

Probing the neutrino mass hierarchy and θ_{13} with supernovae

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Today we know that *neutrinos oscillate* and therefore have masses and mixings.

The following quantities have been measured:

$$|\Delta m_{32}^2| \simeq |\Delta m_{31}^2|$$

$\Delta m_{21}^2, \theta_{12}$ (SNO, KamLand)

What are the next goals?

1) Oscillation : $\theta_{13}, \text{sign}[\Delta m_{31}^2]$

(mass hierarchy)

2) non-oscillation: mass, magnetic moment,
Dirac/Majorana....

Why still these unknowns?

Because of the "conspiracy":

$$|\Delta m_{31}^2| \gg \Delta m_{21}^2$$

$\sin^2 \theta_{13}$ small (< 0.02 , Chooz)

All the situations we are familiar with (solar/atmospheric/reactor ν), reduce to 2ν with small 3ν corrections.

Possibilities:

1. Improve experimental sensitivity
(neutrino factories/superbeams)
2. find a system in which large MSW effects
due to θ_{13} are realized.
supernovae are good candidates!

$$\rho_{res} = \frac{|\Delta m_{31}^2|}{\sqrt{2}G_F E} \sim 10^3 \text{ gcm}^{-3} \quad (E \sim 10 \text{ MeV})$$

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Many ingredients: potential and limitations

$$F_e - F_e^0 = P(\nu_x \rightarrow \nu_e)(F_x^0 - F_e^0)$$

$$P(\nu_x \rightarrow \nu_e) = (1 - e^{-\frac{2|\Delta m_{31}^2|}{E} \sin^2 \theta_{13} (\frac{dn}{ndr})_{res}^{-1}}) \sin^2 \theta_{12}$$

Astrophysics:

original fluxes

density profile

type of progenitor

shock-wave effects

Particle physics:

θ_{12}

Δm_{31}^2

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Order of magnitude estimates:**Thermal production of $\nu_e, \nu_\mu, \nu_\tau, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$:**

$$e^+ e^- \rightarrow \nu \bar{\nu} \quad NN \rightarrow NN \nu \bar{\nu}$$

**Duration of neutrino burst ~ diffusion time:
(random walk)**

$$\tau \sim t_{diff} \sim R^2/\lambda = \mathcal{O}(10) \text{ s}$$

Total energy released ~ core binding energy:

$$E_b \sim \frac{3G_N M^2}{5R} \simeq 1.6 \cdot 10^{53} \text{ ergs} \left(\frac{M}{M_\odot} \right)^2 \left(\frac{10Km}{R} \right)$$

comparable to the whole universe!**Typical kinetic energy of nucleons:****virial theorem:**

$$\langle E_{kin} \rangle \sim \frac{G_N M m_N}{2R} \approx 20 \text{ MeV}$$

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**neutrino energy spectra and luminosities:
a naive argument..**

- emission from a thermal sphere (neutrinosphere):

→ thermal spectra

- different flavors have different interactions with the medium: → "hierarchy" of the spectra

$\nu_e + n$	$\bar{\nu}_e + p$	NC only
$\langle E_e \rangle$	$\langle E_{\bar{e}} \rangle$	$\langle E_{\mu,\tau,\bar{\mu},\bar{\tau}} \rangle$
$\sim 9 - 11 \text{ MeV}$	$\sim 14 - 17 \text{ MeV}$	$\sim 21 - 24 \text{ MeV}$

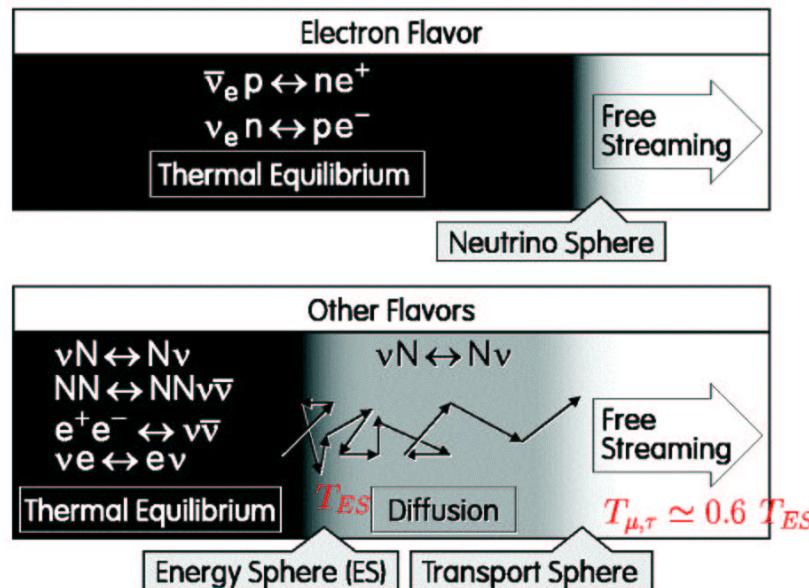
- "equipartition" of luminosities between flavors:

$$L_e \simeq L_{\bar{e}} \simeq L_{\mu,\tau,\bar{\mu},\bar{\tau}}$$

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however, physics is more complicated...

*the non-electron energy sphere is more internal,
but diffusion must be taken into account....*



effective thermal spectra, smaller differences in energies:

$$\langle E_e \rangle < \langle E_{\bar{e}} \rangle < \langle E_{\mu,\tau} \rangle$$

0.8 : 1 : 1.1

Similar luminosities (~ factor of 2)

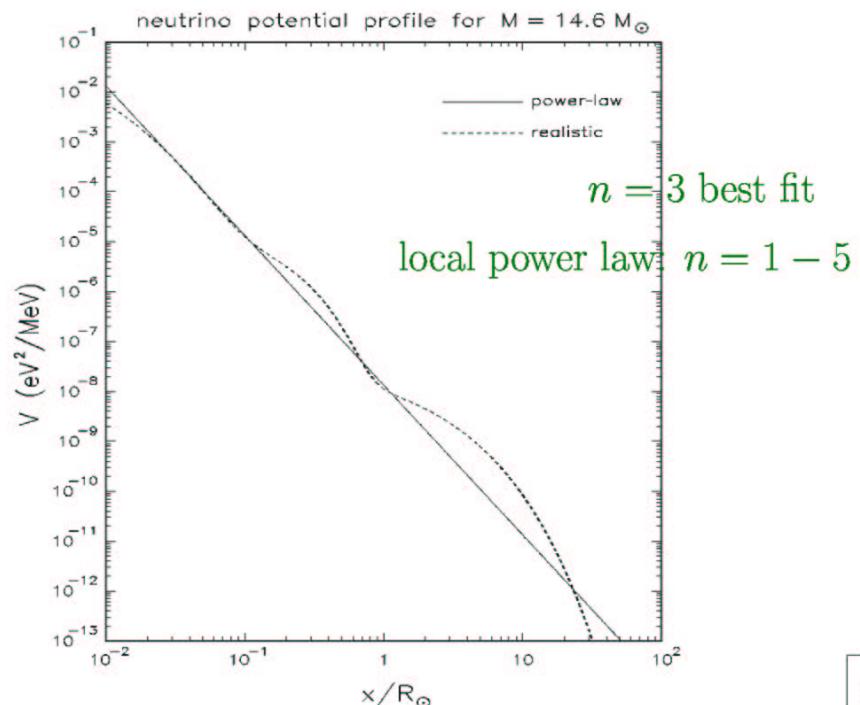
[G.Raffelt, astro-ph/0105250] 4

progenitor density profile:

$$\rho(r) = \rho_0 \left(\frac{r_0}{r}\right)^n \quad n \simeq 3$$

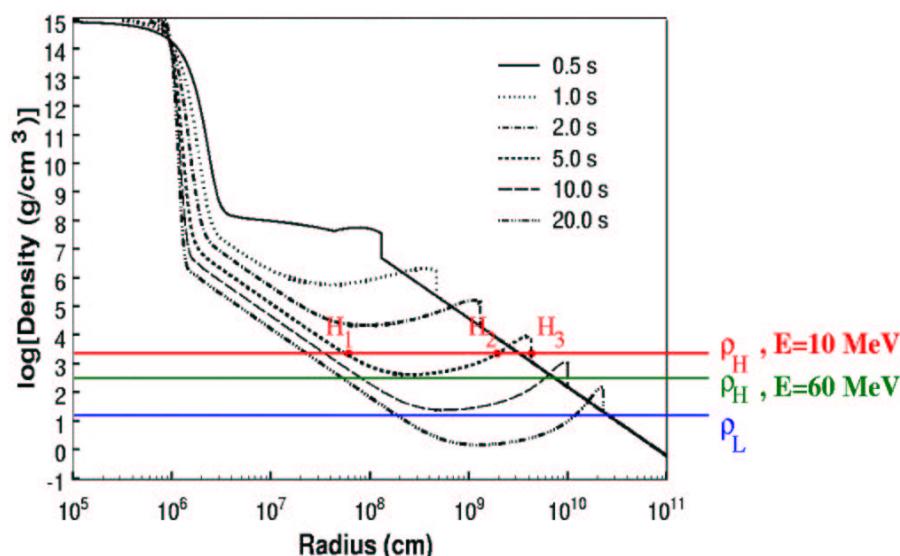
from hydrostatic equilibrium + radiative energy transfer + equation of state + constancy of luminosity and opacity

Deviations due to: convection, non constant luminosity and opacity



Shock-wave effects: how many resonances?

[R.C.Schirato & G.M.Fuller, astro-ph/0205390]



at late times ($t=5 - 10$ s) the shock-wave reaches
the density layers where flavor conversion occurs:
multiple resonances take place

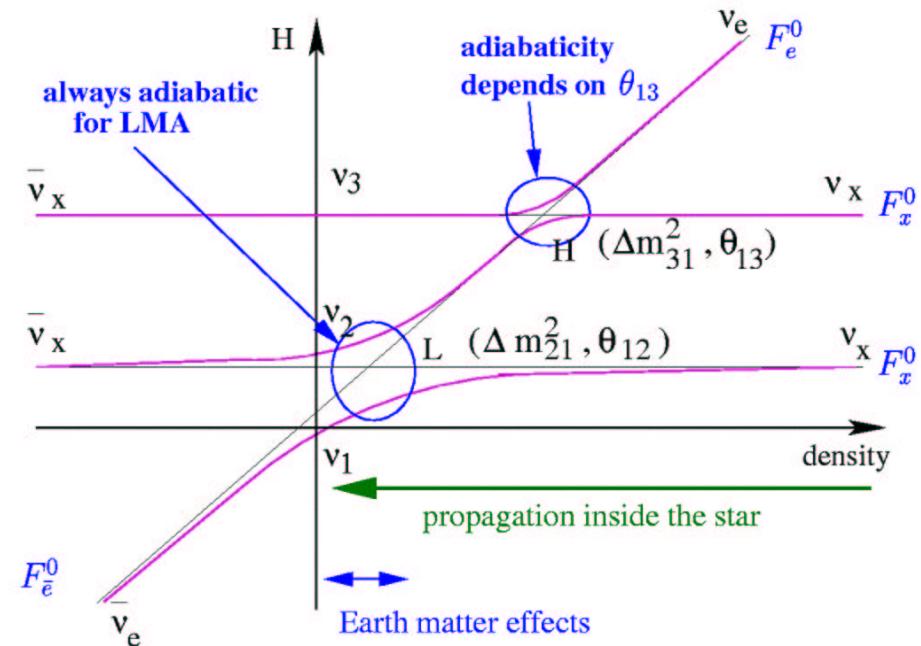
→ significant impact on the conversion effects!
interesting time structure of the signal!

(however effects are suppressed by luminosity decay)

18z

conversion effects: swap of fluxes

for normal hierarchy:



Permutation: (LMA parameters taken)

$$F_e = P_H P_{2e} F_e^0 + (1 - P_H P_{2e}) F_x^0 \quad P_{2e} \equiv P(\nu_2 \rightarrow \nu_e)$$

$$F_{\bar{e}} = P_{1\bar{e}} F_{\bar{e}}^0 + (1 - P_{1\bar{e}}) F_{\bar{x}}^0$$

hardening of spectra

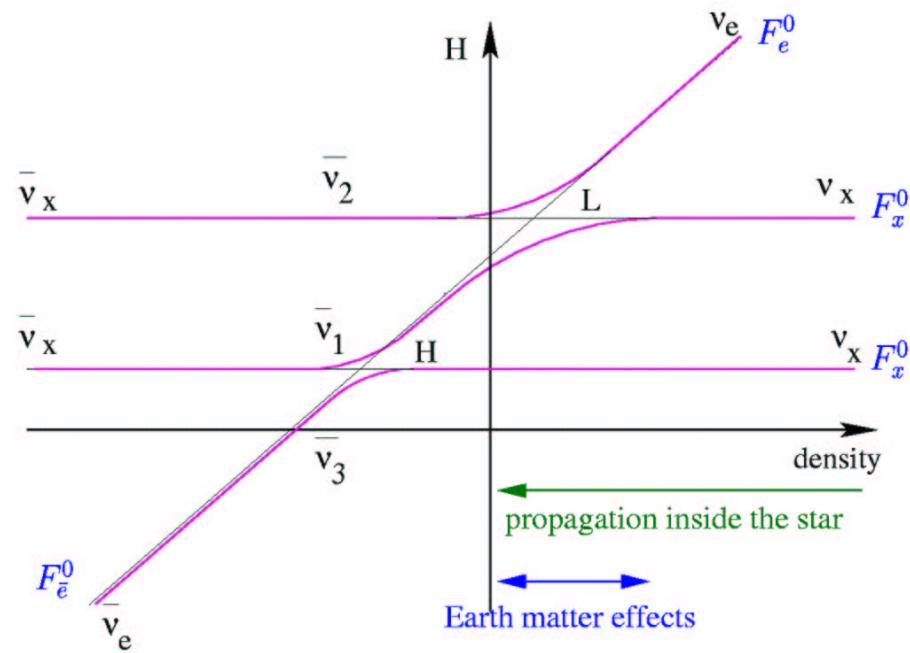
with no Earth crossing:

$$P_{2e} = \sin^2 \theta_{12}$$

$$P_{1\bar{e}} = \cos^2 \theta_{12}$$

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for inverted hierarchy:



$$F_e = P_{2e} F_e^0 + (1 - P_{2e}) F_x^0$$

$$F_{\bar{e}} = P_H P_{1\bar{e}} F_{\bar{e}}^0 + (1 - P_H P_{1\bar{e}}) F_{\bar{x}}^0$$

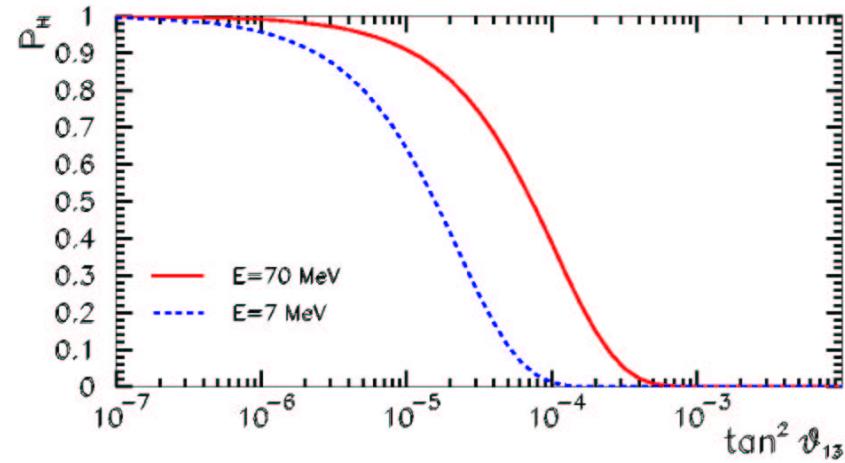
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Jump probability and θ_{13}

taking the density profile of the progenitor star,
 $\rho(r) \propto r^{-3}$, the jump probability in the high density
 resonance has the form:

$$P_H \propto \exp \left[-\sin^2 \theta_{13} \left(\frac{|\Delta m_{31}^2|}{E} \right)^{2/3} \text{const} \right]$$

(Δm_{31}^2 atmospheric mass splitting)



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Detection of supernova neutrinos:

what is needed? high statistics, energy resolution, timing,
 flavor sensitivity, $\nu - \bar{\nu}$ sensitivity,
 directionality, low energy threshold....

what is feasible? examples...

WATER: SUPERKAMIOKANDE [Japan, 32kt volume] (UNO, HyperK..)

$\bar{\nu}_e p \rightarrow e^+ n$	~8000 events (D=10kpc)
$\nu_{e,\mu,\tau} e^- \rightarrow \nu_{e,\mu,\tau} e^-$ (ES)	~200+60+60 events
$\bar{\nu}_e O \rightarrow N e^+$	
$\nu_e O \rightarrow F e^-$	

HEAVY WATER: SNO [Sudbury, Ontario, 1kt volume]

$\nu_e d \rightarrow p p e^-$	~240 events
$\bar{\nu}_e d \rightarrow n n e^+$	~120 events
$\nu_{e,\mu,\tau} d \rightarrow p n \nu_{e,\mu,\tau}$	~490 events

SCINTILLATOR: KamLAND [Japan, 1kt] (Borexino, LVD...)

$\bar{\nu}_e p \rightarrow e^+ n$	~330 events
$\nu p \rightarrow \nu p$	~300 events above 150 keV
$\nu_e {}^{12}C \rightarrow {}^{12}N e^-$	

extracting θ_{13} and the hierarchy:

many (14 !) parameters

$$\langle E_e \rangle, \langle E_{\bar{e}} \rangle, \langle E_x \rangle, \langle E_{\bar{x}} \rangle \quad L_e, L_{\bar{e}}, L_x,$$

$$\eta_e, \eta_{\bar{e}}, \eta_x, \rho_0, n \quad |\Delta m_{31}^2|, \theta_{12}$$

Possibilities:

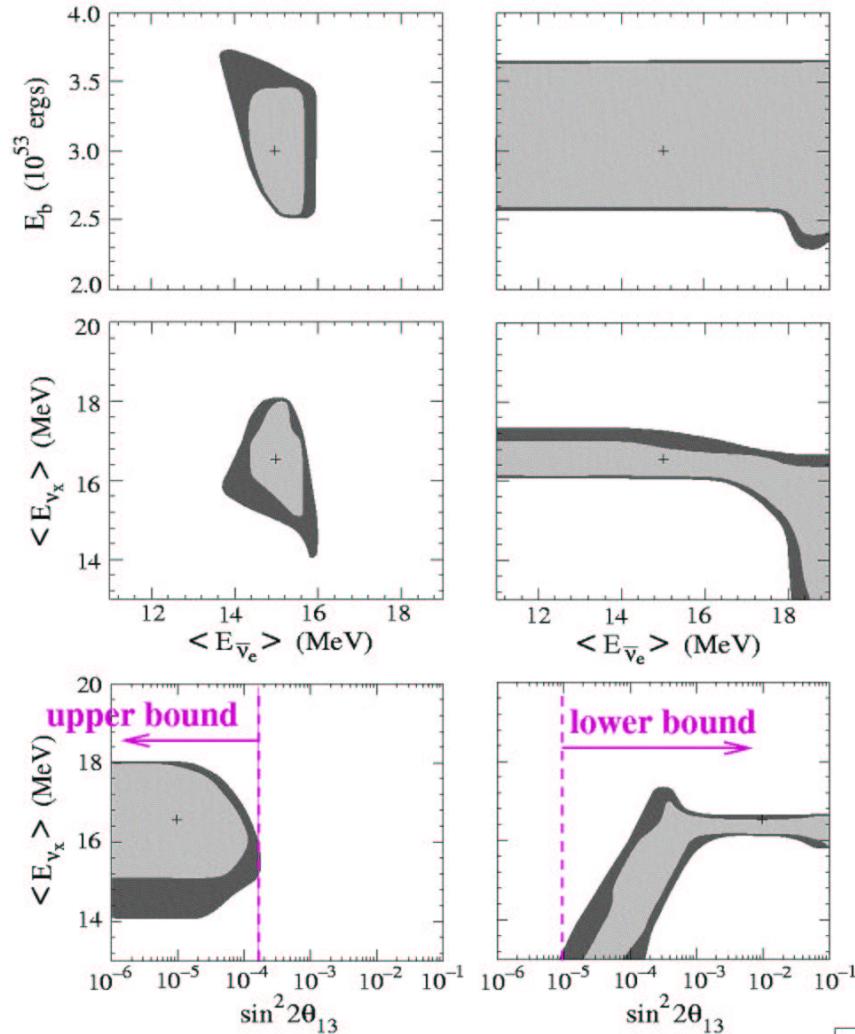
1. Global (astro+particle) fit:

extensive; needs reduction of number of variables

2. find and study (analytically) observables sensitive to θ_{13} and hierarchy

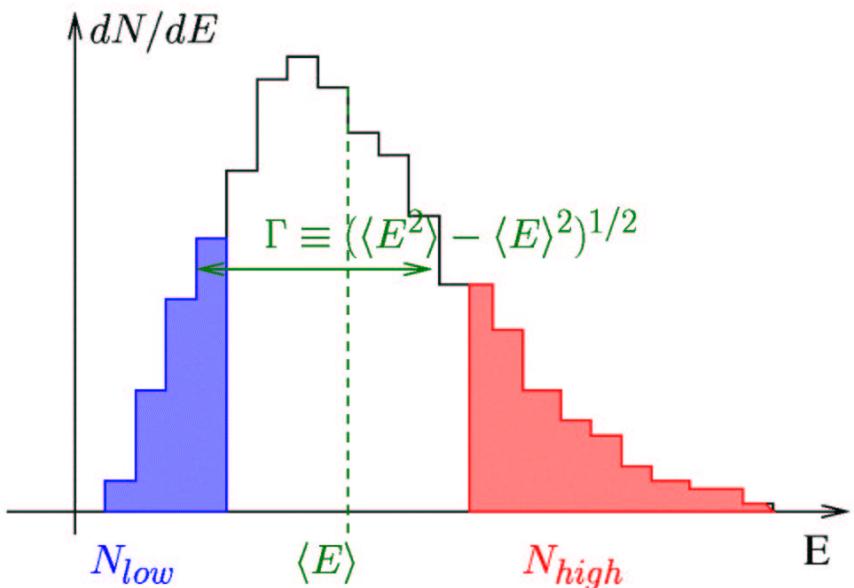
focused; physical insight
 more model-independent

Global fit: variables: L_{tot} , $\langle E \rangle_{\nu_e}$, $\langle E \rangle_{\bar{\nu}_e}$, $\langle E \rangle_{\nu_x}$, θ_{13}
 (Barger et al.)
 for inverted hierarchy:



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relevant observables:



$$R_{low} \equiv \frac{N_{low}(\nu_e)}{N_{low}(\bar{\nu}_e)}$$

$$R_{high} \equiv \frac{N_{high}(\nu_e)}{N_{high}(\bar{\nu}_e)}$$

$$r_\Gamma \equiv \frac{\Gamma(\nu_e)}{\Gamma(\bar{\nu}_e)}$$

$$r_E \equiv \frac{\langle E \rangle_{\nu_e}}{\langle E \rangle_{\bar{\nu}_e}}$$

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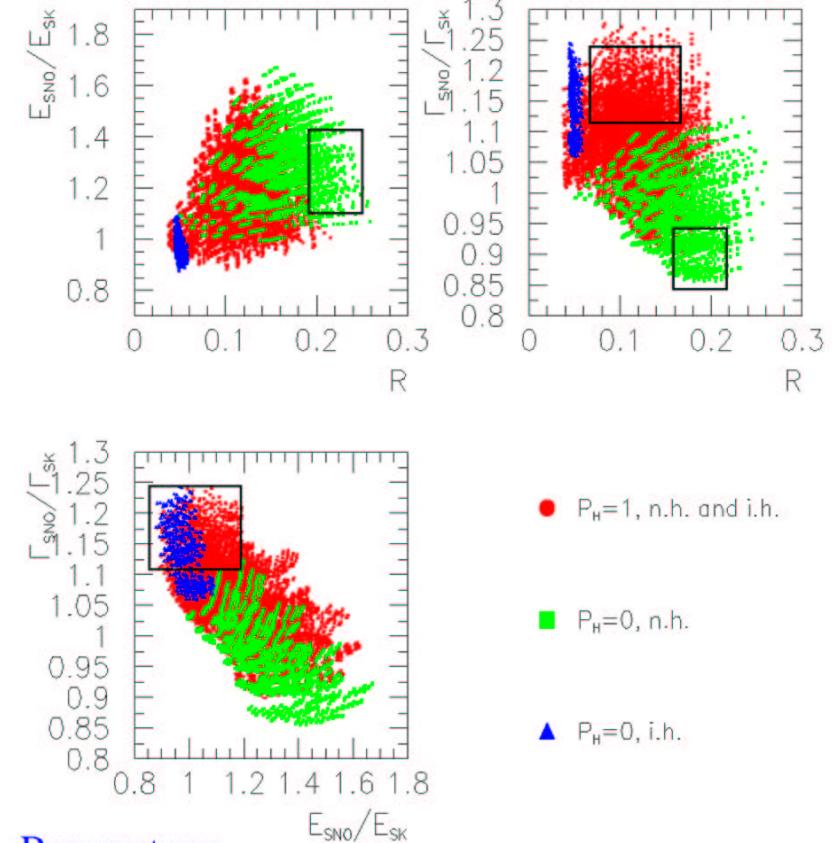
distinguishing between extreme possibilities:

		compositeness	r_Γ	r_E
A	$P_H = 0$ $(s_{13}^2 > 10^{-4})$	ν_e : total $\bar{\nu}_e$: weak	< 1	> 1
B	$P_H = 0$ $(s_{13}^2 > 10^{-4})$	ν_e : strong $\bar{\nu}_e$: total	> 1	< 1
C	$P_H = 1$ $(s_{13}^2 < 5 \cdot 10^{-5})$ both n.h. and i.h.	ν_e : strong $\bar{\nu}_e$: weak	intermediate	intermediate

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scatter plots :

$\nu_e + d$ CC at SNO
 $\bar{\nu}_e + p$ CC at SuperKamiokande



Parameters:

$$\begin{aligned}
 T_e &= 4 - 7 \text{ MeV}, T_x/T_{\bar{e}} = 1.1 - 1.6, T_e/T_{\bar{e}} = 0.5 - 0.8 \\
 L_e/L_x &= 0.5 - 2, L_{\bar{e}}/L_{\bar{x}} = 0.5 - 2, \tan^2 \theta_{12} = 0.34 - 0.42, \\
 \eta_e &= 0 - 3, \eta_{\bar{e}} = 0 - 3, \eta_x = 0 - 2 \\
 T_{\bar{x}} - T_x &= 0.3 - 0.5 \text{ MeV} \quad L_x \simeq L_{\bar{x}}
 \end{aligned}$$

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More on R_{high} :

$$R_{high} = Q \frac{1 - \langle p \rangle + \alpha \langle \bar{p} \rangle}{1 - \langle \bar{p} \rangle + \bar{\alpha} \langle p \rangle}$$

$p = \nu_e$ survival probability

$\bar{p} = \bar{\nu}_e$ survival probability

$$Q = Q(\text{thresholds}, F_x^0, F_{\bar{x}}^0) \quad \text{precisely calculable}$$

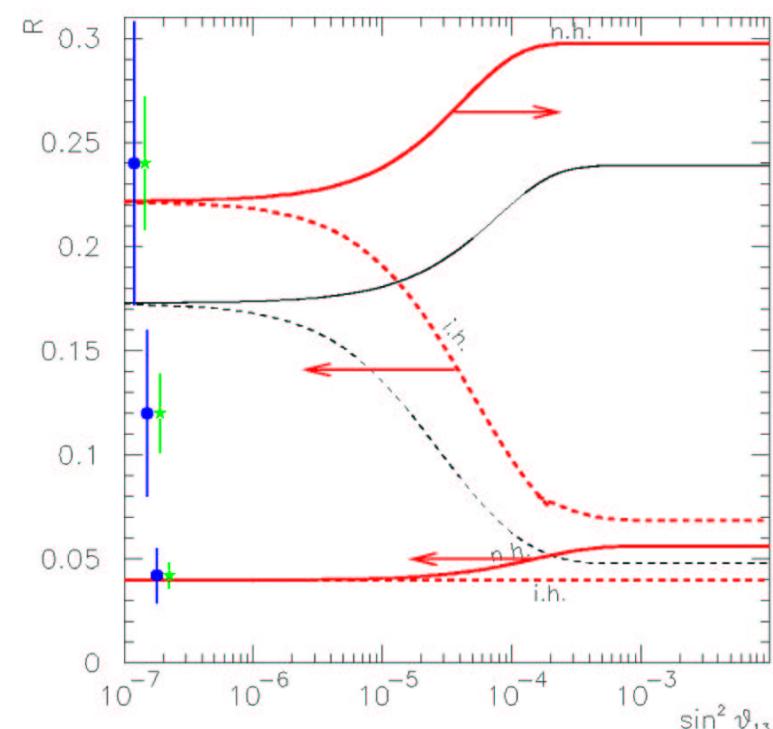
$$\alpha = \alpha(F_x^0, F_e^0) \quad \text{measure overlap of original spectra}$$

$$\bar{\alpha} = \bar{\alpha}(F_{\bar{x}}^0, F_{\bar{e}}^0) \quad \text{small for hierarchy of energies}$$

Astrophysical uncertainties cancel largely, due to:

1. high energy cuts
2. similar fluxes of ν_x and $\bar{\nu}_x$
3. similar energy dependences of $\nu_e + d$ and $\bar{\nu}_e + p$ cross sections ($\sigma \propto E^2$).

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Conclusions:

1. interesting astrophysics and neutrino physics can be studied with supernovae.
2. To probe θ_{13} and the mass hierarchy large astrophysical uncertainties must be taken into account.
3. it is possible to identify and study analytically observables in which the astrophysical uncertainties cancel largely.
4. While the establishment of the mass hierarchy appears difficult, strong (conditional) bounds on θ_{13} can be put.