Ever more singular:
How crack front geometry determines crack front dynamics

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Fracture mechanics in ~1 min

Linear Elastic Fracture Mechanics (LEFM)

Singularity of the stress at the crack tip

Speed limit = $c_R$

Equation of motion $\Leftrightarrow$ Energy balance:

Energy flux into the crack tip = dissipation

$G = \Gamma$

LEFM: ignore what happens within the singular region...

(fine for this talk...)

**Slowing things down:**

*Understanding dynamic fracture through brittle gels*

Fracture of polyacrylamide ➔ dynamic fracture in *slow motion* by reducing sound velocities by **2-3 orders of magnitude**

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus (kPa)</th>
<th>Poisson ratio</th>
<th>$C_R$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gel</td>
<td>100-1000</td>
<td>0.5</td>
<td>5-14</td>
</tr>
<tr>
<td>X% acrylamide</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y% bis-acrylamide</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMMA</td>
<td>3,900,000</td>
<td>0.35</td>
<td>930</td>
</tr>
<tr>
<td>Soda-Lime glass</td>
<td>70,000,000</td>
<td>0.22</td>
<td>3340</td>
</tr>
</tbody>
</table>

Change in the gel’s composition ➔ Change in elastic constants

*Young’s modulus* $E=100$-560 kPa

*Fracture energy* $\Gamma=13$-60 J/m$^2$
A simple crack moving at $0.6C_R \sim 3\text{m/s}$
Gels really probe dynamic fracture: e.g. checking the equation of motion: \( G = \Gamma \)

In an *infinite medium* and constant stress, \( \sigma_\infty \):

\[
\Gamma = G(l, v) \approx \frac{1-v^2}{E} \frac{8l}{\pi} \sigma_\infty^2 \left( 1 - \frac{v}{c_R} \right)
\]


The equation of motion \( G = \Gamma \) works perfectly:

Gels are perfectly representative of brittle materials!

Excellent agreement with Fracture Mechanics for a simple crack

Gels are a convenient testing ground for fracture mechanics

What happens when cracks stop being simple?
The Micro-branching instability

- At a critical velocity a single crack may become unstable to frustrated micro-branches.
- In gels, Micro-branches have the same functional form as in other brittle materials.

\[ V > V_c \]

Micro-branches within a **Crack Front**: Micro-branches are **Energy Sinks** that are:

- **Localized** within the crack front (\(z\) direction)
- **Align** in **chains** along the propagation (\(x\)) direction.
- **Bi-stable** within a crack front

\[
G(v,z) = \Gamma(z)
\]

**Is Energy balance local?**

**What are Crack Front Dynamics when \(z\)-invariance is broken?**
Crack fronts: A transition from 2D to 3D understanding of fracture

The conventional view

- Materials fail at a crack tip because stresses become singular.

\[ \sigma \sim \frac{K}{\sqrt{2\pi \cdot r}} \]

- Crack tip equation of motion:

Challenges to the 2D view – front instabilities

The micro-branching instability –

Localized out-of-plane structures

Other front instabilities

- Front waves (Ramanathan & Fisher ‘97, Sharon et al. ‘01)
- Stepped surfaces (Sommer ‘69, Tanaka et al. ‘98,00’, Baumberger et al. ‘08,’13)
Measuring rapid crack fronts Dynamics

Problem #1: Cracks are fast (~3 km/sec in glass)

Solution: Use gels (~3 m/sec in polyacrylamide)

\[ c \sim \sqrt{\frac{E}{\rho}} \quad E \sim 90 \text{ kPa} \quad c \sim 5 \text{ m/s} \]

Problem #2: How to image a crack front?

Solution: Look *through* the gel

The front becomes a moving shadow across the image
Simple crack

Crack tip imaging:

Front imaging:

The fracture surface *post-mortem*:

Micro-branching crack
Once cracks stop being geometrically simple, their **dynamics** become pretty **complex**.

Can we understand the

Front shapes and dynamics

Velocity fluctuations

Do “simple” **2D Fracture Mechanics work in an intrinsically 3D world??**
The dramatis personae:

Enter \( v(z), K(z), \Gamma(z) \)

Crack tip singularity

\[
\sigma \sim \frac{K}{\sqrt{2\pi \cdot r}}
\]

Transition to front dynamics: Assume the eq of motion is locally valid

\[
v(z) = v(K(z), \Gamma(z))
\]

Perturb around a straight front:

What determines \( \delta K(z) \) in microbranching?

What determines \( \delta \Gamma(z) \) in microbranching?

\[
\frac{\delta v(z)}{\delta K(z)} = \frac{\delta K(z)}{\delta \Gamma(z)} = \frac{\delta \Gamma(z)}{K_0} - \frac{\delta \Gamma(z)}{\Gamma_0}
\]

Fracture Energy

*Rice, '85. First order in \( x'(z) \), neglecting dynamic effects
The life and times of a microbranching event

- Micro-branch **initiation increases** $\delta \Gamma(z) > 0$
- The front is locally stretched as micro-branches progress due to the **inhomogeneity of $\Gamma(z)$**
- **Upon micro-branch arrest**, $\delta \Gamma(z) = 0$ while $\delta K(z) > 0$ $\Rightarrow$ fronts are **locally accelerated**
The crack front is a “slingshot”, cocked by micro-branch initiation.

What determines the magnitude of velocity at release?
What determines the moment of release?
The velocity at the moment of complete release (when $\delta \Gamma \sim 0$)

Immediately after a micro-branch dies $\delta \Gamma \sim 0$

... $\delta v$ and $\delta K$ measured independently become directly correlated

Accumulated data from nine different instances of microbranch death and release

The same data in a $y=x$ plot

Geometry + Fracture Mechanics determines dynamics

$$\frac{2\delta K(z)}{K_0} \sim \frac{\delta v(z)}{V_0}$$
Let's analyze the movie front by front: Front dynamics of one big microbranching event

- At first the front is locally retarded because of micro-branch nucleation + growth (decrease in velocity = increase in fracture energy)
- The curved front collapses into a cusp-like shape
- Cusp formation is immediately followed by release

A finite-time singularity scenario

1. Fronts propagate in the normal direction
2. Diverging curvature means diverging $\delta K$

Develop cusps $\Leftrightarrow$ shocks in curvature
Micro-branches yield to diverging stress $\Rightarrow$ fronts are released from pinning

How to test these assumptions?
Normal propagation + curvature = Burgers equation

\[ \partial_t u + v_n \cdot u(z) \cdot \partial_z u = 0 \]
Burger’s equation for the slope \( u(z) \)

Burger’s Eqn \( \Leftrightarrow \) Finite time singularity

\[ \frac{\partial^2 x}{\partial^2 z} \sim \frac{1}{t^* - t} \]
with \( t^* = \frac{1}{\kappa_0 \cdot v_n} \)
CUSP formation time

\( \partial_t x = v_x(z) = v_n \cos \theta \)
\( \sim v_n \cdot (1 - \theta^2/2) \)
\( \sim v_n \cdot (1 - (\partial x/\partial z)^2/2) \)
taking \( \partial / \partial z \) + using the slope:
\( u(z) \equiv \partial x / \partial z \)
Assumptions for Burger’s equation:

- \( v_n \sim \text{constant} \)
- initial curvature of the front (due to \( \delta \Gamma > 0 \))

No explicit fracture mechanics input required \( \iff \) only geometry!

How constant is the \( v_n \) during the stress buildup?

\[ \frac{(v_n - \langle v \rangle)_{\text{RMS}}}{\langle v \rangle} \]

\( v_n \) varies by \( \sim 10\% \) over stress build-up

\( v_n \) statistics are not far off from the assumptions!
Is *cusp formation* at all related to *micro-branch dynamics*?

\[ t^* = \frac{1}{(\kappa_0 \cdot v_n)} \]

*CUSP formation time*

Front “*stretching*” time, \( \tau \)

\( \tau \sim \) micro-branch *lifetime*

Are \( t^* \) and \( \tau \) related?

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**Geometry**

**Dynamics of cracks**
Are $t^*$ and $\tau$ related? Yes!

Cusp formation $t^* \sim$ Micro-branch lifetimes $\tau$!
Summary: Front geometry drives Front dynamics

Micro-branching provides insight into crack front dynamics

- When a micro-branch is nucleated the front curves due to increased fracture energy
- Crack front curvature spontaneously generates a cusp
- The formation of the cusp $\leftrightarrow$ singular “line tension” $\delta K$
- Front velocities at release are determined by front geometry + Fracture mechanics
- Singular line tension $\Rightarrow$ cusp collapse $\leftrightarrow$ Micro-branch Death

Thank you!
Friction is Fracture: Fracture Processes Drive Frictional Motion

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Review: Fracture
Linear Elastic Fracture Mechanics (LEFM)

- Linear elasticity $\rightarrow$ singularity of the stress at the crack tip
- Energy balance $\rightarrow$ Dissipation $=$ Energy flux into the crack tip
- Important velocity scale $C_R$, Rayleigh wave speed (1255m/s for PMMA)

$\sigma(r) \sim r^{-1/2}$
• Net contact area = $A \ll$ Nominal contact area

• At the transition from stick to slip contacts are being broken and reduce $A$.

We’ll show that:
  Rupture of contacts described by classic Fracture Mechanics

F. Philip Bowden and David Tabor (1950)
I transmitted \( \propto A \)

Frame Rate \( \sim 580,000 \) Frames/sec
Resolution: 1280 Pixels / 200mm

\[ A(x,t) = I(x,t) = \int I(x,y,t) \, dy \]


Strain measurements

2D strain tensor measured at 20 locations
\(~1 \mu V\) signal at 1 MHz rate
What types of rupture events occur upon slip initiation?

**Different ruptures modes:**
are determined by **local pre-stresses** near the interface!

Here we’ll concentrate on ruptures where $C_f < C_s$
“Slow” Rupture Fronts

Each t is a snapshot of the real area of contact across the entire interface (X-t plot)

Block detachment is mediated by propagating crack-like front

Friction $\Leftrightarrow$ Dynamic Fracture Problem
Characterizing “Slow” Ruptures

$0.04C_R < C_f < 0.3C_R$

Spatial profiles of strain collapse to a single functional form.
Does this collapse continue for even higher front velocities?

No data collapse at high velocities!

So, how can we explain this mess?!

LEFM
Comparing Strain Measurements To LEFM

\[ 0.04C_R < C_f < 0.3C_R \]

\[
\Delta \varepsilon_{ij} = \frac{K}{r^{1/2}} \sum'_{i,j}(\theta, c_f)
\]

One free parameter \( K \) fits all of the data well

**Fracture Mechanics:**
\[
\Gamma = \text{energy to break a unit area of contacts}
\]

\[ \Gamma \approx 1 \text{ J/m}^2 \]
Excellent agreement at high velocities.

\[ C_f = 0.96 C_R \]

- No free parameter - using same \( \Gamma \) (Fracture energy)

\[ \Delta \varepsilon_{ij} = \frac{K}{r^{1/2}} \sum'_{ij}(\theta, c_f) \]

\[ \Delta \varepsilon_{xy} \]

\[ \Delta \varepsilon_{xx} \]

\[ \Delta \varepsilon_{yy} \]

Failure at \( x - x_{\text{tip}} > 0 \)
Comparing LEFM to Measurements at All Velocities:

*One free* parameter $\Gamma$ (Fracture energy)

*Great* quantitative agreement for all velocities
Comparing LEFM to Measurements at All Velocities:

**One free parameter** $\Gamma$ (Fracture energy)

*Problem with stress drop prediction*

*Great* quantitative agreement for all velocities
Does the value of $\Gamma \sim 1\text{J/m}^2$ make sense?

Yes! When interface sparseness is taken into account $\Gamma \Leftrightarrow \Gamma_{\text{bulk}}$

Real area of contact - PMMA

Under our conditions: $A \sim 0.005A_0$

$\Gamma_{\text{bulk}} = \Gamma \cdot \frac{A_0}{\Delta A} = \frac{1}{(0.2 \times 0.005)} \sim 1000 \text{J/m}^2$

$\Rightarrow \Gamma_{\text{bulk}} \sim$ the measured bulk fracture energy for PMMA!
Well... What about **friction** (we are talking about friction)?

How is this compatible with a *characteristic* static friction coefficient?

It’s actually not....
In general:

μ_s can vary by ~ a factor of 2 – for the same materials under the same ambient conditions!

Dissipation $\leftrightarrow \Delta A(x,t)$ at the tip of a rupture front

Characterizing the dissipation scale, $X_c$

$A(x)$ characterizes the dissipation at each $x$, $c_f$

$X_c$ contracts as $c_f \rightarrow c_R$!

$X_c$ contracts due to relativistic effects at high front velocities

$$X_c = X_{c0}/f(c_f)$$

$$1/f(c_f) \sim \text{Lorentz contraction of length scales}$$

(for anti-plane $f(c_f) = (1-(c_f/c_s)^2)^{-1/2}$)

J. R. Rice (1980)
Y. Bar Sinai, Efim A. Brener, E. Bouchbinder GRL (2012)
Friction is (really) Fracture

• Singular fields at the rupture tip \( \Leftrightarrow \) Classic Shear Fracture
• **Measured** fracture energy \( (\Gamma \sim 1 \text{J/m}^2) \Leftrightarrow \sim \text{bulk fracture energy} \)
• Cohesive zone size contracts according to “Lorentz Contraction”

Questions:
• As \( C_f \rightarrow C_R \), **classic solution fails** to describe \( \varepsilon_{xy}(x-x_{\text{tip}}>0) \)
• Rupture Nucleation

Thank you!