Single spin dynamics vs. magnetization conserving dynamics in disordered systems

Juan Carlos Andresen Eguiluz

in collaboration with

Helmut G. Katzgraber, Vladimir Dobrosavljevic, Gergely T. Zimanyi
Outline

• Short introduction to spin glasses: from order to disorder
• Avalanches with single flip dynamics: EA and SK model
• Avalanches with magnetization conserving dynamics: Coulomb-glass model
• Discussion and work in progress: single flip vs magn. cons. dynamics
Disordered systems

Ideally ordered:
Disordered systems

Real life disordered:
From order to disorder: From the Ising to the spin-glass model

Ising Model

\[ T > T_c \]
From order to disorder: From the Ising to the spin-glass model

Ising Model

\[ T < T_c \]
From order to disorder: From the Ising to the spin-glass model

Ising Model

\[ S_i = -1, 1 \]

\[ \mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j \]
From order to disorder: From the Ising to the spin-glass model

**Ising Model**

\[ S_i = -1, 1 \]

\[ \mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j \]

**Spin Glass**

\[ T > T_c \]
From order to disorder: From the Ising to the spin-glass model

\[ \mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j \]

\[ S_i = -1, 1 \]

\[ T < T_c \]
From order to disorder: From the Ising to the spin-glass model

Ising Model

Spin Glass

\[ S_i = -1, 1 \]

\[ \mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j \]

\[ \mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} S_i S_j \]
From order to disorder: From the Ising to the spin-glass model

Ising Model

\[ S_i = -1, 1 \]

\[ \mathcal{H} = -J \sum_{\langle i, j \rangle} S_i S_j \]

Spin Glass

\[ \mathcal{H} = - \sum_{\langle i, j \rangle} J_{ij} S_i S_j \]
Characteristics of glassy systems

- Complex energy landscapes
- Slow dynamics, time scales diverge: $\propto e^{\Delta E / T}$
Characteristics of glassy systems

- Complex energy landscapes
- External force, e.g. tilt landscape

Out of equilibrium

$\Delta E$
Applications

Neural networks

Traffic

Power grids

Stock market

Traveling salesman problem
Self-organized criticality (SOC): property of dissipative systems that drive themselves into a scale-invariant state

- Slow driving or energy input
- Fast relaxation events (avalanches, earthquakes,...)
- Power-law distribution of the response with an exponential cutoff that scales with the system size
- No tuning parameter
Avalanches and self-organized criticality

SOC found in different natural systems

- Earthquakes
- Solar flares
- Dislocations flow
- etc.
Avalanches and SOC

To understand better the emergence of SOC

- Use models as simple as possible showing SOC
- So far mostly cellular automata models

Alessandro Vespignani, Enciclopedia della Scienza e della Tecnica (2007)
Avalanches and self-organized criticality in spin models

What about spin models?
Avalanches and self-organized criticality in spin models

What about spin models?

Sherrington-Kirkpatrick model (mean-field of the Edwards-Anderson spin model)

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Pazmandi et al. PRL (1999)
Avalanches and self-organized criticality in spin models

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Configuration space

single flip dynamics
Avalanches and self-organized criticality in spin models

Configuration space

single flip dynamics

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Configuration space
Avalanches and self-organized criticality in spin models

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Configuration space

single flip dynamics
Avalanches and self-organized criticality in spin models

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Configuration space
Avalanches and self-organized criticality in spin models

\[ H = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Configuration space
Avalanches and self-organized criticality in spin models

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Configuration space
Avalanches and self-organized criticality in spin models

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i \]

Configuration space
Self-organized criticality in the SK model

Characteristic avalanche size \( n_N^* \sim \exp(-n/n_N^*) \)
Self-organized criticality in the SK model

Characteristic avalanche size $n_N^* \sim \exp(-n/n_N^*)$
Self-organized criticality in the SK model

Characteristic avalanche size $n_N^* \sim \exp\left(-n/n_N^*\right)$
Self-organized criticality in the SK model

Characteristic avalanche size $n_N^* \sim \exp\left(-\frac{n}{n_N^*}\right)$
Self-organized criticality in the SK model

\[ N \to \infty \text{ extrapolation of } n_N^* \]
Self-organized criticality in the SK model

Similar results for avalanches and magnetization jumps.
What about the EA model?

- SK: Similarities out-of-equilibrium avalanches with static calculations [Le Doussal et al. PRB (2012)]
- Previous work predic SOC in equilibrium avalanches of the 3-dimensional spin-glass model [Le Doussal et al. PRB (2012)]
- What about out-of-equilibrium avalanches of Edward-Anderson model?

\[
\mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} S_i S_j - H \sum_i S_i
\]
What about the EA model?

Same procedure as for the SK model for the 3-dimensional EA model
What about the EA model?

SOC mean-field universality class property?

\[ d_c = 6 \]
What about the EA model?

SOC mean-field universality class property?

\[ d = 8 > d_c \]
Avalanches: from EA to SK

\[
(n^*)^{-1} \bigg|_{d \to \infty} = 0.0023(22)
\]
Does SOC come from the infinite range property?

- Long-range nature of the SK model?
- So far $z = 2d$, such that $d = \infty \rightarrow z = \infty$
- We want to test $d = \infty$, with $z = \text{const.}$
Does SOC come from the infinite range property?

- Long-range nature of the SK model?
- So far $z = 2d$, such that $d = \infty \rightarrow z = \infty$
- We want to test $d = \infty$, with $z = \text{const.}$

Viana-Bray Model!

$$\mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j - H \sum_i S_i$$

Fixed coordination number $z$, random neighbors
Does SOC come from the infinite range property?
Self-organized criticality in spin-glasses

Condition for self-organized criticality:
Diverging number of neighbors in the thermodynamic limit

Andresen et al. PRL (2013)
Other possible spin-models showing scale-free avalanches

\[ \mathcal{H} = \frac{1}{2} \sum_{i,j} \frac{q_i q_j}{|r_i - r_j|} \left( n_i - \frac{1}{2} \right) \left( n_j - \frac{1}{2} \right) + \sum_i (\phi_i + V_i) n_i \]

- \( r_i \) position of charge \( q_i \), \( V_i \) external potential,
- \( \phi_i \) random potential, \( n_i \in \{0, 1\} \)
- and is a charge neutral system (half filled)
Coulomb glass mapping

Mapped onto an Ising spin model with magnetization conserving dynamics:

\[ \mathcal{H} = \sum_{ij} J_{ij} S_i S_j + \sum_i (\varphi_i + V_i) S_i \]

- Long-range interactions
- Disorder
- Out of equilibrium through an external potential
Avalanches in the Coulomb-glass model

- Adiabatically increase the external potential
- Observables
  - Total electron hop
  - Net charge displacement
Avalanches in the Coulomb-glass model

- Adiabatically increase the external potential
- Observables
  - Total electron hop
  - Net charge displacement
Avalanches in the Coulomb-glass model

• Adiabatically increase the external potential

• Observables
  - Total electron hop
  - Net charge displacement
Avalanches in the Coulomb-glass model

- Adiabatically increase the external potential
- Observables
  - Total electron hop
  - Net charge displacement
Avalanches in the Coulomb-glass model

For small fields $\mathcal{E}$ no more than one or two electron hops

$0 < \mathcal{E} < 0.1$

$0.1 < \mathcal{E} < 0.2$
Avalanches in the Coulomb-glass model

For intermediate fields an avalanche size dependence emerges

$0.2 < \mathcal{E} < 0.3$

$0.3 < E < 0.4$
Avalanches in the Coulomb-glass model

Close to the depinning field $\mathcal{E}_{dp} = 0.6$

$0.55 < \mathcal{E} < 0.575$

$0.575 < E < 0.6$
Avalanches in the Coulomb-glass model

Extrapolation of the characteristic avalanche size

\[
\frac{1}{n^*} \bigg|_{L \to \infty} = 0.0049(61)
\]
\[
\omega = 1.79(6)
\]

\[0.5 < E < 0.6\]

Andresen et al. arXiv:1309.2887
Summary: single flip vs.
magnetization conserving?

- Single flip: SOC for spin-glasses when number of neighbors diverge
- Magnetization conserving dynamics: scale-free avalanches only close to the depinning transition

- Is this due to the different spin flip dynamics applied?
Work in progress: single flip vs. magn. cons. dynamics

- Work in progress ...
- Does the dynamic imposed to the system change its behavior out of equilibrium? E.g. presence or absence of self-organized criticality?
Work in progress: single flip vs. magn. cons. dynamics

- **Single flip:**
  Diluted dipolar systems driven by an external magnetic field

- **Magn. cons. dynamics:**
  SK model with zero magnetic field constrain driven by an site-alternating magnetic field

**SOC when** $z \to \infty$?  
**No SOC even when** $z \to \infty$?
Work in progress: single flip vs. magn. cons. dynamics

- **Single flip:**
  Diluted dipolar systems driven by an external magnetic field

- **Magn. cons. dynamics:**
  SK model with zero magnetic field constrain driven by an site-alternating magnetic field

**SOC when**

\[ z \rightarrow \infty \]?

**No SOC even when**

\[ z \rightarrow \infty \]?