On the MSW effect of the neutrino background

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Introduction: why neutrino background?

- Interactions with background matter modify neutrino dispersion relation (*Wolfenstein, 1977*)

  $\rightarrow$ the modification is *flavor-dependent*

  $\rightarrow$ plays an important role in neutrino flavor evolution
- Ordinarily (in Sun, Earth) the interaction is with background electrons and nucleons
- In certain cases, neutrino number density $\gtrsim$ number density of "normal matter"

  $\rightarrow$ neutrino "self-induced" refraction may be important

Supernova & Early Universe

- In the early Universe, $n_\nu \sim n_\gamma \gg n_{baryons}, (n_e - n_{\bar{e}})$
  $\rightarrow$ neutrino self-refraction is relevant for studies of flavor evolution: effects of lepton chemical potential, lepton asymmetry generation through active-sterile conversions, etc.
- In supernova, near the core (r $\sim$ 15 – 30 km), $n_\nu \sim n_{baryons}$
  $\rightarrow$ various applications, e. g., synthesis of heavy elements

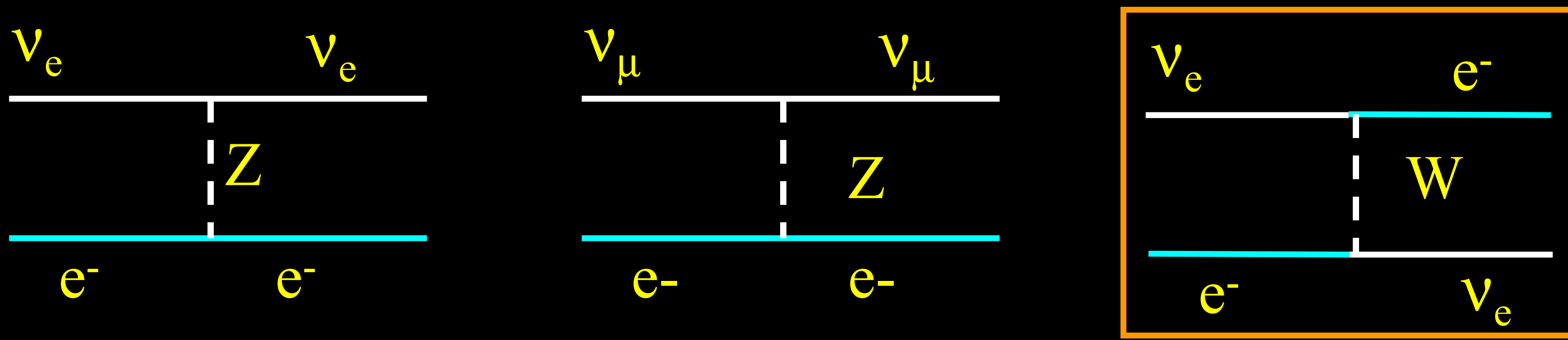
Early Work

Fuller, Mayle, Wilson & Schramm, ApJ., 1987

Notzold & Raffelt, Nucl. Phys., 1988

- Recognized the importance of neutrino self-refraction in supernova
- Treated the problem by analogy with "conventional MSW"

"Conventional MSW"

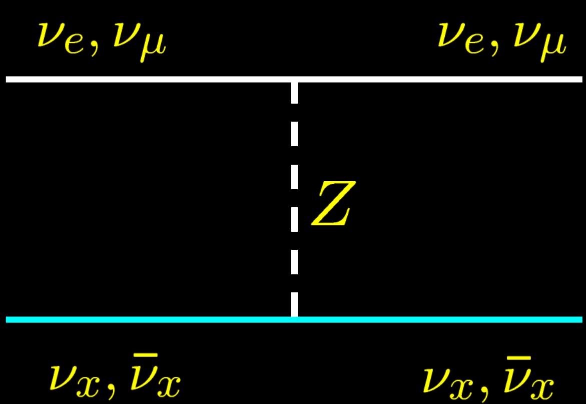


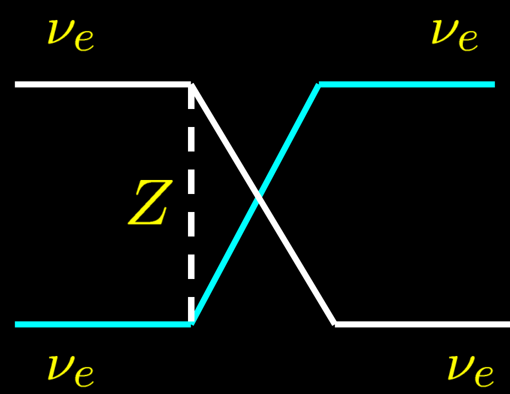
- Evolution Hamiltonian for a single neutrino

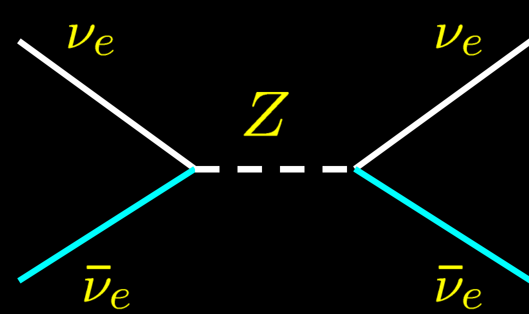
$$H = H_{vac} + \Delta H_{matt}$$

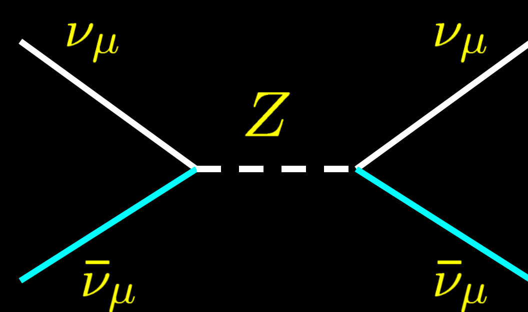
$$\Delta H_{matt} \;=\; \sqrt{2} G_F (n_e - n_{\bar{e}}) \begin{pmatrix} 1 + 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Early works constructed the Hamiltonian due to the neutrino background by analogy









$$H = H_{vac} + \Delta H_{matt} + \Delta H_{\nu\nu}$$

$$\begin{aligned}\Delta H_{\nu\nu} &= \sqrt{2} G_F \langle 1 - \cos\Theta \rangle \\ &\times \Bigg[(n_{\nu_e} - n_{\bar{\nu}_e} + n_{\nu_\mu} - n_{\bar{\nu}_\mu}) \\ &+ \begin{pmatrix} (n_{\nu_e} - n_{\bar{\nu}_e}) & 0 \\ 0 & (n_{\nu_\mu} - n_{\bar{\nu}_\mu}) \end{pmatrix} \Bigg]\end{aligned}$$

Things are not that simple, however…

- The effect of the neutrino background on *active-active* oscillations is qualitatively different
  (J. Pantaleone, PLB 1992, PRD 1992)

- The NC weak interaction Hamiltonian

$$H_{\mathrm{NC}} = \frac{G_F}{\sqrt{2}} \left(\sum_a j_a^\mu \right) \left(\sum_b j_{b\mu} \right)$$

  possesses a U(2) flavor symmetry:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \to U \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

- $\to$ Any result derived from it should also obey U(2)

Flavor off-diagonal terms

- If all neutrino states (both in the "beam" and "background") are rotated, the Hamiltonian in the new basis should be *exactly the same* as in the old basis

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \to U \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \qquad U H_{\nu\nu}(U\nu_x)U^\dagger = H_{\nu\nu}(\nu_x)$$

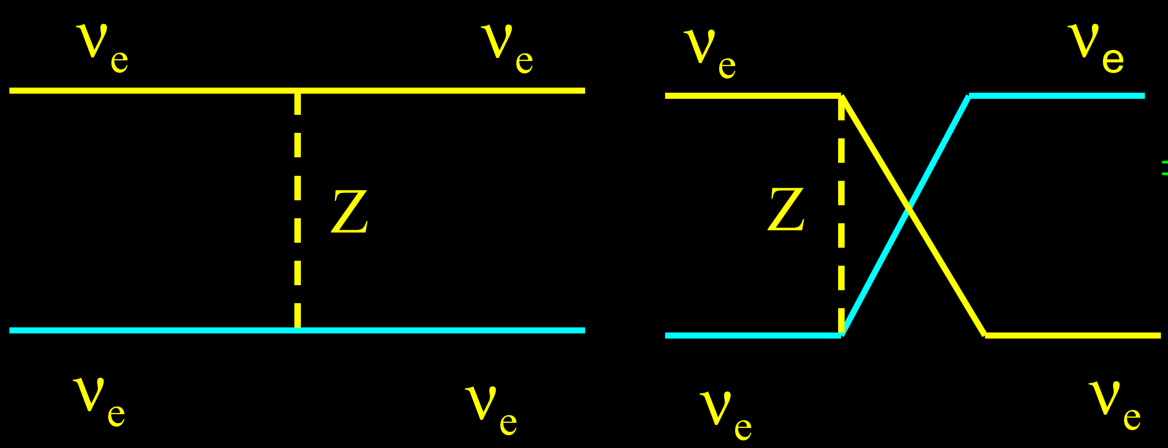
- The diagonal Hamiltonian used in earlier studies was not U(2) invariant

- The Hamiltonian $\Delta H_{\nu\nu}$ generically cannot be *diagonal*

What would be U(2) invariant?

- Consider ν_e in ν_e BG. Since NC cannot change flavor, there will be no flavor transitions

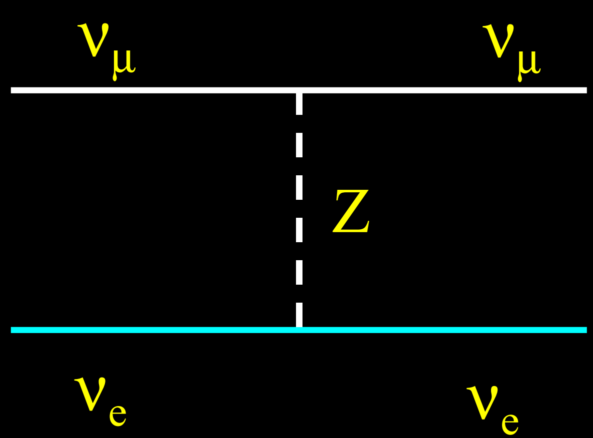
  The two relevant diagrams are



$$\Rightarrow H_{ee} = 2\sqrt{2}G_F n_\nu$$
$$H_{e\mu} = 0$$
$$(\nu_e \ backgr)$$

- Consider now ν_μ in ν_e BG. Assume that Hamiltonian is again flavor-diagonal.

Only one flavor-diagonal diagram



$$\Rightarrow H_{\mu\mu} = \sqrt{2} G_F n_\nu \quad (\nu_e \;\; backgr)$$

- Putting things together, one gets for ν_e BG

$$H_{\nu\nu} = \sqrt{2} G_F n_\nu \langle 1 - \cos\Theta \rangle \left[1 + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right].$$

Density matrix of the background neutrino

- Now, if we rotate the background $\nu_e \rightarrow \nu_x$ and use U(2), we find

$$H^{(i)}_{\nu\nu} = \sum_j \sqrt{2} G_F n^{(j)}_\nu (1 - \cos\Theta_{ij}) \left[1 + \begin{pmatrix} |\nu^{(j)}_e|^2 & \nu^{(j)}_e \nu^{(j)*}_\mu \\ \nu^{(j)*}_e \nu^{(j)}_\mu & |\nu^{(j)}_\mu|^2 \end{pmatrix} \right]$$

Pantaleone argued that

1. Under certain assumptions, neutrino ensemble can be described by a system of single particle equations
2. The Hamiltonian for each neutrino mode is given by $H_{\nu\nu}$ above

- This result was also later rederived by
  - Sigl & Raffelt, Nucl. Phys. B, 1993
  - McKellar & Thomson, PRD, 1994

in the context of a more general analysis of the flavor evolution of a neutrino ensemble (collisions and well as refraction, Pauli blocking, etc)

- Subsequent studies used the density matrix νν Hamiltonian as a starting point

- Neutrino evolution in the early Universe (equilibration of flavors)
  - Lunardini & Smirnov, PRD 2001
  - Pastor, Raffelt & Semikoz, PRD 2002
  - Dolgov, Hansen, Pastor, Petcov & Raffelt, Nucl Phys B, 2002
  - Wong, Y. Y., PRD 2002
  - Abazajian, Beacom & Bell, PRD 2002

  …….
- …and in the supernova core (*r*-process)
  - Qian & Fuller, PRD 1995
  - Pantaleone, PLB, 1995
  - Sigl, PRD, 1995
  - McLaughlin, Fetter, Balantekin & Fuller, PRC 1999
  - Pastor & Raffelt, PRL,2002

  …….

Questions

- What is the physical mechanism behind the density matrix Hamiltonian?

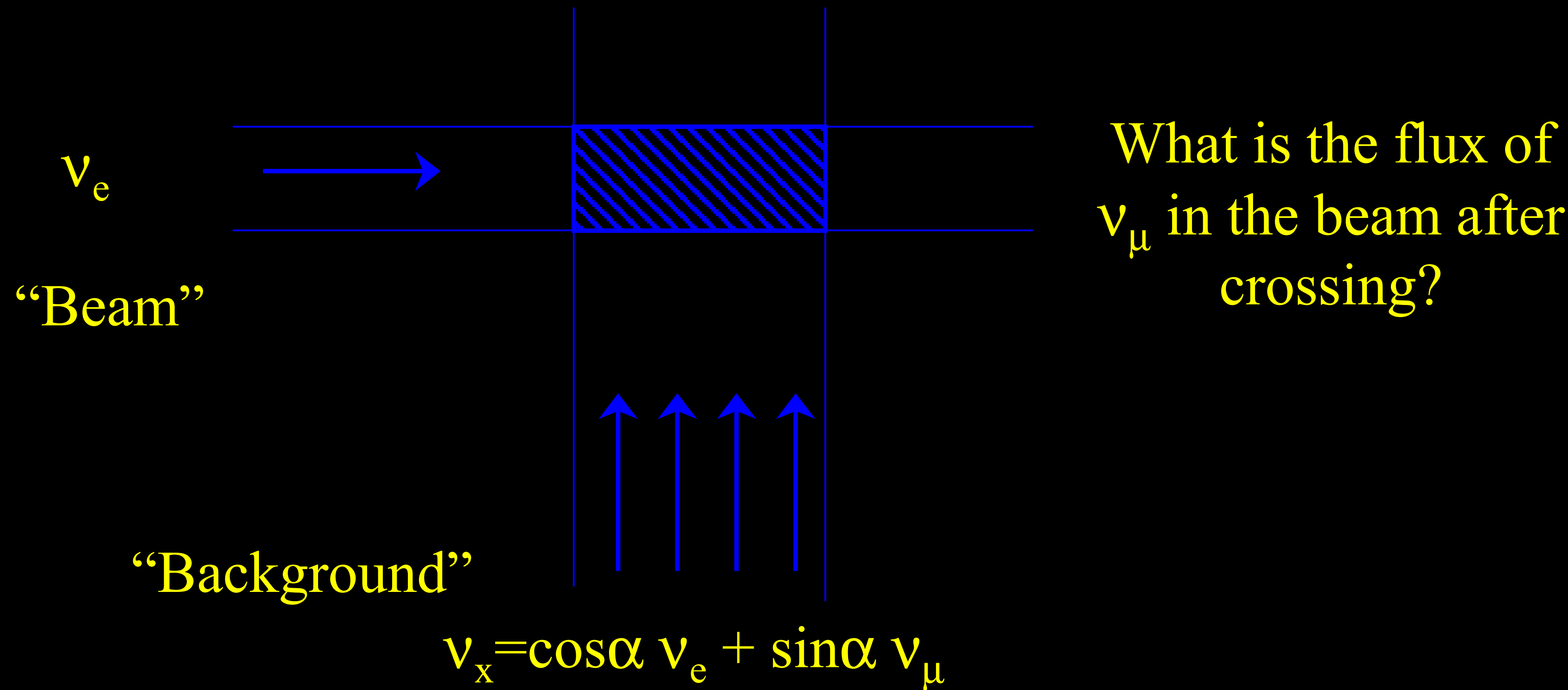
  Can one have a simple picture, from first principles?

- What physical assumptions go into the derivation?

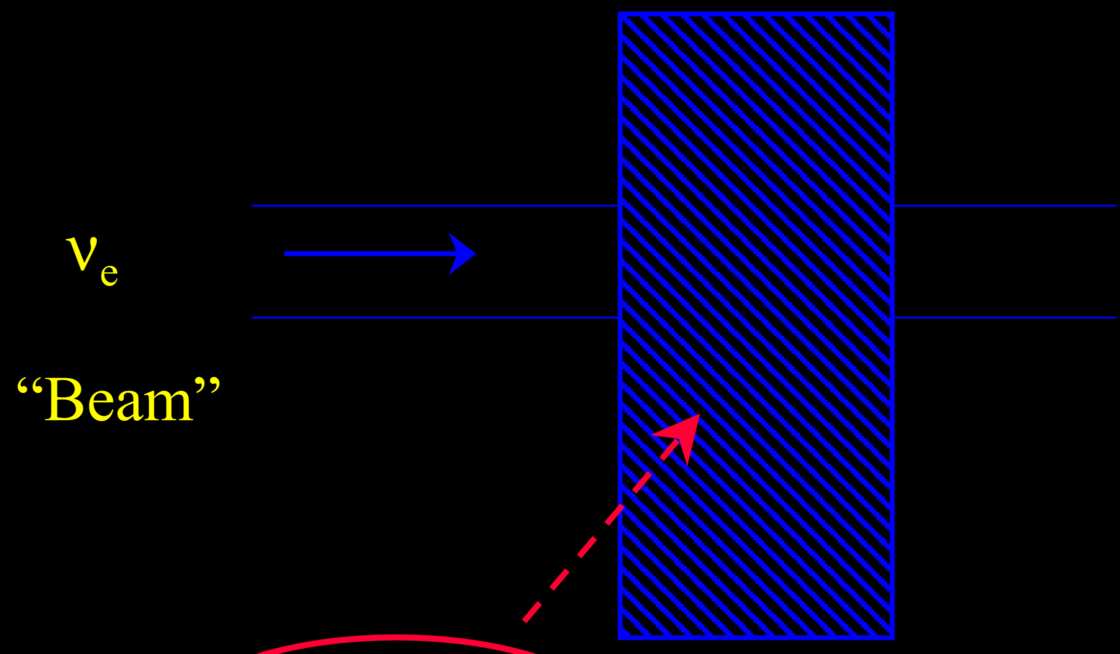
- What is the justification for using the single-particle approach? (*a priori* a multi-particle problem)

Naïve attempt to "derive" result from first principles

- Consider toy problem: two intersecting beams



Similar to FCNC problem?



What is the flux of ν_μ in the beam after crossing?

Material with Flavor changing interactions

$$H_{FCNC} = \frac{\sqrt{2} G_F n_2}{2} \left[const + \begin{pmatrix} \epsilon & \epsilon' \\ \epsilon' & -\epsilon \end{pmatrix} \right]$$

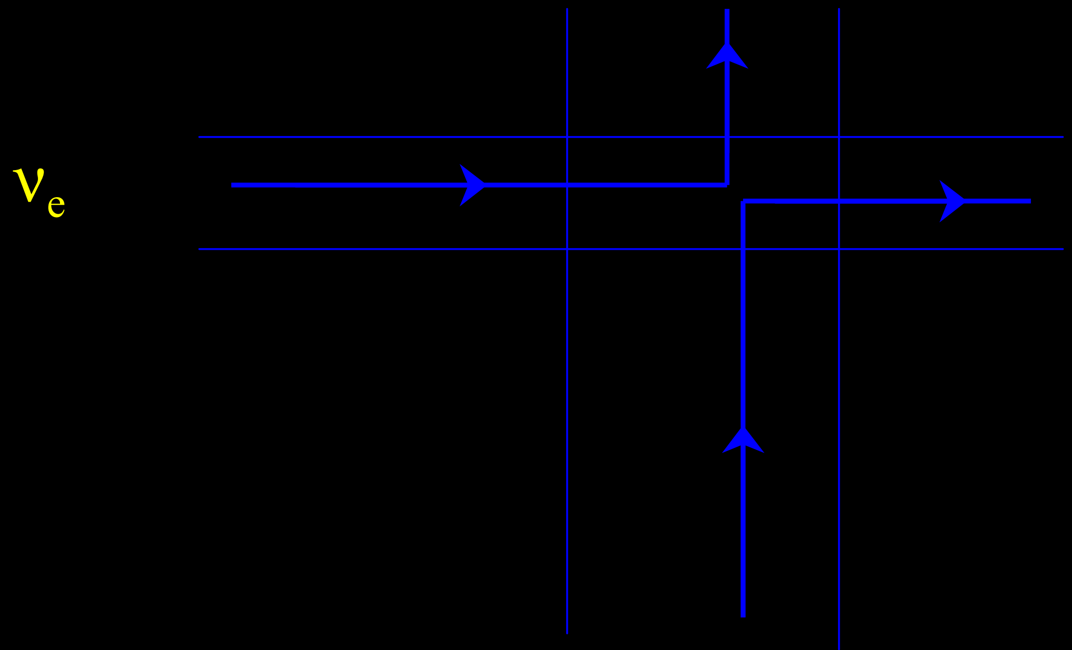
The FCNC case is easy to understand

- Usual (incoherent) scattering: flux(ν_μ) $\propto$ ε'^2 N
- Coherent scattering (forward direction): *amplitudes add up*
  flux(ν_μ)$\propto$ $|\varepsilon'$ N$|^2$

- $\Rightarrow$ prescription:
  - Take amplitude for elementary process
  - Multiply by # of scatterers
  - Square to find the rate

Elementary event: two neutrino system

- NC interactions conserve flavor



ν_μ can only appear in the beam if it came from the background

$\nu_x = \cos\alpha\ \nu_e + \sin\alpha\ \nu_\mu$

Elementary event: two neutrino system

- Beam and BG neutrinos exchange momenta

basis:

$$H_{2\nu} = \sqrt{2}\frac{G_F}{V}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \qquad \begin{pmatrix} \nu_e(p)\nu_e(k) \\ \nu_e(p)\nu_\mu(k) \\ \nu_\mu(p)\nu_e(k) \\ \nu_\mu(p)\nu_\mu(k) \end{pmatrix}$$

Initial state: $|\psi\rangle = |\nu_e\rangle|\nu_x\rangle = (\cos\alpha, \sin\alpha, 0, 0)$

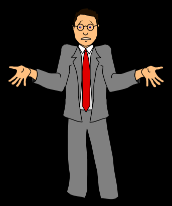
Small-t evolution: conversion probability agrees with intuition

$$t \to t + \delta t$$

$$P(e, x \to \mu, any) \propto \sin^2\alpha$$

- Amplitude of measuring neutrino ν_x as ν_ν is $\propto \sin\alpha$
- Multiply by number of scattering events
- Find that the flux of ν_μ goes like $\propto \sin^2\alpha$
- But the density matrix Hamiltonian yields

$$P_{\nu_e \to \nu_\mu} \propto \sin^2 2\alpha$$



The puzzle

- The result P $\propto \sin^2 2\alpha$ looks paradoxical

  - The conversion amplitude for an elementary process has a maximum for $\alpha=\pi/2$ (background composed of pure ν_μ states)

    But the density matrix Hamiltonian predict no conversion in this case! Why?

Hidden physical assumptions?

- Maybe the density matrix Hamiltonian is only valid under some physical assumptions?
- Maybe the result could be understood only once those assumptions are included?

Pantaleone, 1992

- For general conditions, the flavor evolution of massive neutrinos is a many-body phenomenon

- Massive neutrinos: require averaging. The diagrams diagonal in the propagation (mass) eigenstate sum coherently but the exchange diagrams do not.

Sigl & Raffelt; McKellar & Thomson:

- No such assumptions mentioned

Changing background?

- Additional problem: usually coherent scattering assumes that the scatterers are unchanged (one cannot say on what particle the scattering occurred)
- But in our case, the background definitely changes, to conserve flavor
- How to take it into account?

Key point

- Let's consider the change of the background more carefully

- Consider the beam neutrino ν_e ($|e\rangle$) scattering from several background neutrinos ν_x ($|$ x x x …$\rangle$)

$$|e\rangle|xxx...xx\rangle \Rightarrow |e\rangle|xxx...xx\rangle + ia|Exch\rangle,$$

where $a \propto G_F dt$ and

$$\begin{aligned} |Exch\rangle &= |x\rangle|exx...xx\rangle + |x\rangle|xex...xx\rangle \\ &+ |x\rangle|xxe...xx\rangle + ... \,. \end{aligned}$$

- Compute the expectation value of the "ν_μ number" operator in the final state, $|\mu\rangle\langle\mu|$, to determine the flux of ν_μ

$$\langle F|\hat{\mu}|F\rangle$$

$\hat{\mu}$ only acts on the states of the beam

$$\begin{aligned}\langle F|\hat{\mu}|F\rangle &= a^2\langle x|\hat{\mu}|x\rangle \\ &\times \underbrace{(\langle exx\ldots x| + \langle xex\ldots x| + \ldots)(|exx\ldots x\rangle + |xex\ldots x\rangle + \ldots)}\end{aligned}$$

$$a^2 \sin^2\alpha \quad \times \quad (N_2^2 - N_2)\cos^2\alpha + N_2$$

- In large N limit, $\propto N^2 \sin^2 2\alpha$, precisely as expected!

Lessons:

- As a result of the elementary scattering event, the background changes

- Only terms that are $\propto$ N^2 should be kept; terms proportional to N correspond to incoherent scattering

- There is no conversion $\propto$ N^2 in the ν_μ background because the states $|e\mu\mu\ldots\rangle$, $|\mu e\mu\ldots\rangle$, $|\mu\mu e\ldots\rangle$, etc are *mutually orthogonal*

- The part of the changed background that gets coherently amplified is *the projection on the initial state*

$$|x\ldots xex\ldots\rangle = \langle x|e\rangle|x\ldots xxx\ldots\rangle + \langle y|e\rangle|x\ldots xyx\ldots\rangle$$

Two beams

- Consider interactions between two beams, with N_1 and N_2 particles, treating beam and BG symmetrically

$$|eee...\rangle|xxx...\rangle \Rightarrow |eee...\rangle|xxx...\rangle + ia|Exch\rangle,$$

where

$$|Exch\rangle = \overbrace{(|xee...e\rangle + |exe...e\rangle + |eex...e\rangle + ...)}^{N_1} \times \underbrace{(|exx...x\rangle + |xex...x\rangle + |xxe...x\rangle + ...)}_{N_2}.$$

- $N_1 N_2$ terms

- In $|\text{Exch}\rangle$, project each of the states on the initial direction and orthogonal directions, for example

$$|xee...\rangle = \langle e|x\rangle|eee...\rangle + \langle\mu|x\rangle|\mu ee...\rangle.$$

- Do this for the N_1N_2 terms and add the result

$$\begin{aligned}
|Exch\rangle &= N_1N_2\langle e|x\rangle\langle x|e\rangle|eee...\rangle|xxx...\rangle \\
&+ N_2\langle\mu|x\rangle\langle x|e\rangle(|\mu ee...\rangle + |e\mu e...\rangle + ...)|xxx...\rangle \\
&+ N_1\langle e|x\rangle\langle y|e\rangle|eee...\rangle(|yxx...\rangle + |xyx...\rangle + ...) \\
&+ \langle\mu|x\rangle\langle y|e\rangle(|\mu ee...\rangle + |e\mu e...\rangle + ...) \\
&\quad (|yxx...\rangle + |xyx...\rangle + ...)
\end{aligned}$$

- Incoherent piece. If dropped, the rest good be rewritten …

… to first order in a, as a product of single particle rotated states

$$|eee\ldots ee\rangle|xxx\ldots xx\rangle \xrightarrow{t\to t+\delta t} |e'e'e'\ldots e'e'\rangle|x'x'x'\ldots x'x'\rangle.$$

$$\begin{aligned}|e'\rangle &= |e\rangle + iN_2 a[1/2\times|\langle x|e\rangle|^2|e\rangle + \langle\mu|x\rangle\langle x|e\rangle|\mu\rangle],\\ |x'\rangle &= |x\rangle + iN_1 a[1/2\times|\langle e|x\rangle|^2|x\rangle + \langle e|x\rangle\langle y|e\rangle|y\rangle].\end{aligned}$$

- This looks close to what is predicted by the density matrix Hamiltonian, but not exactly!

$$H_{\nu\nu} = aN_2\begin{pmatrix}\cos^2\alpha & \sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha & \sin^2\alpha\end{pmatrix}\Rightarrow$$

$$|e\rangle \xRightarrow{t\to t+\delta t} |e\rangle + iN_2 a[\cos^2\alpha|e\rangle + \sin\alpha\cos\alpha|\mu\rangle],$$

- The difference is the factor of ½. It came about because the term $\propto N_1N_2$ had to be split between beams

- But this makes sense, because doing otherwise amounts to counting the interaction energy twice!

- Writing

$$|e'\rangle = |e\rangle + iN_2 a[|\langle x|e\rangle|^2|e\rangle + \langle\mu|x\rangle\langle x|e\rangle|\mu\rangle],$$
$$|x'\rangle = |x\rangle + iN_1 a[|\langle e|x\rangle|^2|x\rangle + \langle e|x\rangle\langle y|e\rangle|y\rangle],$$

would give

$$|Exch\rangle = 2N_1 N_2\langle e|x\rangle\langle x|e\rangle|eee...\rangle|xxx...\rangle + ...$$

- Does this have any physical effect?

Correct evolution equation

- To find out, we need to get the correct evolution equation.
- We have the result of the evolution for small δt

$$
\begin{aligned}
|\psi(t+\delta t)\rangle - |\psi(t)\rangle &= i\sqrt{2}G_F N_2/V \\
&\times \left[|\phi\rangle\langle\phi|\psi\rangle - 1/2 \times |\langle\phi|\psi\rangle|^2|\psi\rangle\right]
\end{aligned}
$$

This result is independent of the basis $\rightarrow$ can be used for any t

$$\boxed{i\psi_i' = \sqrt{2}G_F n_2(\phi_i\phi_j^*\psi_j - 1/2|\phi_j\psi_j^*|^2\psi_i)}$$

- U(2) invariant

Solution

- The equation is of the form

$$i\psi' = (H_0 + C(|\phi\psi^*|)\mathbb{I})\psi$$

- The solution is

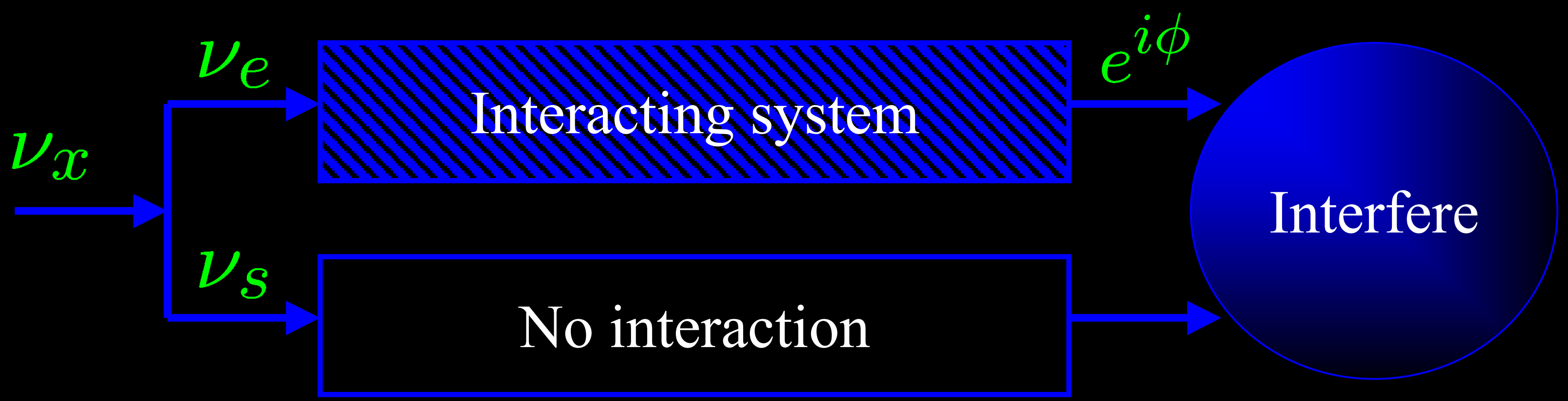
$$\psi_1(t) = \exp\left[-i\int^t C(|\phi_0(\tilde{t})\psi_0(\tilde{t})^*|)d\tilde{t}\right]\psi_0(t)$$

where ψ_0 solves $i\psi' = H_0\psi$

- Precession in the flavor space according to H_0; the *C* term gives an overall phase
- This phase depends on the relative angles between beam and background, $|\phi_j\ \psi_j^*|^2 = \cos^2\theta_{relative}$

How to probe the phase?

- The phase appears as a result of interaction between neutrinos
- *Idea:* try to probe this phase by constructing a superposition state, part of which interacts with the medium and part doesn't
- Consider active-sterile oscillations



Active-sterile case

- The standard Hamiltonian for the active-sterile oscillations is

$$H_{es} = 2\sqrt{2} G_F n_2 \begin{pmatrix} \cos^2\alpha & 0 \\ 0 & 0 \end{pmatrix} = 2\sqrt{2} G_F n_2 |\langle x|e\rangle|^2 |e\rangle\langle e|$$

- Carrying out an analysis similar to the active-active case, we get

$$H_{es} = 2\sqrt{2} G_F n_2 [|\langle x|e\rangle|^2 |e\rangle\langle e| - 1/2 |\langle z|e\rangle|^2 |\langle x|e\rangle|^2]$$

- Just like in the active-active case, this evolution Hamiltonian also contains *an additional term*
- This term is, once again, *an overall phase*
- It is *nonlinear*, so the naïve interference argument does not apply

Does the extra term ever become important?

- What are the conditions for it to be an overall phase?

- Maybe it never matters for neutrino flavor evolution?

- To understand this, consider the case when the background is in a quantum superposition state

Entangled background

- Standard formula assumes each neutrino in the ensemble has its own wavefunction (Hartree approximation, no quantum entanglement)

- What about the entangled background, say

  $|x\ x\ x \ldots\rangle + |y\ y\ y \ldots\rangle$?

- Our method is general, can be used even for this case

- Repeat the toy experiment, but with the background $|x\ x\ x\ \ldots\rangle + |y\ y\ y\ \ldots\rangle$. What is the ν_μ flux?

- Just as before, perform exchanges

$$|eee\ldots\rangle(|xxx\ldots\rangle + |yyy\ldots\rangle) \Rightarrow$$
$$|eee\ldots\rangle(|xxx\ldots\rangle + |yyy\ldots\rangle) + ia|Exch\rangle,$$

$$\begin{aligned} |Exch\rangle &= |xee\ldots\rangle|xee\ldots\rangle + |xee\ldots\rangle|xex\ldots\rangle + \ldots \\ &+ |yee\ldots\rangle|eyy\ldots\rangle + |yee\ldots\rangle|yey\ldots\rangle + \ldots \end{aligned}$$

- The flux of ν_μ is nonzero

- States of the type $\exp[i\phi_1]|e\ e\ e\ \ldots\rangle|\ x\ x\ x\ \ldots\rangle + \exp[i\phi_2]|e\ e\ e\ \ldots\rangle|y\ y\ y\ \ldots\rangle$ form. The phase is now *relative*, and has a physical effect

Conclusions

- $\nu\nu$ refraction Hamiltonian can be simply derived from first principles as an interference effect, once the change in the background state is properly included
- No special assumptions, i.e. decoherence between certain states, are necessary
- The standard formalism overcounts neutrino interaction energy, but…
- …The correct equation differs only by an overall phase, both for active-active and active-sterile oscillations, with no effect on oscillation physics under normal conditions