Temporal correlations in avalanching processes

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...or what power law distributions don’t tell you....

- Power laws in natural phenomena
- Earthquakes and solar flares
- Temporal clustering
- Time-energy correlations
- Understanding the physical mechanisms
- ... Also in brain activity...
- Homeostatic balance between excitation and inhibition
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Power laws in natural hazards

Forest fires in Ontario (Canada) 1976-1996
Turcotte & Malamud 2004

Exponent 1.1

Malamud 2004

Areas covered by lava in volcanic eruptions
(Springerville, Arizona) Lahaie & Grasso 1998

\[ f = 5.5 A_F^{1.38} \]
\[ r^2 = 0.996 \]
Earthquakes in the world during 1973-2003
How big can an earthquake be?

**Gutenberg-Richter Law (1954)**

\[ P(>M) \sim 10^{-b M} \quad (b \sim 1) \]

**Seismic moment**

\[ M_0 = \mu A \Delta u \]

\[ M = (2/3) \log(M_0) - 6 \]

(Kanamori, Anderson 1975)

\[ P(>M_0) \sim M_0^{-\alpha} \]

**Energy**

\[ M = (2/3) \log(E) + \text{cost} \]

\[ P(>E) \sim E^{-\alpha} \]

**Universality of** \( \alpha \sim 0.7 \)
Temporal correlations: Sequences of aftershocks

**Omori law** *(JCSIUT,1894)*

\[ n_{AS}(t) \sim (c + t)^{-p} \quad p \sim 1 \]

- \( c \) depends on \( M \) main shock and \( M \) lower cutoff
  

**Productivity law**

\[ N_{AS}(M) \sim 10^{\alpha M} \quad \alpha \sim b \]


At time \( t \) after a main shock at \( t=0 \)
Intertime distribution

Probability distribution of intertimes $\Delta t$ between consecutive events

- $P(\Delta t)$ is an exponential for a Poisson process

- It exhibits a more complex structure as temporal correlations are present in the process

Barabasi, Nature 2005
Corral (PRL, 2004) rescaling $\Delta t$ by the average rate in the area obtained a universal scaling law for the probability density

$$D(\Delta t, M_c) = R(M_c) f(R(M_c)\Delta t)$$

holds also for Japan, Spain, New Zeland... scaling function not universal
(different areas are characterized by different rates)
Solar flares

Sudden rearrangement of stressed magnetic field lines gives rise to energetic bursts from solar corona.

These phenomena take place in active regions identified by sunspots, dark-looking due to the effects of intense magnetic field.
Solar flares and Earthquakes

\[ \alpha = 1.65 \pm 0.1 \]

LdA, Godano, Lippiello, Nicodemi PRL 2006

Graphs showing various data sets for solar flares and Earthquakes, with different log-log plots comparing occurrence rates and time intervals.
Is the occurrence of two phenomena as different as earthquakes and solar flares driven by the same physical mechanism?

Can statistical properties discriminate?
Magnitude correlations

Evaluating the $<M_i M_j> - <M_i>^2$ gives values comparable with statistical noise.

Red data represent the correlations evaluated in a catalog where magnitudes are reshuffled with respect to occurrence time.
We define for any couple of successive events of the catalog:

The time distance \( \Delta t_i = t_{i+1} - t_i \) and the magnitude difference

\[ \Delta m_i = m_{i+1} - m_i \]

\[ \Delta m_i^* = m_{i+1} - m_i^* \]

for a catalog where we reshuffle the previous magnitude \( i^* \neq i \)

We evaluate the conditional probability

\[ P(\Delta m_i < m_0 \mid \Delta t_i < t_0) = \frac{N(m_0, t_0)}{N(t_0)} \]

# couples of subsequent events with both \( \Delta m_i < m_0, \Delta t_i < t_0 \)

# couples of subsequent events with \( \Delta t_i < t_0 \)
We calculate the conditional probabilities in the California catalog and for $10^4$ realizations of the reshuffled catalog (Gaussian distributed)

$$P(\Delta m_i < m_0 \mid \Delta t_i < t_0)$$

If $|\partial P(m_0 \mid t_0)| = P(\Delta m_i < m_0 \mid \Delta t_i < t_0) - Q(m_0 \mid t_0) > \sigma(m_0 \mid t_0)$

Evidence for time-energy correlations
Earthquakes in California

The next earthquake tends to have magnitude

\[ \frac{d\delta P(m_0 | t_0)}{dm_0} \]

For \( m_0 < 0 \) the probability is larger in the real than in the reshuffled catalog, where magnitudes are uncorrelated

Maximum for \( m_0 \) in \([-1,-0.5]\)

Experimental data
Numerical data

The next earthquake tends to have magnitude close but smaller than the previous one
Branching model for seismicity

We treat seismicity as a **point process in time**, where \( \{m_i(t_i)\} \) is the history of past events.

Given the history, one assumes that each event can trigger future ones according to a two-point conditional rate. The rate of events of magnitude \( m \) at time \( t \) is

\[
\rho(m(t) \mid \{m_i(t_i)\}) = \sum_{i : t_i < t} \rho(m(t) \mid m_i(t_i)) + \mu P(m)
\]

where \( \mu \) is a constant rate of independent sources, \( P(m) \) their magnitude distribution.

In the ETAS model (Ogata, JASA 1988) the magnitude \( m \) is independent of previous events

\[
\rho(M_i(t) \mid M_i(t_i)) = P(m_i) g(t_i - t_j; m_j) \propto 10^{-bm_i} 10^\alpha_n (t_i - t_j + c)^{-p}
\]
We assume that the magnitude difference fixes a characteristic time

\[ \tau_{ij} = \tau_0 10^{b(m_j - m_i)} \]

where \( \tau_0 \) is a constant measured in seconds

and that \( \rho \left( m_i(t_i) \mid m_j(t_j) \right) \) is invariant for \( \Delta t \rightarrow \lambda \Delta t = \frac{\Delta t}{\tau} \)

This time represents the temporal scale for correlations:

A \( m=2 \) earthquake is correlated to a previous \( m=6 \) event over a time scale of about 2 years

A \( m=5 \) earthquake is correlated to a previous \( m=6 \) event over a time scale of few days
Therefore the conditional rate becomes with time rescaled by $t_{ij}$

$$\rho(m_i(t_i) \mid m_j(t_j)) = F\left(\frac{t_i - t_j}{\tau_{ij}}\right)$$

where $F(x)$ is a normalizable function.

On the basis of this scaling hypothesis we recover the GR law:

$$\int_{t_0}^{\infty} \rho(m(t) \mid m_0(t_0))dt = \tau_0 10^{-b(m-m_0)} \int_0^\infty F(x)dx$$

Total number of aftershocks

and the Omori law:

$$\rho(m, t-t_0) = \int_{-\infty}^{\infty} \rho(m(t) \mid (m_0(t_0)) P(m_0) dm_0$$

$$\propto \frac{10^{-bm}}{t-t_0} \int_{-\infty}^{\infty} F(z)dz$$

Rate of $m$ events at time $t$
The next earthquake tends to have magnitude close but smaller than the previous one.
In good agreement with models implementing avalanching:

Olami-Feder-Christiensen model

Non conservative spring-block model, where $\alpha<0.25$ is the degree of dissipation

Successive instabilities generated by the stress redistribution

$\alpha \in [0.175 : 0.2]$  

Lippiello et al EPL 2013
Spatio-temporal organization of foreshocks and aftershocks

Earthquakes tend to concentrate towards the future mainshock (foreshocks)
Then they spread out after mainshock occurrence (aftershocks)

R = Average distance from the mainshock
1/R

main magnitude
2
3
4
The maximum distance between the main and aftershocks occurring in

\[ [\Delta t, \Delta t(1 + \varepsilon)] \]

increases as a power law as long as events are 90% aftershocks.

Aftershock triggering is controlled by stress diffusion.

\[ R_{\text{MAX}} (\Delta t) \propto \Delta t^H \]

\[ H = 0.54 \pm 0.05 \]

Lippiello, Godano LdA, PRL 2009
Correlations in solar flare occurrence

\[ \delta P(\lambda | t_{th}) = P(\frac{E_{i+1}}{E_i} > \lambda | \Delta t_i < t_{th}) - Q(\lambda | t_{th}) > \sigma(\lambda | t_{th}) \]

In consecutive flares (occurring within 3 hours) the energy of the second flare is close but larger than the energy of the previous one.
Understanding flare triggering

• A flare is due to the reconnection of magnetic flux tubes

• Footprints of magnetic flux tubes are anchored in the photosphere
  i.e. plasma in turbulent flow
  • Magnetic flux tubes follow the local velocity field
    and are twisted by the vorticity

• A flare is released as soon as a tube reaches a critical twist
  (scale free energy distribution)

Tube-tube interactions:
  Reconnection of one tube affects the surrounding magnetic flux tubes

For the size distribution, Evolution according to fluid velocity is sufficient (even without twisting).

For the intertime distribution, Tube interactions are necessary.
In order to observe that for close-in-time flares the energy of the second flare is close but larger than the energy of the previous one.

Reconnection heats up the surrounded plasma increasing the local coronal pressure and the “critical” twist of the surrounding tubes.

Rather than avalanching, this leads to a stabilizing effect. Following flare larger than the previous one!

\[ \mathcal{P}(\lambda | t_{th}) = P(E_{i+1} / E_i > \lambda | \Delta t_i < t_{th}) - Q(\lambda | t_{th}) + \sigma(\lambda | t_{th}) \]
Neuronal avalanches

Beggs & Plenz (J. Neuroscience 2003, 2004) have measured spontaneous local field potentials continuously using a 60 channel multielectrode array in mature organotypic cultures of rat cortex in vitro and in vivo (rat & monkey) (PNAS 2008, 2009)

- dissociated neurons (V. Pasquale et al, Neurosci. 2008;
  A. Mazzoni et al PLoS ONE 2007)

- Avalanche size distribution is a power law with an exponent close to -3/2
- Avalanche duration distribution is a power law with an exponent close to -2.0

Critical state optimizes information transmission
We introduce the main ingredients of neural activity:

Threshold firing, Neuron refractory period, Activity dependent synaptic plasticity

We assign to each neuron a potential $v_i$ and to each synapse a strength $g_{ij}$

A neuron fires when the potential is at or above threshold $v_{\text{max}}$ (-55mV)

Synapses can be excitatory or inhibitory

After firing a neuron is set to zero resting potential (-70mV) and remains quiescent for one time step (refractory period)

Activity dependent (Hebbian) plasticity and pruning

Activity is triggered by random stimulation of a single neuron
After training the network by plastic adaptation, we apply a sequence of stimuli at random to trigger avalanche activity.

- different $\alpha$
- regular, small world, scale-free networks
- excitatory and inhibitory synapses

$1.5\pm0.1$ & $2.1\pm0.1$ for avalanche size & duration

Levina, Herrmann, Geisel, Nat Phys 2007
Millmann, Mihalas, Kirkwood, Niebur, Nat Phys 2010

Avalanche inter-time distribution

Experiments performed by D. Plenz (Lombardi et al PRL 2012)

Spontaneous neuronal activity can exhibit slow oscillations between bursty periods, up-states, and quiet periods, down-states.

Small correlated avalanches, neurons depolarized after firing.

Disfacilitation period after large avalanche
Neurons hyperpolarized after firing.
Implementation of up and down states

- **Down-state**
  After an avalanche with \( s \geq s_{\min} \)
  all neurons active in the last avalanche become hyperpolarized depending on their own activity
  \( v_i = v_i - h \delta v_i \)
  \( h > 0 \) is a hyper-polarization constant
  short term memory at neuron level

  System is stimulated by a small constant random drive

- **Up-state**
  After an avalanche with \( s < s_{\min} \)
  all neurons active in the last avalanche become depolarized depending on the last avalanche size
  \( v_i = v_{\max} \left( 1 - \frac{s}{s_{\min}} \right) \)
  the smaller the last avalanche the closer the potential to the firing threshold
  Memory at the network level

  System is stimulated by a random drive
  (network effect which sustains the up-state) \( \in ]0, \frac{s_{\min}}{s} [ \)
Expressing the balance between excitation and inhibition is the unique parameter controlling the distribution

\[ R = \frac{h}{s_{\text{min}}} \]

\textbf{Homeostatic regulatory mechanism}
P(Δt; s_c) for avalanches with s > s_c

- Remove avalanches smaller than a given threshold s_c
- Evaluate new P(Δt; s_c)
- Fixed point at θ period
Avalanches and oscillations

- Hierarchical structure corresponding to nested $\theta$-$\gamma$ oscillations
- Large avalanches occur with $\theta$ frequency and trigger smaller ones related to $\gamma$
- Sizes related to $\theta$ cycles fall within the blue region of $P(s)$
- Sizes related to $\gamma$ cycles fall within the green region of $P(s)$
- The relationship between avalanches and oscillations does not imply a characteristic size

In fMRI data from 7 healthy humans we analyse extreme activity \((B > B_c)\)

\[ s_i(t) = B(\vec{r}_i, t + \delta t) - B(\vec{r}_i, t) \]

activity variation at each voxel \(i\)

We evaluate the conditional probability

\[ P(\Delta s < s_0 | \Delta t < t_0) \]

with \(\Delta t = t' - t\) and \(\Delta s = s_i(t') - s_m(t)\)

Both in the real and in a reshuffled catalog where \(B\) are uncorrelated

We monitor the conditional probability difference

\[ \delta P(s_0 | t_0) = P(\Delta s < s_0 | \Delta t_i < t_0) - Q(s_0 | t_0) \]

\[ > \sigma(m_0 | t_0) \quad \text{correlations!} \]
The derivative \( \frac{d\Delta P(s_0 \mid t_0)}{ds_0} \) represents the probability difference to observe \( \Delta s = s_0 \) with \( \Delta t < t_0 \)

Brain tends to realize activity balance

depressions are compensated by successive enhancements and vice versa

\[ P(\Delta s < s_0 \mid \Delta t < t_0) \] is different than zero

Consecutive variations with opposite sign are correlated

A local increase in activity induces a close-in-time activity depression
CONCLUSIONS

- Power law behaviour for event size distribution in many natural phenomena → critical activity
- Complex temporal correlations
- Scaling properties of energy/time distributions unable to discriminate among different phenomena
- Conditional probability analysis
- Aftershock triggering controlled by stress diffusion
- Solar flare occurrence driven by kink instability due to turbulent flow
- Balance between excitation and inhibition controls temporal organization in brain activity