Avalanches in Turbulent Confined Plasmas

P.H. Diamond
CMTFO, CASS, Dept. of Physics, UCSD
WCI Center for Fusion Theory, NFRI

KITP Avalanches Program; 2014
Outline:

• A very brief primer on tokamak turbulence and transport
• Avalanches in turbulent transport
• Zonal flows and the secondary pattern selection problem
  • ExB staircase and avalanches
    • Staircase as a heat flux jam
• Discussion
What is a Tokamak?

N.B. No advertising intended…
Tokamak: the most intensively studied magnetic confinement device

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>ITER</th>
<th>KSTAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius</td>
<td>6.2m</td>
<td>1.8m</td>
</tr>
<tr>
<td>Minor radius</td>
<td>2.0m</td>
<td>0.5m</td>
</tr>
<tr>
<td>Plasma volume</td>
<td>830$\text{m}^3$</td>
<td>17.8$\text{m}^3$</td>
</tr>
<tr>
<td>Plasma current</td>
<td>15MA</td>
<td>2.0MA</td>
</tr>
<tr>
<td>Toroidal field</td>
<td>5.3T</td>
<td>3.5T</td>
</tr>
<tr>
<td>Plasma fuel</td>
<td>H, D-T</td>
<td>H, D-D</td>
</tr>
<tr>
<td>Superconductor</td>
<td>Nb$_3$Sn, NbTi</td>
<td>Nb$_3$Sn, NbTi</td>
</tr>
</tbody>
</table>
Basic of Magnetic Fusion

What is required for ignition?

- Fuel: D, T
- Amount/density \( n \)
- Ignition temperature \( T \)
- Energy confinement time \( \tau_E \)

Fusion power \( \sim n^2 T^2 \left( \sim \beta^2 B^4 \right) \geq \) Loss power \( \sim \frac{nT}{\tau_E} \)

\[ n \cdot T \cdot \tau_E \geq 3 \times 10^{28} \text{ m}^{-3}\text{Ks} \]

Lawson criterion for D-T fusion

\( \Rightarrow \) Good confinement required for ignition!

\( \beta = \frac{P_{Th}}{P_{B^2}} \)

Limited by stability
Tokamak Turbulence and Transport

→ How do plasmas form a profile?
→ What limits gradients?
Primer on Turbulence in Tokamaks I

- Strongly magnetized
  - Quasi 2D cells
  - Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance)

- $\vec{V}_\perp = + \frac{c}{B} \vec{E} \times \hat{z}$

- $\nabla T_e, \nabla T_i, \nabla n$ driven

- Akin to thermal Rossby wave, with: $g \rightarrow$ magnetic curvature

- Resembles to wave turbulence, not high $Re$ Navier-Stokes turbulence

- $Re$ ill defined, $K \leq 1$
Primer on Turbulence in Tokamaks II

[Klasky, ORNL; Ethier, Wang, PPPL]

S. Ku et al, EPS/ICPP 2012
Primer on Turbulence in Tokamaks III

- $\nabla T, \nabla n$, etc. driver
- Quasi-2D, elongated cells aligned with $B_0$
- Characteristic scale $\sim$ few $\rho_i$
- Characteristic velocity $v_d \sim \rho_*c_s$

Transport scaling: $D \sim \rho v_d \sim \rho_*D_B \sim D_{GB}$

i.e. Bigger is better! $\Rightarrow$ sets profile scale via heat balance

Reality: $D \sim \rho_*^\alpha D_B, \alpha < 1 \Rightarrow$ why??

2 scales:
- $\rho \equiv$ gyro-radius
- $a \equiv$ cross-section
- $\rho_* \equiv \rho/a \Rightarrow$ key ratio
L→H Transition → Transport Barrier Formation

- A Remarkable Phenomenon: Plasma Spontaneously Self-Organizes to Improved Confinement
  - L→H Transition – jam forms at edge
  - Transport bifurcation, ‘phase transition’ ⇒ $P_{\text{thresh}}$, hysteresis, etc.
  - Characterized by reduction of transport, turbulence in localized edge layer
  - Likely related to $V_{\text{ExB}}$ shear suppression of turbulent transport in edge layer

A. Hubbard et. al. 2002
Coupling of Transport Bifurcation to turbulence, $\langle u_E \rangle'$ suppression

Non-linear Fick’s Law, extension

$$Q = -\frac{\chi T}{1 + \alpha v'_E} \nabla T - \chi_{neq} \nabla T$$

Shearing feedback

$$v'_E = -\frac{\partial}{\partial r} \left( \frac{c}{eB n_0} \nabla p \right)$$

Profile Bifurcation

Heat flux $S$-curve induced by profile-dependent shearing feedback

FIG. 2. Temperature profiles near the power threshold (arbitrary units):
(a) $Q(a) = 0.99Q_c$; (b) $Q(a) = 1.01Q_c$. 

FIG. 4. Power hysteresis in the energy confinement time (arbitrary units): (a) increasing power; (b) decreasing power.
Avalanches in Turbulent Transport
Basic Phenomenology of CA Models – and Transport

• See: P.D. and Hahm, PoP’95; Newman, et al. PoP’96

• Avalanches happen:

  ➔ broad spectrum of inward, outward propagating avalanches evident

• What is an avalanche?
  – sequence of correlated toppling or eddy over-turning events
  – akin to fall of dominos
  – typically: \( \Delta_c < l_{aval} < L_p \) → meso-scale
• Cells “pinned” by magnetic geometry

• Remarkable similarity:

<table>
<thead>
<tr>
<th>Turbulent transport in toroidal plasmas</th>
<th>Sandpile model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Localized fluctuation (eddy)</td>
<td>Grid site (cell)</td>
</tr>
<tr>
<td><strong>Local turbulence mechanism:</strong></td>
<td><strong>Automata rules:</strong></td>
</tr>
<tr>
<td>Critical gradient for local instability</td>
<td>Critical sandpile slope ($Z_{ct}$)</td>
</tr>
<tr>
<td>Local eddy-induced transport</td>
<td>Number of grains moved if unstable ($N_f$)</td>
</tr>
<tr>
<td>Total energy/particle content</td>
<td>Total number of grains (total mass)</td>
</tr>
<tr>
<td>Heating noise/background fluctuations</td>
<td>Random rain of grains</td>
</tr>
<tr>
<td>Energy/particle flux</td>
<td>Sand flux</td>
</tr>
<tr>
<td>Mean temperature/density profiles</td>
<td>Average slope of sandpile</td>
</tr>
<tr>
<td>Transport event</td>
<td>Avalanche</td>
</tr>
<tr>
<td>Sheared electric field</td>
<td>Sheared flow (sheared wind)</td>
</tr>
</tbody>
</table>

Automaton toppling ↔ Cell/eddy overturning
Are avalanches a consequence of the toy CA model? NO!

- Avalanches observed, studied in flux driven simulations
  - First: Carreras, et. al. PoP'96 → resistive interchanges
  - GK: GYSELA, GT5D, XGC1p ...

- Comment:
  - flux tube and $\delta f$ simulations and those which artificially constrain $\nabla P$, will not capture (full) avalanche dynamics
  - avalanching not captured in quasi-linear models
Transport: Local or Non-local?

• 40 years of fusion plasma modeling
  - local, diffusive transport
    \[ Q = -n\chi(r)\nabla T \]

• 1995 → increasing evidence for:
  - transport by avalanches as in sand pile/SOCs
  - turbulence propagation and invasion fronts
  - non-locality of transport
    \[ Q = -\int \kappa(r, r')\nabla T(r')\,dr' \]

• Physics:
  - Levy flights, SOC, turbulence fronts…

• Fusion:
  - gyro-Bohm breaking
    (ITER: significant \( \rho_* \) extension)
  → **fundamentals of turbulent transport modeling?**

Guilhem Dif-Pradalier et al. PRL 2009
What Do Profiles Look Like?

• SOC profile $\neq$ linearly marginal profile

• For moderate drive, SOC occupation profile $<$ marginal profile

• N.B. Important
  – Observe SOC profile approaches marginal profile near boundary
  – Flip intensity largest near boundary $\rightarrow$ losses
  – As deposition increases, edge gradient steepens
  
  $\Rightarrow$ with bi-stable flux, transport bifurcation naturally initiated first, at boundary

Newman PoP96
Heat avalanche dynamics model (Continuum)

Hwa+Kardar ’92, P.D. + Hahm ’95, Carreras, et al. ’96, ... GK simulation, ... Dif-Pradalier ’10

- $\delta T$: deviation from marginal profile $\rightarrow$ conserved order parameter

- Heat Balance Eq.: $\partial_t \delta T + \partial_x Q[\delta T] = 0$ $\rightarrow$ up to source and noise

- Heat Flux $Q[\delta T]$ $\rightarrow$ utilize symmetry argument, ala’ Ginzburg-Landau

  - Usual: $\rightarrow$ joint reflectional symmetry (Hwa+Kardar’92, Diamond+Hahm ’95)

\[ Q = Q_0(\delta T) = \frac{\lambda}{2} \delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T \]

lowest order $\rightarrow$ Burgers equation

\[ \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T \]
• External Shear

- How is transport suppressed?
  ➔ shear decorrelation!

Back to sandpile model:

2D pile + 
sheared flow of 
grains

Shearing flow decorrelates
Toppling sequence

Avalanche coherence destroyed by shear flow

FIG. 10. A cartoon of the sandpile with a shear flow zone. The whole pile is flowing to the right at the top and to the left at the bottom connected by a variable sized region of sheared flow.

FIG. 11. Time evolution of the overturning sites (like Fig. 4). The avalanches do not appear continuous in time because only every 50th time step is shown. (a) The shear-free case shows avalanches of all lengths over the entire radius. (b) The case with sheared flow shows the coherent avalanches being decorrelated in the shear zone in the middle of the pile.
• Implications:

Spectrum of Avalanches

**FIG. 12.** (a) Frequency spectra with and without a shear flow region. This shows a marked decrease in the low-frequency power (with shear) and a commensurate increase in high-frequency power. (b) The insert shows the decorrelation time ($\tau_d = 1/\sigma$) as a function of the shear parameter (the product of the shearing rate and the size of the shear zone).

**FIG. 11.** Time evolution of the overturning sites (like Fig. 4). The avalanches do not appear continuous in time because only every 50th time step is shown. (a) The shear-free case shows avalanches of all lengths over the entire radius. (b) The case with sheared flow shows the coherent avalanches being decorrelated in the shear zone in the middle of the pile.

**FIG. 14.** The slopes of a sandpile with a shear region in the middle, including all the shear effects (diamonds) and just the transport decorrelation and the linear effect (circles).
Concept of a Transport Bifurcation
i.e. how generate the sheared flow??

N.B. Edge sheared flow / transport barrier $\Rightarrow$ L$\rightarrow$H transition

→ First Theoretical Formulation of L$\rightarrow$H Transition as an

- Transport Bifurcation
- $\langle E_T \rangle'$ Bifurcation

$$Q = -\frac{\chi}{1 + \alpha \langle V_E \rangle^2} \nabla T - \chi_0 \nabla T$$

→ Appearance of S-curve in a Physical Model of L$\rightarrow$H Transition

→ Formulation of Criticality Condition (Threshold) for Transport Bifurcation

→ Theoretical Ideas on Hysteresis, ELMs, Pedestal Width, .....
L→H Transition

• Now try bi-stable toppling rule, i.e. if $Z_i - Z_{i+1}$ large enough
  ⇒ reduced or no toppling

• Obvious motivation is $Q = -\frac{\chi \nabla P}{1+\alpha \nabla E^2}$ and $V_E \approx \frac{c}{eBn} \frac{\nabla P}{n}$

• Hard gradient limit imposed

• Transitions happen, pedestal forms!
Note

• Critical deposition level required to form pedestal ("power threshold")
• Pedestal expands inward with increasing input after transition triggered
• Now, including ambient diffusion (i.e. neoclassical)
  – $N_F$ threshold evident
  – Asymmetry in $L \rightarrow H$ and $H \rightarrow L$ depositions
Hysteresis Happens!

- Hysteresis loop in mean flux-gradient relation appears for $D_0 \neq 0$
- Hysteresis is consequence of different transport mechanisms at work in “L” and “H” phases
- Diffusion ‘smoothes’ pedestal profiles, allowing filling limited ultimately by large events

\[ \Gamma(R) = \text{Flux} \]
\[ Z(R) = \text{Mean Slope} \]

Gruzinov PoP2003
Zonal Flows and the Secondary Pattern Selection Problem
• Zonal Flows Ubiquitous for:
  ~ 2D fluids / plasmas \( R_0 < 1 \)
  Rotation \( \vec{\Omega} \), Magnetization \( \vec{B}_0 \), Stratification
  Ex: MFE devices, giant planets, stars…
What is a Zonal Flow?

- $n = 0$ potential mode; $m = 0$ (ZFZF), with possible sideband (GAM)
- toroidally, poloidally symmetric $E \times B$ shear flow

Why are Z.F.’s important?

- Zonal flows are secondary (nonlinearly driven):
  - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. ‘78)
  - modes of minimal damping (Rosenbluth, Hinton ‘98)
  - drive zero transport ($n = 0$)
- natural predators to feed off and retain energy released by gradient-driven microturbulence
Zonal Flows I

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
  → Zonal flow in magnetized plasma / QG fluid
  - Kelvin’s theorem is ultimate foundation

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge
    \[ \rho^2 \nabla^2 \phi = n_{i,GC}(\phi) - n_e(\phi) \]
    (ion GC, electron density)

  - so \( \Gamma_{i,GC} \neq \Gamma_e \) → \( \rho^2 \left\langle \tilde{v}_{rE} \nabla^2 \phi \right\rangle \neq 0 \) ‘PV transport’

  - If 1 direction of symmetry (or near symmetry):
    \[ -\rho^2 \left\langle \tilde{v}_{rE} \nabla^2 \phi \right\rangle = -\partial_r \left\langle \tilde{v}_{rE} \tilde{v}_{\perp E} \right\rangle \] (Taylor, 1915)

  \[ -\partial_r \left\langle \tilde{v}_{rE} \tilde{v}_{\perp E} \right\rangle \] Reynolds force → Flow

What sets cross-phase?
Shearing I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree’66, BDT’90)
  - radial scattering + $\langle V_E \rangle'$ $\rightarrow$ hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1 / \tau_c$
  - shaping, flux compression: Hahm, Burrell ’94

- Other shearing effects (linear):
  - spatial resonance dispersion: $\omega - k_{||} v_{||} \Rightarrow \omega - k_{||} v_{||} - k_\theta \langle V_E \rangle' (r - r_0)$
  - differential response rotation $\rightarrow$ especially for kinetic curvature effects

$\rightarrow$ N.B. Caveat: Modes can adjust to weaken effect of external shear

(Carreras, et. al. ‘92; Scott ‘92)
Shearing II

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. ‘98, et. seq.)
  Coherent interaction approach (L. Chen et. al.)

\[ \frac{dk_r}{dt} = -\partial (\omega + k_\theta V_E) / \partial r ; \quad V_E = \langle V_E \rangle + \tilde{V}_E \]

Mean shearing
\[ k_r = k_r^{(0)} - k_\theta V_E' \tau \]

Zonal
\[ \left\langle \delta k_r^2 \right\rangle = D_k \tau \]

Random shearing
\[ D_k = \sum_q k^2 \left| \tilde{V}'_{E,q} \right|^2 \tau_{k,q} \]

- Mean Field Wave Kinetics
\[ \frac{\partial N}{\partial t} + (\tilde{V}_g + \tilde{V}) \cdot \nabla N - \frac{\partial}{\partial r} (\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial k} = \gamma_k N - C\{N\} \]

\[ \Rightarrow \frac{\partial}{\partial t} \langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N \rangle = \gamma_k \langle N \rangle - \langle C\{N\} \rangle \]

- Wave ray chaos (not shear RPA) underlies \( D_k \rightarrow \) induced diffusion
- Induces wave packet dispersion
- Applicable to ZFs and GAMs

Zonal shearing
Shearing III

- Energetics: Books Balance for Reynolds Stress-Driven Flows!

- Fluctuation Energy Evolution – Z.F. shearing

\[
\int dk \omega \left( \frac{\partial}{\partial t} \langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N \rangle \right) \Rightarrow \frac{\partial}{\partial t} \langle \varepsilon \rangle = -\int dk V_{gr} (\tilde{k}) D_k \frac{\partial}{\partial k_r} \langle N \rangle
\]

Point: For \( d\langle \Omega \rangle / dk_r < 0 \), Z.F. shearing damps wave energy

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

\[
\partial_t \delta V_\theta + \partial \left( \delta \langle \tilde{V}_r \tilde{V}_\theta \rangle \right)/\partial r = -\gamma \delta V_\theta
\]

N.B.: Wave decorrelation essential: Equivalent to PV transport (c.f. Gurcan et. al. 2010)

- Bottom Line:
  - Z.F. growth due to shearing of waves
  - “Reynolds work” and “flow shearing” as relabeling \( \rightarrow \) books balance
  - Z.F. damping emerges as critical; MNR ‘97
Feedback Loops I

- Closing the loop of shearing and Reynolds work
- Spectral ‘Predator-Prey’ equations

Prey → Drift waves, $\langle N \rangle$
\[
\frac{\partial}{\partial t} \langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N \rangle = \gamma_k \langle N \rangle - \frac{\Delta \omega_k}{N_0} \langle N \rangle^2
\]

Predator → Zonal flow, $|\phi_q|^2$
\[
\frac{\partial}{\partial t} |\phi_q|^2 = \Gamma_q \left[ \frac{\partial \langle N \rangle}{\partial k_r} \right] |\phi_q|^2 - \gamma_d |\phi_q|^2 - \gamma_{NL} [ |\phi_q|^2 ] |\phi_q|^2
\]
Feedback Loops II

- Recovering the ‘dual cascade’:
  - Prey → \( <N> \sim <\Omega> \Rightarrow \) induced diffusion to high \( k_r \) \( \Rightarrow \) Analogous → forward potential
    enstrophy cascade; PV transport
  - Predator → \( |\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle \) \( \Rightarrow \) growth of \( n=0, m=0 \) Z.F. by turbulent Reynolds work
    \( \Rightarrow \) Analogous → inverse energy cascade

- Mean Field Predator-Prey Model
  (P.D. et. al. ’94, DI^2H ‘05)
  \[
  \frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta \omega N^2 \\
  \frac{\partial}{\partial t} V^2 = \alpha NV^2 - \gamma_d V^2 - \gamma_{NL} (V^2)V^2
  \]

**System Status**

<table>
<thead>
<tr>
<th>State</th>
<th>No flow</th>
<th>Flow ((\alpha_2 = 0))</th>
<th>Flow ((\alpha_2 \neq 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N) (drift wave turbulence level)</td>
<td>( \frac{\gamma}{\Delta \omega} )</td>
<td>( \frac{\gamma_d}{\alpha} )</td>
<td>( \frac{\gamma_d + \alpha_2\gamma\alpha^{-1}}{\alpha + \Delta \omega \alpha_2 \alpha^{-1}} )</td>
</tr>
<tr>
<td>(V^2) (mean square flow)</td>
<td>0</td>
<td>( \frac{\gamma}{\alpha} )</td>
<td>( \frac{\gamma - \Delta \omega \gamma \alpha^{-1}}{\alpha + \Delta \omega \alpha_2 \alpha^{-1}} )</td>
</tr>
<tr>
<td>Drive/excitation mechanism</td>
<td>Linear growth</td>
<td>Linear growth</td>
<td>Linear growth Nonlinear damping of flow</td>
</tr>
<tr>
<td>Regulation/inhibition mechanism</td>
<td>Self-interaction</td>
<td>Random shearing, self-interaction</td>
<td>Random shearing, self-interaction</td>
</tr>
<tr>
<td>Branching ratio (\frac{V^2}{N})</td>
<td>0</td>
<td>( \frac{\gamma}{\gamma_d} )</td>
<td>( \frac{\gamma - \Delta \omega \gamma \alpha^{-1}}{\gamma_d} )</td>
</tr>
<tr>
<td>Threshold (without noise)</td>
<td>( \gamma &gt; 0 )</td>
<td>( \gamma &gt; \Delta \omega \gamma \alpha^{-1} )</td>
<td>( \gamma &gt; \Delta \omega \gamma \alpha^{-1} )</td>
</tr>
</tbody>
</table>
A Central Question: Secondary Pattern Selection

• Two secondary structures suggested
  – Zonal flow $\rightarrow$ quasi-coherent, regulates transport via shearing
  – Avalanche $\rightarrow$ stochastic, induces extended transport events
• Nature of co-existence?
Staircases and Traffic Jams

Single Barrier → Lattice of Shear Layers

→ Jam Patterns
Observation of ExB staircases

→ Failure of conventional theory
  
  (emergence of particular scale???)

Model extension from Burgers to telegraph

\[ \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T \]

\[ \Rightarrow \tau \partial_t^2 \delta T + \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T \]

finite response time → like drivers’ response time in traffic

Analysis of telegraph eqn. predicts heat flux jam

- scale of jam comparable to staircase step
Motivation: ExB staircase formation (1)

- ExB flows often observed to self-organize in magnetized plasmas eg.) mean sheared flows, zonal flows, ...

- ExB staircase’ is observed to form
  
  (G. Dif-Pradalier, P.D. et al. Phys. Rev. E. ’10)

- flux driven, full f simulation

- Quasi-regular pattern of shear layers and profile corrugations

- Region of the extent \( \Delta \gg \Delta_c \) interspersed by temp. corrugation/ExB jets
  
  \( \rightarrow \) ExB staircases

- so-named after the analogy to PV staircases and atmospheric jets

- Step spacing \( \rightarrow \) avalanche outer-scale
**ExB Staircase (2)**

- Important feature: co-existence of shear flows and avalanches

- Seem mutually exclusive ?!?
  - strong ExB shear prohibits transport
  - avalanches smooth out corrugations

- Can co-exist by separating regions into:
  1. avalanches of the size $\Delta \gg \Delta_c$
  2. localized strong corrugations + jets

- How understand the formation of ExB staircase???
  - What is process of self-organization linking avalanche scale to ExB step scale?
    i.e. how explain the emergence of the step scale ???
Staircases, cont’d

• The point:
  – fit: \( Q = -\int dt' \kappa(r, r') \nabla T(r') \) \( \kappa(r, r') \sim \frac{S^2}{(r - r')^2 + \Delta^2} \) \( \rightarrow \) some range in exponent
  – \( \Delta \gg \Delta_c \) i.e. \( \Delta \sim \) Avalanche scale \( \gg \Delta_c \sim \) correlation scale
  – Staircase ‘steps’ separated by \( \Delta \) ! \( \rightarrow \) stochastic avalanches produce quasi-regular flow pattern!?

N.B.

• The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking…)

• What IS new is the connection to stochastic avalanches, independent of geometry

  – What is process of self-organization linking avalanche scale to zonal pattern step?
    i.e. How extend predator-prey feedback model to encompass both avalanche
    and zonal flow staircase? Self-consistency is crucial!
Corrugation points and rational surfaces – no relation!

Step location not tied to magnetic geometry structure in a simple way.
Staircases build up from the edge

→ staircases may not be related to zonal flow eigenfunctions

→ How describe generation mechanism??

(GYSELA simulation)
Towards a model

- How do we understand quasi-regular pattern of ExB staircase, generated from stochastic heat avalanche???

- An idea: jam of heat avalanche

  corrugated profile ↔ ExB staircase

  → corrugation of profile occurs by ‘jam’ of heat avalanche flux

  * → time delay between $Q[\delta T]$ and $\delta T$ is crucial element

  like drivers’ response time in traffic

- How do we actually model heat avalanche ‘jam’??? → origin in dynamics?
Traffic jam dynamics: ‘jamiton’

• A model for Traffic jam dynamics → Whitham

\[ \rho_t + (\rho v)_x = 0 \]
\[ v_t + vv_x = -\frac{1}{\tau} \left\{ v - V(\rho) + \frac{v}{\rho} \rho_x \right\} \]

→ Instability occurs when \( \tau > \frac{\nu}{(\rho_0 V_0')^2} \)

\[ D_{eff} = v - \tau \rho_0^2 V'_0^2 < 0 \rightarrow \text{clustering instability} \]

→ Indicative of jam formation

• Simulation of traffic jam formation

http://math.mit.edu/projects/traffic/

→ Jamitons (Flynn, et.al., ’08)

n.b. I.V.P. → decay study
Heat avalanche dynamics model (`the usual')

Hwa+Kardar ’92, P.D. + Hahm ’95, Carreras, et al. ’96, ... GK simulation, ... Dif-Pradalier ’10

- \( \delta T \): deviation from marginal profile → conserved order parameter

- Heat Balance Eq.: \( \partial_t \delta T + \partial_x Q[\delta T] = 0 \) → up to source and noise

- Heat Flux \( Q[\delta T] \) → utilize symmetry argument, ala’ Ginzburg-Landau

- Usual: → joint reflectional symmetry (Hwa+Kardar’92, Diamond+Hahm ’95)

\[
\delta T \leftrightarrow -\delta T \quad x \leftrightarrow -x
\]

lowest order → Burgers equation

\[
Q = Q_0(\delta T) = \frac{\lambda}{2} \delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T
\]

hyperdiffusion

\[
\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T
\]
An extension of the heat avalanche dynamics

• An extension: a finite time of relaxation of $Q$ toward SOC flux state

$$\partial_t Q = -\frac{1}{\tau} (Q - Q_0(\delta T))$$

$$Q_0[\delta T] = \frac{\lambda}{2} \delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

(Guyot-Krumhansl)

$\rightarrow$ In principle $\tau(\delta T, Q_0)$ $\Leftrightarrow$ large near criticality ($\sim$ critical slowing down)

i.e. enforces time delay between $\delta T$ and heat flux

• Dynamics of heat avalanche:

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \chi_4 \partial_x^4 \delta T - \tau \partial_t^2 \delta T$$

$\rightarrow$ Burgers

(P.D. + T.S.H. ’95)

New: finite response time

$\rightarrow$ Telegraph equation

n.b. model for heat evolution

diffusion $\rightarrow$ Burgers $\rightarrow$ Telegraph
Relaxation time: the idea

- What is \( \tau \) physically? \( \rightarrow \) Learn from traffic jam dynamics

- A useful analogy:

<table>
<thead>
<tr>
<th>heat avalanche dynamics</th>
<th>traffic flow dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>temp. deviation from marginal profile</td>
<td>local car density</td>
</tr>
<tr>
<td>heat flux</td>
<td>traffic flow</td>
</tr>
<tr>
<td>mean SOC flux (ala joint reflection symmetry)</td>
<td>equilibrium, steady traffic flow</td>
</tr>
<tr>
<td>heat flux relaxation time</td>
<td>driver’s response time</td>
</tr>
</tbody>
</table>

- driver’s response can induce traffic jam
- jam in avalanche \( \rightarrow \) profile corrugation \( \rightarrow \) staircase?!?
- Key: instantaneous flux vs. mean flux
Time delay: microscopic foundation?

- Relaxation by plasma turbulence = mixing of phase space density

\[
\frac{df}{dt} = 0 \Rightarrow \partial_t \langle \delta f(1) \delta f(2) \rangle + \frac{1}{\tau_{mix}} \langle \delta f(1) \delta f(2) \rangle = -\langle \tilde{v}_r, \delta f \rangle \langle f \rangle '
\]

phase space density correlation = ‘phasetrophy’

turbulent mixing
production due gradient relaxation

i.e. PV mixing time sets delay

- Energy moment leads to heat flux evolution equation (Gurcan ’13)

\[
\partial_t Q = -\frac{1}{\tau_{mix}} (Q - Q_0) \quad Q_0 = -\chi_{turb} \nabla T
\]

→ Heat flux relaxes toward the mean value, in the mixing time

The delay time is a natural consequence of phase space density mixing. The delay time is typically in the order of mixing time.
Heat flux dynamics: when important?

• Heat flux evolution: \[ \partial_t Q = -\frac{1}{\tau_{mix}} (Q - Q_0) \rightarrow \text{time delay, when important?} \]

Conventional Transport Analysis

\[ \tau_{mix} \ll \text{time scale of interest} \]

→ Heat flux relaxes to the mean value immediately

\[ Q = Q_0 \]

→ Profile evolves via the mean flux

\[ \partial_t T + \partial_x Q_0 = 0 \]

then

diff. \[ \partial_t T = \chi \partial_x^2 T \]

Burgers \[ \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T \]

New approach for transport analysis

→ mixing time can be long, so \[ \tau_{mix} \sim \text{time scale of interest mesoscale} \]

→ Heat evo. and Profile evo. must be treated self-consistently

\[ \begin{cases} 
\partial_t Q = -\frac{1}{\tau} (Q - Q_0) \\
\partial_t \delta T + \partial_x Q[\delta T] = 0 
\end{cases} \]

then telegraph equation:

\[ \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \tau \partial_t^2 \delta T \]
Brief summary on model extension

- Physical idea: analogy to traffic dynamics, drivers’ response time
- Microscopic foundation: mixing of phase space density
- Finite response time → Heat dynamics described by telegraph eqn.
  \[ \sqrt{\frac{\chi_2}{\tau}} \]
- Connects avalanche dynamics to elasticity in/of turbulence
Analysis of heat avalanche dynamics via telegraph

- How do heat avalanches jam?

- Consider an initial avalanche, with amplitude $\delta T_0$, propagating at the speed $v_0 = \lambda \delta T_0$

  $\Rightarrow$ turbulence model dependent

- Dynamics:

  \[ \partial_t \delta T + v_0 \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \chi_4 \partial^4_x \delta T - \tau \partial^2_t \delta T \]

  - Pulse
  - ‘Heat flux wave’: $\sqrt{\frac{\chi_2}{\tau}}$
telegraph $\Rightarrow$ wavy feature
  - Two characteristic propagation speeds

  $\Rightarrow$ In short response time (usual) heat flux wave propagates faster

  $\Rightarrow$ In long response time, heat flux wave becomes slower and pulse starts overtaking.

What happens???
Analysis of heat avalanche jam dynamics

- In large tau limit, what happens? → Heat flux jams!!

- Recall plasma response time akin to driver’s response time in traffic dynamics

- negative heat conduction instability occurs (as in clustering instability in traffic jam dynamics)

\[
\partial_t \delta T + v_0 \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \chi_4 \partial^4_x \delta T - \tau \partial_t^2 \delta T
\]

- \( \rightarrow (\chi_2 - v_0^2 \tau) \partial_x^2 \delta T - \chi_4 \partial_x^4 \delta T \)

- \(<0 \text{ when overtaking} \)

- \( \rightarrow \text{clustering instability} \)

n.b. akin to negative viscosity instability of ZF in DW turbulence

- instead ZF as secondary mode in the gas of primary DW

- \( \Rightarrow \) Heat flux ‘jamiton’ as secondary mode in the gas of primary avalanches
Analysis of heat avalanche jam dynamics

• Growth rate of the jamiton instability

\[ \gamma = -\frac{1}{2\tau} + \frac{1}{2\tau} \sqrt{\frac{r + 1}{2} - 2\tau \chi_2 k^2 \left( 1 + \frac{\chi_4 k^2}{\chi_2} \right)} \]

\[ r = \sqrt{\left\{ 4\tau \chi_2 k^2 \left( 1 + \frac{\chi_4 k^2}{\chi_2} \right) - 1 \right\}^2 + 16v_0^2 k^2 \tau^2} \]

• Threshold for instability

\[ \tau > \frac{\chi_2}{v_0^2} \left( 1 + \frac{\chi_4 k^2}{\chi_2} \right) \quad \text{n.b.} \quad 1/\tau = 1/\tau[E] \]

→ clustering instability strongest near criticality

→ critical minimal delay time

• Scale for maximum growth

\[ k^2 \approx \frac{\chi_2}{\chi_4} \sqrt{\frac{\chi_4 v_0^2}{4\chi_2}} \quad \text{from} \quad \frac{\partial \gamma}{\partial k^2} = 0 \implies 8\tau \frac{\chi_4^2}{\chi_2} k^6 + 4\tau \chi_4 k^4 + 2\frac{\chi_4}{\chi_2} k^2 + 1 - \frac{v_0^2 \tau}{\chi_2} = 0 \]

→ staircase size, \[ \Delta_{stair}^2(\delta T), \delta T \] from saturation: consider shearing
Scaling of characteristic jam scale

• Saturation: Shearing strength to suppress clustering instability

Jam growth $\rightarrow$ profile corrugation $\rightarrow$ ExB staircase $\rightarrow$ $u'_E \times B$

$\rightarrow$ estimate, only

$\rightarrow$ saturated amplitude: $\frac{\delta T}{T_i} \sim \frac{1}{v_{thi} \rho_i} \sqrt{\frac{\chi_4}{\tau}}$

• Characteristic scale

$\Delta^2 \sim k^{-2}(\delta T) \sim \frac{2v_{thi}}{\chi T_i} \rho_i \sqrt{\chi_2 \tau}$

- Geometric mean of $\rho_i$ and $\sqrt{\chi_2 \tau}$: ambient diffusion length in 1 relaxation time
- ‘standard’ parameters: $\Delta \sim 10\Delta_c$
Jam growth qualitatively consistent with staircase formation

Dif-Pradalier ’13 caveat: based on model with compressional waves

outer radius:
large chi
→ smear out instability
or
→ heat flux waves propagate faster
→ harder to overtake, jam

good agreement in early stage

Dif-Pradalier ’13 caveat: based on model with compressional waves
Summary

• A model for ExB staircase formation
  - Heat avalanche jam $\rightarrow$ profile corrugation $\rightarrow$ ExB staircase
  - model developed based on analogy to traffic dynamics $\rightarrow$ telegraph eqn.

• Analysis of heat flux jam dynamics
  - Negative conduction instability as onset of jam formation
  - Growth rate, threshold, scale for maximal growth
  - Qualitative estimate: scale for maximal growth $\Delta \sim 10\Delta_c$
    $\rightarrow$ comparable to staircase step size
Ongoing Work

• This analysis ↔ set in context of heat transport

• Implications for momentum transport? ➔
  – consider system of flow, wave population, wave momentum flux
  – time delay set by decay of wave population
    correlation due ray stochastization ➔ elasticity
  – flux limited PV transport allows closure of system
Aside: FYI – Historical Note

→ Collective Dynamics of Turbulent Eddy

→ ‘Aether’ I – First Quasi-Particle Model of Transport?!

Kelvin, 1887

XLV. On the Propagation of Laminar Motion through a turbulently moving Inviscid Liquid. By Sir William Thomson, LL.D., F.R.S.*

1. IN endeavouring to investigate turbulent motion of water between two fixed planes, for a promised communication to Section A of the British Association at its coming Meeting in Manchester, I have found something seemingly towards a solution (many times tried for within the last twenty years) of the problem to construct, by giving vortex motion to an incompressible inviscid fluid, a medium which shall transmit waves of laminar motion as the luminiferous æther transmits waves of light.

2. Let the fluid be unbounded on all sides, and let \( u, v, w \) be the velocity-components, and \( p \) the pressure at \( (x, y, z, t) \). We have

\[
\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad \ldots \ldots \ldots \quad (1),
\]

* Communicated by the Author, having been read before Section A of the British Association at its recent Meeting in Manchester.
$R^2 \sim \langle \tilde{v}^2 \rangle$

21. Eliminating the first member from this equation, by (34), we find
\[
\frac{d^2 f}{dt^2} = \frac{2}{3} R^2 \frac{d^2 f}{dy^2} \quad \ldots \ldots \quad (51).
\]

Thus we have the very remarkable result that laminar disturbance is propagated according to the well-known mode of waves of distortion in a homogeneous elastic solid; and that the velocity of propagation is \( \frac{\sqrt{2}}{3} R \), or about \( \cdot 47 \) of the average velocity of the turbulent motion of the fluid.

→ time delay between Reynolds stress and wave shear introduced
→ converts diffusion equation to wave equation
→ describes wave in ensemble of vortex quasi-particles