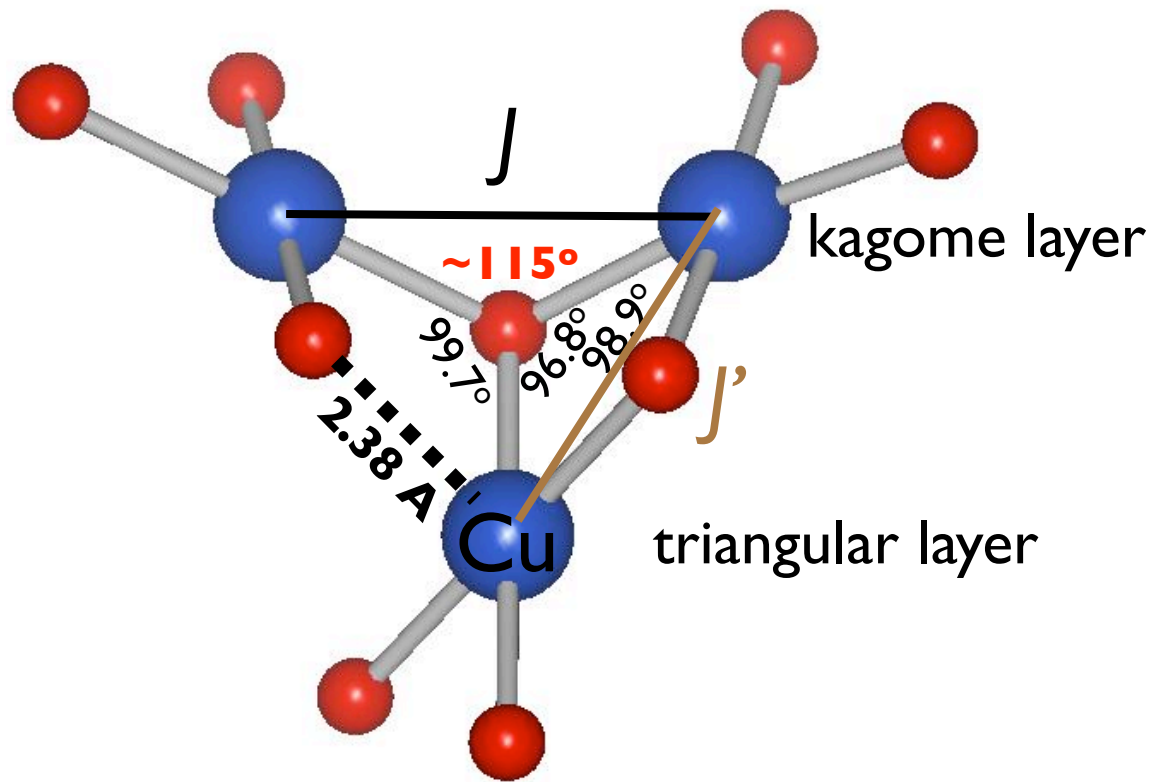


## Rhombohedral, R-3m

Atom	Site	fraction	x	y	z
Cu1	3b	0.3723	0	0	1/2
Cu2	9e	0.8927			
Zn1	3b	0.6277			
Zn2	9e	0.1073			
Cl	6c	1	0	0	0.195773
O	36i	1	0.206226	0.412451	0.061618
D	36i	0.9935	0.131562	0.263119	0.090799

Kagome lattice,  
magnetically 90% filled

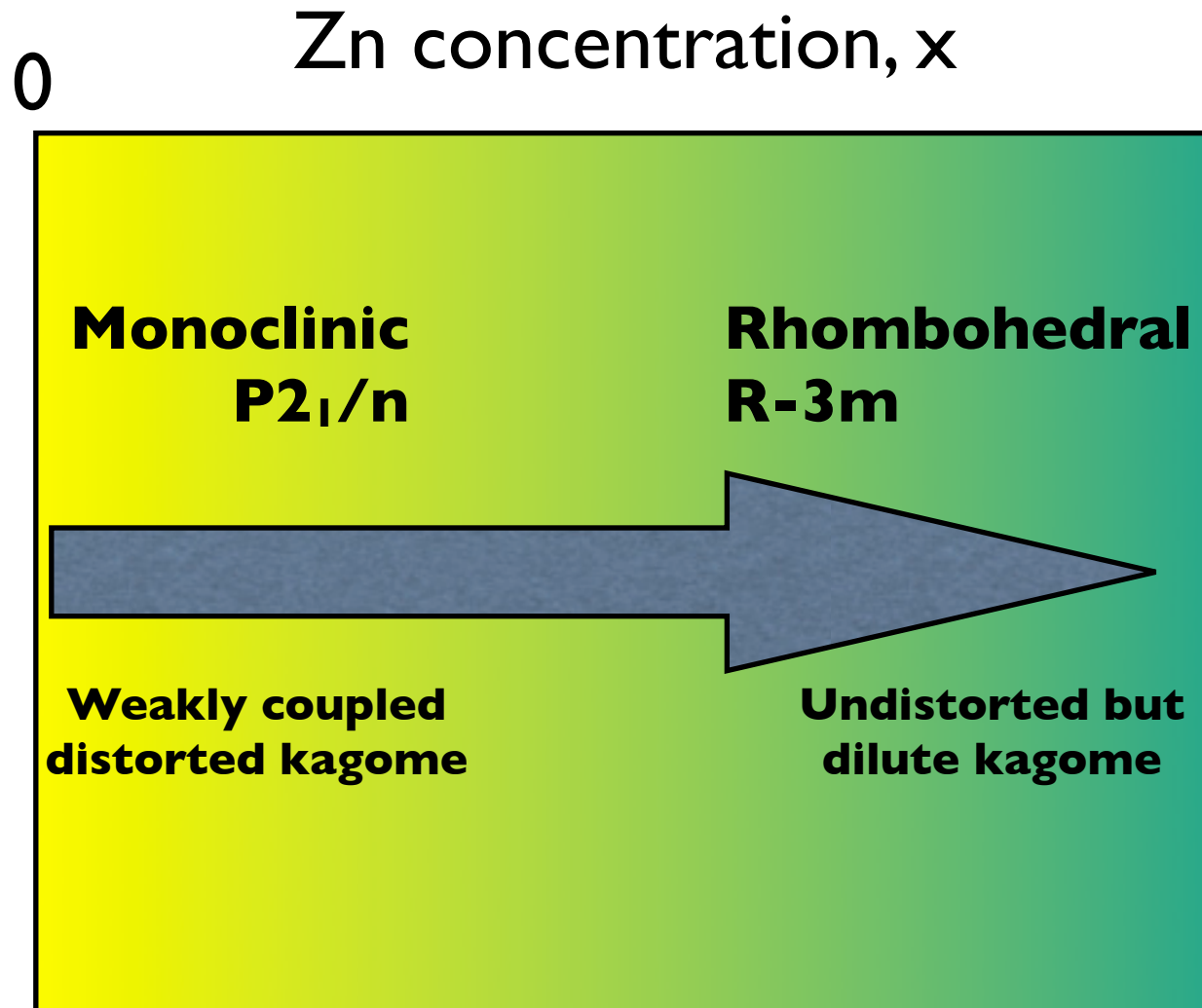
# $\text{Cu}_4(\text{OD})_6\text{Cl}_2$ : superexchange paths



- Jahn-Teller distortion around the doped  $\text{Cu}^{2+}$  ion
- Bond angle of  $\text{Cu}^{2+}-\text{O}^{2-}-\text{Cu}^{2+} > 110^\circ$  between kagome spins:  
strong AFM  $J$  in the kagome layer
- Bond angles of  $\text{Cu}^{2+}-\text{O}^{2-}-\text{Cu}^{2+} < 100^\circ$  between kagome and triangular spins:  
**weak** coupling between kagome and triangular layers
- In kagome layer, two bonds with different bond angles and lengths

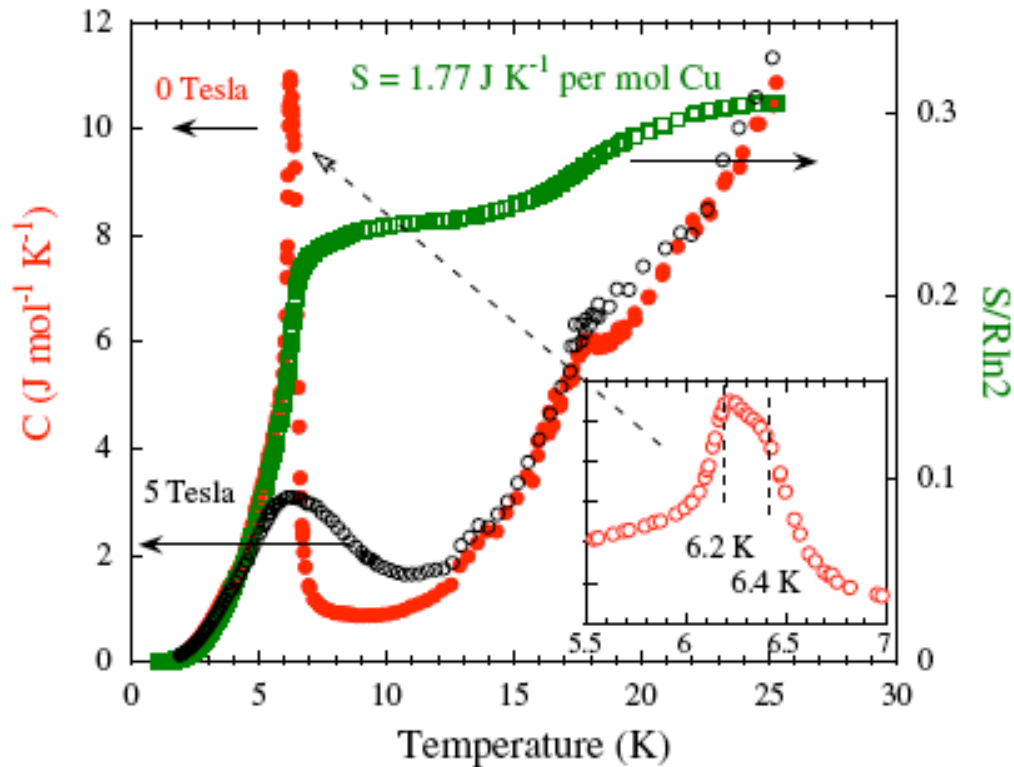
Weakly coupled **distorted** kagome system

# Phase Diagram of $\text{Zn}_x\text{Cu}_{4-x}(\text{OD})_6\text{Cl}_2$





# $\text{Cu}_4(\text{OD})_6\text{Cl}_2$ : Phase Transitions at Low Temperatures

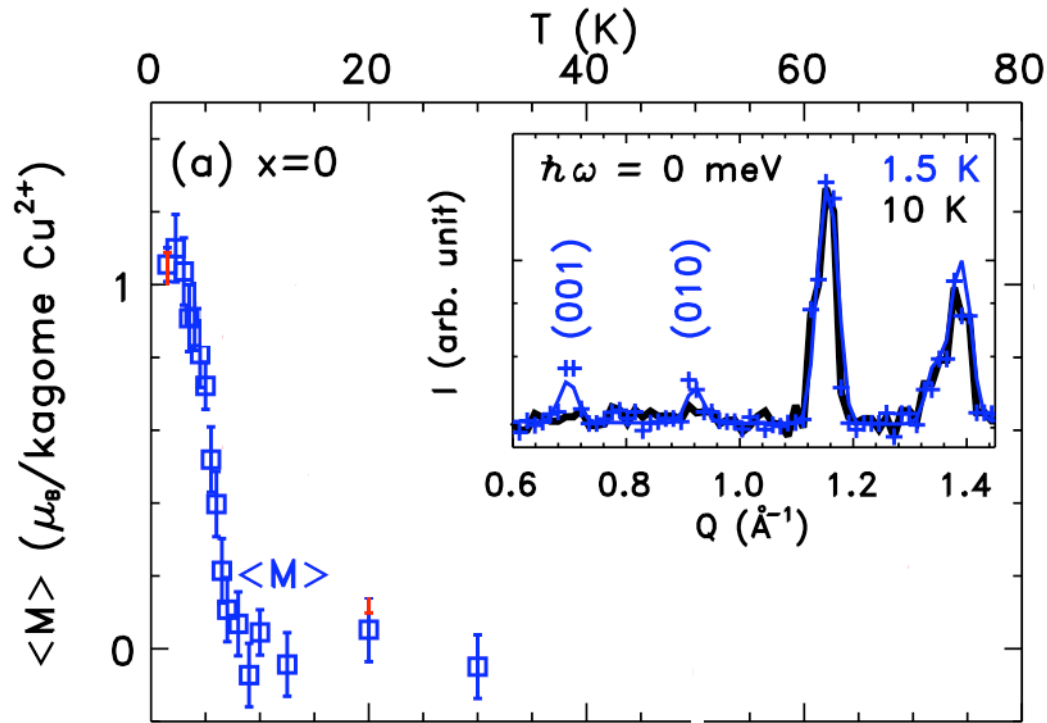


Zheng *et al.*, PRL (2005)

- Two transitions at  $\sim 18$  K and  $\sim 6$  K.
- From  $\mu\text{SR}$  data, they interpreted the 18K transition involves a long range order.
- At 6K, it transits to a metastable state with strong fluctuations.
- But.. the entropy released at 18 K is only  $0.05 R\ln 2/\text{Cu}$ , surprisingly small for a Neel ordering ( $S R\ln 2/\text{Cu}$ ).
- The large amount of entropy is released at 6 K.

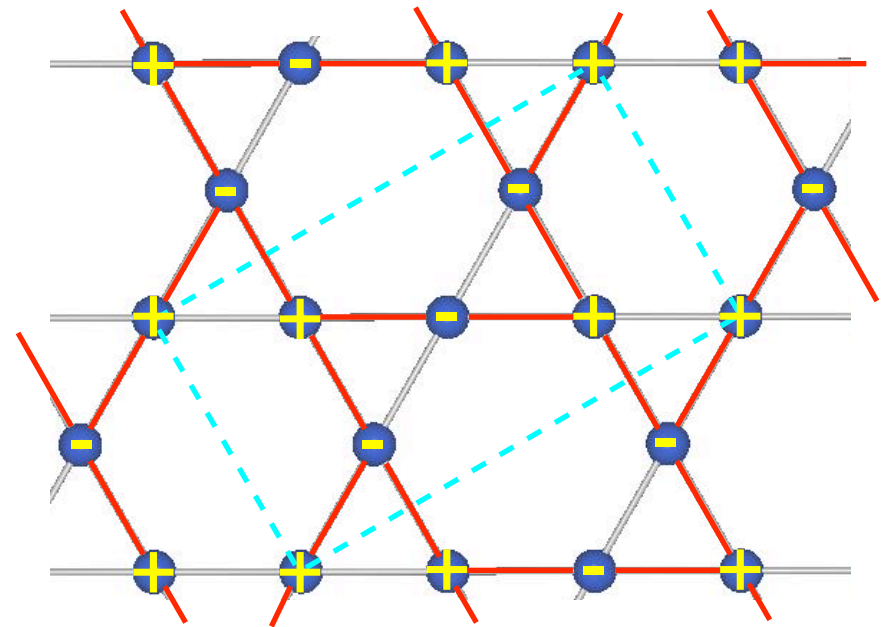
# $\text{Cu}_4(\text{OD})_6\text{Cl}_2$ : Elastic neutron scattering and Neel state

S.-H. Lee *et al.*, Nature Materials (2007)



- Neel ordering occurs at  $\sim 7$  K.
- $Q_M = (001)$ .
- consistent with the specific heat data that showed the large amount of entropy was released around 6K.

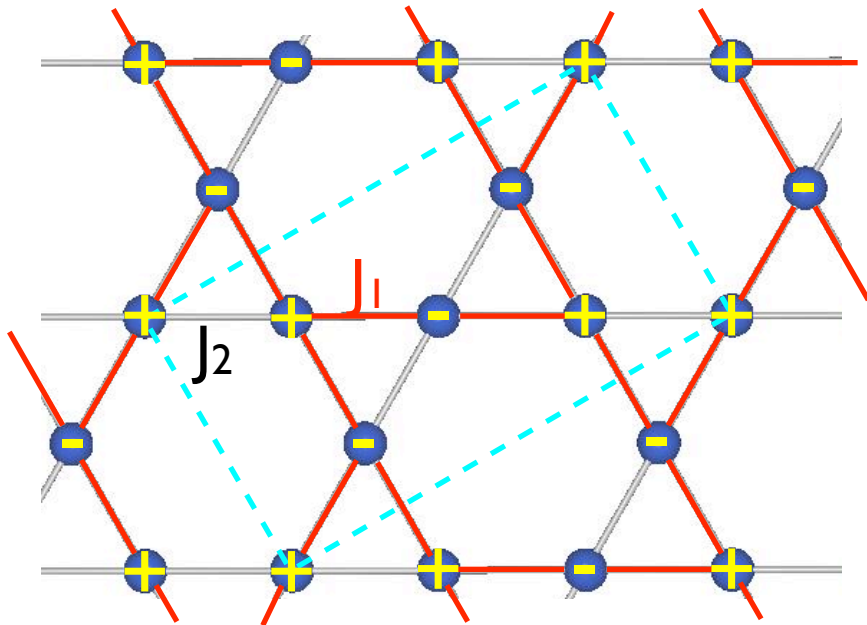
- The Neel state is collinear, rather than 120 degree configuration.



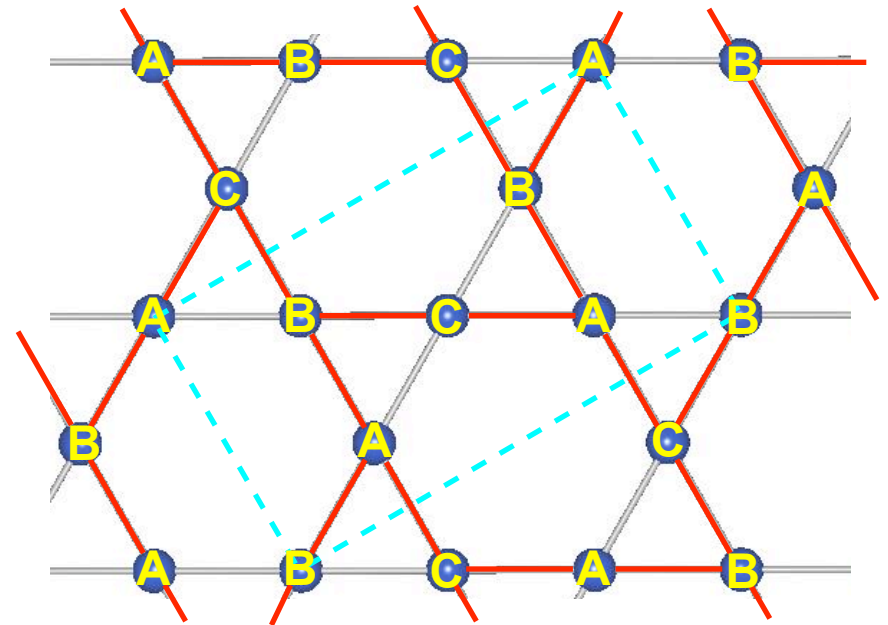
Collinear

vs

120° configuration



$$E(\text{coll}) = 4 J_2 - 8 J_1$$

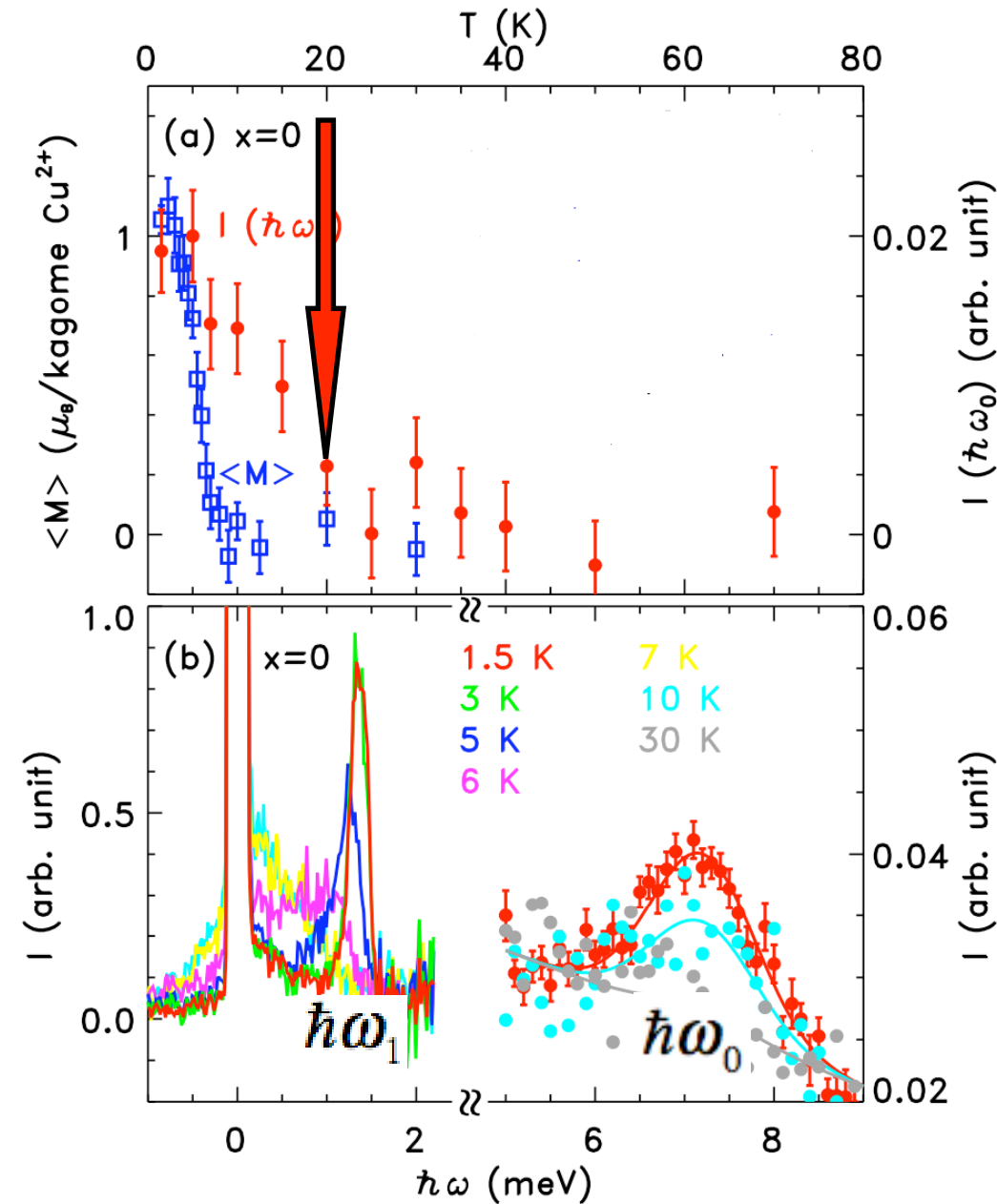


$$E(120^\circ) = -2 J_2 - 4 J_1$$

$E(\text{coll}) < E(120^\circ) \Rightarrow J_2 < 2/3 J_1$
---

- If we take Goodenough's prediction,  $J \sim \cos^4\theta/d^7$ ,  $J_2 \sim 1/3 J_1$  for  $\text{Cu}_4(\text{OD})_6\text{Cl}_2$ .

# Cu<sub>4</sub>(OD)<sub>6</sub>Cl<sub>2</sub> : Phase Transitions at Low Temperatures

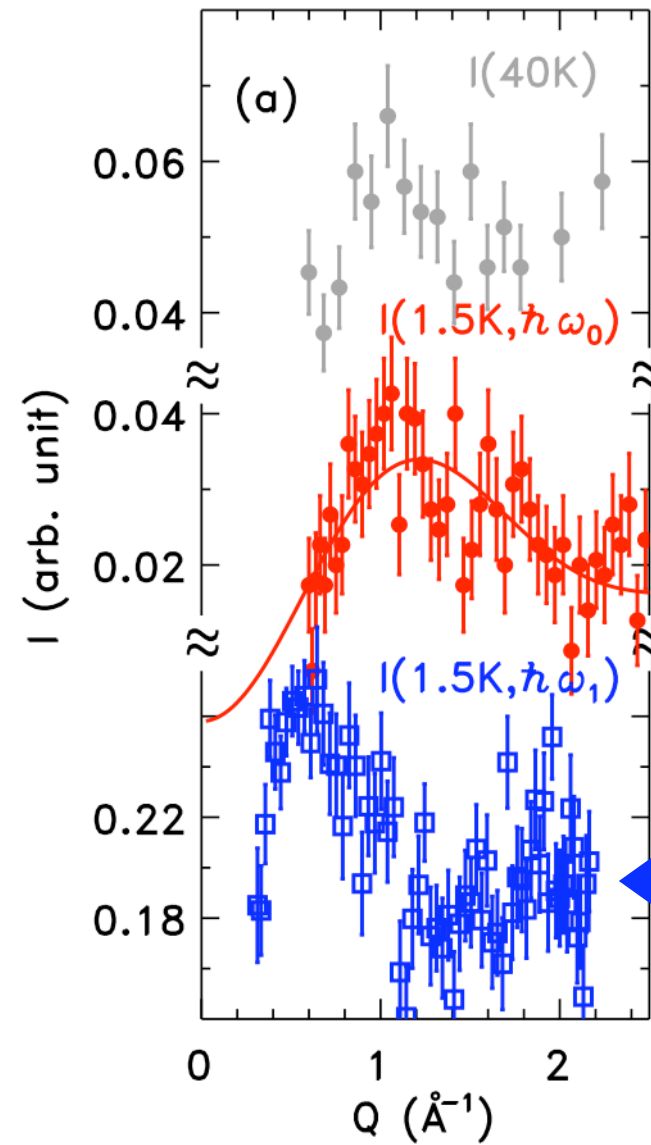


- The 7 meV mode survives well above 7K, gradually diminishes around 20K, coincides with the 18K transition in the Cv.
- The energy of the 7 meV mode does not change with T

- Two excitations at 1.5K at 1.3 meV and 7 meV.
- With warming, the 1.3meV mode shifts to lower energies and above the Neel transition, it becomes quasielastic continuum.

# Cu<sub>4</sub>(OD)<sub>6</sub>Cl<sub>2</sub> : Q-dependence of the magnetic fluctuations

x=0



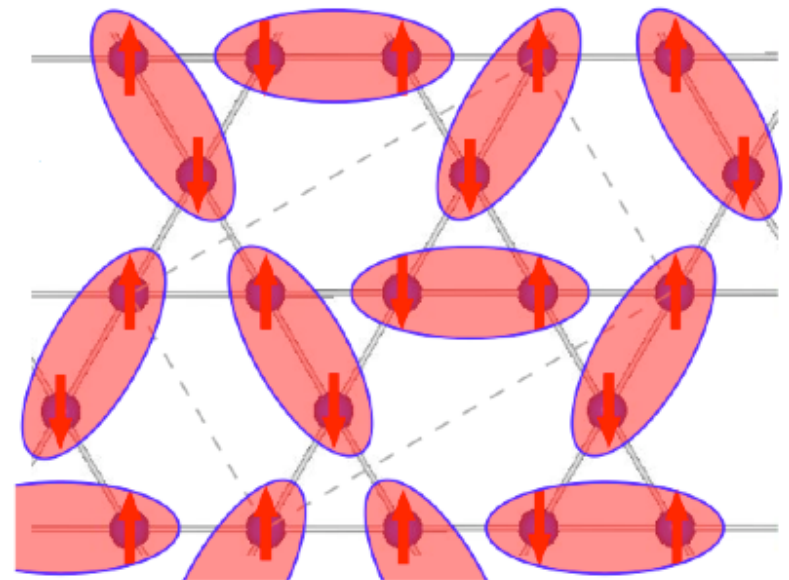
- The two excitations also have different Q dependence

$\hbar\omega_0$  mode

← Spin dimers:  $1 - \frac{\sin(Qr_0)}{Qr_0}$  with  $r_0 = 3.41 \text{ \AA}$

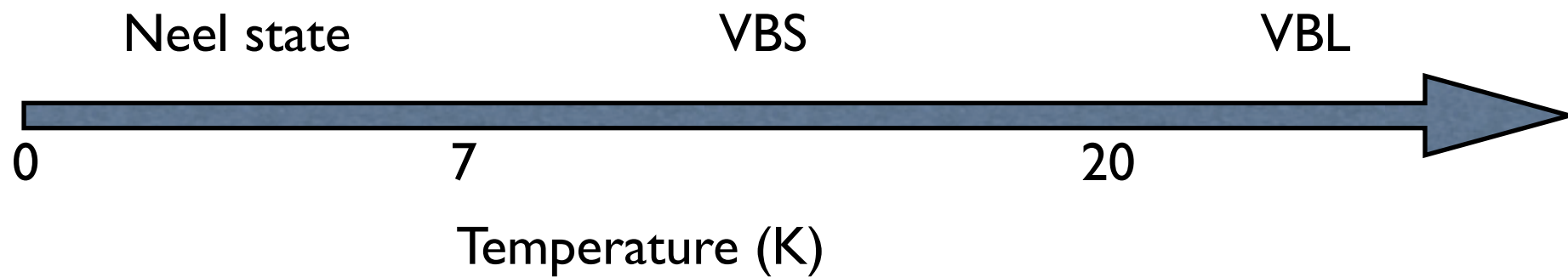
$\hbar\omega_1$  mode

← Spin waves

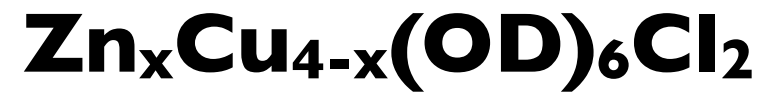


# Summary on $\text{Cu}_4(\text{OD})_6\text{Cl}_2$

- Specific heat: two transitions at  $\sim 18$  K and  $\sim 6$  K.
- Their interpretation of the  $\mu\text{SR}$  data:
  - The 18K transition: LRO
  - The 6K transition is to a metastable state with strong fluctuations.
- But.. the entropy released at 18 K is only  $0.05 R \ln 2 / \text{Cu} \ll S R \ln 2 / \text{Cu}$ .
- The large amount of entropy is released at 6 K.
  
- Elastic neutron scattering:
  - LRO occurs below  $T_N \sim 7\text{K}$ .
  - No evidence for another LRO above  $T_N$ .
- Inelastic neutron scattering:
  - The energy of spin waves shifts from 1.3 meV at 1.5K to lower energies as  $T$  increases to  $T_N$ , and it becomes a gapless continuum.
  - The 2nd excitation mode at 7 meV gradually weakens and disappears  $\sim 20\text{K}$ .
  - The energy of the 7 meV mode does not shift with increasing  $T$ .
  - $S(Q, \hbar\omega=7\text{meV})$  resembles that of spin dimers.
  - Above 20K,  $S(Q)$  at low energies resembles that of spin dimers.

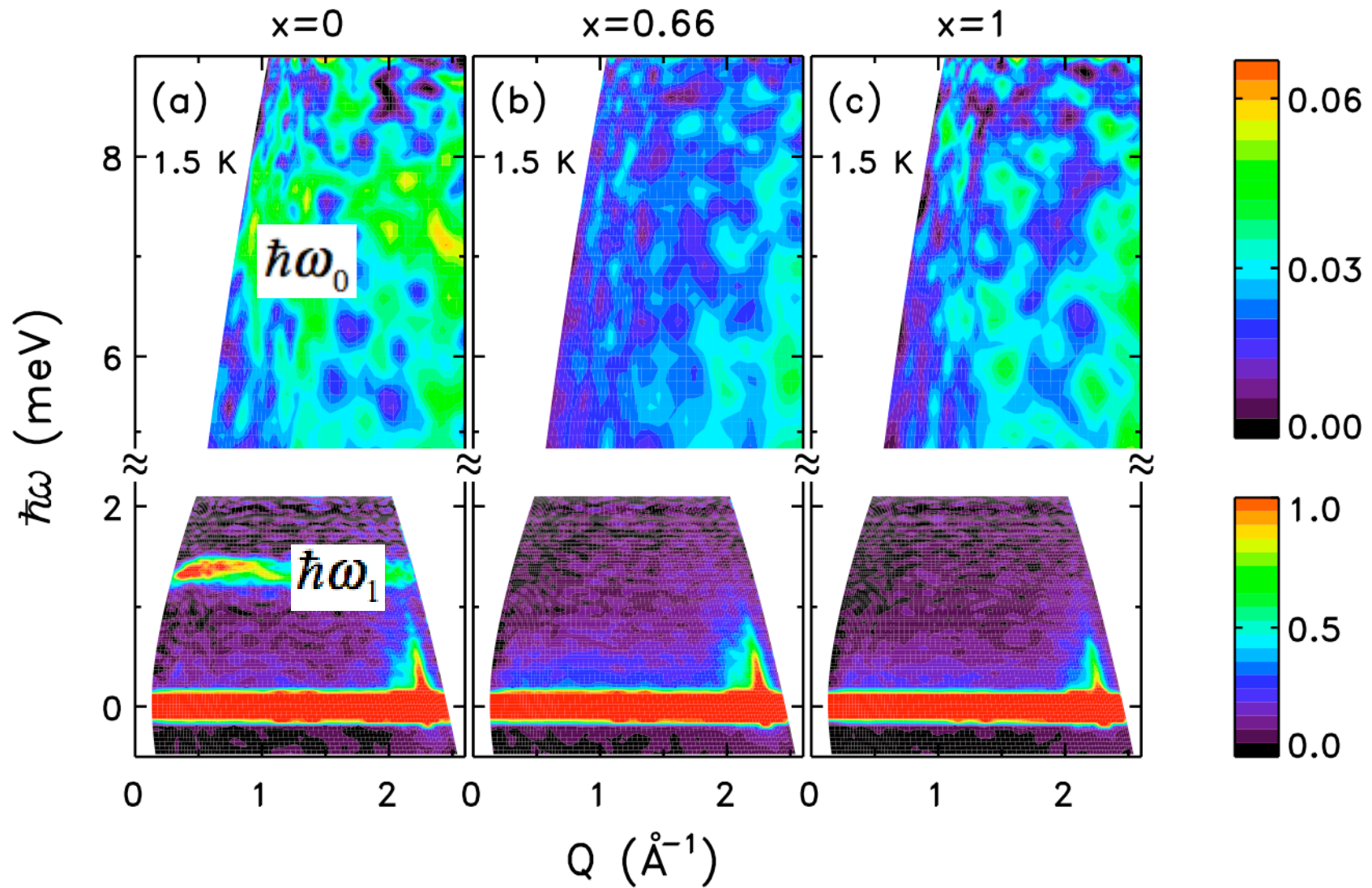


**Zn doping effects on**

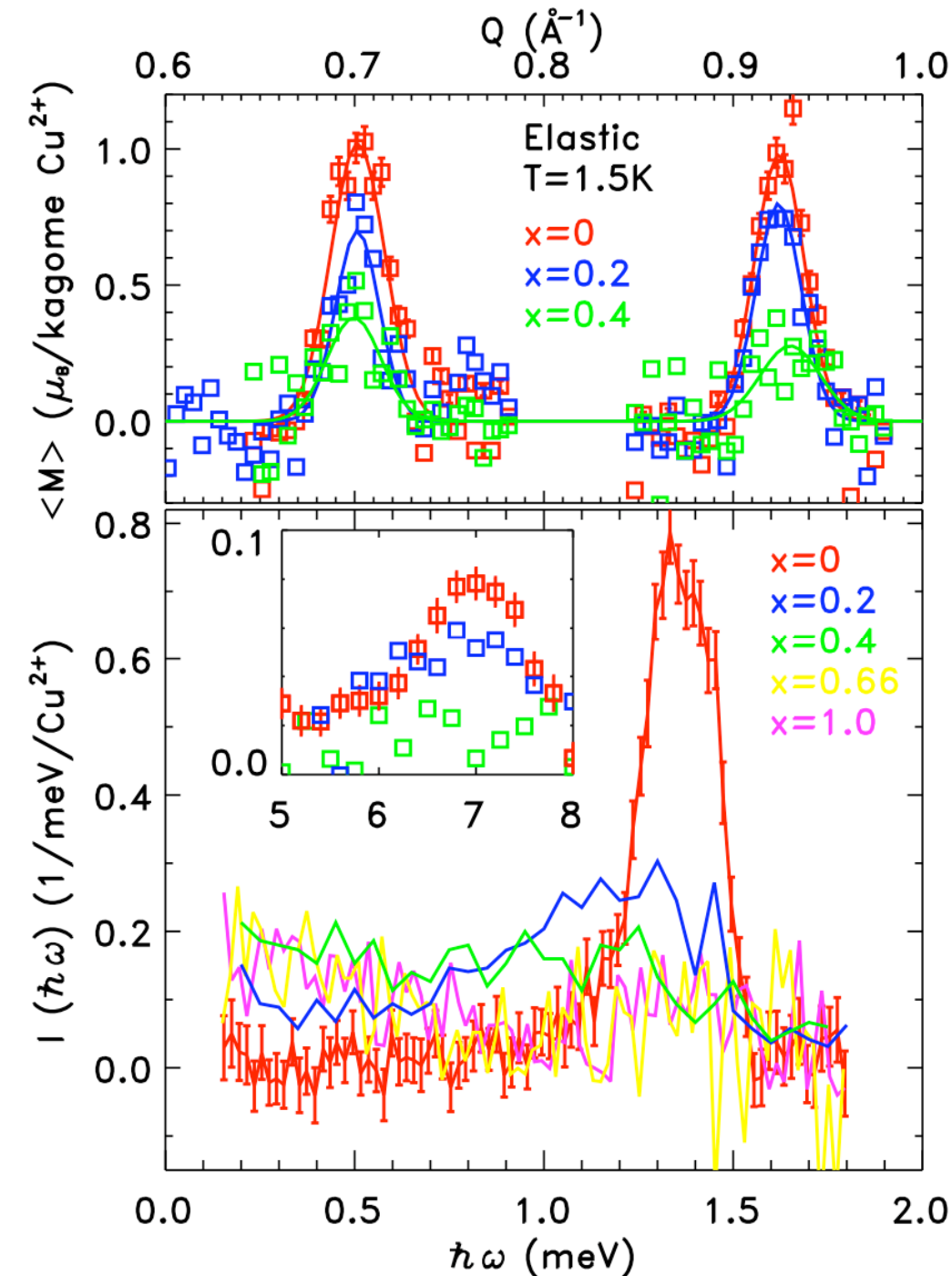




# $\text{Zn}_x\text{Cu}_{4-x}(\text{OD})_6\text{Cl}_2$ : Spin Fluctuations with $x$



# $\text{Zn}_x\text{Cu}_{4-x}(\text{OD})_6\text{Cl}_2$ : Spin correlations vs $x$



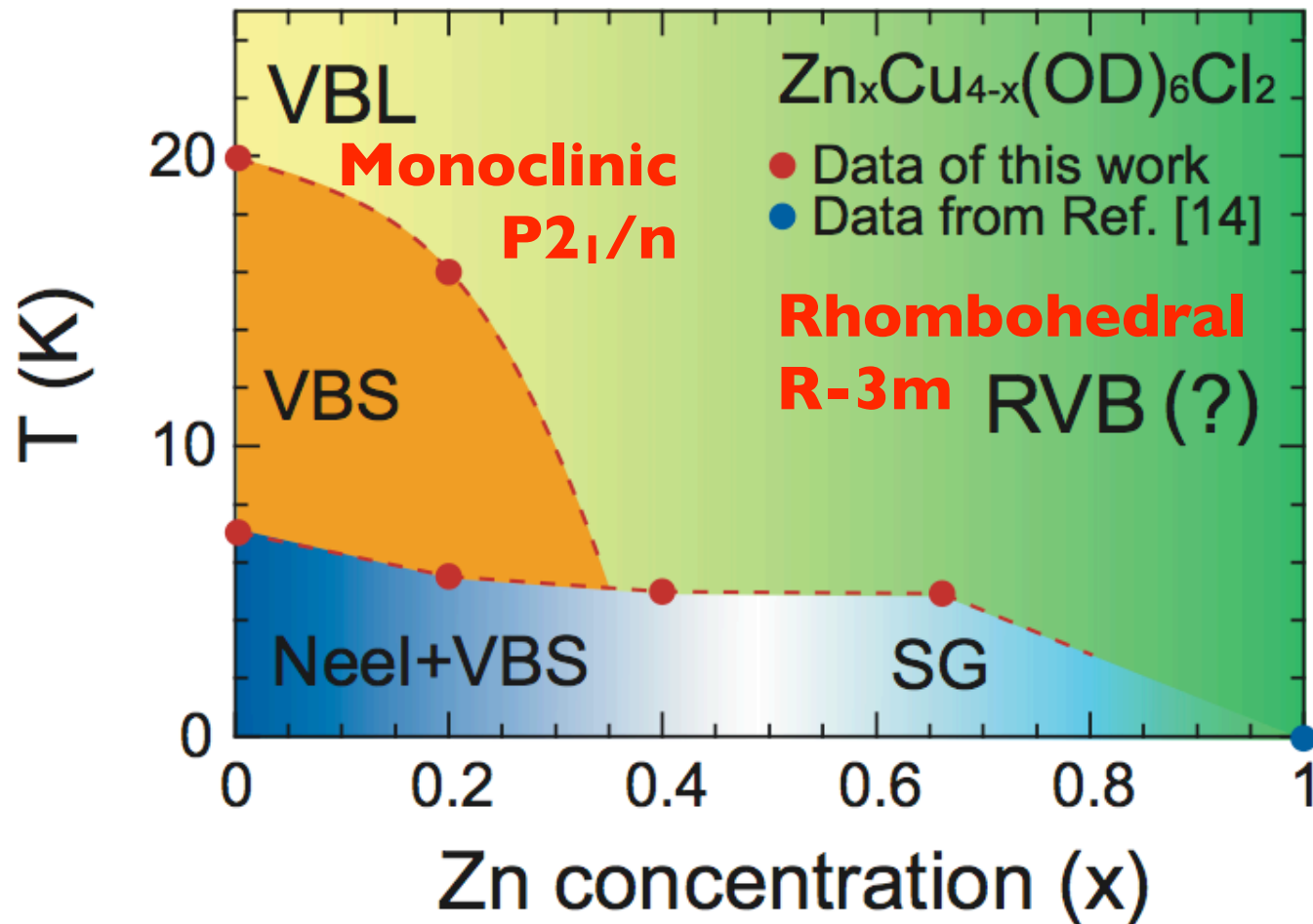
## **Elastic scattering at 1.5 K**

- The Neel state weakens upon Zn doping.
- It disappears just above  $x = 0.4$ .

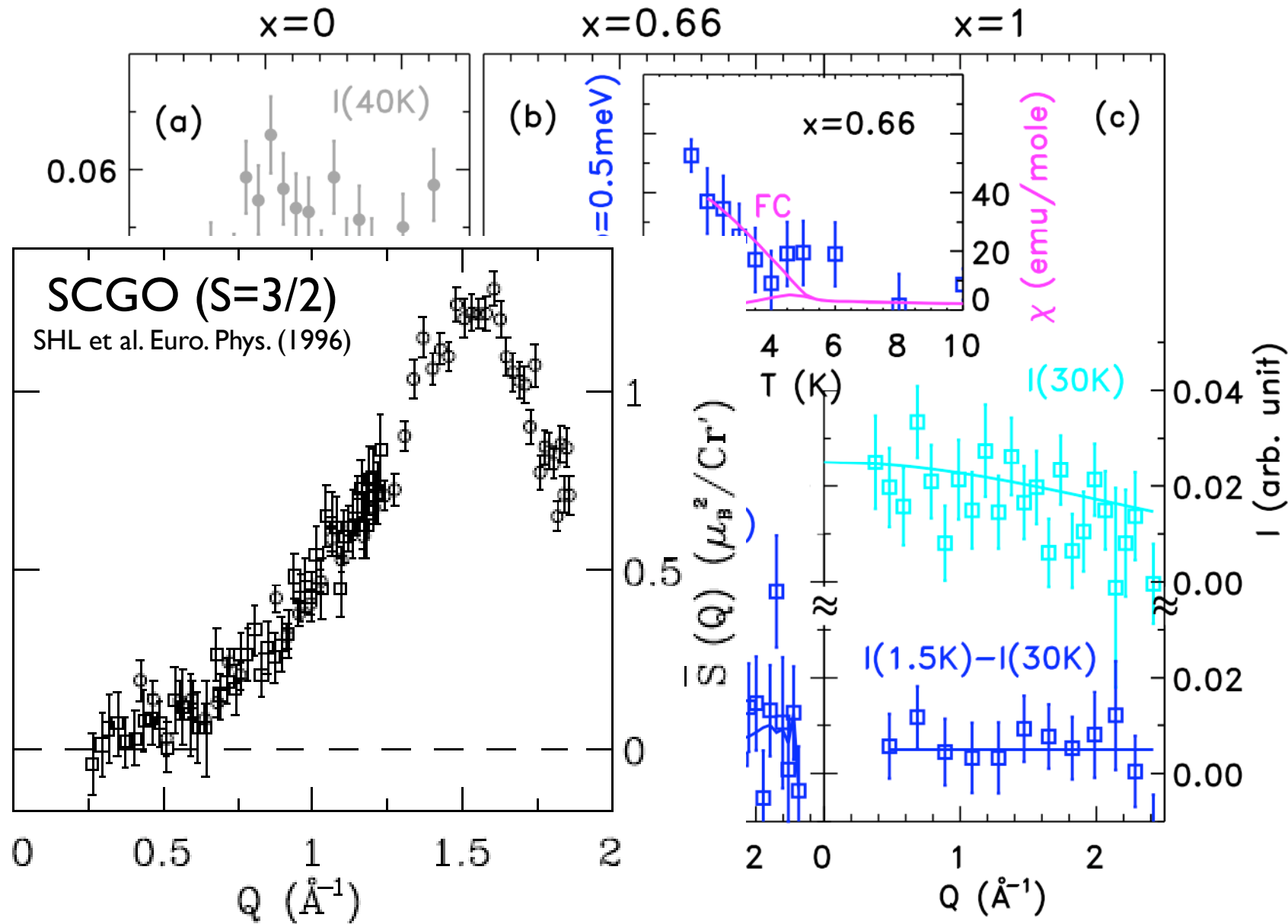
## **Inelastic scattering at 1.5 K**

- The  $\hbar\omega_0 = 7$  meV mode disappears at  $x \sim 0.4$ .
- The  $\hbar\omega_1 = 1.3$  meV mode weakens and shifts to low energies with  $x$ .
- The energy excitation spectrum becomes a featureless continuum for  $x > 0.4$ .

# Summary: Phase Diagram of $\text{Zn}_x\text{Cu}_{4-x}(\text{OD})_6\text{Cl}_2$

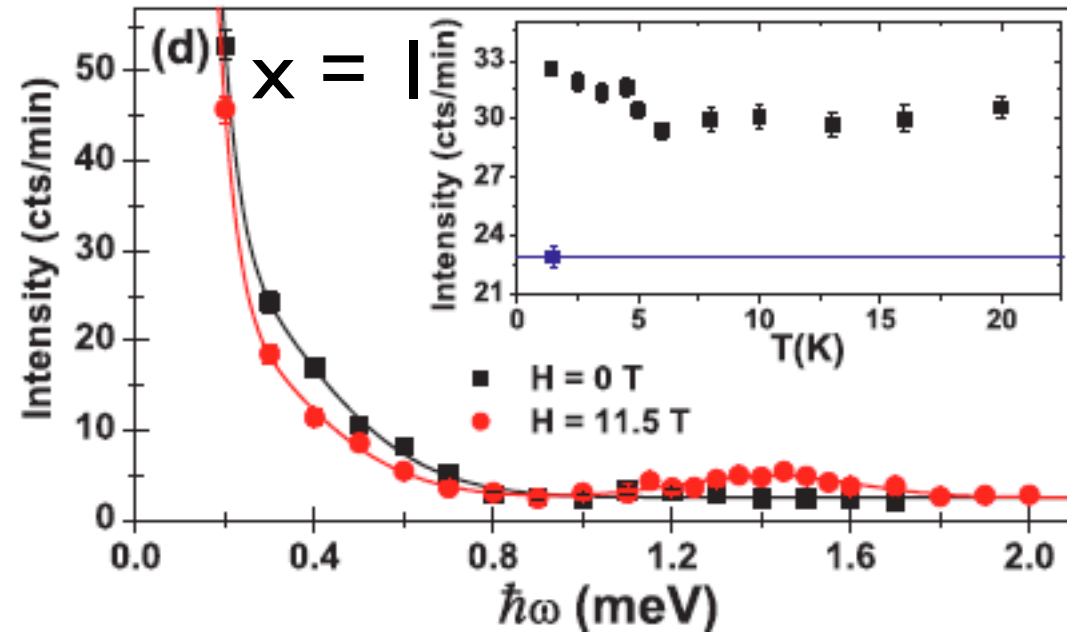


# Zn<sub>x</sub>Cu<sub>4-x</sub>(OD)<sub>6</sub>Cl<sub>2</sub>: Spin Fluctuations vs x

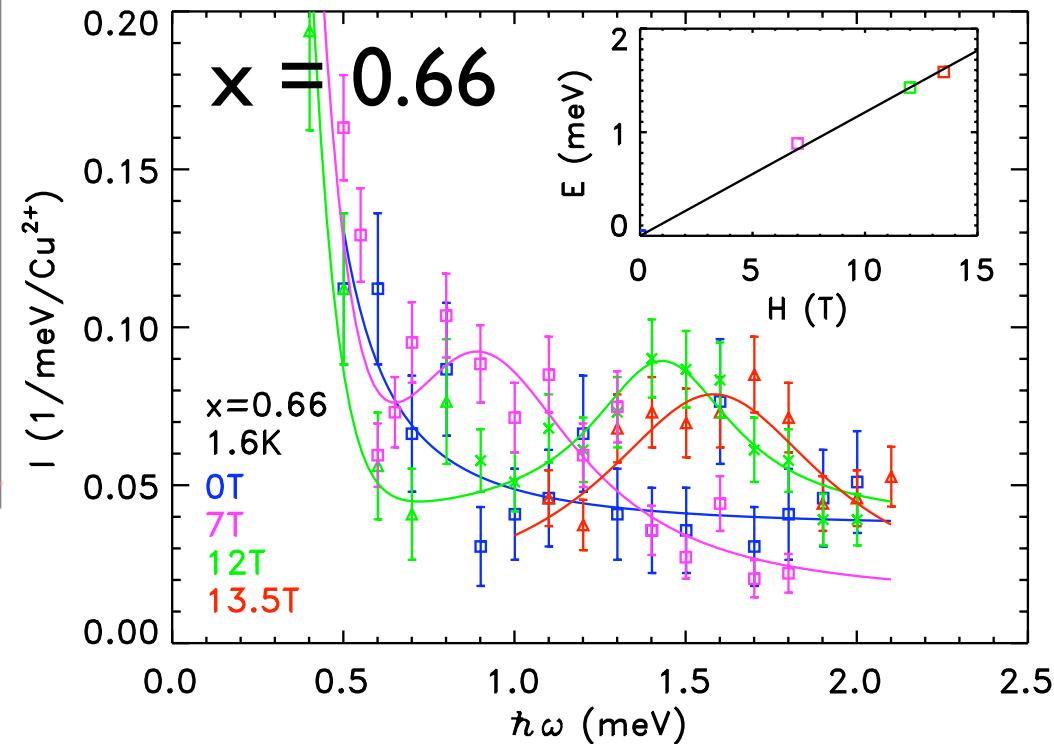


# $Zn_xCu_{4-x}(OD)_6Cl_2$ ( $x > 0.66$ ): Magnetic field effects on spin fluctuations

J.S. Helton/Y. Lee *et al.*, PRL (2007)



S.-H. Lee *et al.*, Nature Materials (2007)



$$E \text{ (meV)} = 0.12(3)H \text{ (T)} \approx g\mu_B H$$

No evidence for any field induced static ordering.

# ZnCu<sub>3</sub>(OD)<sub>6</sub>Cl<sub>2</sub>: Characteristics of Spin Fluctuations

- Featureless in energy and in momentum
- Under H,  $E$  (meV) =  $gu_B H$
- For  $x = 1$ , 10% of Cu<sup>2+</sup> ions at triangular sites.
- Are those free spins responsible for the spin fluctuations in the  $x = 1$  system?

- Assuming that all Cu<sup>2+</sup> ions contribute to the observed intensity,

$$\int_{0.2 \text{ meV}}^{1.7 \text{ meV}} I(\hbar\omega) d(\hbar\omega) = 0.11(3) / \text{Cu}^{2+}$$

whereas if only the 10% free spins contribute,

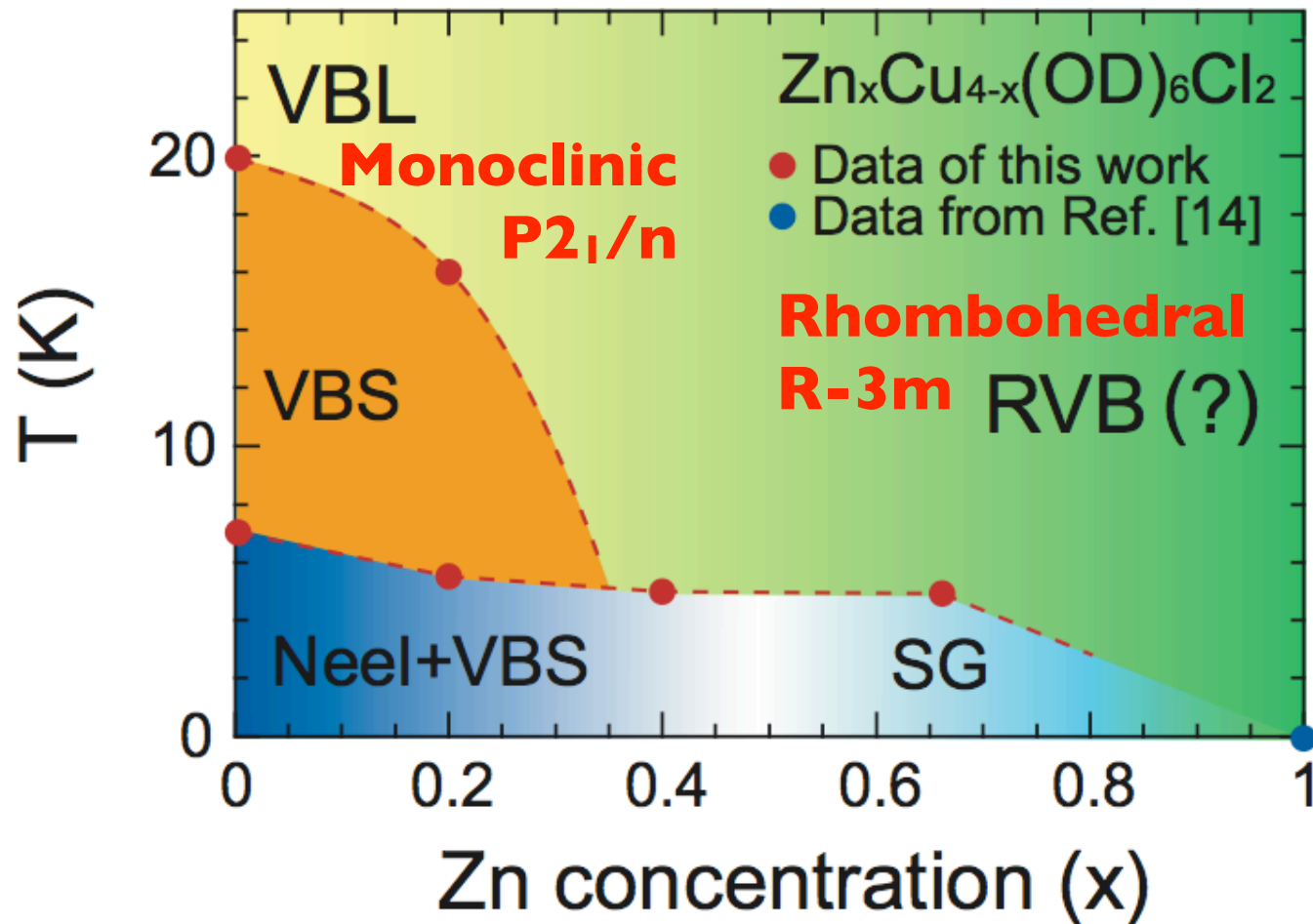
$$\int_{0.2 \text{ meV}}^{1.7 \text{ meV}} I(\hbar\omega) d(\hbar\omega) = 1.1(3) / \text{Cu}^{2+}$$

$$> S(S+1) / \text{Cu}^{2+} = 0.75 / \text{Cu}^{2+}$$

- As  $x$  increases, the spin fluctuations gradually shifts from the collective spin waves of  $x = 1$  to the low energy continuum of  $x = 0$ .

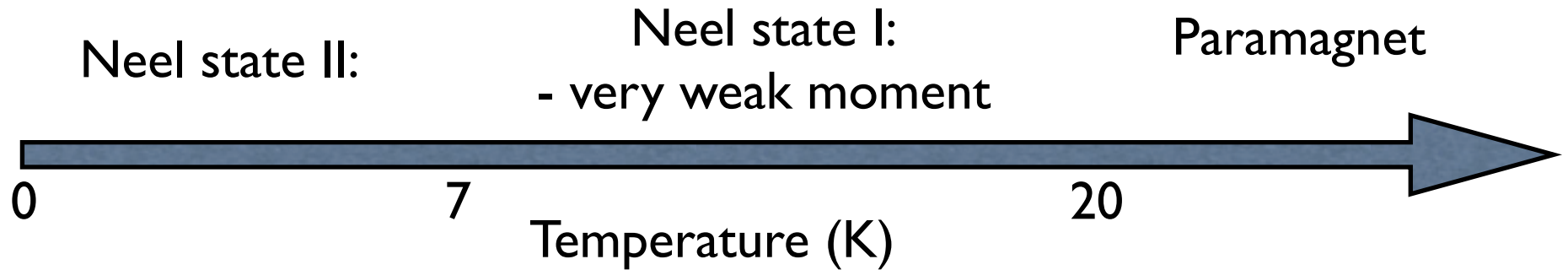
We conclude that the low energy continuum of ZnCu<sub>3</sub>(OD)<sub>6</sub>Cl<sub>2</sub> is due to collective excitations of kagome Cu<sup>2+</sup> ions.

# Summary: Phase Diagram of $\text{Zn}_x\text{Cu}_{4-x}(\text{OD})_6\text{Cl}_2$



# Scenarios for $\text{Cu}_4(\text{OD})_6\text{Cl}_2$

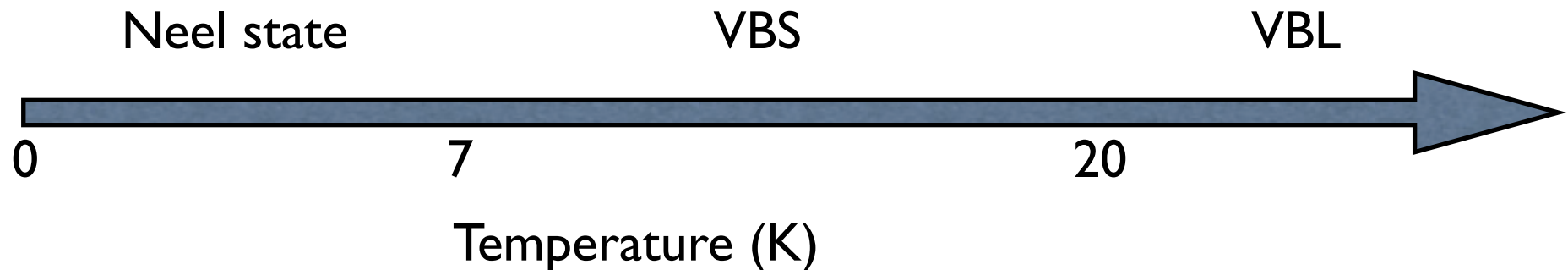
## - Scenario I



but..

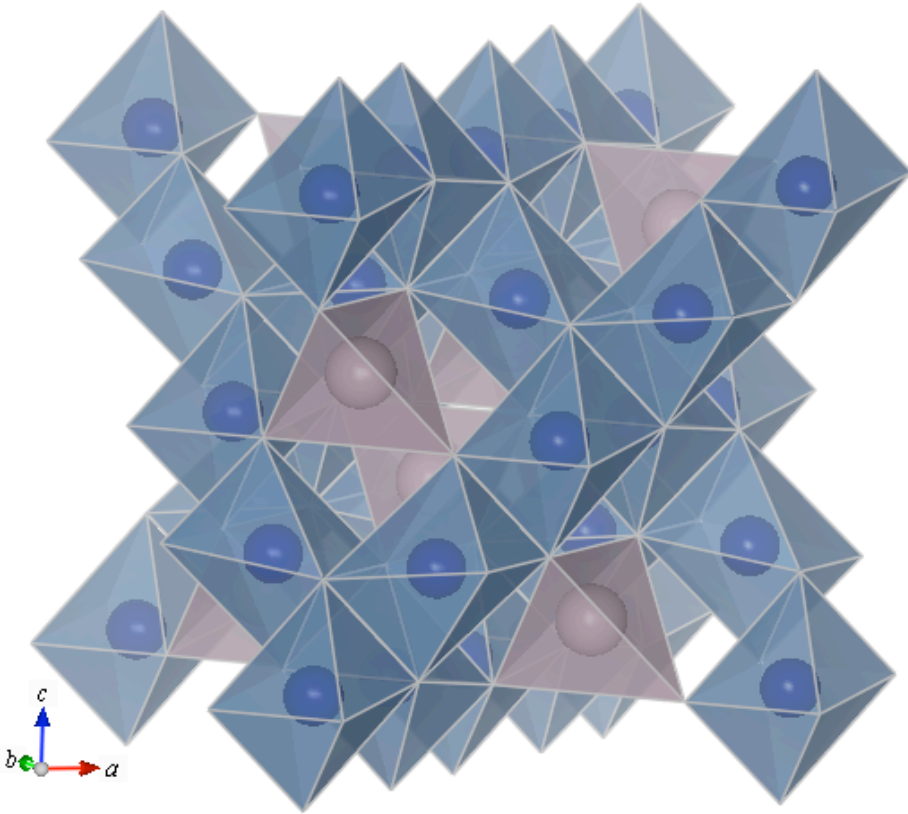
- The spin wave gap is proportional to the frozen moment.
- The 7 meV energy does not shift with increasing T.

## - Scenario II





# Spinel $AB_2O_4$ : Crystal Structure



- At high temperature,  
 $Fd3m$ , cubic  
 $a=6.1637\text{\AA}$ ,  $b=6.8166$ ,  $c=9.114\text{\AA}$   
 $\alpha=\beta=\gamma=90^\circ$
- $AO_4$  tetrahedra
- $BO_6$  Octahedra
- The octahedra are **edge sharing**
- B ions form a pyrochlore lattice

# Guide map of “simple” spinel oxides

From Takagi's talk '06

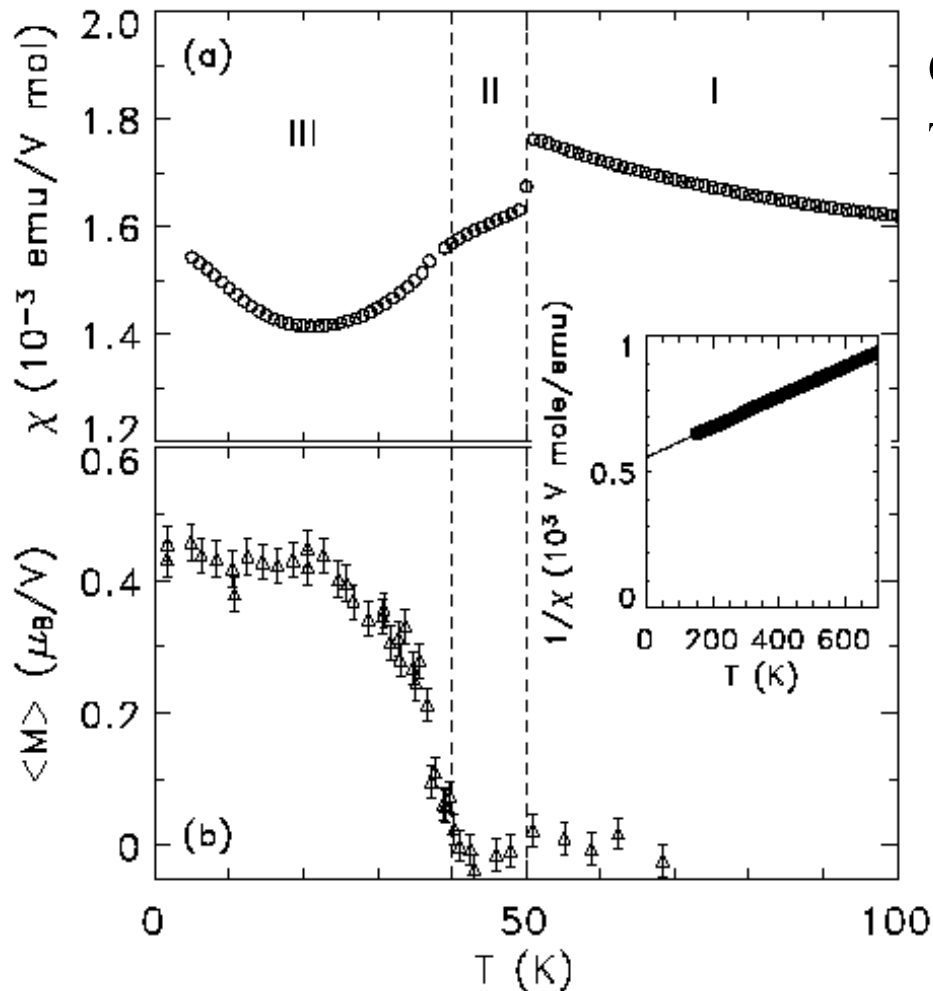
Transition Metal <b>M</b> =	<b>Ti</b>	<b>V</b>	<b>Cr</b>	<b>Mn</b>
	<b>d<sup>0.5</sup></b>	<b>d<sup>1.5</sup></b>	<b>d<sup>2.5</sup></b>	<b>d<sup>3.5</sup></b>
<p><b>Li<sup>1+</sup>M<sub>2</sub><sup>3.5+</sup>O<sub>4</sub></b></p> <p>“<b>Charge</b>” frustration 1:1 3+ &amp; 4+, mixed valent</p>	<p><b>LiTi<sub>2</sub>O<sub>4</sub></b></p> <p>BCS SC</p>	<p><b>LiV<sub>2</sub>O<sub>4</sub></b></p> <p>Heavy Fermion</p>	<p><b>(LiCr<sub>2</sub>O<sub>4</sub>)</b></p>	<p><b>LiMn<sub>2</sub>O<sub>4</sub></b></p>
	<b>d<sup>1</sup></b>	<b>d<sup>2</sup></b>	<b>d<sup>3</sup></b>	<b>d<sup>4</sup></b>
<p><b>Zn<sup>2+</sup>M<sub>2</sub><sup>3+</sup>O<sub>4</sub></b></p> <p>(<b>Mg<sup>2+</sup>, Cd<sup>2+</sup>, Hg<sup>2+</sup></b>)</p> <p>Mott insulator “<b>Spin</b>” frustration all 3+, AF interactions</p>	<p><b>MgTi<sub>2</sub>O<sub>4</sub></b></p>	<p><b>ZnV<sub>2</sub>O<sub>4</sub></b> <b>MgV<sub>2</sub>O<sub>4</sub></b> <b>CdV<sub>2</sub>O<sub>4</sub></b> <b>MnV<sub>2</sub>O<sub>4</sub></b></p>	<p><b>ZnCr<sub>2</sub>O<sub>4</sub></b> <b>CdCr<sub>2</sub>O<sub>4</sub></b> <b>HgCr<sub>2</sub>O<sub>4</sub></b> <b>CoCr<sub>2</sub>O<sub>4</sub></b></p>	<p><b>ZnMn<sub>2</sub>O<sub>4</sub></b> <b>MnMn<sub>2</sub>O<sub>4</sub></b></p>
		<p><b>Spin liquid</b> <b>orbital degeneracy</b> <i>3D spin-Peierls transition</i> <b>spin-orbit-lattice coupling</b> Field-induced plateau states <b>effective 1D physics</b> <b>Multiferroic behaviors</b></p>		

# $S = 1$ pyrochlore AFM with orbital degrees of freedom: $AV_2O_4$ ( $A = Zn, Cd$ )

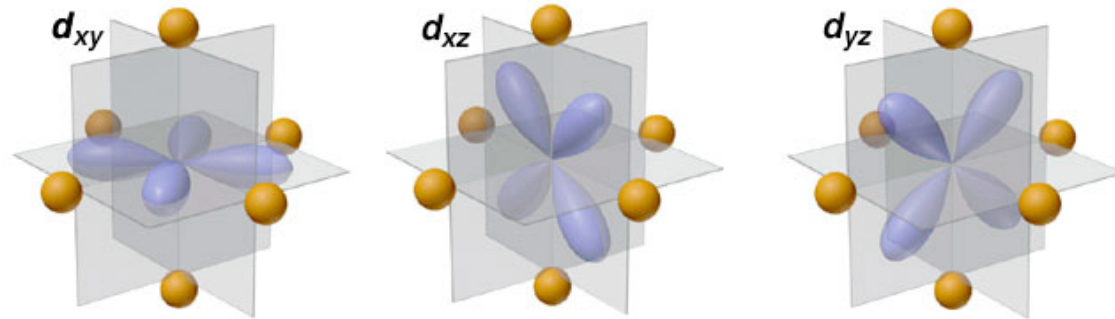
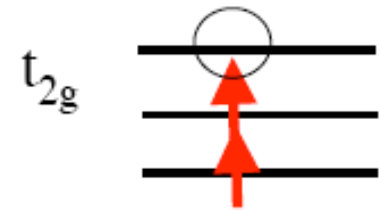
Additional ingredient: orbital degrees of freedom

$V^{3+} (d^2, S=1)$

*Y. Ueda et al., (1997)*



$\Theta_{CW} = -1000$  K  
 $T_N = 40$  K



Two transitions

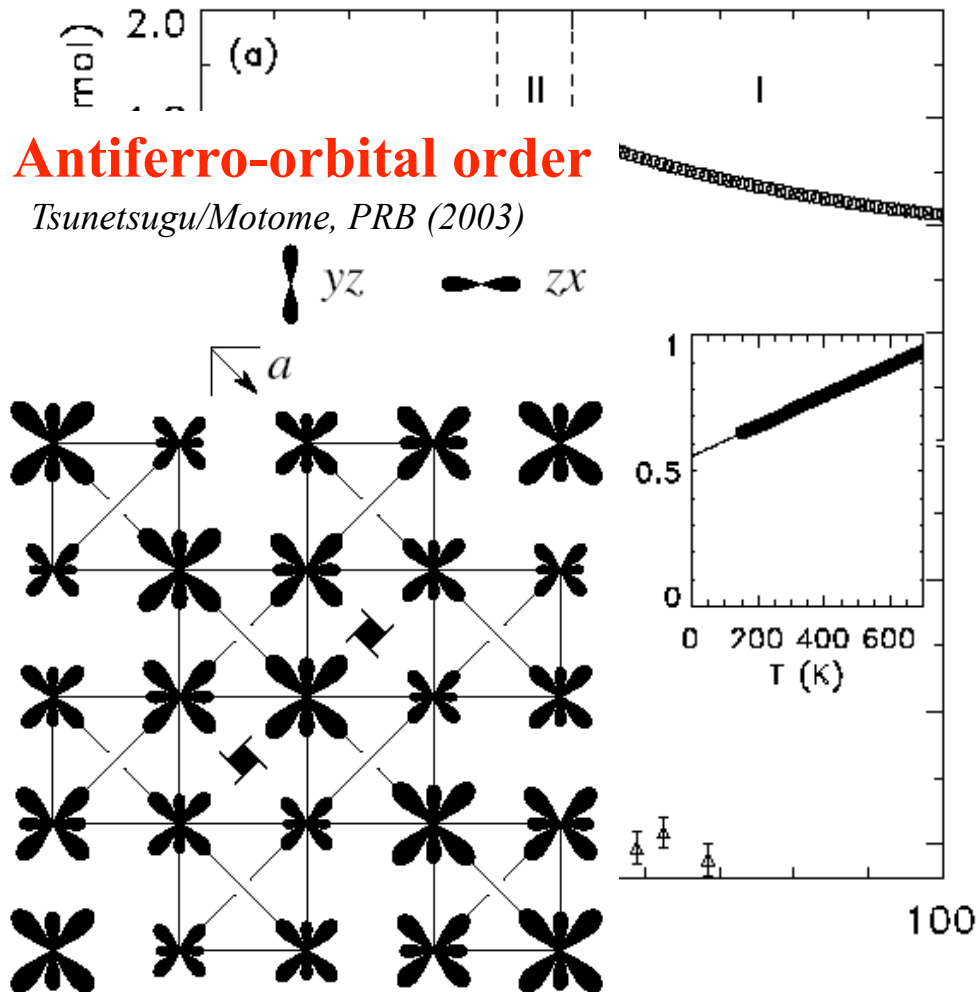
$T_N \neq T_S$

# Effect of the $A^{2+}$ -ion on orbital ordering in $AV_2O_4$

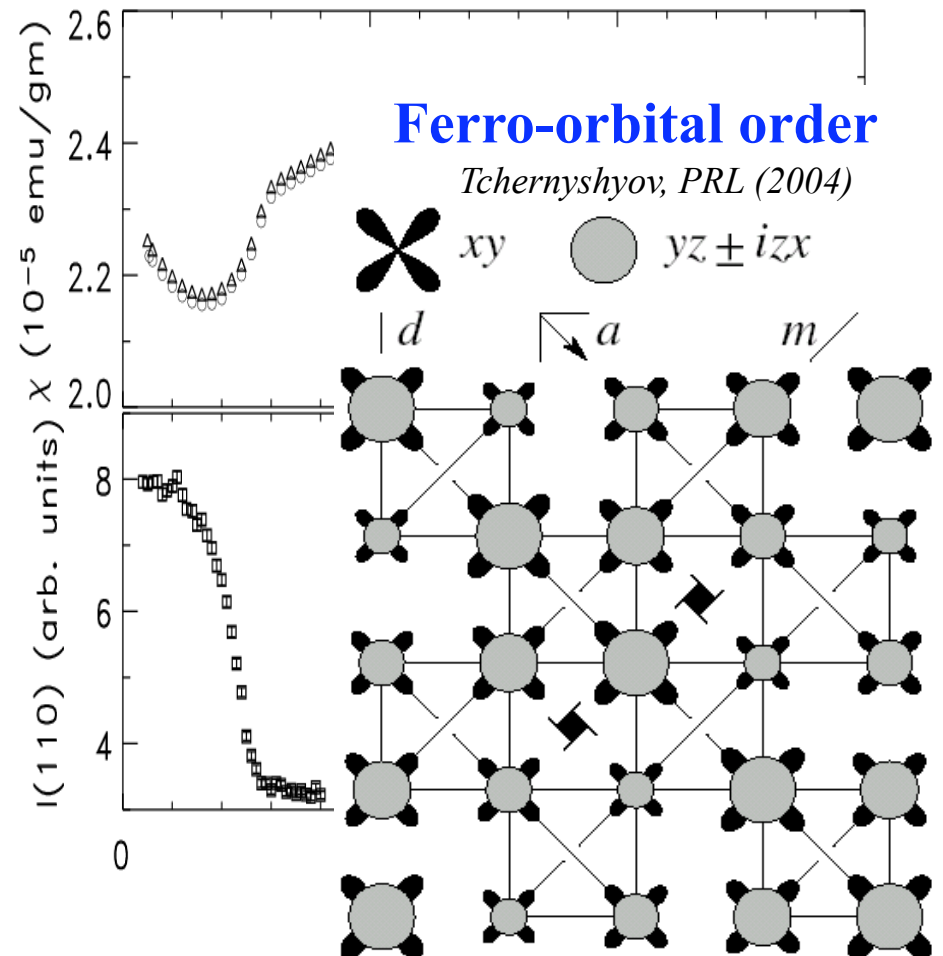
SHL et al., PRL (2004)

Zhang/Louca/SHL et al., PRB (2006)

$ZnV_2O_4$



$CdV_2O_4$

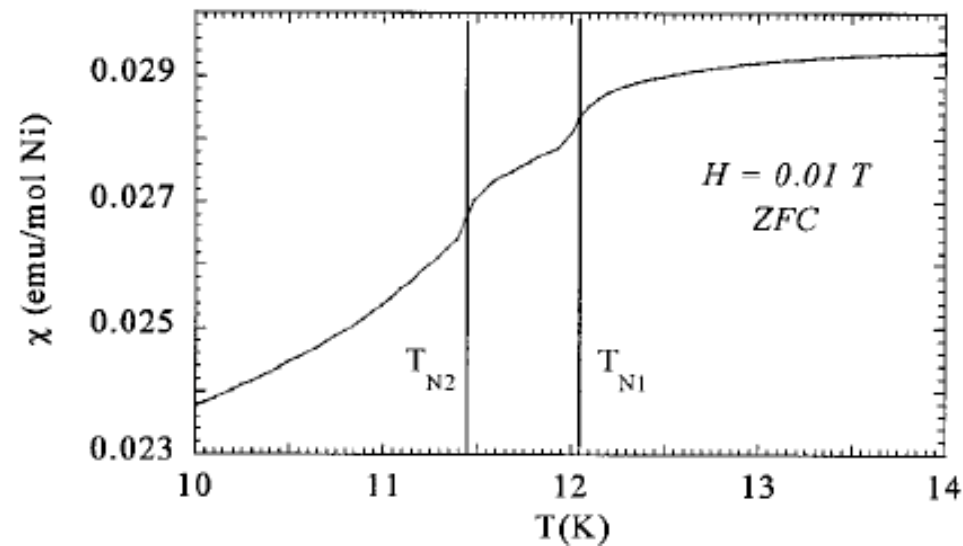
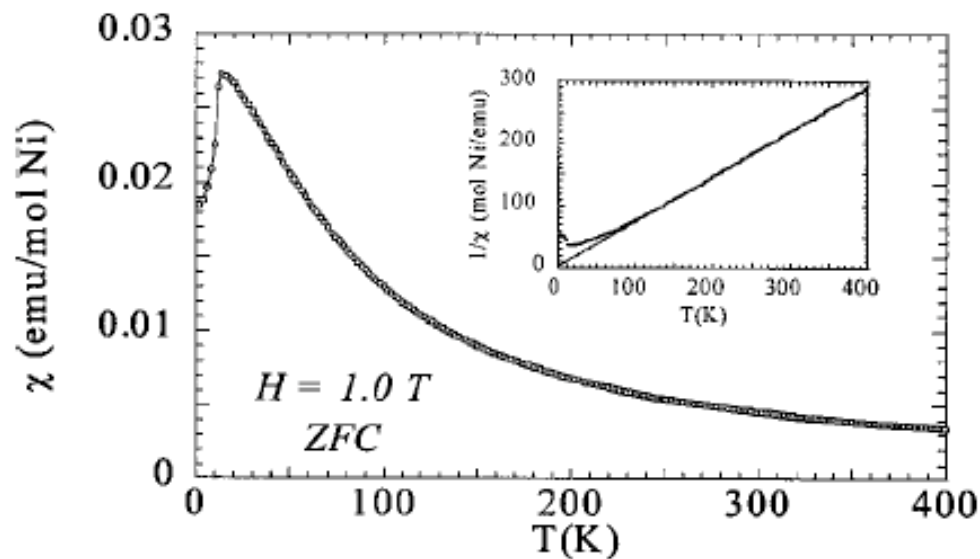


**A Spinel with  $e_g$  electrons**

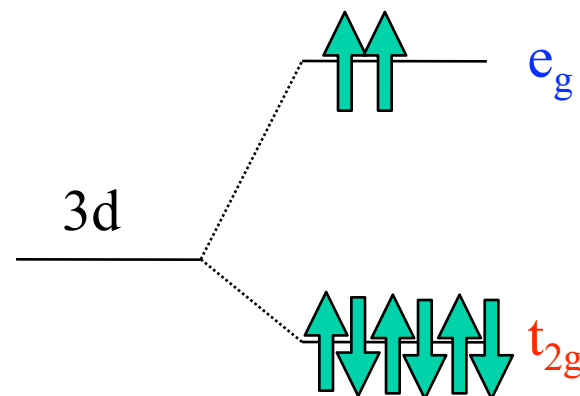
**$\text{GeNi}_2\text{O}_4$  ( $S=1$ )**

# GeNi<sub>2</sub>O<sub>4</sub>: An S=1 spinel

Takagi's group, unpublished (2002)  
M.K. Crawford *et al.*, PRB (2003)



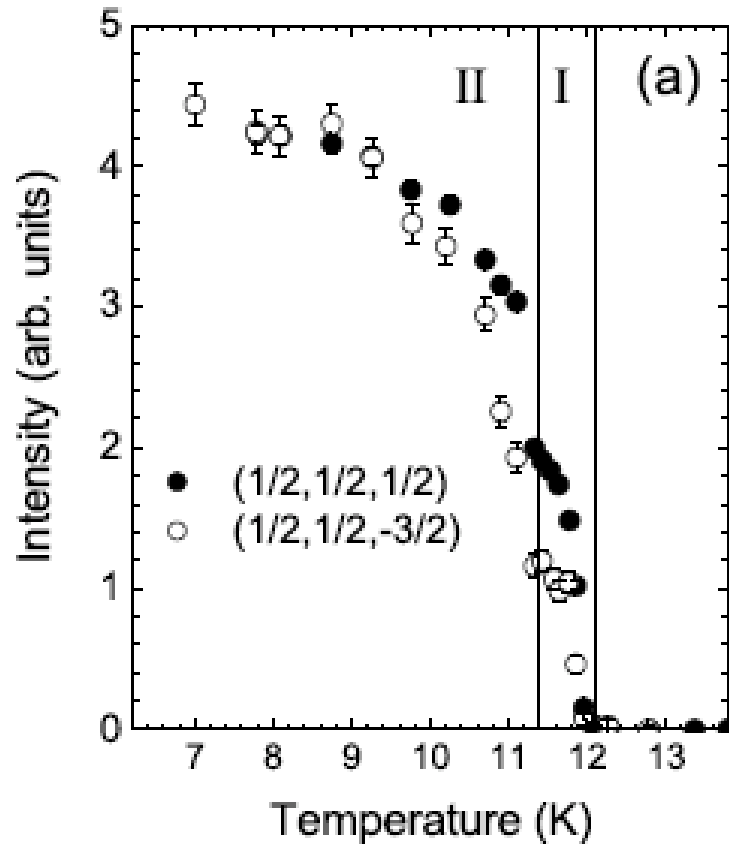
Ni<sup>2+</sup> (3d<sup>8</sup>): no orbital degeneracy



- $\Theta_{CW} = -4.4 K$
- Two transitions at 12.1 K and 11.4 K
- No structural transition
- The two transitions are purely magnetic
- $T_N > |\Theta_{CW}|$  : AFM and FM Js

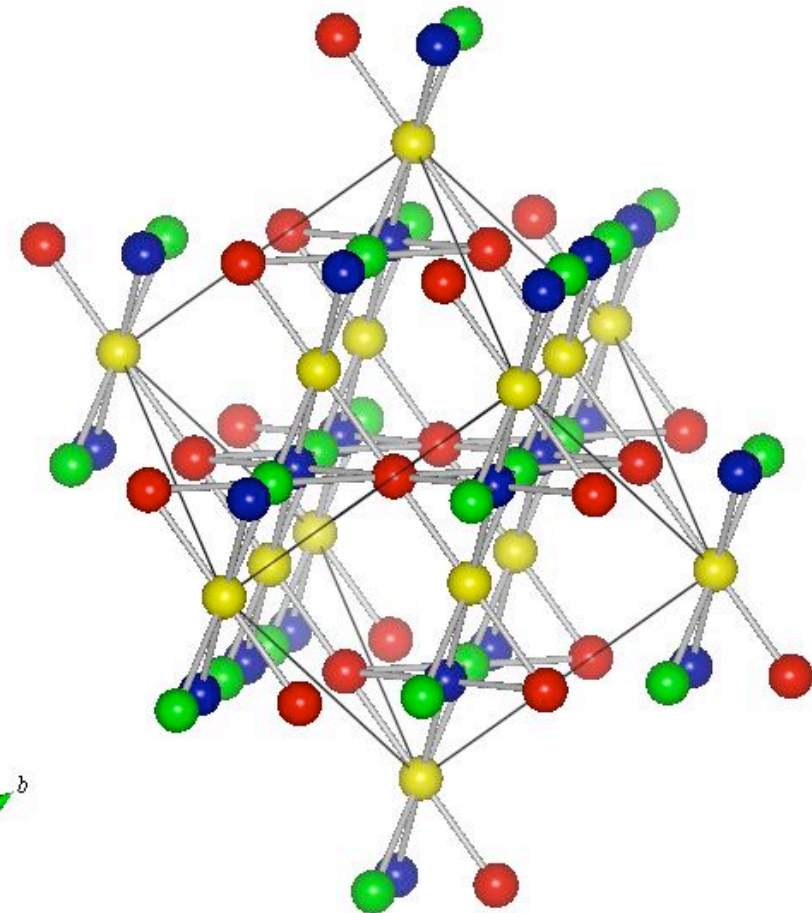
# GeNi<sub>2</sub>O<sub>4</sub>: Neutron diffraction from a single crystal

M. Matsuda *et al.*, cond-mat/0708.3162 (2007)



- $I(1/2, 1/2, 1/2) / I(1/2, 1/2, -3/2)$  is different in the two phases
- The two phases have different magnetic structures

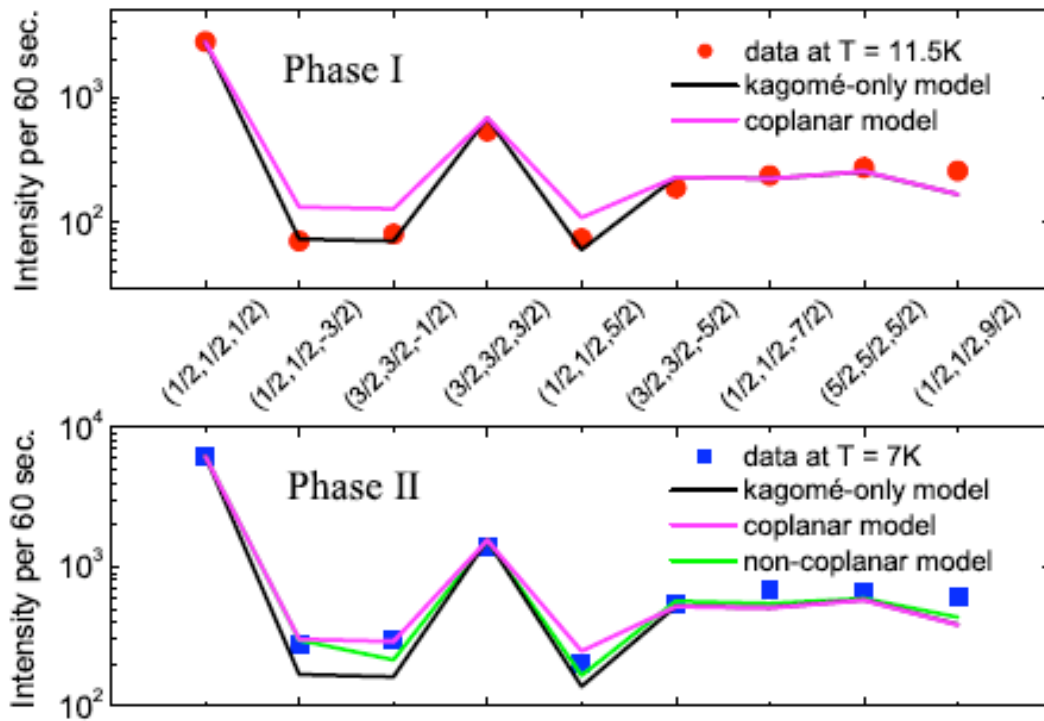
- Four sublattices each of which forms an FCC lattice
- Along  $\langle 111 \rangle$ , three sublattices form kagome planes and one forms triangular planes



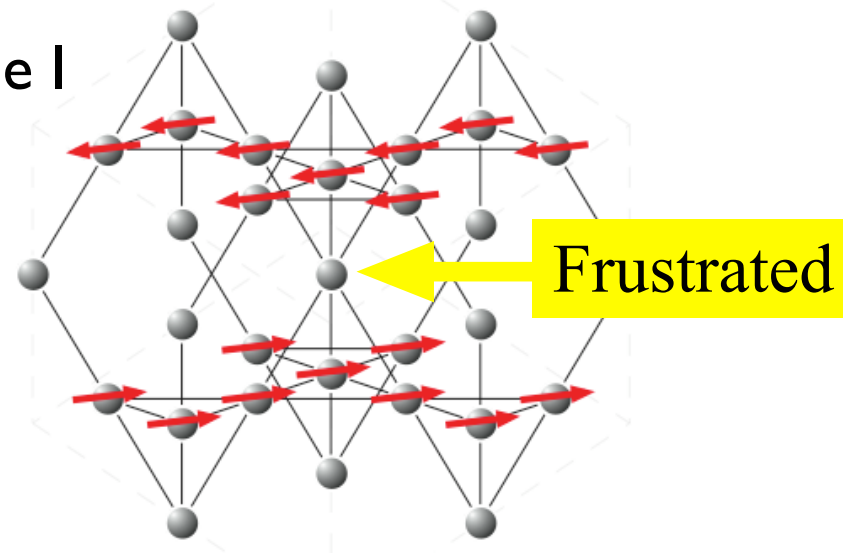


# GeNi<sub>2</sub>O<sub>4</sub>: Magnetic structures of the two phases

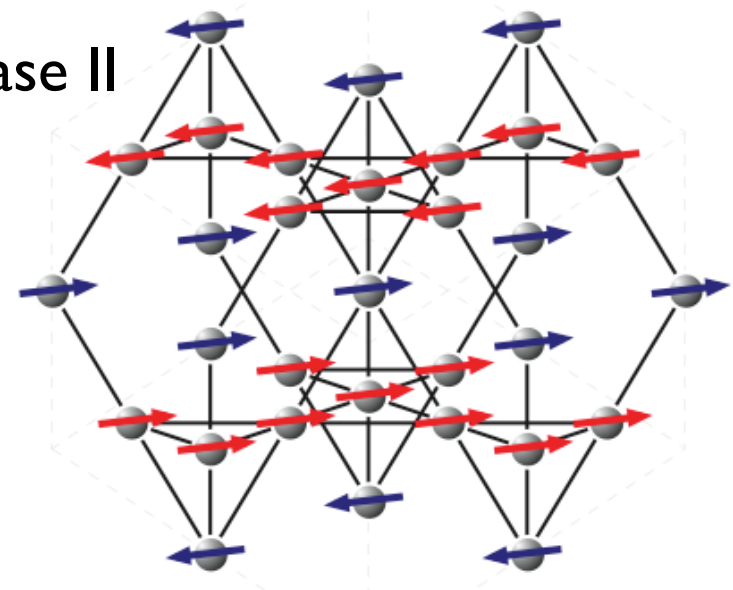
M. Matsuda *et al.*, cond-mat/0708.3162 (2007)



Phase I



Phase II

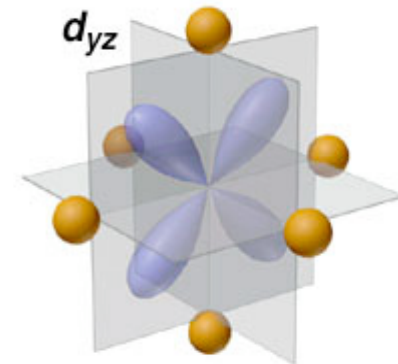
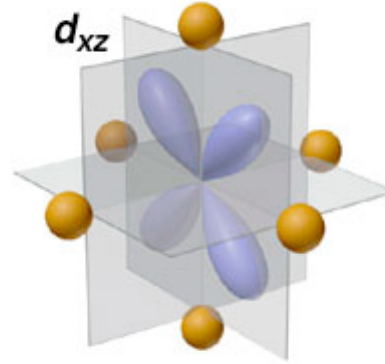
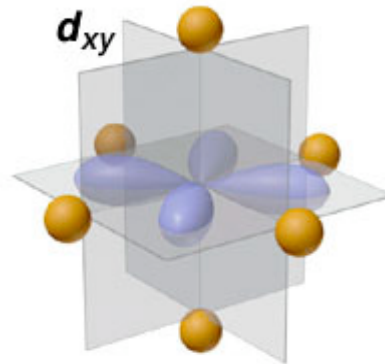
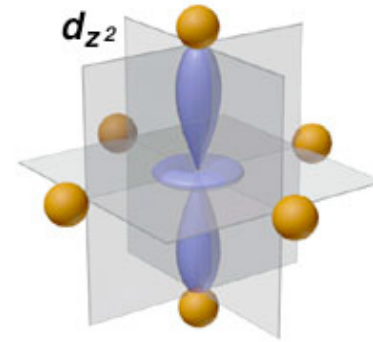
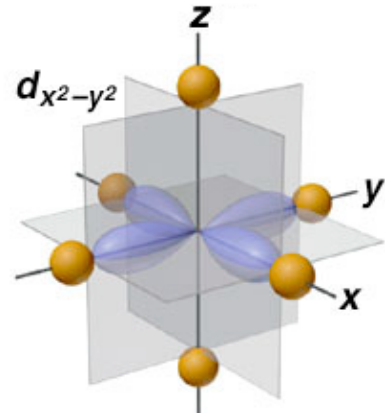
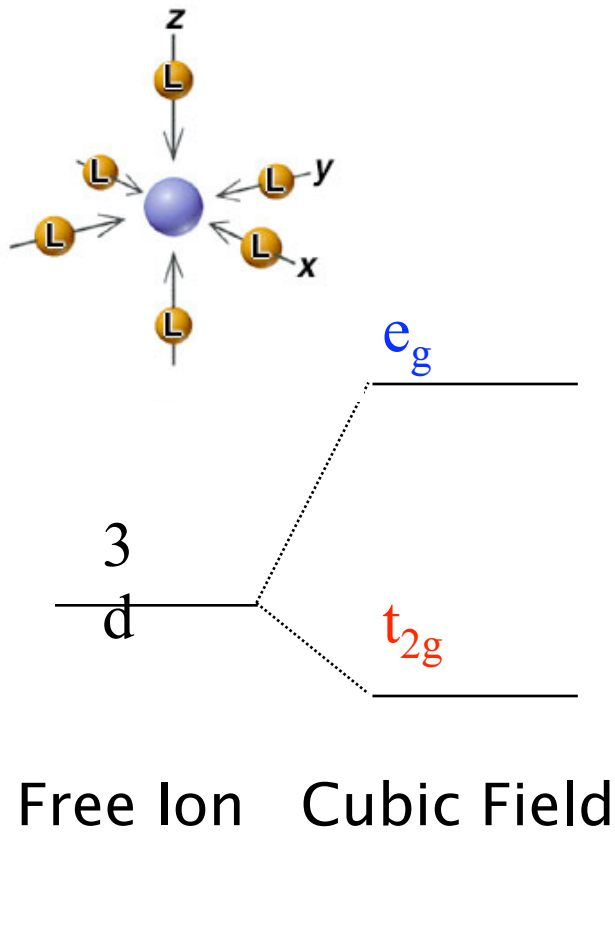


- In phase I, only kagome spins are ordered with  $\langle M \rangle = 1.3 \mu_B$
- In phase II, kagome and triangular spins are ordered with  $\langle M \rangle = 1.8 \mu_B$
- kagome and triangular spins lie on the  $\langle 111 \rangle$  plane



# Spinel $AB_2O_4$ with $B = 3d$ transition metal ions

## *Electronic states vs Magnetism*



lie along the ligand axes

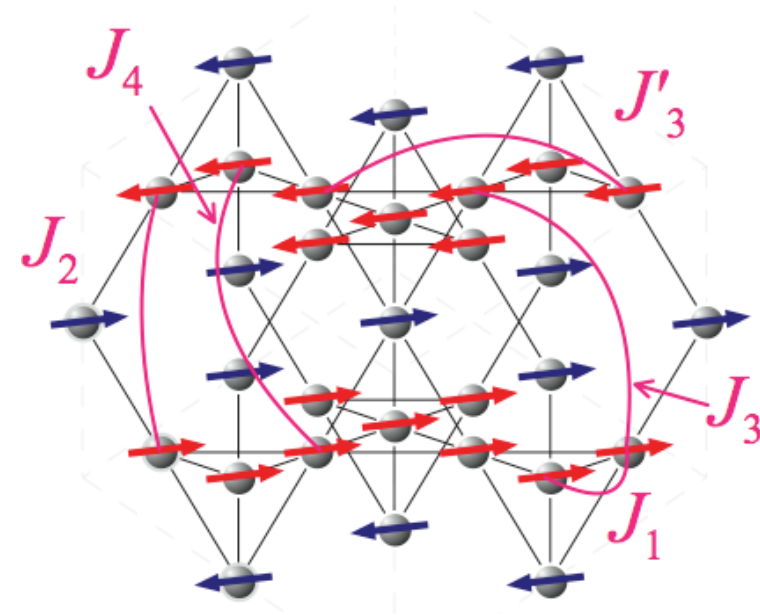
B-O-B superexchange  
--> 2nd or 3rd neighbor Js

lie between the ligand axes

B-B direct overlap  
--> AFM NN J

# GeNi<sub>2</sub>O<sub>4</sub>: The origin of the two magnetic phase transitions

Superexchange interactions that involve one to two superexchange steps



$J$	$d$	Path	$\theta$	$n_B$	$Z_K$	$Z_T$
$J_1$	$\sqrt{2}$	BOB	$90^\circ$	2	4 (I), 2 (T)	6 (K)
$J_2$	$\sqrt{6}$	BOAOB	$125^\circ, 125^\circ$	1	4 (I), 4 (K), 4 (T)	12 (K)
		BOBOB	$90^\circ, 90^\circ$	4		
$J_3$	$\sqrt{8}$	BOAOB	$125^\circ, 125^\circ$	2	2 (I), 4 (K)	6 (I)
$J_3'$	$\sqrt{8}$	BOBOB	$90^\circ, 90^\circ$	4	4 (I), 2 (K)	6 (T)
$J_4$	$\sqrt{10}$	BOAOB	$125^\circ, 125^\circ$	1	8 (K), 4 (T)	12 (K)

Mean-field energy for the observed (1/2, 1/2, 1/2) structure

$$E_{\text{kag}} = 2J_1 - J_3 + J_3' - 4J_4;$$

$$E_{\text{tri}} = 3J_3 - 3J_3',$$

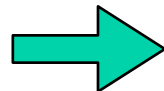
➔

$$E_{\text{tot}} = \frac{3}{4}E_{\text{kag}} + \frac{1}{4}E_{\text{tri}} = \frac{3}{2}J_1 - 3J_4$$

Mean-field energy for (1,0,0) and (1,1,0) structures

$$E_{\text{tot}}^* = -J_3 - J_3' + 3J_6 \pm |J_1 - 2J_2 + 2J_4|$$

$$E_{\text{tot}} < E_{\text{tot}}^*$$



$$J_4 > \frac{5}{2}J_1 - 2J_2 + J_3 + J_3'$$

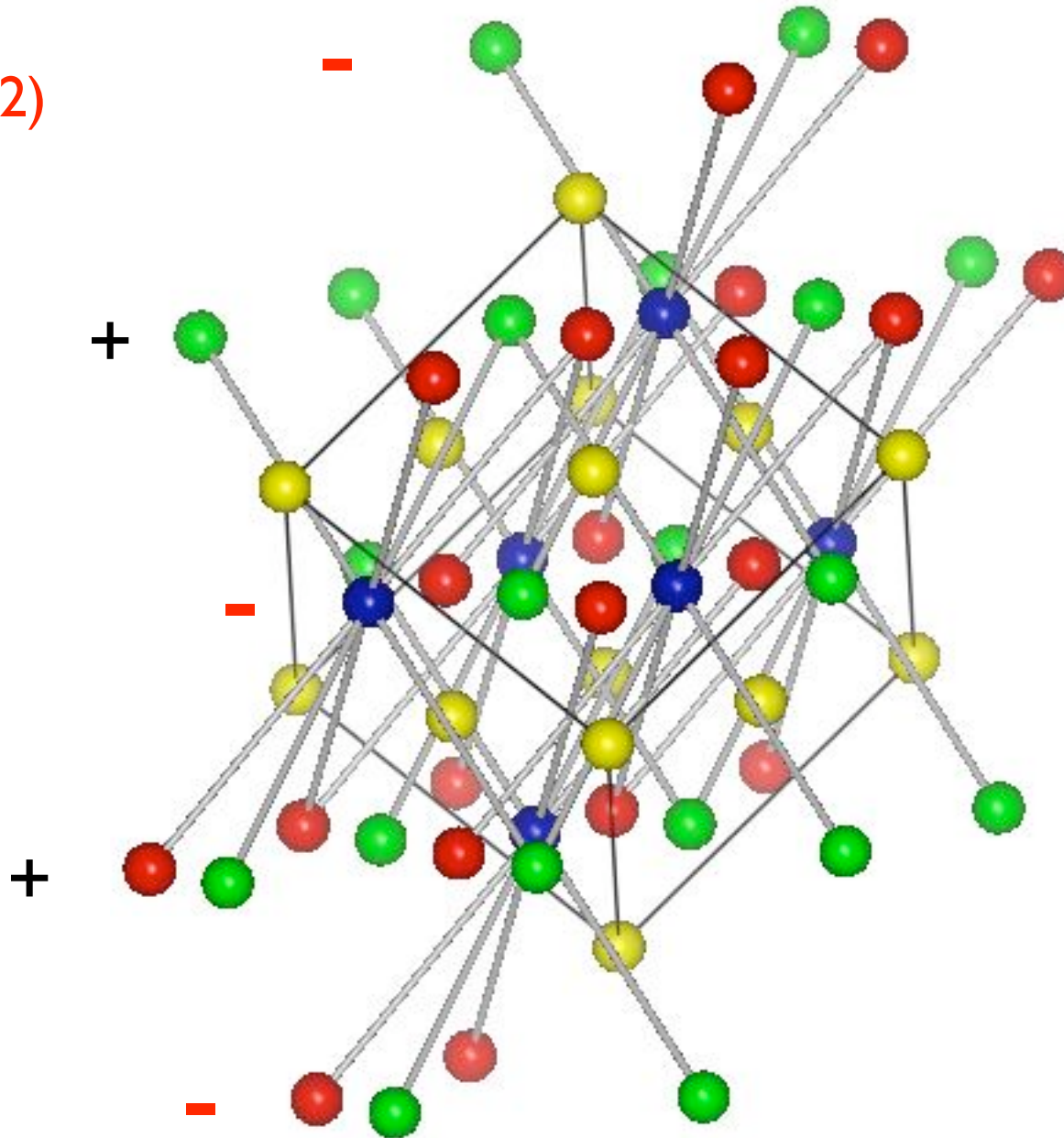
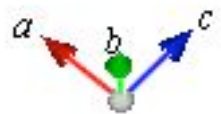
:AFM  $J_4$ , FM  $J_1$

Consistent with the Goodenough-Kanamori rules for GeNi<sub>2</sub>O<sub>4</sub>

# $J_4$ bondings among kagome spins

$Q=(1/2,1/2,1/2)$

$-8 J_4$



Average total E due to  $J_4$  per spin  
 $= 3 * (-8 J_4) / 4 = -6 J_4$

# GeNi<sub>2</sub>O<sub>4</sub>: The origin of the two magnetic phase transitions

Mean-field energy for kagome and triangular spins

$$\begin{aligned}
 E_{\text{kag}} &= 2J_1 - J_3 + J'_3 - 4J_4; & \longrightarrow & & k_B T_{N1} &= \frac{2}{3}|E_{\text{kag}}| \\
 E_{\text{tri}} &= 3J_3 - 3J'_3, & & & k_B T_{N2} &= \frac{2}{3}|E_{\text{tri}}|
 \end{aligned}$$

$$k_B T_{N2} \lesssim k_B T_{N1} \longrightarrow J_4 - J_1/2 \gtrsim J'_3 - J_3$$

$J'_3 > J_3$ ;  $J'_3$  may be antiferromagnetic and  $J_3$  ferromagnetic  
*consistent with the Goodenough-Kanamori rules?*

$J$	$d$	Path	$\theta$	$n_B$	$Z_K$	$Z_T$
$J_1$	$\sqrt{2}$	BOB	$90^\circ$	2	4 (I), 2 (T)	6 (K)
$J_2$	$\sqrt{6}$	BOAOB	$125^\circ, 125^\circ$	1	4 (I), 4 (K), 4 (T)	12 (K)
		BOBOB	$90^\circ, 90^\circ$	4		
$J_3$	$\sqrt{8}$	BOAOB	$125^\circ, 125^\circ$	2	2 (I), 4 (K)	6 (I)
$J'_3$	$\sqrt{8}$	BOBOB	$90^\circ, 90^\circ$	4	4 (I), 2 (K)	6 (T)
$J_4$	$\sqrt{10}$	BOAOB	$125^\circ, 125^\circ$	1	8 (K), 4 (T)	12 (K)

# Summary

## GeNi<sub>2</sub>O<sub>4</sub>

- The two magnetic phase transitions are due to the existence of two types of spins for a given  $k$ : majority kagome and minority triangular spins.
- At 12.1K, kagome spins order due to AFM  $J_4$  interactions.
- At 11.4K, triangular spins order. Exchange theory containing only  $J_s$  calls for AFM  $J_3'$  and FM  $J_3$ , which is not consistent with the Goodenough-Kanamori rules.