What is coherent in neutrino oscillations

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Abstract

The standard treatment of neutrino oscillations recalls the story of the mathematics lecturer who said "It is obvious that...." then stopped and said; "Is it obvious?", went out for a half hour, and returned to say; "Yes it is obvious". The textbook neutrino-oscillation wave function, a coherent linear combination of states with different energies, is not found in any real experiments. Its application to reality is obvious. But the interval between "Is it obvious?" and "Yes it is obvious" is filled with many wrong arguments, many wrong papers, and more papers showing that the wrong arguments are wrong. We clarify this issue by describing the passage of a neutrino from source to detector as a multipath experiment where knowing the path destroys coherence, considering the beam and the detector as a correlated quantum system and applying this approach to Bragg scattering by X-rays as well as neutrinos. Amplitudes with the same energy and different masses are detected coherently and produce oscillations. Amplitudes with different energies are incoherent. Quantum mechanics alone shows the existence of a neutrino mass difference to be required to explain the observed Super-Kamiokande data.

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Neutrino Oscillations and Coherence

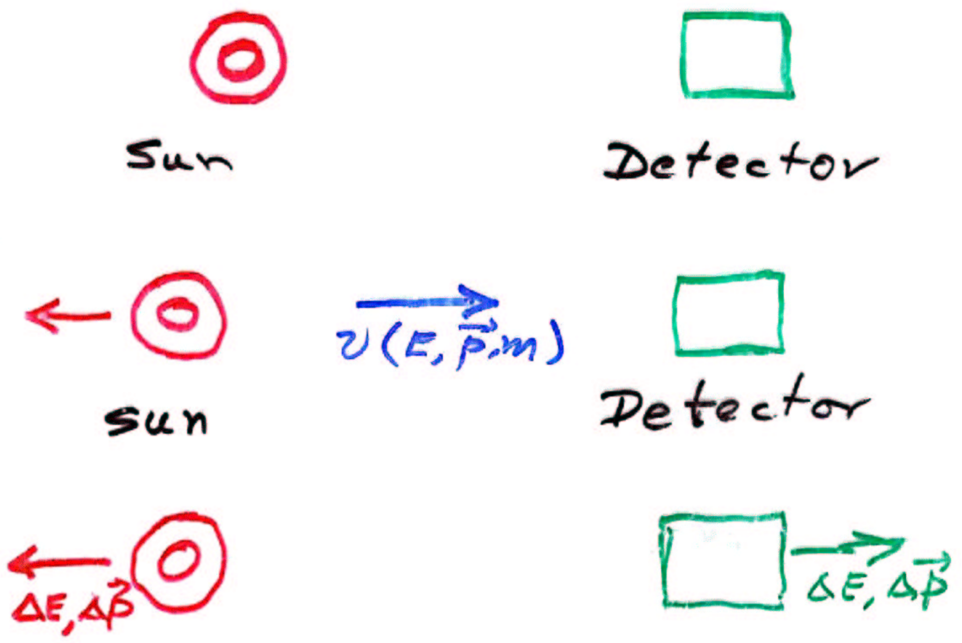
Who needs this talk?

All said years ago by Boris!

But none are so blind
as those who don't want to see
And want to write wrong papers.

Need Lipkin to explain
Boris for Pedestrians

QM of Solar Neutrinos

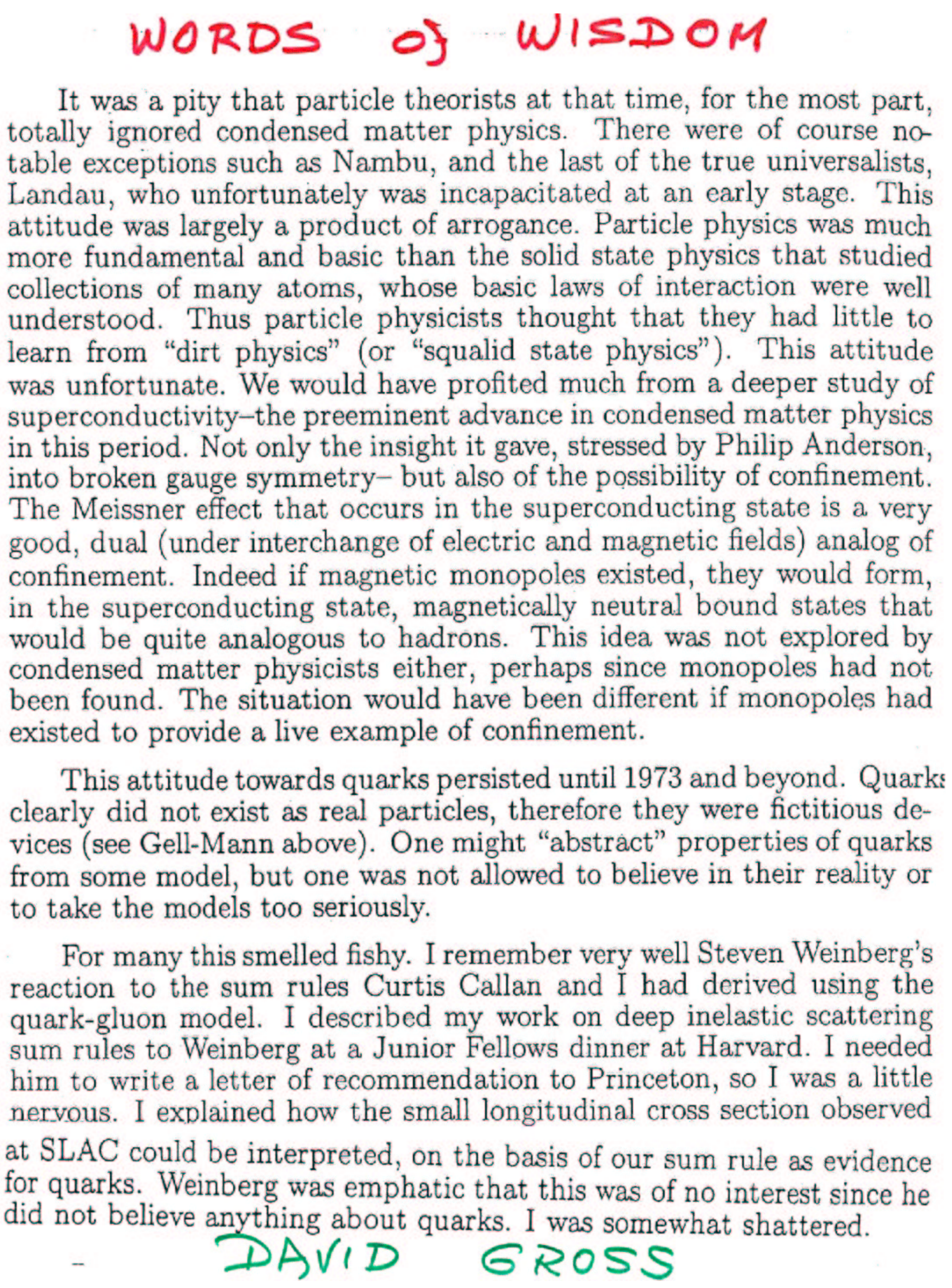


Which Mass Carried Momentum?

If we Know — No coherence

If we don't Know, - Is it ignorance or is it Q.M.?

Two Slit experiment over 150 million Km!



WORDS of WISDOM

It was a pity that particle theorists at that time, for the most part, totally ignored condensed matter physics. There were of course notable exceptions such as Nambu, and the last of the true universalists, Landau, who unfortunately was incapacitated at an early stage. This attitude was largely a product of arrogance. Particle physics was much more fundamental and basic than the solid state physics that studied collections of many atoms, whose basic laws of interaction were well understood. Thus particle physicists thought that they had little to learn from "dirt physics" (or "squalid state physics"). This attitude was unfortunate. We would have profited much from a deeper study of superconductivity–the preeminent advance in condensed matter physics in this period. Not only the insight it gave, stressed by Philip Anderson, into broken gauge symmetry– but also of the possibility of confinement. The Meissner effect that occurs in the superconducting state is a very good, dual (under interchange of electric and magnetic fields) analog of confinement. Indeed if magnetic monopoles existed, they would form, in the superconducting state, magnetically neutral bound states that would be quite analogous to hadrons. This idea was not explored by condensed matter physicists either, perhaps since monopoles had not been found. The situation would have been different if monopoles had existed to provide a live example of confinement.

This attitude towards quarks persisted until 1973 and beyond. Quarks clearly did not exist as real particles, therefore they were fictitious devices (see Gell-Mann above). One might "abstract" properties of quarks from some model, but one was not allowed to believe in their reality or to take the models too seriously.

For many this smelled fishy. I remember very well Steven Weinberg's reaction to the sum rules Curtis Callan and I had derived using the quark-gluon model. I described my work on deep inelastic scattering sum rules to Weinberg at a Junior Fellows dinner at Harvard. I needed him to write a letter of recommendation to Princeton, so I was a little nervous. I explained how the small longitudinal cross section observed at SLAC could be interpreted, on the basis of our sum rule as evidence for quarks. Weinberg was emphatic that this was of no interest since he did not believe anything about quarks. I was somewhat shattered.

DAVID GROSS

Bragg Scattering by Solid

Coherent Scattering from all atoms

Scattering by Quantum Solid

Atom remains in SAME LEVEL

No information identifies scattering atom!

Can't Possibly Know which atom scattered photon

Unavailable Info - Key to coherence

Bragg Scattering by Solid

Coherent Scattering from all atoms

Scattering by Classical Solid

Recoil identifies scattering atom

No coherence!

Experimental set up and quantum mechanics must forbid knowledge necessary to determine the neutrino mass.

Instructive Example - Bragg scattering by a crystal

Coherence from incomplete momentum information on scattering from different atoms produces constructive interference at Bragg angles and peaks in angular distribution.

Single scattered photon transfers momentum to scattering atom. Detecting recoil momentum would identify scattering atom and destroy coherence.

QM prevents measurement of individual atom momenta

QM of Crystal dynamics and incident photon interactions allow elastic scattering. Photon scattered by single atom in crystal but crystal quantum state unchanged.

Purely quantum effect. Classical momentum transfer to an atom in classical crystal changes atom momentum and motion. Allows identification of scattering atom.

Simple toy model - each atom bound to equilibrium position by harmonic oscillator potential.

Atom scattering the photon initially in definite discrete energy level $|i\rangle$.

Cannot absorb the momentum transfer according to the energy and momentum kinematics of free particles.

Final state $|f\rangle$ must be allowed energy level.

Finite probability that $|f\rangle = |i\rangle$ (elastic scattering)

Which atom scattered photon? Information unavailable.

Coherent scattered amplitudes from all scattering atoms

Amplitudes arising from different processes which would be classiscally distinguishable can be coherent.

The quantum mechanics of localized states can conceal the information which would be classically available from energy-momentum conservation for free particles.

Same effect conceals neutrino mass in π decay

No problem in measuring decay muon momentum

Initial p_π information must be incomplete

Not strictly at rest; localized in some energy level $|i\rangle$ of the material where it stopped.

02 QUANTUM MECHANICS OF SOLAR NEUTRINOS

Emitted neutrinos carry energy and momentum

Sun recoils with conservation of energy and momentum

Emission and propagation of neutrinos follow QM

Examine transition $|i\rangle \equiv |S(E,p)\rangle \rightarrow |f\rangle \rightarrow |SD\rangle$,

$$|f\rangle \equiv c_1 \cdot |S(E-E_1, p-p_1)\rangle \cdot |\nu(E_1,p_1)\rangle +$$

$$+c_2 \cdot |S(E-E_2, p-p_2)\rangle \cdot |\nu(E_2,p_2)\rangle$$

$$|\langle SD|\, T(\nu)\, |f\rangle\,|^2 = |c_1 \cdot \langle D|\, T_1(\nu)\, |\nu\rangle\,|^2 + |c_2 \cdot \langle D|\, T_2(\nu)\, |\nu\rangle\,|^2$$

Sun states drop out of the relation because of orthogonality.

$$\langle S(E-E_1, p-p_1)|\, S(E-E_2, p-p_2)\rangle = 0$$

Interference term vanishes. Missing mass experiment.

No coherence beween two mass eigenstates.

CORRECTION

Sun is wave packet in momentum space.

$$|i\rangle = \int g(p)dp\,|S(E,p)\rangle \equiv \Psi_S(X)$$

X denotes the center of mass co-ordinate of the sun.

$$|f\rangle \equiv c_1 \cdot e^{-ip_1X}\Psi_S(X) \cdot e^{ip_1x_1} + c_2 \cdot e^{-ip_2X}\Psi_S(X) \cdot e^{ip_2x_2}$$

x_1 and x_2 co-ordinates of neutrinos - masses m_1 and m_2

Consider two neutrino states with same energy.

$$|\langle SD|\, T(\nu)\, |f\rangle\,|^2 = |c_1 \, \langle D|\, T_1(\nu)\, |\nu\rangle\,|^2 + |c_2 \, \langle D|\, T_2(\nu)\, |\nu\rangle\,|^2 +$$

$$+\{c_1^* c_2 F(\delta p)\, \langle \nu|\, T_1(\nu)\, |D\rangle\, \langle D|\, T_2(\nu)\, |\nu\rangle + c.c.\}$$

$$F(\delta p) = \int dX \Psi_S^*(X) e^{i\delta pX}\Psi_S(X) \approx 1 - (1/2)\cdot \delta p^2 \langle X^2\rangle$$

Interference term no longer vanishes

Interference term - proportional to “solar form factor”

$$F(\delta p) = \int dX \Psi_S^*(X) e^{i\delta p X} \Psi_S(X) \approx 1 - \frac{2\pi^2 \langle X^2 \rangle}{\lambda^2} \approx 1$$

$\lambda = 2\pi/\delta p$ wave length of the neutrino oscillation produced by momentum difference δp

Departure from coherence in interference proportional to the ratio of the mean square quantum fluctuation in the position of the sun to the square of the oscillation wave length and is clearly negligible for wave lengths of the order to the sun-earth distance.

Lipkin’s Principle for oscillation coherence

If you can measure it you can measure it!

PROOF

Any sensible experiment must have $x_s << \lambda$

$$\lambda >> x_s; \quad \delta p_s \approx \frac{\hbar}{x_s} >> \frac{\hbar}{\lambda} \approx \delta p_{osc}$$

Any sensible experiment will have δp coherence

Initial state $|i\rangle$ has pion coherent linear combination of different momentum eigenstates with sharp energy. Muon energy determines neutrino energy but not momentum.

At neutrino detector, amplitudes with same energy and different momenta produced from the different coherent momentum components in the initial pion wave function can be coherent with a definite relative phase.

Amplitudes with different energies not coherent

This can explain why no electrons are observed at a short distance from the detector.

If neutrino amplitudes propagate as free particles, the relative phase is completely determined between the amplitudes for neutrinos having the same energy but different masses and different momenta

This produces neutrino oscillations with the same relation between mass differences and phase differences given by the standard treatments.

"Which Path" - With a Quantum Detector

When are amplitudes for two paths coherent?

Quantum Mechanics gives the answer!

Two amplitudes $|L(x)\rangle$ and $|R(x)\rangle$ for two paths

With no detector, wave function at point x on screen

$$\Psi(x) = |L(x)\rangle + |R(x)\rangle$$

With no detector, intensity at point x on screen

$$I(x) = |\Psi(x)|^2 = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2Re[\langle L(x)\,|R(x)\rangle]$$

With quantum detector in "R" path; $D_i \rightarrow D_f$

Wave function at point x on screen

$$\Psi(x, D) = |L(x), D_i\rangle + \left|R(x), D_f\right\rangle$$

With quantum detector, intensity at point x

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2Re[\langle L(x)\,|R(x)\rangle \cdot \langle D_i\,|D_f\rangle]$$

Interference term with quantum detector

Additional factor - detector overlap $\langle D_i\,|D_f\rangle$

Can add phase of $\langle D_i\,|D_f\rangle$

A Toy Model for a Quantum Detector

A spin 1/2 nucleus rotated 180^o about z axis

$$|D_f\rangle = e^{i\pi s_z}\,|D_i\rangle = e^{i\pi\sigma_z/2}\,|D_i\rangle$$

$$\langle D_i\,|D_f\rangle = \langle D_i|\,e^{i\pi\sigma_z/2}\,|D_i\rangle = \langle D_i|\,i\sigma_z\,|D_i\rangle = i\langle\sigma_z\rangle_i$$

With no detector, intensity at point x on screen

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2Re[|\langle L(x)\,|R(x)\rangle\,| \cdot e^{i\theta(x)}]$$

$\theta(x)$ is relative phase of $|L(x)\rangle$ and $|R(x)\rangle$

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2|\langle L(x)\,|R(x)\rangle\,|\cos\theta(x)$$

With quantum detector in "R" path

Wave function at point x on screen

$$\Psi(x, D) = [|L(x), D_i\rangle + i\sigma_z\,|R(x), D_i\rangle]$$

With quantum detector, intensity at point x

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 - 2|\langle L(x)\,|R(x)\rangle\,|\sin\theta(x)] \cdot \langle\sigma_z\rangle_i$$

Interference term with quantum detector

Additional factor $\langle D_i|\,\sigma_z\,|D_i\rangle = \langle\sigma_z\rangle_i$

With extra 90^o phase.

Detailed Quantum Mechanics of Neutrino Detector

Initial state of neutrino and detector

$$\Psi_i(\nu, D) = \sum_{k=1}^{N_\nu} \sum_{\vec{P}_k} \left| \nu(E_\nu, m_k, \vec{P}_k), D_i(E_i) \right\rangle$$

N_ν neutrino mass states

$E_\nu, m_k, \vec{P}_k$ neutrino energy, mass and momentum

$D_i(E_i)$ initial state of the detector - energy E_i.

Final muon detector state after absorption of neutrino with mass m_k; emission of a $\mu^\pm$ with energy and momentum E_μ and $\vec{P}_\mu$

$$\Psi_f(\mu^\pm, D) = \sum_{k=1}^{N_\nu} \sum_{\vec{P}_k} \left| \mu^\pm(E_\mu, \vec{P}_\mu), D^\mp_{kf}(E - E_\mu) \right\rangle$$

$D^\mp_{kf}$ is final detector state produced in "path k"

$E = E_\nu + E_i$ is total conserved energy

Transition in detector on nucleon, co-ordinate $\vec{X}$, charge exchange $I_\mp$; momentum transfer $\vec{P}_k - \vec{P}_\mu$.

$$\left\langle D^\mp_{kf} \right| T^\mp \left| D_i \right\rangle = \left\langle D^\mp_{kf} \right| I_\mp e^{i(\vec{P}_k - \vec{P}_\mu)\cdot\vec{X}} \left| D_i \right\rangle$$

Detector overlap between absorbing m_k and m_j

$$\langle D^\mp_{kf} \left| D^\mp_{jf} \right\rangle = \langle D_i | \, e^{i(\vec{P}_j - \vec{P}_k)\cdot\vec{X}} \, | D_i \rangle$$

If quantum fluctuations in active nucleon position in detector initial state small in comparison with oscillation wave length, $\hbar/(\vec{P}_j - \vec{P}_k)$

$$|\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i | \, |\vec{X}^2| \, | D_i \rangle \ll 1$$

$$\langle D^\mp_{kf} \left| D^\mp_{jf} \right\rangle \approx 1 - (1/2) \cdot |\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i | \, |\vec{X}^2| \, | D_i \rangle \approx 1$$

Full overlap after absorbing neutrinos with same energy and different momenta

Neutrinos with different energies - no coherence

Two Reasons Why a talk is needed on Coherence

1. There may be some interesting new physics.
2. Confusion arises from misunderstanding simple QM.

First Reason - Possible New Physics?

We now know that after a weak interaction on the sun, neutrino waves with at least two different masses leave the sun and arrive on earth.

Standard model says these two waves remain coherent after traversing over 100 million kilometers.

It is like a two-slit experiment where an electron goes through two slits and produces an interference pattern on a screen over 100 million kilometers away.

There is no other experiment showing preservation of quantum-mechanical correlations over such large distances.

Whether the relative phase of these amplitudes remains coherent or whether there is some dephasing is worth experimental investigation.

Possible New Physics?

CP violation was first observed in an unexpected and unpredicted CP violating phase.

Many years passed without a single piece of additional evidence for CP violation.

Will the next clue to new physics beyond the standard model also show up in an unexpected phase many years before additional evidence for this new physics is seen alsewhere?

Experiments apparently showing phase preservation of quantum-mechanical correlations over very large distances should be carefully checked.

Whether the relative phase of these amplitudes remains coherent or whether there is some dephasing from new physics is worth further theoretical and experimental investigation.

Second Reason - Total Confusion

Standard textbook coherence treatment misleading

Textbooks describe gedanken oscillations in time

Between states with different energies

1. No known source of coherent different energy neutrinos
2. Real neutrino detectors don't see oscillations in time
3. Ambiguous handwaving interprets gedanken oscillation

Confusion arises from misunderstanding simple QM.

Many wrong papers on the subject.

Irrelevant Lorentz invariance and field theory

All experiments detect neutrinos with detectors

1. At rest in the laboratory system
2. In thermal equilibrium with their environment
   - A. Described by a density matrix diagonal in energy
   - B. Unable to observe relative phases

   Between states with different energies
3. Localized in space in a region

   Tiny compared with the distance to the source

All experiments detect neutrinos with detectors

1. At rest in the laboratory system
   - A. Forget about Lorentz Invariance
   - B. Nobody needs Lorentz frame with moving detector
2. In thermal equilibrium with their environment
   - A. Described by a density matrix diagonal in energy
   - B. Unable to observe relative phases

   Between states with different energies
3. Localized in space in a region

   Tiny compared with the distance to the source
   - A. Described by a wave function or density matrix not diagonal in momentum
   - B. Well defined relative phases between eigenstates with different momenta
   - C. Able to observe coherence between neutrinos

With same energy and different momenta

Many papers do not correctly describe the detector

05 THE RIGHT WAY TO TREAT FLAVOR OSCILLATIONS

WHAT IS THE PROBLEM?

An amplitude with definite flavor is created at a source

A coherent mixture of amplitudes from mass eigenstates

Neutrinos propagate freely from source to detector

Mass eigenstates propagate independently - no interactions

Relative phases of mass eigenstates change during propagatior

The amplitude flavor is measured at a remote detector

WHAT IS THE SOLUTION?

1. Solve the free Schroedinger or Dirac Equation
2. Introduce the proper initial conditions at the source
3. Get the answer for what is observed at the detector

The free Dirac or Schroedinger Equation is trivial

No need for fancy field theory or Feynman diagrams

No need for Lorentz transformations

Mixtures of noninteracting mass amplitudes - no problem

WHY DOESN'T EVERYONE DO THIS?

Interpreting the Standard Textbook Wave Function

Real & Gedanken ν-oscillation Experiments

Source creates particle mixture - two or more mass eigenstate:

Different mixture observed in detector

Flavor eigenstate with sharp momentumm - different energies

Oscillates in time with well-defined oscillation period

Flavor eigenstate with sharp energy - different momenta

Oscillates in space with well-defined oscillation wave length

Confusion in Description of Flavor Oscillations

Sharp momentum or sharp energy - “Gedanken” experiments

Conventional Wisdom - Oscillations in Time

For simplicity assume 45^o mixing angle

$$|\nu_e\rangle = (1/\sqrt{2})(|\nu_1\rangle + |\nu_2\rangle); \qquad |\nu_\mu\rangle = (1/\sqrt{2})(|\nu_1\rangle - |\nu_2\rangle)$$

ν_e produced at t=0 with momentum p and energies

$$E_1^2 = p^2 + m_1^2; \qquad E_2^2 = p^2 + m_2^2$$

$|\nu_e\rangle$ and $|\nu_\mu\rangle$ components oscillate in time

$$\left|\frac{\langle\nu_\mu\,|\nu_e(t)\rangle}{\langle\nu_e\,|\nu_e(t)\rangle}\right| = \left|\frac{e^{iE_1t} - e^{iE_2t}}{e^{iE_1t} + e^{iE_2t}}\right| = \tan\left(\frac{(E_1 - E_2)t}{2}\right) =$$

$$= \tan\left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)}\right)$$

This is a “non-experiment”. Real experiment measures space

Now Comes the Hand Waving - Method A

Convert time into distance

$$x = vt = \tfrac{p}{E}\cdot t$$

$$\left|\frac{\langle\nu_\mu\,|\nu_e(t)\rangle}{\langle\nu_e\,|\nu_e(t)\rangle}\right| = \tan\left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)}\right) \approx \tan\left(\frac{(m_1^2 - m_2^2)x}{4p}\right)$$

Problems with Hand Waving - Method A again

$$x = vt = \tfrac{p}{E}\cdot t$$

$$\left|\frac{\langle\nu_\mu\,|\nu_e(t)\rangle}{\langle\nu_e\,|\nu_e(t)\rangle}\right| = \tan\left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)}\right) \approx \tan\left(\frac{(m_1^2 - m_2^2)x}{4p}\right)$$

A Different Hand Waving - Method B

But ν_1 and ν_2 states have different velocities

$$x = v_1t_1 = \tfrac{p}{E_1}\cdot t_1 = v_2t_2 = \tfrac{p}{E_2}\cdot t_2$$

$$\left|\frac{\langle\nu_\mu\,|\nu_e(x)\rangle}{\langle\nu_e\,|\nu_e(x)\rangle}\right| = \left|\frac{e^{iE_1t_1} - e^{iE_2t_2}}{e^{iE_1t_1} + e^{iE_2t_1}}\right| = \tan\left(\frac{(E_1t_1 - E_2t_2)}{2}\right) =$$

$$= \tan\left(\frac{(m_1^2 - m_2^2)x}{2p}\right)$$

Differs by factor of 2 in oscillation wave length. Which is correc

A Real Calculation Without Hand Waving (?)

All confusion avoided by direct use of real experiment

ν_e produced at x=0 with energy E

Only neutrinos with SAME ENERGY can be coherent at detect

$|\nu_e\rangle$ and $|\nu_\mu\rangle$ components oscillate in space

$$\left|\frac{\langle\nu_\mu|\nu_e(x)\rangle}{\langle\nu_e|\nu_e(x)\rangle}\right| = \left|\frac{e^{ip_1x}-e^{ip_2x}}{e^{ip_1x}+e^{ip_2x}}\right| = \tan\left(\frac{(p_1-p_2)x}{2}\right) =$$

$$= \tan\left(\frac{(m_1^2-m_2^2)x}{2(p_1+p_2)}\right)$$

Simple argument is right

Treatment is completely relativistic

Needs no discussion of time dependence or “proper times”

03 Quantum Mechanics Guide to Flavor Oscillations

Why classical particle description is wrong

Energy-momentum kinematics

Example of pion decay at rest $\pi \to \mu\nu$

$$E_\pi = M_\pi; \qquad p_\pi = 0$$

$$E_\nu = M_\pi - E_\mu; \qquad p_\nu = -p_\mu$$

$$M_\nu^2 = (M_\pi - E_\mu)^2 - p_\mu^2$$

Missing Mass Experiment - M_ν known. No interference

Space-time measurements

Source Detector

Neutrino created at $(x=0,\ t=0)$ with momentum p

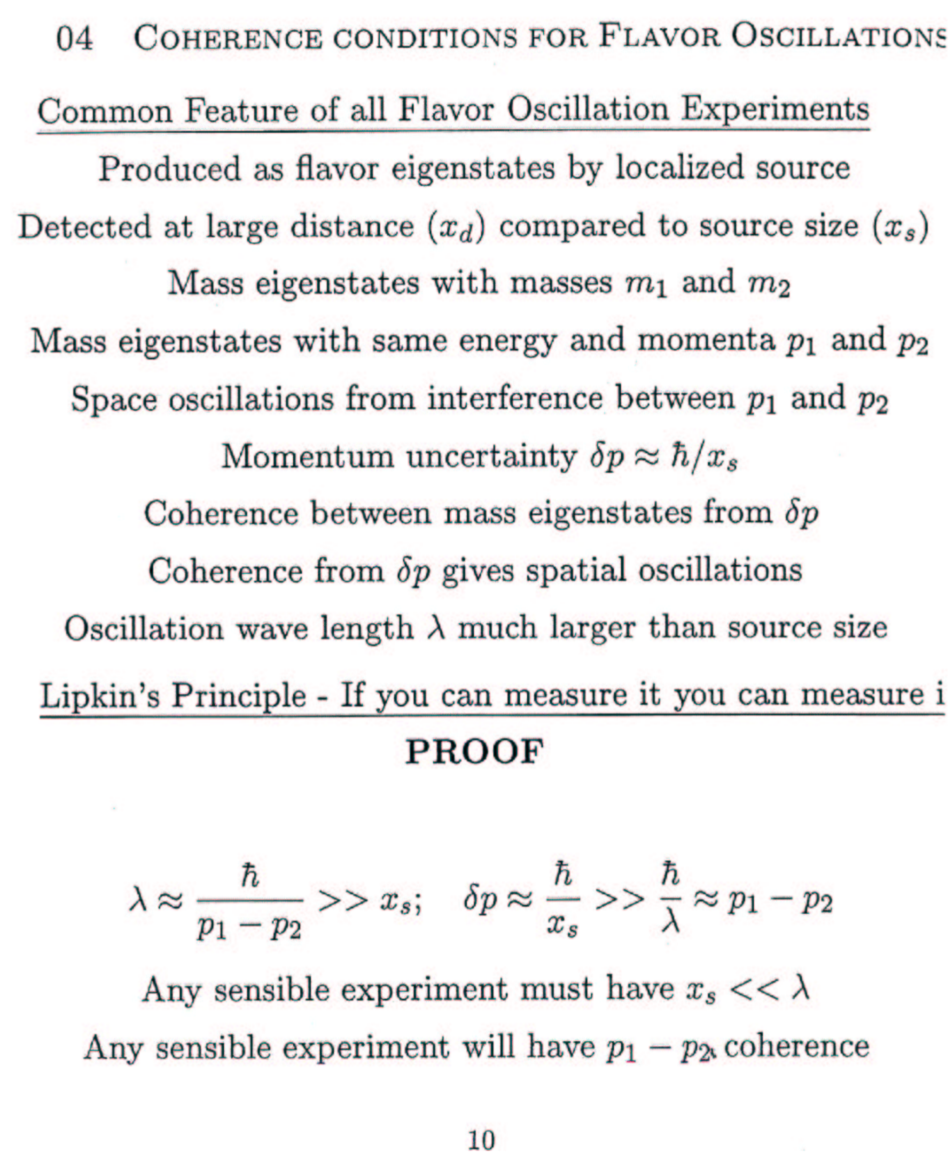
Neutrino detected at $(x=x_d,\ t=?)$

Neutrino velocities different $v_1 = \frac{p}{m_1}$; $v_2 = \frac{p}{m_2}$

Arrival times different - $t_1 = \frac{x_d \cdot m_1}{p}$ $t_2 = \frac{x_d \cdot m_2}{p}$

No coherence; no interference

Solutions to Paradoxes - Wave-particle duality



04 COHERENCE CONDITIONS FOR FLAVOR OSCILLATIONS

Common Feature of all Flavor Oscillation Experiments

Produced as flavor eigenstates by localized source

Detected at large distance (x_d) compared to source size (x_s)

Mass eigenstates with masses m_1 and m_2

Mass eigenstates with same energy and momenta p_1 and p_2

Space oscillations from interference between p_1 and p_2

Momentum uncertainty $\delta p \approx \hbar / x_s$

Coherence between mass eigenstates from δp

Coherence from δp gives spatial oscillations

Oscillation wave length λ much larger than source size

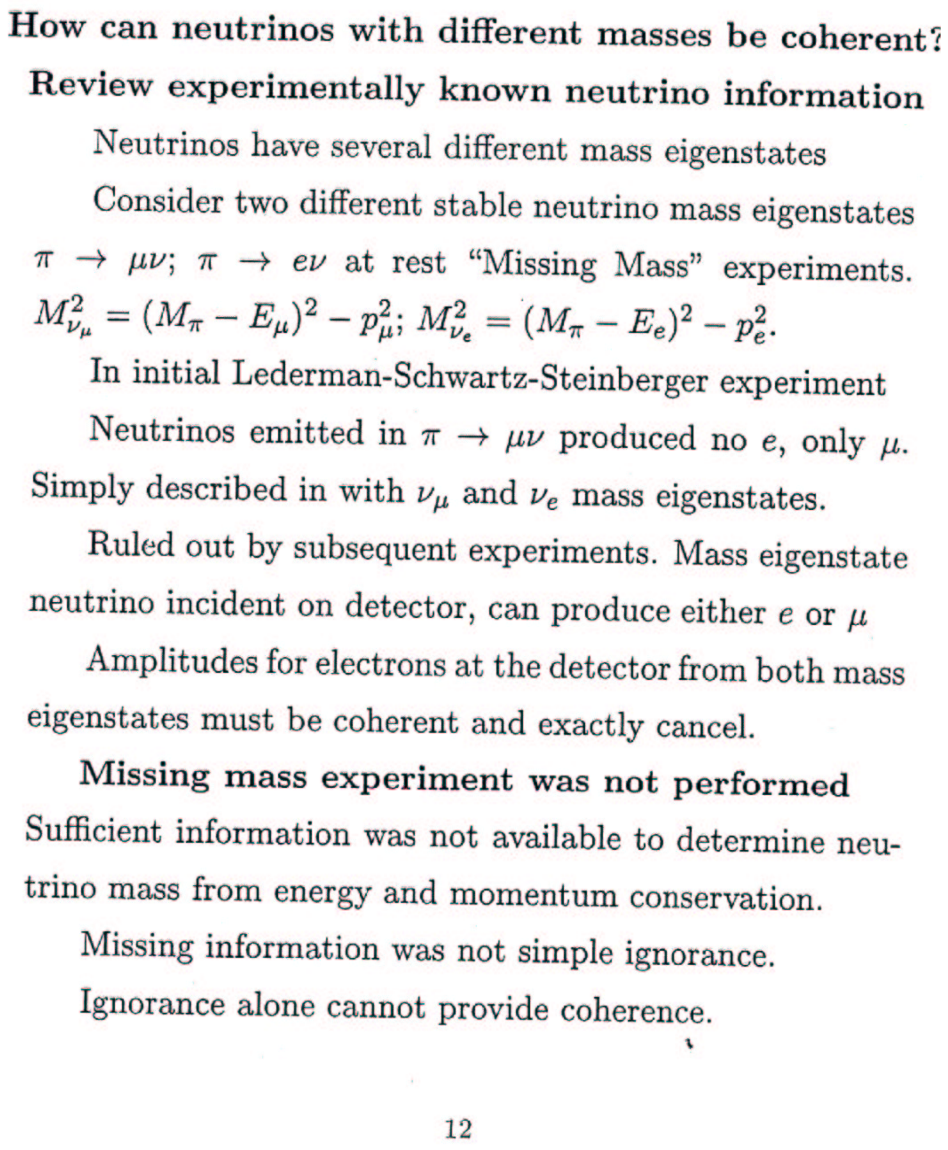
Lipkin's Principle - If you can measure it you can measure i

PROOF

$$\lambda \approx \frac{\hbar}{p_1 - p_2} >> x_s; \quad \delta p \approx \frac{\hbar}{x_s} >> \frac{\hbar}{\lambda} \approx p_1 - p_2$$

Any sensible experiment must have $x_s << \lambda$

Any sensible experiment will have $p_1 - p_2$ coherence



How can neutrinos with different masses be coherent?

Review experimentally known neutrino information

Neutrinos have several different mass eigenstates

Consider two different stable neutrino mass eigenstates

$\pi \rightarrow \mu\nu$; $\pi \rightarrow e\nu$ at rest "Missing Mass" experiments.

$M^2_{\nu_\mu} = (M_\pi - E_\mu)^2 - p^2_\mu$; $M^2_{\nu_e} = (M_\pi - E_e)^2 - p^2_e$.

In initial Lederman-Schwartz-Steinberger experiment

Neutrinos emitted in $\pi \rightarrow \mu\nu$ produced no e, only μ.

Simply described in with ν_μ and ν_e mass eigenstates.

Ruled out by subsequent experiments. Mass eigenstate neutrino incident on detector, can produce either e or μ

Amplitudes for electrons at the detector from both mass eigenstates must be coherent and exactly cancel.

Missing mass experiment was not performed

Sufficient information was not available to determine neutrino mass from energy and momentum conservation.

Missing information was not simple ignorance.

Ignorance alone cannot provide coherence.