Magnetic domain wall dynamics and correlated activated events below the depinning threshold

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Magnetic domain wall dynamics and correlated activated events below the depinning threshold

- Driven domain walls in thin film ferromagnets as a paradigmatic example of the universal dynamics of driven disordered elastic systems (in particular below the depinning threshold).

- New phenomena → theory?
- New predictions → experiments?
- Thermal Effects?. Universal activated dynamics?
- How good is the minimalistic elastic interface model for concrete systems?, Dynamic universality class of domain wall dynamics?
- “Irreversible collective jumps”: statics, depinning, and in between...
Pinning dependent field driven domain wall dynamics and thermal scaling in an ultrathin Pt/Co/Pt magnetic film

[J. Gorchon et al, PRL 113, 027205, July 2014]

- J. Gorchon, J. Ferre, V. Jeudy [LPS, Orsay]
- S. Bustingorry, A.B. Kolton [Bariloche]
- T. Giamarchi [Geneva]
Disorder vs Elasticity
A dynamic competition

Response to H in the UP direction?

Thin films
c \approx 0.5nm
Mean velocity $V$ vs applied field $H$ and Temperature $T$ ?

$V(H,T) = ?$
V(H,T)?... does it matter?

Interfaces Motion control → Applications

Disorder vs Elasticity → Universality
[in collective transport]
Similar Effective Physics

Contact lines in partial wetting
Moulinet, Rolley (Paris).

Fractures
Bonamy, Ponson, Santucci (Paris, Lyon)

Laurent Ponson's talk
Earthquakes models
Jagla, Rosso

Ferroelectric Domain Walls
P. Paruch et al., (Geneva).
Elastic String in a Random Medium

equation of motion for $u(x,t)$ ?
A minimal model

... to capture the competition between disorder and elasticity and to predict the response to a driving field

\[ \gamma \partial_t u(x, t) = c \partial_x^2 u(x, t) + F_p(u, x) + f + \eta(x, t) \]

[Note: far more simple than a micro-magnetic model... too simple?]
Do magnetic domain walls in thin films behave as simple elastic strings in disordered media?
Qualitatively OK!

![Graph showing velocity vs. magnetic field for Pt/Co/Pt films.

Metaxas et al, PRL 2007
Pt/Co/Pt films

Three-reference nonequilibrium steady states

... and quantitatively?

Scaling Arguments
Functional renormalization group
Numerical Simulations
Ultra-slow dynamics: creep

- Scaling Arguments
- Functional renormalization group
- Numerical Simulations

\[
\begin{align*}
\ln v &\sim f^{-\mu} \\
T > 0 &\quad v \sim (f-f_c)^\beta \\
T = 0 &\quad v \sim T^\psi
\end{align*}
\]
Creep Law

- 1987 Ioffe-Vinokur, Nattermann

- Beautiful scaling arguments, based on Anderson-Kim (1964), Langer (1967) ideas, and under strong assumptions (to be discussed later on!).

\[ v \sim \exp \left[ -\frac{U(f)}{T} \right] = \exp \left[ -\frac{U_c}{T} \left( \frac{f_c}{f} \right)^\mu \right] \]

- Sublinear Universal Response

- Nonlinear Transport exponent

From Statics (geometry)

\[ \mu = \frac{d - 2 + 2\zeta_{eq}}{2 - \zeta_{eq}} \]

- Thermal nucleous scaling with force

From Statics (geometry)

\[ \nu_{eq} = \frac{1}{2 - \zeta_{eq}} \]

**Divergentes Barriers (nucleous) → glassy nature of the ground state**
Creep Law

- **1987 Ioffe-Vinokur, Nattermann**

\[ v \sim \exp \left[ -\frac{U_c}{T} \left( \frac{f_c}{f} \right)^\mu \right] \]

\[ \mu = \frac{d - 2 + 2\zeta_{eq}}{2 - \zeta_{eq}} \]

\[ \nu_{eq} = \frac{1}{2 - \zeta_{eq}} \]

\[ d = 1, \quad \zeta_{eq} = 2/3 \]

Huse-Henley (1985), Kardar (1985)

Quantitative precise predictions

\[ \mu = 1/4 \quad \nu_{eq} = 3/4 \]

Magnetic DW

\[ H \leftrightarrow f \]

EXPERIMENTAL TESTS?
A crucial experiment

Interdimensional universality of dynamic interfaces

Kab-Jin Kim\textsuperscript{1}, Jae-Chul Lee\textsuperscript{1,2}, Sung-Min Ahn\textsuperscript{1}, Kang-Soo Lee\textsuperscript{1}, Chang-Won Lee\textsuperscript{3}, Young Jin Cho\textsuperscript{3}, Sunae Seo\textsuperscript{3}, Kyung-Ho Shin\textsuperscript{2}, Sug-Bong Choe\textsuperscript{1} & Hyun-Woo Lee\textsuperscript{4}
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Interdimentional universality of dynamic interfaces

Kab-Jin Kim¹, Jae-Chul Lee¹,², Sung-Min Ahn¹, Kang-Soo Lee¹, Chang-Won Lee³, Young Jin Cho³, Sunae Seo³, Kyung-Ho Shin², Sug-Bong Choe¹ & Hyun-Woo Lee⁴

\[ \ln v \sim -H^{-\mu} \sim -L_{opt}^\theta \quad \text{iff} \quad L_{opt} < w \quad \text{[wire width]} \]

If \( L_{opt} > w \) the interface behaves as a (single) particle

So far...

- The velocity-Force characteristics of Pt/Co/Pt thin film domain walls is *qualitatively* consistent with the d=1 elastic string model.

- In particular, CREEP motion of Pt/Co/Pt thin film domain walls is *quantitatively* consistent with the prediction for the d=1 elastic string model:

\[ v \sim e^{\frac{V_c}{T}} \left( \frac{f_c}{f} \right)^{1/4}, \quad L_{opt} \sim f^{-3/4} \]

- **Universality**: micromagnetic descriptions not necessary (nor practical) for *precisely* predicting some relevant observables.
New Questions

- Temperature dependence of the creep law?
  \[ v \sim e^{\frac{U_c}{T}} \left( \frac{f_c}{f} \right)^\mu, \quad L_{opt} \sim f^{-\nu} \]
- Quantitative analysis at the higher forces?
New data!
Velocity vs Field vs Temperature

[J. Gorchon etal, PRL July 2014]
Creep vs temperature

\[ H \ll H_{dep} \]

Non-universal intrinsic T dependencies

\[ T_{dep}(T), \quad H_{dep} = H_{dep}(T), \quad v_0(T) \]

[J. Gorchon et al, PRL July 2014]
New (old?) regime: “TAFF” \( H \lesssim H_{dep} \)

Effective barriers vanish linearly as \( H \to H_{dep} \)

\[ v = v_{dep} e^{-\frac{T_{dep}}{T} \left( 1 - \frac{H}{H_{dep}} \right)} \]

[J. Gorchon et al., PRL July 2014]
Theory?

\[ \nu \sim e^{-\frac{T_{dep}}{T} \left( \frac{H_{dep}}{H} \right)^\mu} \]

\[ \nu \sim T^\psi G \left[ \frac{(H - H_{dep})^\beta}{T^\psi} \right] \]

\[ \nu \sim e^{-\frac{T_{dep}}{T} \left( 1 - \frac{H}{H_{dep}} \right)} \]

**Diagram:***
- **Creep:** \( \ln \nu \sim f^{-\mu} \)
- **Thermal Rounding:** \( \nu(f_c) \sim T^\psi \)
- **Depinning:** \( T > 0 \)
- **Fast Flow:** \( T = 0 \)
  \( \nu \sim (f - f_c)^\beta \)

- **Equilibrium**
- **Depinning point** \( f_c \)
- **Fast flow** \( (f \rightarrow \infty) \)
Theory?

$\nu \sim e^{-\frac{T_{\text{dep}}}{T}(1 - \frac{H}{H_{\text{dep}}})}$

To illustrate:
$d$ dimensional interfaces in Periodic potentials, near $f_c$:

$$U(f) \sim (1 - \frac{f}{f_c})^{\frac{6-d}{4}}$$


d-independent linear dependence in a disordered potential (SR):
Theory?

\[ v \sim e^{-\frac{T_{\text{dep}}}{T} \left(1 - \frac{H}{H_{\text{dep}}}\right)} \]

\[ T=0^+ \quad \text{Simulations} \]

\[ v \sim e^{-\overline{U}(f)/T} \]

Matching with depinning

\[ \nu(H, T) \approx \nu(H_{\text{dep}}, T) e^{-\frac{T_{\text{dep}}}{T} \left( 1 - \frac{H}{H_{\text{dep}}} \right)} \]

\[ \nu(H_{\text{dep}}, T) \sim T^{\psi} \]

\[ \nu(f_c) \sim T^{\psi} \]

\[ T=0 \]
\[ \nu \sim (f-f_c)^{\beta} \]

(b) equilibrium  depinning  fast flow (f \to \infty)

creep \ln \nu \sim f^{-\mu}

thermal rounding
Matching with depinning

\[ \nu(H, T) \approx \nu(H_{\text{dep}}, T)e^{-\frac{T_{\text{dep}}}{T} \left(1 - \frac{H}{H_{\text{dep}}}\right)} \]

\[ \nu \sim T^\psi e^{-\frac{T_{\text{dep}}}{T} \left(1 - \frac{H}{H_{\text{dep}}}\right)} \]

[J. Gorchon et al., PRL July 2014]
Thermal rounding exponent of the depinning transition of an elastic string in a random medium

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We study numerically thermal effects at the depinning transition of an elastic string driven in a two-dimensional uncorrelated disorder potential. The velocity of the string exactly at the sample critical force is shown to behave as $V \sim T^\psi$, with $\psi$ the thermal rounding exponent. We show that the computed value of the thermal rounding exponent, $\psi = 0.15$, is robust and accounts for the different scaling properties of several observables both in the steady state and in the transient relaxation to the steady state. In particular, we show the compatibility of the thermal rounding exponent with the scaling properties of the steady-state structure factor, the universal short-time dynamics of the transient velocity at the sample critical force, and the velocity scaling function describing the joint dependence of the steady-state velocity on the external drive and temperature.
Universal magnetic domain wall dynamics beyond the creep regime, up to the depinning threshold. Consistency with the 1d-Quenched-Edwards-Wilkinson universality class.

To expose this universality in a concrete system we need to first disentangle extrinsic and intrinsic temperature dependences.

Open question: vanishing of barriers behaviour near threshold and its possible connection with the thermal rounding of the depinning transition...
Looking closer at the universal creep motion

- **creep**: $\ln v \sim f^{-\mu}$
- **thermal rounding**: $v(f_c) \sim T^\psi$
- **Equilibrium**: $T > 0$
- **Depinning**: $v \sim (f - f_c)^\beta$
- **Fast flow**: $f \to \infty$
Creep Law

• 1987 Ioffe-Vinokur, Nattermann

i. Static description of the interface

ii. Energy barriers scale as energy minima

iii. Existence of typical Barriers

\[ H[u] = \frac{c}{2} \int d^d x (\nabla u)^2 + \int d^d x V_p(u, x) - \int d^d x u \]

\[ u_{\text{static}} \sim L^{\zeta_{eq}} \quad E_{\text{static}}(L) \sim L^{d-2+2\zeta_{eq}} \]

\[ E[L] \sim E_{\text{static}}(L) - f L^{d+1} \rightarrow L_{\text{min}} \sim \left( \frac{1}{F} \right)^{\frac{1}{2-\zeta_{eq}}} \]

\[ E_{\text{min}} \sim \left( \frac{1}{f} \right)^{\frac{d-2+2\zeta_{eq}}{2-\zeta_{eq}}} \rightarrow v \sim e^{-\beta E_{\text{min}}} \sim e^{-\beta U_c(f_c/f)^\mu} \]

Divergentes Barriers \rightarrow glassy nature of the ground state
Creep Law Assumptions

- **1987** Ioffe-Vinokur, Nattermann
- *Equal scaling:* energy barriers scale as energy minima $\rightarrow$ **OK**
Creep Law Assumptions

- *Static* description of the interface?
- PRB2000 Chauve – Giamarchi – Le Doussal

**FRG picture:**

- *Depinning like behaviour for length-scales larger than the typical thermal nucleous.*
- *Ultra-slow far from equilibrium motion*
Creep Law Assumptions

- Static description of the interface? → **Only below thermal nucleous**

**Numerics:**

- Confirms FRG picture (down to d=1):
  Depinning roughness at large scales

- The depinning transition is not a “standard” phase transition
  [eg. Ising model]
Creep Law Assumptions

- **1987** Ioffe-Vinokur, Nattermann

  - *Typical Independent jumps assumption:*

  *How are the distribution and correlations of activated jumps?***

- E. E. Ferrero [Grenoble]
- L. Foini, T. Giamarchi [Geneva]
- A. Rosso [Orsay]
Futile motion problem for $f < f_c$

\[ \gamma \partial_t u(x, t) = c \partial_x^2 u(x, t) + F_p(u, x) + f + \eta(x, t) \]
Creep dynamics of elastic manifolds via exact transition pathways

Alejandro B. Kolton,1,* Alberto Rosso,2,† Thierry Giamarchi,3,‡ and Werner Krauth4,§

\[ E = \sum_{i} \frac{1}{2}(u_{i+1} - u_i)^2 - f \ u_i + V(i, u_i) \]

**Exact** pathways of an extended system?
Creep dynamics of elastic manifolds via exact transition pathways

Alejandro B. Kolton,1,* Alberto Rosso,2,† Thierry Giamarchi,3,‡ and Werner Krauth4,§

Optimal path: minimal barriers, relaxing in valleys and connecting two metastable states $\alpha$ and $\gamma$, such that $E_\alpha > E_\gamma$.

ABK, A. Rosso, W. Krauth, T. Giamarchi, PRL (2006),
A sequence of metastable states

Is it unique?  Does it represent the Low $T$ Steady state?
Unique ordered sequence of metastable states ($T=0+$ attractor)
Irreversible Activated Jumps (thermal nuclei)
Creep Jumps Distribution

\[ P(s) \sim s^{-\tau} G\left(\frac{s}{S_{\text{cut}}}\right) \]

\[ S_{\text{cut}} \sim f^{-\frac{1+\zeta_{\text{eq}}}{2-\zeta_{\text{eq}}}} \]
Creep theory critical thermal nucleous

\[ P(s) \sim s^{-\tau} G(s/S_{cut}) \]
\[ P(l) \sim l^{-\tau'} G(s/L_{cut}) \]

\[ S_{cut} \sim f^{\frac{1+\zeta_{eq}}{2-\zeta_{eq}}} \sim L_{cut}^{1+\zeta_{eq}} \]
\[ L_{cut} \sim f^{\frac{1}{2-\zeta_{eq}}} \sim L_{opt} \]

The velocity is controlled but the cutoff of a powerlaw distributed activated events
Irreversible collective jumps

\[ E = \sum_i \frac{1}{2} (u_{i+1} - u_i)^2 - f u_i + V(i, u_i) \]

**CREEP JUMPS:**
between metastable states visited by the optimal path in the \( T \rightarrow 0 \) limit

\[ E = \sum_i \frac{1}{2} (u_{i+1} - u_i)^2 + \frac{m^2}{2} (w - u_i)^2 + V(i, u_i) \]

**AVALANCHEs:**
Between metastable states visited at \( T=0 \) (Middleton attractor) by quasistatically moving \( w \)

**STATIC SHOCKS:**
Between global minima visited at \( T=0 \) by quasistatically moving \( w \).
Temporal correlations between jumps

Static Jumps
\( (f=0, \ T=0) \)

Depinning Avalanches
\( (f=f_c, \ T=0) \)

Creep Jumps
\( (f<fc, \ T>0) \)
Creep jumps
Static shocks
Depinning avalanches
Geometry & Transport

Critical Fluctuations here?

Conclusions

- **Universality** in Domain Walls Dynamics (Pt/Co/Pt films) below the depinning threshold. Consistency with the QEW universality class.

- **Creep law:** Typical thermal nucleous controlling the creep velocity identified as the cutoff of power-law distributed thermally activated jumps.

- **Correlations:** Activated jumps have strong temporal and spatial correlations, compared with depinning avalanches or static shocks...
T→ 0 Steady State: one particle

- At Equilibrium, \( f = 0 \), Boltzmann impose \( P(GS) \to 1 \) for \( T \to 0 \):

- Occupation probabilities also exist for the \( 0 < f < f_c \) steady-state dynamics in a finite system. The \( T \to 0 \) limit imposes a dominant configuration.
$T \to 0$ Steady State: one particle

- $P(X^*) \to 1$, when $T \to 0$

transparent for a particle on a 1D ring [Derrida (83); Le Doussal, Vinokur (95)]
Low Temperature Steady State: one particle

Is the same valid for the elastic interface?

Determina todas las propiedades del estado estacionario a baja T en un sistema periódico finito
$T \to 0$ Steady State: one particle

Can we do the same for the elastic interface?

- $P(X^*) \to 1$, when $T \to 0$

transparent for a particle on a 1D ring [Derrida (83); Le Doussal, Vinokur (95)]
Theorems: interfaces of dimension $d$ in $d+1$ with convex elastic energy, not necessarily harmonic nor local

- **Theorem 1**: If there is no configuration which lowers the energy of $\alpha$ in the backward direction, the coarse-grained dynamics starting from $\alpha$ is always forward-directed.

- **Theorem 2**: Let $\alpha$ be any metastable configuration escaping into a configuration $\gamma$ with $h^\gamma \geq h^\alpha$ and $\gamma'$ any configuration such that $h^\gamma' \geq h^\alpha$ and having an energy barrier $E^\gamma_{esc} > E^\alpha_{esc}$: all $\gamma'$ then satisfy $h^\gamma' \geq h^\gamma$.

- **Practical Consequences**:
  - The dynamics is *periodic* after a single pass of the line around the system, and we only have to consider forward motion.
  - The metastable configuration with the largest barrier (dominant) is *always* encountered, independently of the initial condition.
The dominant configuration is the only statistically relevant configuration of the $T \to 0$ steady state dynamics: Under the conditions of Arrhenius activation the system will spend much more time on it than in any other configuration.
A theory for Elastic Manifolds in Random Media

- Scaling Arguments
- Functionar Renormalization group
- Numerical Simulations

\[ \begin{align*}
\mu &= 1/4 \\
\psi &\approx 0.15 \\
\beta &\approx 0.24 \\
\end{align*} \]

String, \( d=1 \)

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E. Ferrero, S. Bustingorry, ABK, A. Rosso, Comptes Rendus Physique (2013)
Creep Law (more rigorous)

- **2000** Chauve, Giamarchi, Le-Doussal

\[
\int D\hat{u} \delta \left( \eta \frac{\partial u}{\partial t} + \frac{\delta H}{\delta u(r, t)} - F - \zeta(r, t) \right) = \int D\hat{u} D\hat{u} \exp \left[ i\hat{u} \left( \eta \frac{\partial u}{\partial t} + \frac{\delta H}{\delta u(r, t)} - F - \zeta(r, t) \right) \right]
\]

Average over thermal and quenched disorder can be done → Field theory action

\[
S(u, \hat{u}) = \int_{rt} i\hat{u}_{rt} \left( \eta \partial_t - c\nabla^2 \right) u_{rt} - \eta T \int_{rt} i\hat{u}_{rt} i\hat{u}_{rt} - F \int_{rt} i\hat{u}_{rt} - \frac{1}{2} \int_{rtt'} i\hat{u}_{rt} i\hat{u}_{rt'} \Delta(u_{rt} - u_{rt'}),
\]

donde \( F_p(u, r) F_p(u', r') = \Delta(u - u') \delta(r - r') \). This function is renormalized → FRG (D. Fisher)
Polar magneto optical Kerr effect

Figure 3.1: PMOKE microscope setup. Non-optical components are shown in red. The light path is shown in blue.

Peter Metaxas, PhD thesis (2007)
Bloch Wall
Thermally Assisted Flux Flow

- **1964 Anderson-Kim [TAFF]**

Vortices drive $F \leftrightarrow J$ (current)

\[ v \propto e^{-\beta(U_b - Fa/2)} - e^{-\beta(U_b + Fa/2)} \approx e^{-\beta U_b} F \]

Exponentially small response but *lineal* ...
Polar magneto optical Kerr effect

Figure 3.1: PMOKE microscope setup. Non-optical components are shown in red. The light path is shown in blue.
Sputter deposited by Romacq and Balts (Grenoble)

Figure 4.7: Cross-sectional schematic of the Pt/Co($t_{Co}$)/Pt films.

Figure 4.8: Hysteresis loops measured using polar Kerr rotation for the Pt/Co(0.5 nm)/Pt and Pt/Co(0.8 nm)/Pt films. The field was applied perpendicular to the plane of the films and swept at a rate of 0.44 kOe/s.

> MS, mejor señal Kerr. Hc es porporcional al desorden.
Figure 3.2: The blue line represents the value of the applied field, $H$, as time, $t$, progresses during a displacement measurement. The film is first negatively saturated, and the PMOKE image (rectangle) exhibits no magnetic contrast. A short but intense positive field pulse is then applied to nucleate a positively magnetised domain (grey circle). This domain is then expanded by applying another positive field pulse (propagation). Subtraction of the remanent PMOKE images obtained following nucleation and propagation steps yields an image showing the region swept out by the domain wall during the latter.
**Figure 3.3:** Difference image where the PMOKE image obtained following propagation has been subtracted from the PMOKE image obtained following nucleation. The black region is the area swept out by the domain wall and the white arrows indicate the direction of wall motion. To determine the wall displacement, an intensity profile is calculated in a region with minimal wall curvature (dotted box). The distance is then taken as the full-width half maximum in the intensity profile.
Current Driven Domain Walls

\[ T = 95 \text{K} \]

\[ J \]

\[ J \]

\[ 2 \mu \text{m track (B8P47)} \]

J. Curiale et al, PRL 2013
A mechanism for spatial and temporal earthquake clustering

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[1] The Gutenberg-Richter law states that the size-frequency distribution of earthquakes follows a power law. This trend is usually justified using spring-block models, where slips with the appropriate statistics of sizes have been numerically observed. However, prominent spatial and temporal clustering features of earthquakes, as those implied by the Omori law of aftershocks, are not accounted for by this kind of model unless they are complemented with ad hoc assumptions, such as stress recovery laws after slip events, or the phenomenological rate-and-state equations to describe friction. We show that when a mechanism of structural relaxation is incorporated into a spring-block model, realistic earthquake patterns following the Gutenberg-Richter and Omori laws are obtained. Moreover, features well known from laboratory friction experiments, such as velocity weakening and increase of static friction with contact time, appear as a consequence of the relaxational mechanism as well, without making any a priori assumptions on the velocity dependence of the friction force in the model. In this way, our model shows that a single physical mechanism may be a unifying concept behind the Gutenberg-Richter and Omori laws and the rate-and-state equations of rock friction.

A mechanism for spatial and temporal earthquake clustering

\[ H = \int d^2r [V(u(r) - u_0(r), r) + \frac{k_1}{2} (\partial_r u)^2 + \frac{k_0}{2} (u(r) - Vt)^2] \]

Interface (fast)
\[ \partial_t u = -\lambda \frac{\delta H}{\delta u} \]

Relaxation or contact aging (slow)
\[ \partial_t u_0 = R \nabla^2 \frac{\delta H}{\delta u_0} \]

Disorder seen only by \( r' \) block
\( r \in \text{lattice} \)
Original Motivation: Frictional properties of the model

Rock Friction Experiments (Lab scale)

Modified Depinning Model
A mechanism for spatial and temporal earthquake clustering

EVENTS

\[ M = \frac{2}{3} \log_{10} S \]

Gutenberg-Richter

\[ N(M) \sim 10^{-bM} \]

Omori

\[ N(t) = \frac{A}{(t + c)^p} + N_0 \]

Figure 7. Number of events after main shocks in our model (solid symbols) for parameters and system size as in Figure 3a, normalized to the background seismicity (i.e., \( N(t \rightarrow \infty) = 1 \)). We stacked the events after about 450 main shocks with \( M > 3 \), and the lower magnitude cutoff used to count aftershocks is \( M_0 = 1.5 \). For comparison, the same analysis for data in the California region is presented with open symbols. In this case we sum over 7 events of magnitude larger than 6.0. For this case \( M_0 = 2.0 \). For reference, the continuous lines correspond to decays following the Omori law, with exponents \( p = 1.0 \) and \( p = 1.5 \).