From continuous to crackling plasticity

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The classical view of plastic flow in textbooks: Smooth and homogeneous

Fluctuations are assumed to be mild
Jerkyness of plastic « flow »
(Becker and Orowan, Z. Phys., 1932)

« The jerky extension of Zinc single crystals »

« A single glide process, started obviously from one point in the crystal, produces gliding by thousands or even millions of atomic distances; in other words, a first impulse gives rise to a whole avalanche of dislocations »
“Crackling” plasticity: single crystals of ice
Miguel et al., *Nature*, 2001)
“Crackling” plasticity: single crystals of ice
“Crackling” plasticity: single crystals of ice

Power law distribution of avalanche amplitudes and energies, $p(E) \sim E^{-\tau_E}$

Fluctuations are wild
Micro- and nano-pillars (Dimiduk et al., 2006) and many others

Nickel single crystals

Fluctuations are wild
Classical vs Crackling plasticity

Classical view of plasticity

Scale-free, intermittent plasticity

Mild fluctuations

Wild fluctuations

Are they compatible?
Acoustic emission from dislocation motion:
« Continuous » vs « discrete » AE

Continuous AE = « classical noise »
Discrete AE = « Crackling noise »

AE recorded during the plastic deformation of Al
(Imanaka et al., 1973)

« Datastreaming »: full signal sampled at 5 MHz

Cyclic loading: pure Al
AE burst

- AE maximal amplitude $A_0$
- Duration $\delta t$
- AE energy $E = \int_{\delta t} A^2(t) dt$
Discrete AE: Source model (Rouby et al., 1983)
Collective dislocation motion: Avalanches

- velocity sensors

\[ A(t) \sim b \ n(t) \ L \ v(t) \sim b \ \frac{dS}{dt} \]

- \( b \): Burger’s vector
- \( n \): number of dislocations
- \( v \): dislocations velocity

Decay hypothesis: \[ v(t) = v(t_0) \exp[-\alpha(t - t_0)] \]

\[ \Rightarrow A_0 \sim S \sim \varepsilon_p \]
\[ \Rightarrow E \sim A_0^2 \sim \varepsilon_p^2 \]
Discrete AE: Source model (Rouby et al., 1983)
Collective dislocation motion: Avalanches

\[ E \sim A_0^2 \sim \varepsilon_p^2 \]
Continuous AE as the result of classical plasticity
(Rouby et al., 1983; Slimani et al., 1992)

Acoustic power = sum of numerous, small, and uncorrelated motions
(mild fluctuations)

⇒ Energies (instead of amplitudes) add up

\[
\frac{dE}{dt} \sim \dot{\varepsilon}_p
\]
Ice

~ 100%

Cadmium

98 to 100%

Copper

10 to 50%

Aluminum

<2%
Is continuous AE the signature of *mild* fluctuations?
Continuous AE as the result of classical plasticity
(Rouby et al., 1983; Slimani et al., 1992)

Acoustic power = sum of numerous, small, and uncorrelated motions
(\textit{mild fluctuations})

\[
\frac{dE}{dt} \sim \dot{\varepsilon}_p
\]

Cyclic loading of Aluminum
Acoustic “noise” analysis

- Gaussian distribution of local maxima
  (skewness $\zeta = 0.02$, excess kurtosis $\kappa = -0.27$)

- Multifractal analysis $\rightarrow$ No significant intermittency

$\Rightarrow$ All the attributes of gaussian noise
Coexistence of continuous emission and acoustic bursts
Power law statistics of avalanches

→ Wild and mild fluctuations can coexist
Non universal exponent
(determined from a maximum likelihood method; Clauset et al., 2009)

\[ \text{Ice} \rightarrow \text{Cadmium} \rightarrow \text{Copper} \rightarrow \text{Aluminum} \]

\[ <\tau_E> = 1.4 \pm 0.03 \rightarrow 1.45 \pm 0.05 \rightarrow 1.55 \pm 0.08 \rightarrow 2.0 \pm 0.05 \]
Hexagonal vs Face Centered Cubic

- Anisotropic glide (single-slip)
- Long-range elastic interactions dominate
- Kinematic hardening

- Isotropic glide (multi-slip)
- Short-range interactions
- Isotropic hardening
- Pattern formation

Crackling plasticity favoured

« Gaussian » plasticity favoured

Copper under fatigue (© J. Zhang)
A simple model

Deterministic equation: \[ \frac{d\rho_m}{d\gamma} = a - c\rho_m \]

- \(a\): nucleation rate
- \(c\): multiplication / annihilation rate
- \(\gamma\): controlled strain

→ Evolution towards an equilibrium density \(\rho_c = a/c\)

Stochastic equation:

\[ \frac{d\rho_m}{d\gamma} = a - c\rho_m + \sqrt{2D} \xi(\gamma) \]

- \(\xi\): internal “noise” \(\langle \xi(\gamma) \rangle = 0, \langle \xi(\gamma_1), \xi(\gamma_2) \rangle = \delta(\gamma_1 - \gamma_2)\)
- \(D\): intensity parameter (“effective temperature”)

→ Gaussian fluctuations only
A simple model

Starting point:
\[ \frac{d\rho_m}{d\gamma} = a - c\rho_m \]

Gaussian fluctuations only:
\[ \frac{d\rho_m}{d\gamma} = a - c\rho_m + \sqrt{2D} \xi(\gamma) \]

Multiplicative noise: dislocations react collectively to perturbations
\[ \frac{d\rho_m}{d\gamma} = a - c\rho_m + \sqrt{2D}\rho_m \xi(\gamma) \]

\[ p(\rho_m) \sim \exp \left( -\frac{a}{D\rho_m} \right) \rho_m^{-(1+c/D)} \]

Power law tail + gaussian-like distribution at small scales

The power law exponent depends on \( c/D \)
Interpretation of data

Exponent from the model

\[ \alpha = 1 + c/D \]

Link with acoustic emission exponent

\[ \alpha = \tau_A \]

Avalanche structure

\[ A_0 \sim E^{1/2} \]

\[ \tau_A = 2\tau_E - 1 \]

For Ice c/D is smaller than for Al
A simple model

$c/D \sim 100$
Mild fluctuations dominate

$c/D < 1$
Wild fluctuations dominate
Micro- and nano-pillars

(From Brinckmann, PRL, 2008)

For non-bulk crystals $c/D << 1$
→ intermittent plasticity even for FCC

Smaller is wilder!