Boris Kayser Day!

Neutrino Mass Matrix and Hierarchy

Peter Kaus
U.C. Riverside
and
Sydney Meshkov
Caltech

KITP at UCSB
Neutrino Conference
March 3 – 7, 2003

FERMILAB-Pub-73-026-E, **Inclusive pi - Distributions from 205 GeV/c pp Interactions**, Y. Cho, et al.

FERMILAB-Conf-73/027-T, **Theoretical Interpretations of Neutrino Experiments**, Emmanuel A. Paschos

FERMILAB-Conf-73/028-T/E, **The Impact Picture for Strong Interactions**, J. K. Walker

FERMILAB-Pub-73-029-T, **A New Approach to Relations Between Deep Inelastic Structure Functions**, H. J. Lipkin and E.A. Paschos

FERMILAB-Conf-73/030-E, **v-p Interactions at 200 GeV/c**, D. Bogert, et al.

FERMILAB-Conf-73/031-E, **pp interactions at 303 GeV/c: Inclusive Studies of Gamma, K0(S), Lambda0, PI+- and Proton Production**, F. T. Dao, et al.

FERMILAB-Pub-73-032-E, **Adjustment of the Crystal Diffraction Measurement of the Pion Mass**, Robert E. Shafer

FERMILAB-Pub-73-033-T, **Impact Picture and Diffractive Dissociation**, Hung Cheng, James K. Walker and Tai Tsun Wu

FERMILAB-Pub-73-034-T, **Ratio of Real to Imaginary Parts for PP, P bar P, pi± P and K±P Forward Elastic Scattering Amplitudes from 50 to 5000 GeV**, Hung Cheng, James K. Walker and Tai Tsun Wu

FERMILAB-Pub-73-036-T, **A Field Theory Formulation of the Parton Model**, A. I. Sanda

FERMILAB-Pub-73-037-E **Study Of Charged Multiplicity Distributions in High Energy Particle Collisions**, F. T. Dao, J. Lach, and J. Whitmore

FERMILAB-Pub-73-038-E, **pp Interactions at 303 GeV/c: Separation of Diffractive and Nondiffractive Components of the Multiplicity Distribution**, F. T. Dao, et al.

FERMILAB-Conf-73-039-T, **Introduction to Conformal Invariance**, C. Callan

FERMILAB-Pub-73/040-T, **Implications of Parton Model Concepts for Large Transverse Momentum Production of Hadrons**, S. D. Ellis, et al.

FERMILAB-Pub-73-041-T, **The Complex Helicity Plane**, R. C. Brower, et al.

FERMILAB-Pub-73-042-T, **Completely Unitary Multiparticle Models**, C. S. Lam

FERMILAB-Pub-73-043-T, **SU(3) Symmetry In Electron-Positron Annihilation**, Harry J. Lipkin

FERMILAB-Pub-73-044-T, **Tests of Higher Symmetries**, B. Kayser, H. J. Lipkin and S. Meshkov

FERMILAB-Pub-73/045-T, **Positivity and the Axial-Vector Current in Quantum Electrodynamics**, R. J. Crewther, et al.

FERMILAB-Pub-73/046-T, **Patterns of Diffractive Production**, D. Horn, et al.

FERMILAB-Pub-73-047-E, **Study of Neutral -Charged Particle Correlations, in High-Energy Collisions**, F. T. Dao and J. Whitmore

FERMILAB-Pub-73-048-T, **I=1/2 Contributions to nu(mu) + N ---> nu(mu) + N + pi0 in the Weinberg Weak-Interaction Model**, Stephen L. Adler

Information Resources Department

Preprints, Theses and Proposals

1973 Technical Publications

Preprints
73/001 through 73/050

Looking for a particular preprint, but not sure of the reference number, title or author? Search the Library Online Catalog for help. There you will find references to all Fermilab preprints. Log on to the catalog here to begin your

NOTE: All of the 1973 preprints are available in PDF (portable document format) only. You will need Adobe Acrobat Reader to view these files. A free Reader can be downloaded from Adobe's Website:

| 73/001 - 73/050 | 73/051 - 73/100 | 73/101 - 73/150 | 1973 preprints homepage

FERMILAB-Pub-73-013-T, **Do Light Pions Necessarily Obey PCAC?**, Roger Dashen and David J. Gross

FERMILAB-Pub-73-014-T, **Applications of Dual Resonance Models to Inclusive Reactions**, Martin B. Einhorn

FERMILAB-Pub-73-015-T, **Multiplicity Distribution in Diffractive Dissociation at High Energies**, William R. Frazer and Dale R. Snider

FERMILAB-Pub-73-016-E, **Sigma- Production In High Energy Proton Interactions**, V. Hungerbuehler, et al.

FERMILAB-Conf-73/017-T, **Dynamical Issues in Hadron Physics**, Henry D.I. Abarbanel

FERMILAB-Pub-73-018-T, **Physics at the ISR: A Review of Recent Results**, M. Jacob

FERMILAB-Pub-73-019-T, **Testing the Adler and Gross - Llewellyn Smith Sum Rules in High Energy Neutrino Reactions**, D. Cline and E. A. Paschos

FERMILAB-Pub-73/020-E, **pp Interactions at 303 GeV/c; Tests of Scaling in Single Particle pi +- and Proton Distributions**, F. T. Dao, et al.

FERMILAB-Pub-73-021-E, **pp Interactions at 303 GeV/c: Inclusive Studies of gamma, KS0 and lambda0 Production**, F. T. Dao, et al.

FERMILAB-Pub-73-022-E, **pp Interactions at 303 GeV/c: Analysis of Diffraction Excitation in the Reaction pp ---> pX**, F. T. Dao, et al.

FERMILAB-Pub-73/023-T, **Neutrino Cross Sections in a Quark Parton Model with the Weinberg Neutral Current**, Carl H. Albright

FERMILAB-Pub-73-024-E, **pp Interactions at 303 GeV/c: Production Angle Distribution**, F.T. Dao, et al.

FERMILAB-Conf-73/025-E, **pp Interactions At 205 GeV/c**, S. Barish et al.

FERMILAB-Pub-73-049-T, **Asymptotically Free Gauge Theories - I***, David J. Gross and Frank Wilczek

FERMILAB-Pub-73-050-T, **Inclusive Decay Amplitudes**, D. Horn

| 73/001 - 73/050 | 73/051 - 73/100 | 73/101 - 73/150 | 1973 preprints homepage

Home Preprints Online Catalog Services Reference Instructions & Policies Forms Journals Drawing Numbers Site Index Mailto

(1)

Neutrino Model

1. Assume Strict Hierarchy
   Analagous to quarks, charged leptons.
2. Use known Data: Δ_{sol}, Δ_{atm}, Mixing $\angle$s from ν Oscillation Expts.
3. Assuming 1. is a statement about mass ratios

$\boxed{1,2,3} \rightarrow$ M_ν Mass Matrix
U Mixing Matrix

4. M_ν, U matrix elements given in powers of Λ
5. $\underset{\text{neutrinos}}{\Lambda}$ = analog of $\underset{\text{quarks, charged leptons}}{\lambda}$
6. $\Lambda \simeq 1.6\,\lambda$
7. $m_{\beta\beta 0\nu} \leq 0.004$ eV $(s_{13}=0)$
   ≤ 0.006 eV $(s_{13} \leq .13)$

hep-ph/0211338
(revised to include KamLAND)

(2)

Quarks + Charged Leptons

Hierarchical Model Successful!

$M, U \equiv f(\lambda = \sqrt{m_2/m_3})$ $0.22 < \lambda < 0.25$

Can we do analogue for Neutrinos?

Big Problem!

Know only 2 mass squared differences

$\Delta_{sol} = m_2^2 - m_1^2 \simeq 7.1 \times 10^{-5}\,eV^2$

$\Delta_{atm} = m_3^2 - m_2^2 \simeq 2.7 \times 10^{-3}\,eV^2$

This allows m_2/m_3 to range from 1 (degeneracy) to small ~.1-.2 (hierarchy)

To determine 3 masses,
Need one more Eqn.

Provided by Hierarchy Assumption.

Mass Matrix M

(3)

$$M = \begin{vmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{vmatrix}$$

m_1, m_2, m_3 = Neutrino mass eivs (Maj.)

M can be undiagonalized by U

$M_\nu = U M U^{-1}$

See Ma hep-ph 0208097

M_ν has flavor eigenstates

$\psi_{\nu_e}, \psi_{\nu_\mu}, \psi_{\nu_\tau}$

Mixing Matrix U

(4)

U rotates mass (Maj) eigenstates $\psi_{1,2,3}$ into flavor eigenstates $\psi_{\nu_e, \nu_\mu, \nu_\tau}$

$$U = \begin{bmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{-i\partial} \\ s_{12}c_{23} + c_{12}s_{13}s_{23}e^{i\partial} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\partial} & -c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\partial} & c_{12}s_{23} + s_{12}s_{13}c_{23}e^{i\partial} & c_{13}c_{23} \end{bmatrix}$$

Let $\delta = 0$ (No CP violation)

Let $S_{23} = C_{23} = \frac{1}{\sqrt{2}}$ (Maximal mixing for Atmospheric Oscillations)

$$U = \begin{bmatrix} Cc_{13} & -Sc_{13} & s_{13} \\ \frac{1}{\sqrt{2}}S + \frac{1}{\sqrt{2}}Cs_{13} & \frac{1}{\sqrt{2}}C - \frac{1}{\sqrt{2}}Ss_{13} & -\frac{1}{\sqrt{2}}c_{13} \\ \frac{1}{\sqrt{2}}S - \frac{1}{\sqrt{2}}Cs_{13} & \frac{1}{\sqrt{2}}C + \frac{1}{\sqrt{2}}Ss_{13} & \frac{1}{\sqrt{2}}c_{13} \end{bmatrix}$$

θ_{13} small

Fogli et al

$S_{13} \leq 0.13$ (No lower limit yet)

From Kamland Data Analysis

Assume $S_{13} = 0$ [Will do finite value analysis later.]

$\therefore C_{13} = 1$

5

$$U = \begin{bmatrix} C & -S & 0 \\ S/\sqrt{2} & C/\sqrt{2} & -1/\sqrt{2} \\ S/\sqrt{2} & C/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

where $S = \sin(\theta_{12}), C = \cos(\theta_{12})$ and we have set the rotation angle $\theta_{23} = \pi/4$ (maximal mixing) and the angle $\theta_{13} = 0$, no CP violation.

To lowest order in S (expanding C in Eq. (3) in terms of S), U is given by

$$U_1 = \begin{bmatrix} 1 & -S & 0 \\ S/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ S/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

which, except for the extreme θ_{23} mixing, is much like the quark mixing matrix. In fact, if we consider U_1 to be the result of two successive rotations, $U_1 = v_1 v_0$, we get

$$v_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

and

$$v_1 = \begin{bmatrix} 1 & -S/\sqrt{2} & -S/\sqrt{2} \\ S/\sqrt{2} & 1 & 0 \\ S/\sqrt{2} & 0 & 1 \end{bmatrix}$$

Suggests that an appropriate expansion parameter for U is $\varepsilon = S/\sqrt{2}$

$$\varepsilon = S/\sqrt{2} = \sin(\theta_{12})/\sqrt{2}$$

Currently: $\tan^2(\theta_{12}) = T^2 \simeq 0.45$

$\theta_{12} \simeq 34°$, $S^2 = 0.31$

$\therefore \varepsilon^2 \simeq S^2/2 \simeq 0.16$

Barger+Marfatia,
Bahcall et al,
Pascoli,
Petcov,
Rodejohann

Diagonal Mass Matrix

6

Define conventional hierarchical mass pattern by

$$m_3 : m_2 : m_1 = 1 : \Lambda^2 : \rho\Lambda^4$$

$\rho = 1 \Rightarrow$ strict hierarchy

ρ expresses our ignorance of m_1

Three eigenvalues of M

$$M = m_3 \begin{bmatrix} \rho\Lambda^4 & 0 & 0 \\ 0 & \Lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

m_3

$m_2 = \Lambda^2 m_3$

$m_1 = \rho\Lambda^4 m_3$

Hierarchy expansion parameter Λ

$$\Lambda \equiv \sqrt{m_2/m_3}$$

7

Evaluation of Λ

$\Lambda^2 \equiv m_2/m_3$ in hierarchical expansion

$\Delta_{sol} = m_2^2 - m_1^2 \simeq 7.1 \times 10^{-5}\ eV^2$

$\Delta_{atm} = m_3^2 - m_2^2 \simeq 2.7 \times 10^{-3}\ eV^2$ Pascoli et al, Fogli et al

$$\sqrt{\frac{\Delta_{sol}}{\Delta_{atm}}} = \sqrt{\frac{m_2^2 - m_1^2}{m_3^2 - m_2^2}} = \sqrt{\frac{\Lambda^4 m_3^2 - \rho^2 \Lambda^8 m_3^2}{m_3^2 - \Lambda^4 m_3^2}}$$

$$= \Lambda^2 + \frac{1}{2}\Lambda^6(1-\rho^2) \quad \left(\text{Expanding in } \Lambda^2\right)$$

$\therefore$ To order Λ^4

$$\Lambda^2 = \sqrt{\frac{\Delta_{sol}}{\Delta_{atm}}} = 0.16$$

$$\varepsilon^2 = S^2/2 = 0.16$$

So: Phenomenologically, There is a close relation between Λ and ε, or between Λ and θ_{12}!

8

M and U as $M(\Lambda), U(\Lambda)$

Assume that $\Lambda = S/\sqrt{2}$, $\Lambda^2 = S^2/2$

Express M and U as functions of one parameter $\Lambda = \sqrt{m_2/m_3}$ in analogy with the Wolfenstein parametrization for quarks and charged leptons.

$$U \doteq \begin{bmatrix} \sqrt{(1-2\Lambda^2)} & -\sqrt{2}\Lambda & 0 \\ \Lambda & \sqrt{(1/2)(1-2\Lambda^2)} & -1/\sqrt{2} \\ \Lambda & \sqrt{(1/2)(1-2\Lambda^2)} & -1/\sqrt{2} \end{bmatrix}$$

All expansions are made in terms of $\Lambda^2 = 0.16$

$$M_\nu = U M U^{-1}$$

(9)

$$M = m_3 \begin{bmatrix} \Lambda^4\rho & 0 & 0 \\ 0 & \Lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To order Λ^4, M_ν, is given by

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\rho+2) & -\Lambda^3 & -\Lambda^3 \\ -\Lambda^3 & -\Lambda^4 + (1/2)\Lambda^2 + 1/2 & -\Lambda^4 + (1/2)\Lambda^2 - 1/2 \\ -\Lambda^3 & -\Lambda^4 + (1/2)\Lambda^2 - 1/2 & -\Lambda^4 + (1/2)\Lambda^2 + 1/2 \end{bmatrix}$$

Determine m_3, m_2, m_1 to order Λ^4.

Doing this depends on the Hierarchy Assumption, but is independent of U. $\Lambda = \sqrt{\frac{m_2}{m_3}} = \sqrt{.16} = 0.4$

$$m_3 = \sqrt{\Delta_{atm} + \Delta_{sol}} = 5.2 \times 10^{-2}\,eV$$

$$m_2 = \Lambda^2 m_3 = \sqrt{\Delta_{sol}/\Delta_{atm}}\, m_3 = 8.3 \times 10^{-3}\,eV$$

$$m_1 = \rho\Lambda^4 m_3 = \rho(\Delta_{sol}/\Delta_{atm}) m_3 = 1.3\rho \times 10^{-3}\,eV$$

ρ is a parameter of order 1.

It expresses our ignorance of the lightest neutrino mass, m_1.

(10)

Effective Neutrino Mass $\langle m \rangle$ measured in $\beta\beta 0\nu$

$\langle m \rangle$ is given by $M_{\nu_{ee}}$, the $(1,1)$ matrix element of M_ν.

Using U and M, to order Λ^6 we obtain:

$$\langle m \rangle = M_{\nu_{ee}} = m_3 \Lambda^4 [\rho(1-2\Lambda^2)+2]$$
$$= (0.7\rho + 2) \times 1.3 \times 10^{-3}\,eV$$

Various limits:

Upper limit – (for $S_{13}=0$) $\rho = 2$, $m_1 = 2.6 \times 10^{-3}\,eV$

$$\langle m \rangle \le 4 \times 10^{-3}\,eV$$

Lower limit – $\rho = -2$, $m_1 = -2.6 \times 10^{-3}\,eV$

$$\langle m \rangle \simeq 8 \times 10^{-4}\,eV$$

$$\therefore \quad 10^{-3}\,eV < \langle m \rangle < 4 \times 10^{-3}\,eV$$

(11)

Finite $S_{13} = \sin(\theta_{13})$

Scale S_{13}: $S_{13} = d\ell\,\Lambda^2$ $|d\ell| \le 1$ Ma, Giunti + Tanimoto

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\kappa^2+\rho+2) & -\Lambda^3 - \frac{1}{\sqrt{2}}\Lambda^2\kappa & -\Lambda^3 + \frac{1}{\sqrt{2}}\Lambda^2\kappa \\ -\Lambda^3 - \frac{1}{\sqrt{2}}\Lambda^2\kappa & -\frac{1}{2}\Lambda^4(\kappa^2+1) + \frac{1}{2}\Lambda^2 + \frac{1}{2} & \frac{1}{2}\Lambda^4(\kappa^2+1) + \frac{1}{2}\Lambda^2 - \frac{1}{2} \\ -\Lambda^3 + \frac{1}{\sqrt{2}}\Lambda^2\kappa & \frac{1}{2}\Lambda^4(\kappa^2-1) + \frac{1}{2}\Lambda^2 - \frac{1}{2} & -\frac{1}{2}\Lambda^4(\kappa^2+1) + \frac{1}{2}\Lambda^2 + \frac{1}{2} \end{bmatrix}$$

2 parameters ρ, $d\ell$, each of order 1

ρ = defined by $m_1 = \rho\Lambda^4 m_3$

$d\ell$ = " " $S_{13} = d\ell\,\Lambda^2$

Three interesting regimes for $d\ell$

1. $d\ell = S_{13} = 0$. Discussed before.

$M_{\nu_{ee}} = \langle m \rangle = \Lambda^4(\rho+2)m_3$

$M_{\nu_{e\mu}} = M_{\nu_{e\tau}} = -\Lambda^3 m_3$

2. $d\ell = 1$, $S_{13} = \Lambda^2$

$M_{\nu_{ee}} = \langle m \rangle = \Lambda^4(\rho+2+d\ell^2)m_3 = \Lambda^4(\rho+3)m_3$

$M_{\nu_{e\mu}} = M_{\nu_{e\tau}} = \pm\frac{1}{\sqrt{2}}\Lambda^2 m_3$

3. $d\ell = d\ell'\Lambda$, $S_{13} = d\ell'\Lambda^3$

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\rho+2) & -\Lambda^3\left(1+\frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^3\left(1-\frac{\kappa'}{\sqrt{2}}\right) \\ -\Lambda^3\left(1+\frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^4+\Lambda^2/2+\frac{1}{2} & -\Lambda^4+\Lambda^2/2-\frac{1}{2} \\ -\Lambda^3\left(1-\frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^4+\Lambda^2/2-\frac{1}{2} & -\Lambda^4+\Lambda^2/2+\frac{1}{2} \end{bmatrix}$$

$$M_{\nu_{e\mu}} = -\Lambda^3\left[1 + d\ell'/\sqrt{2}\right]m_3$$

$$M_{\nu_{e\tau}} = -\Lambda^3\left[1 - d\ell'/\sqrt{2}\right]m_3$$

Case 3 is the only case that allows a $\underline{0}$ in an off diagonal element.

$\Rightarrow$ Texture Zero

(12)

Texture Zeros

Have been considered as a source of hierarchies and mixing angles. (Ramond, Ling, Frampton, Glashow, Marfatia, King, Desai...)

Looking at $M_{\nu_{4,3}}$ only $d\ell = d\ell'\Lambda$ provides the possibility of having $\ge$ 2 texture 0's.

Can we get more 0's ?

Try $\rho = -2$ (Kills Mee)

$d\ell' = \pm\sqrt{2}$ (Either $M_{\nu_{e\mu}}$ or $M_{\nu_{e\tau}}$) = 0

to order Λ^4

More Texture 0

(13)

Let $\rho = -2$, $d\ell' = -\sqrt{2}$

$$M_{\nu_4} = m_3 \begin{bmatrix} 0 & 0 & -2\Lambda^3 \\ 0 & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} \\ -2\Lambda^3 & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} \end{bmatrix}$$

In terms of masses, $m_2 = \Lambda^2 m_3$, $m_1 = -2\Lambda^4 m_3$ ρ

$$M_{\nu_4} = \begin{bmatrix} 0 & 0 & -\sqrt{(-2m_1m_2)} \\ 0 & \frac{1}{2}(m_1+m_2+m_3) & \frac{1}{2}(m_1+m_2-m_3) \\ -\sqrt{(-2m_1m_2)} & \frac{1}{2}(m_1+m_2-m_3) & \frac{1}{2}(m_1+m_2+m_3) \end{bmatrix}$$

Identical to matrix derived by Desai et al, who categorize all mass matrices with 2 texture 0s

To compare with model of hierarchy generation by Ramond et al, Let $\rho = -2$, Let $S_{13} = d\ell'\Lambda^3$. Keep leading order of Λ in each matrix element. We obtain:

$$M_{\nu_6} = m_3 \begin{bmatrix} \Lambda^6(4+\kappa'^2) & -\Lambda^3(1+\frac{1}{\sqrt{2}}\kappa') & -\Lambda^3(1-\frac{1}{\sqrt{2}}\kappa') \\ -\Lambda^3(1+\frac{1}{\sqrt{2}}\kappa') & \frac{1}{2} & -\frac{1}{2} \\ -\Lambda^3(1-\frac{1}{\sqrt{2}}\kappa') & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

The leading orders of Λ in each M.E.. are of the orders indicated in Ramond et al and "tuned" by Fishbane and Kaus. This model (a SUSY extension of the SM) predicts Λ is the same for all family sectors. Its not; $\Lambda \simeq 0.4$, $\lambda \simeq 0.25$.

Conclusions

(14)

1. The data and the assumed hierarchy pattern imply:

   Mixing Matrix U } Expressed in Powers of Expansion Parameter Λ
   Mass Matrix M_ν }

   AND 2 parameters $\rho + d\ell$ (of order 1)

   ρ expresses ignorance of lightest mass m_1

   $m_1 = 1.3\rho \times 10^{-3}$ eV

   $d\ell$ scales S_{13} to upper exp. limit

   $S_{13} \simeq d\ell\Lambda^2 \simeq 0.16 d\ell$

2. $\Lambda \triangleq$

   $\Lambda^2 = \frac{m_2}{m_3} = \sqrt{\frac{\Delta_{sol}}{\Delta_{atm}}} \simeq 0.16$

   Λ is identical in spirit to λ but $\simeq 1.6\lambda$

3. $m_{\beta\beta0\nu} \leq 4 \times 10^{-3}$ eV ($S_{13} = 0$)

   $\leq 6 \times 10^{-3}$ eV (S_{13} maximal)

References

[1] H. Fritzsch and Z. Xing, Prog. Part. Nucl. Phys., **45** (2000) 1, and Ref. 51 therein; P. Fishbane and P. Kaus, J. Phys., **G26** (2000) 295.

[2] E. Ma, arXiv: hep-ph/0208097.

[3] S. Pascoli and S.T. Petcov, Phys. Lett., **B544** (2002) 239.

[4] See, for example, F. S. Ling and P. Ramond, Phys. Lett., **B543** (2002) 29; and earlier work, Irges, S. Lavignac and P. Ramond, Phys. Rev., **D58** (1998) 035003.

[5] S. F. King, arXiv: hep-ph/0208266.

[6] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys., **28** (1962) 870; Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett., **86** (2001) 5651-5656.

[7] S. Fukuda et. al., Phys. Rev. Lett., **85** (2000) 3999 and references therein.

[8] M. Appolonio et al., Phys. Lett., B **466** (1999) 415 (CHOOZ Collaboration.

[9] F. Boehm et al., Phys. Rev., **D64** (2001) 112001 (PaloVerde Collaboration).

[10] G.L. Fogli, et al., arXiv: hep-ph/021247.

[11] K. Eguchi, et al., Phys. Rev. Lett., **90** (2003) 021802.

[12] J.N. Bahcall, M.C. Gonzales-Garcia and C. Pena-Garay, arXiv: hep-ph/0212147.

[13] V. Barger and D. Marfatia, arXiv: hep-ph/0212126.

[14] S. Pascoli, S.T. Petcov and W. Rodejohann, arXiv: hep-ph/0212113.

[15] Q. R. Ahmed et al., Phys. Rev. Lett., **89** (2002) 011301, 011302.

[16] L. Wolfenstein, Phys. Rev. Lett., **51** (1983) 1945.

[17] G.L. Fogli, et al., arXiv: hep-ph/0208026.

[18] C. Giunti and M. Tanimoto, Phys. Rev., **D66** (2002) 053013.

[19] P. Frampton, S. Glashow, and D. Marfatia, Phys. Lett., **B536** (2002) 79.

[20] B. R. Desai, D. P. Roy, and A. R. Vaucher, arXiv: hep-ph/0209035.

[21] P. Fishbane and P. Kaus, J. Phys., **G25** (1999) 1629.