Superconductivity and Mottness: Exact Results

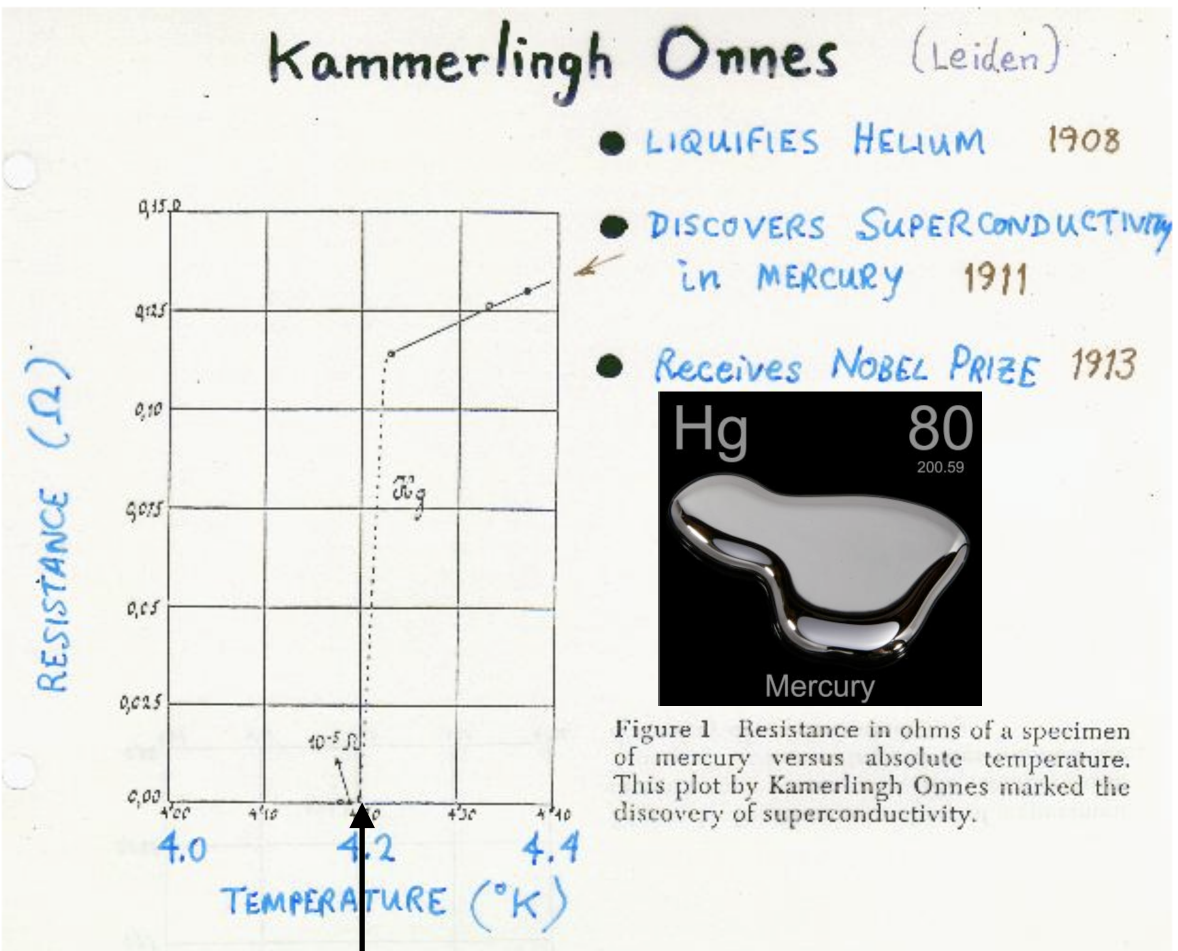
arXiv: 1912.01008

https://www.nature.com/articles/s41567-020-0988-4

with N&V by J. Zaanen

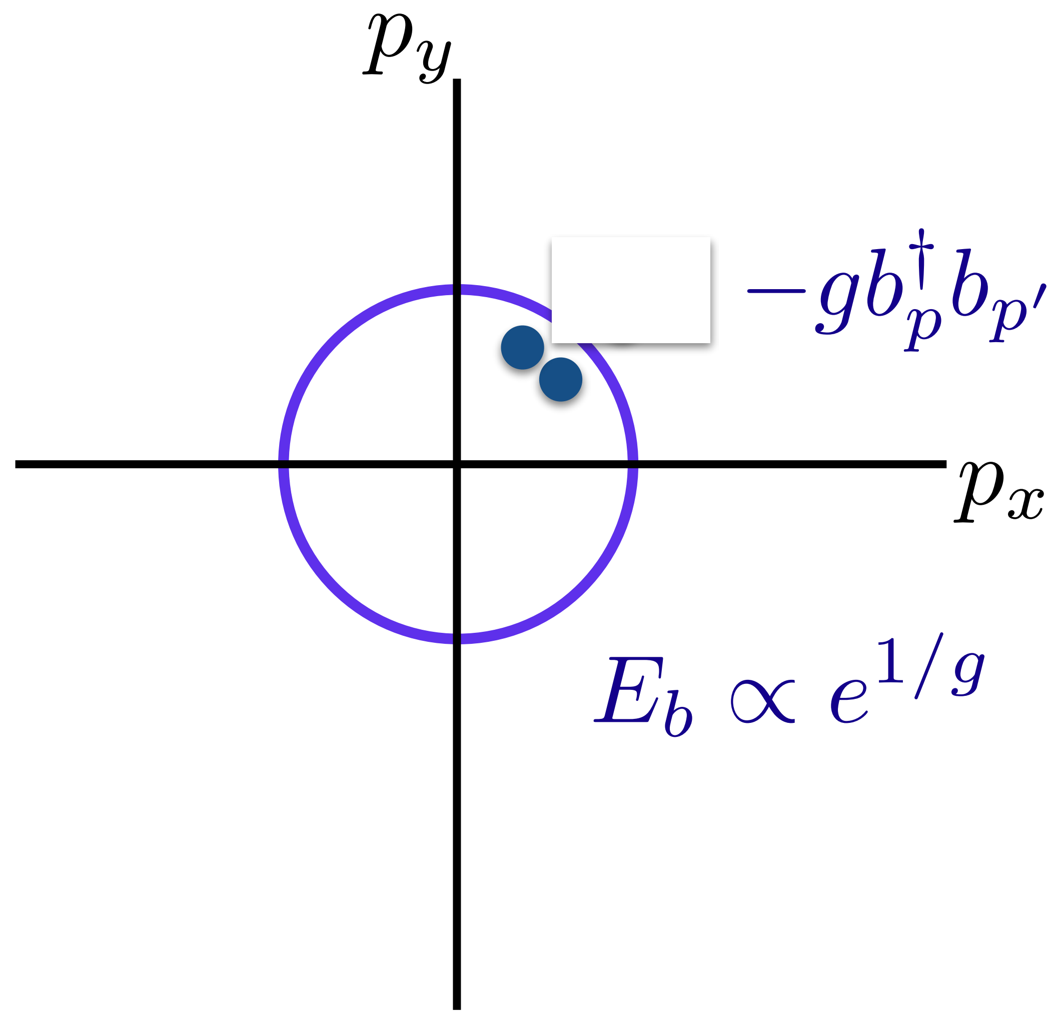
Luke Yeo

Edwin Huang



$$T_c$$

Cooper instability

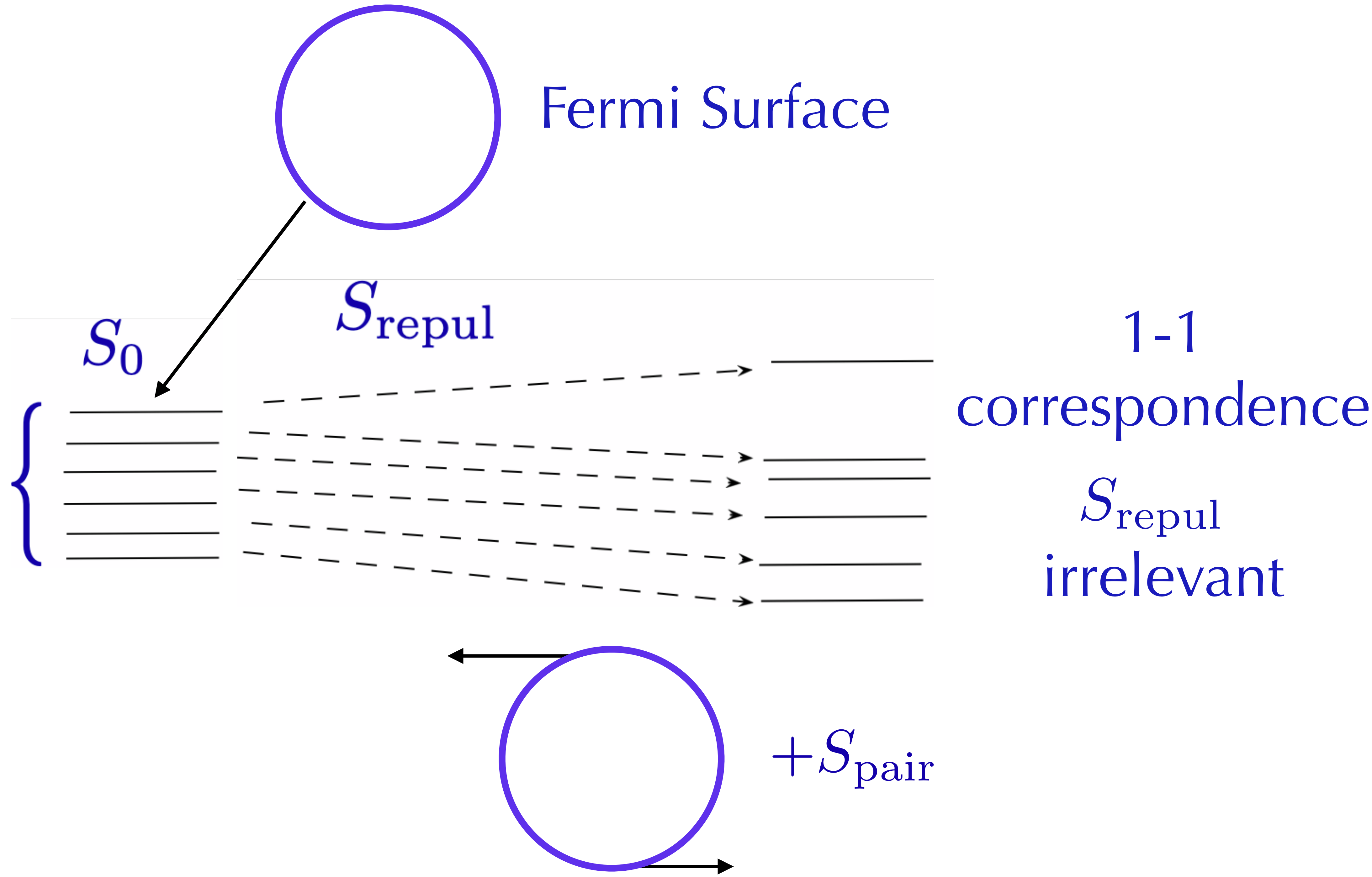


$$-g b_p^\dagger b_{p'}$$

$$E_b \propto e^{1/g}$$

$$\langle \psi^\dagger \psi^\dagger \rangle = \Delta e^{i\phi}$$

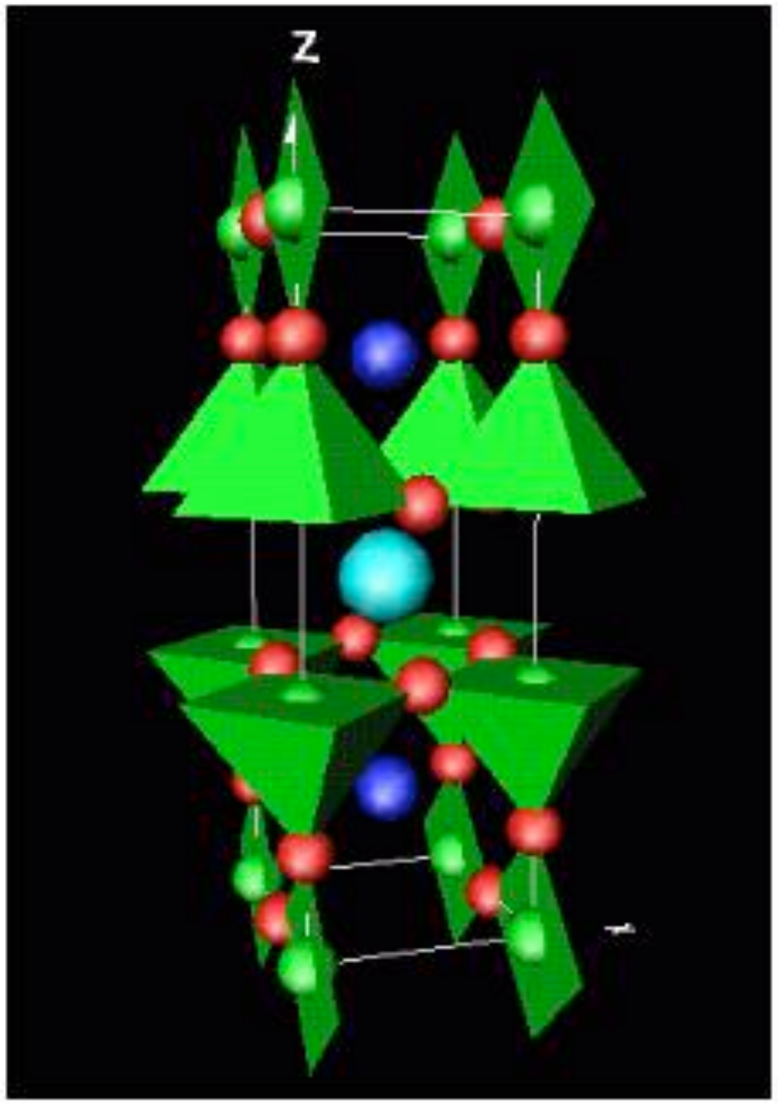
$\mathrm{FL} \to \mathrm{BCS}$



superconductivity $\quad \dfrac{2\Delta}{T_c} = 3.5$

Is there physics
beyond
BCS?

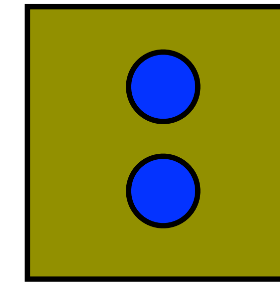
fixed
point beyond
FL?



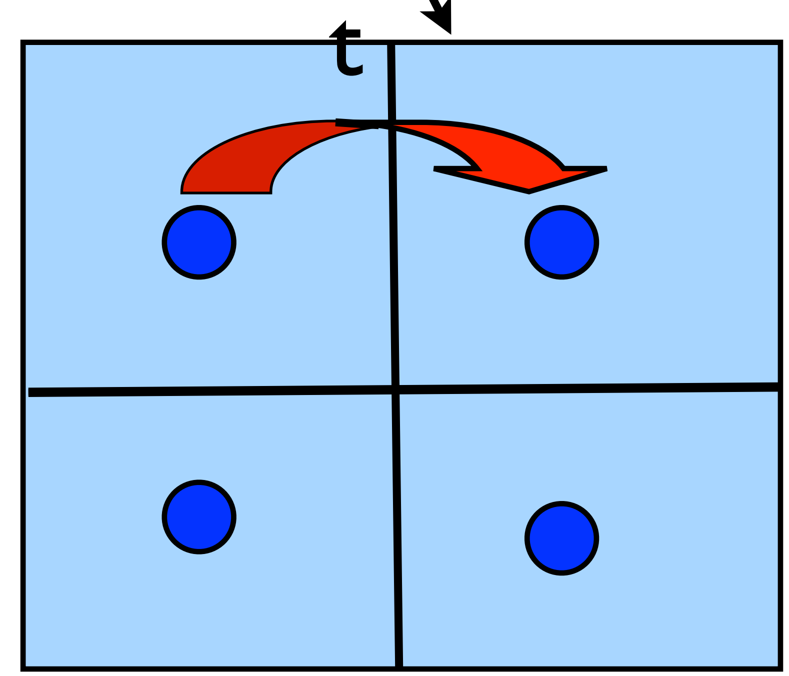
$YBa_2Cu_3O_7$
Cuprate Superconductors

NiO insulates
d^8 ?

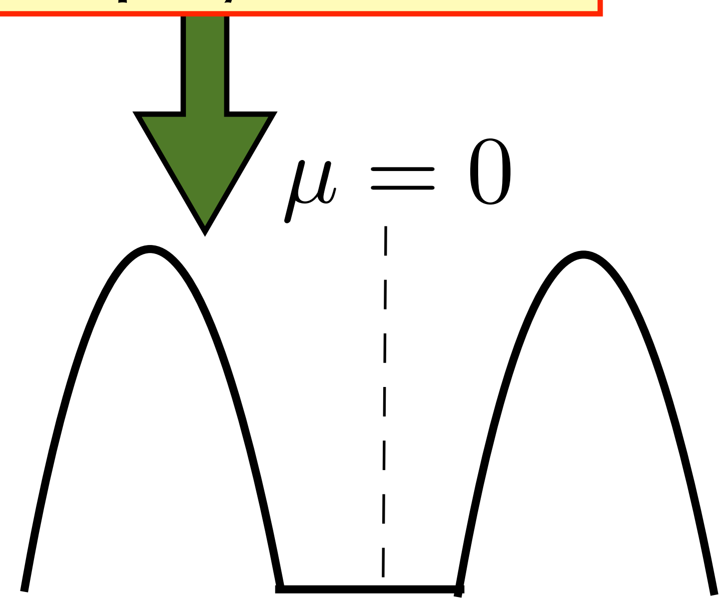
perhaps this costs energy



$U \gg t$



local real-space physics



no change in size of Brillouin zone

solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT

QMC

disputes

Sept. 1997

A Critique of Two Metals

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idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997

A Critique of "A Critique of Two Metals"

Philip W. Anderson and G. Baskaran

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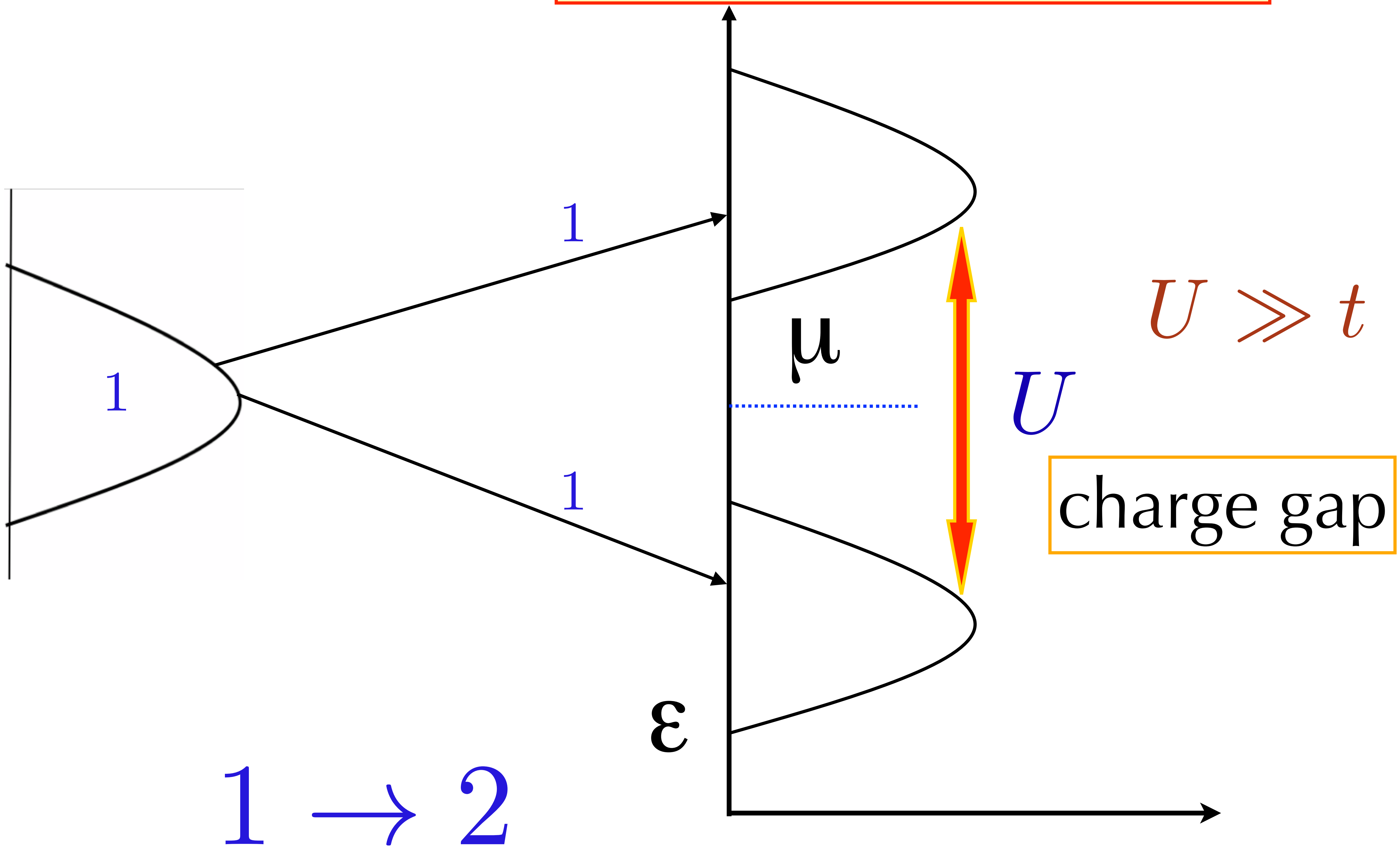
The fundamental argument is presented in the second paragraph: "Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration"...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso "at zero temperature" is added, because of course most Mott

concern. It is the tragedy of Mott that although he almost certainly won his Nobel prize for the Mott insulator, Slater, who couldn't think clearly about finite temperature, won the publicity battle.

Laughlin's objection:

gap with no symmetry breaking not demonstrated!!



$-$

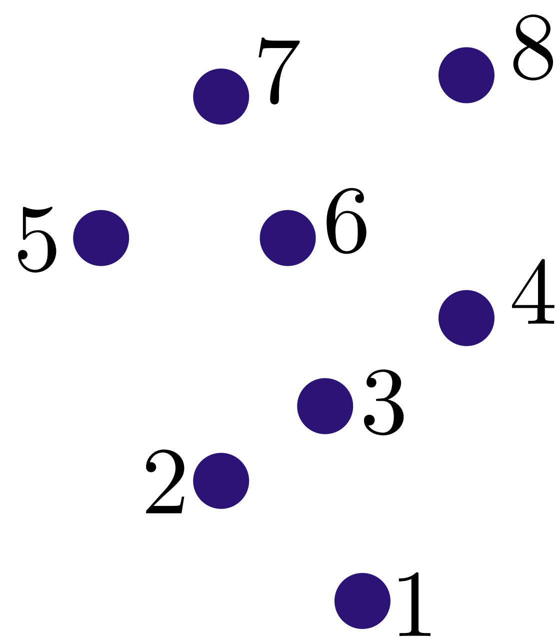
$= \boxed{\text{Mottness}}$

$\boxed{\text{zeros}}$

$= 0$ $\neq 0$

$$\mathrm{DetReG}(\omega = 0, \mathrm{p})$$

counting particles



is there a more efficient way?

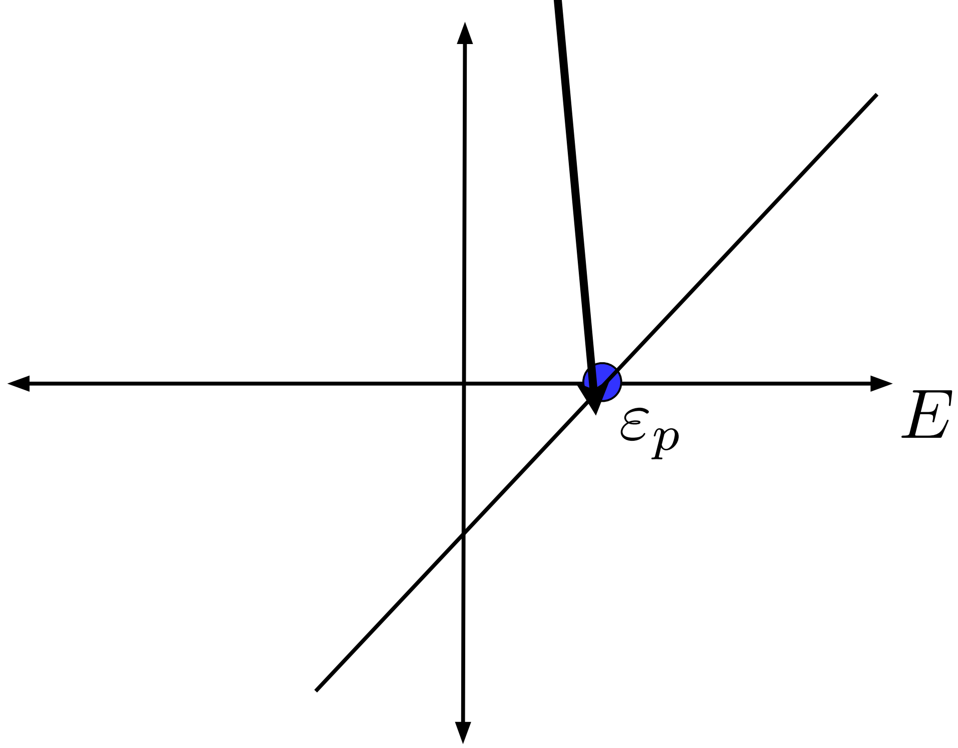
Luttinger counting theorem

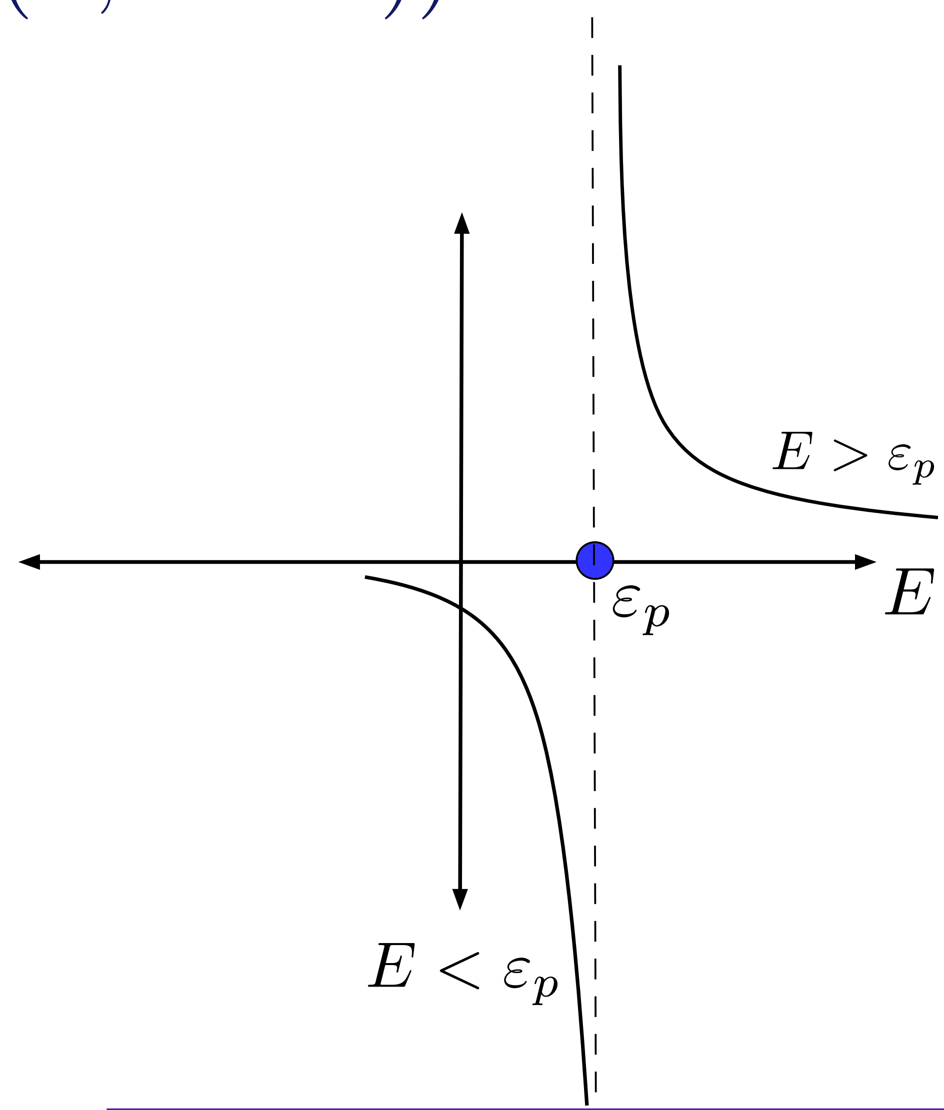
$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing

$$\mathrm{Det}\, G(\omega = 0, \vec{p}) = 0$$





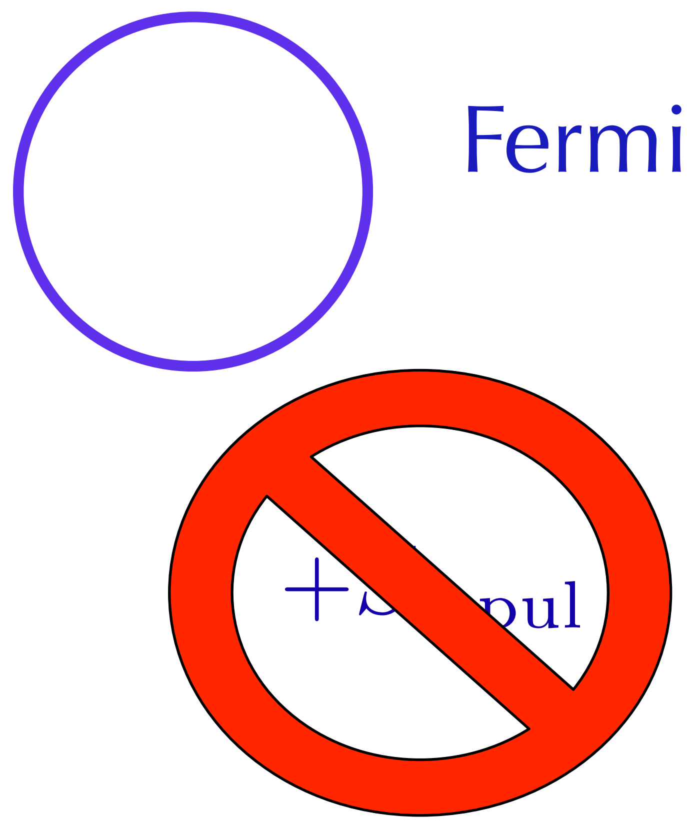
counting poles (qp)

zeros

breakdown of particle concept

Mottness

Where's Mottness??



Fermi Surface

irrelevant

What went wrong?

Mott insulator
in momentum
space???

Hatsugai-Khomoto Model (1992)

$$H_{\mathrm{HK}} = -t \sum_{\langle j,l \rangle, \sigma} \left(c_{j\sigma}^{\dagger} c_{l\sigma} + h.c. \right) - \mu \sum_{j\sigma} c_{j\sigma}^{\dagger} c_{j\sigma}$$

$$c_{k\sigma} = \sum_{j} e^{ikj} c_{j\sigma}$$

$$H_{\mathrm{HK}} = \sum_{k} H_k = \sum_{k} \left(\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow} \right).$$

$$\xi_k = \epsilon_k - \mu$$

relevant interaction

FL+ $Un_{k\uparrow}n_{k'\downarrow} = M.I.$

Hubbard band operators

$c^\dagger_{k\sigma}$

$\zeta_{k\sigma} = c^\dagger_{k\sigma}(1 - n_{k\bar{\sigma}})$ $\quad + \quad$ $\eta_{k\sigma} = c^\dagger_{k\sigma} n_{k\bar{\sigma}}$

$\langle n_{k\sigma} \rangle = \frac{1}{2}$

$$G^R_\sigma(k,\omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \frac{(U/2)^2}{\omega + i0^+ - (\xi_k + U/2)}}$$

$\Sigma(k,\omega)$

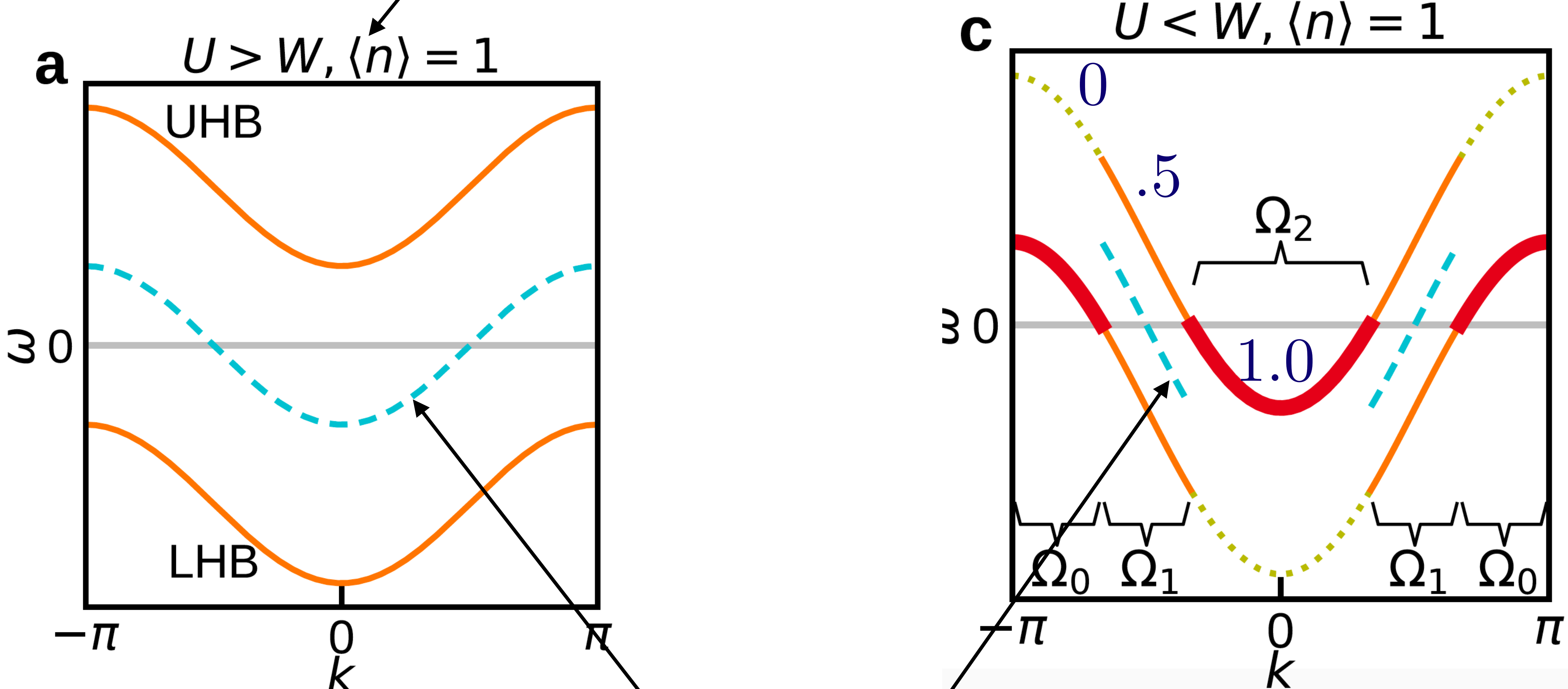
$\omega = \xi_k + U/2$

$\Re\Sigma = \Im\Sigma = \infty$

~~Fermi liquid QP~~

Mott transition: composite excitations

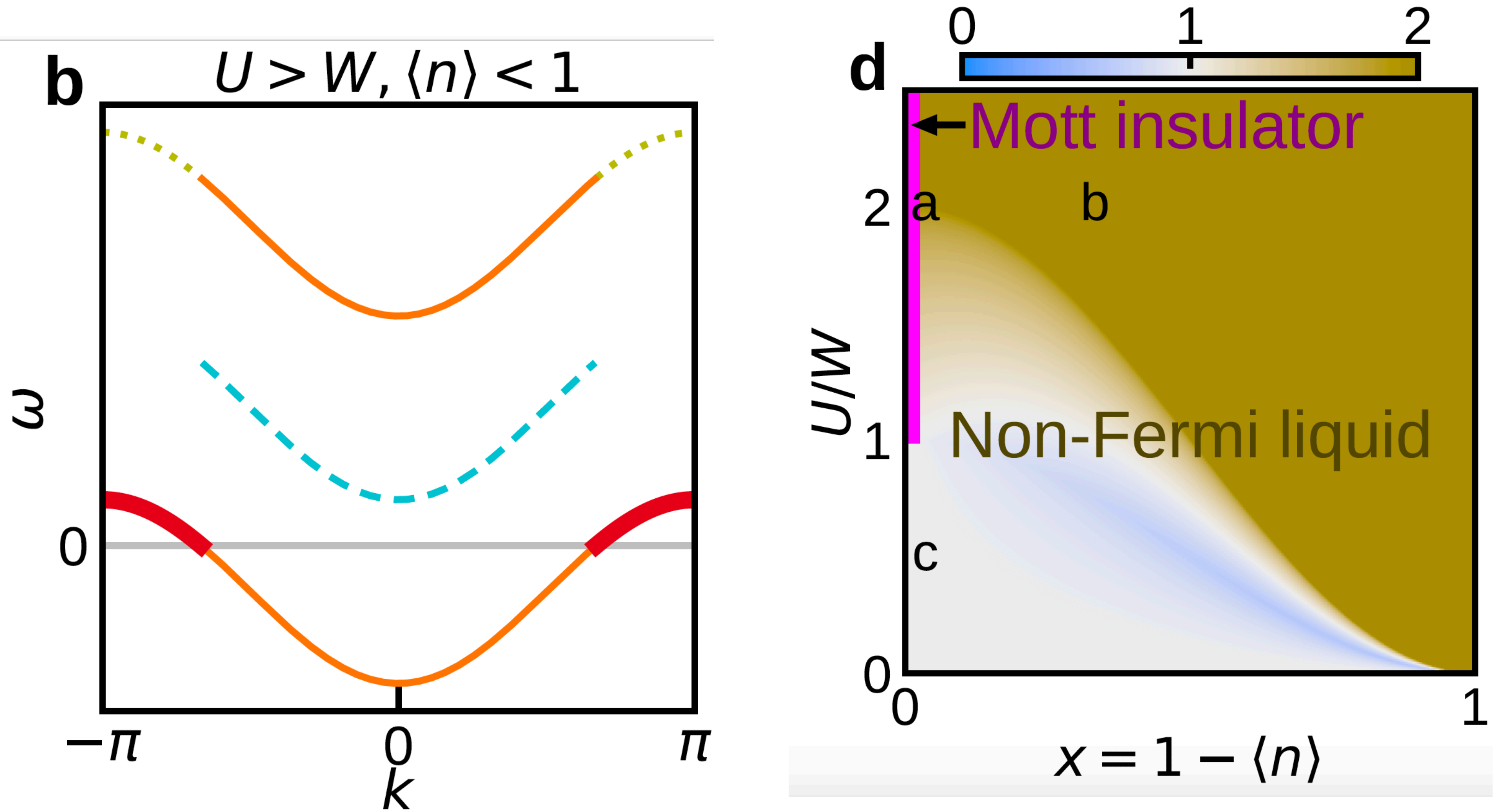
$$\Delta E = U - 4dt = U - W$$



insulator

Luttinger Surface

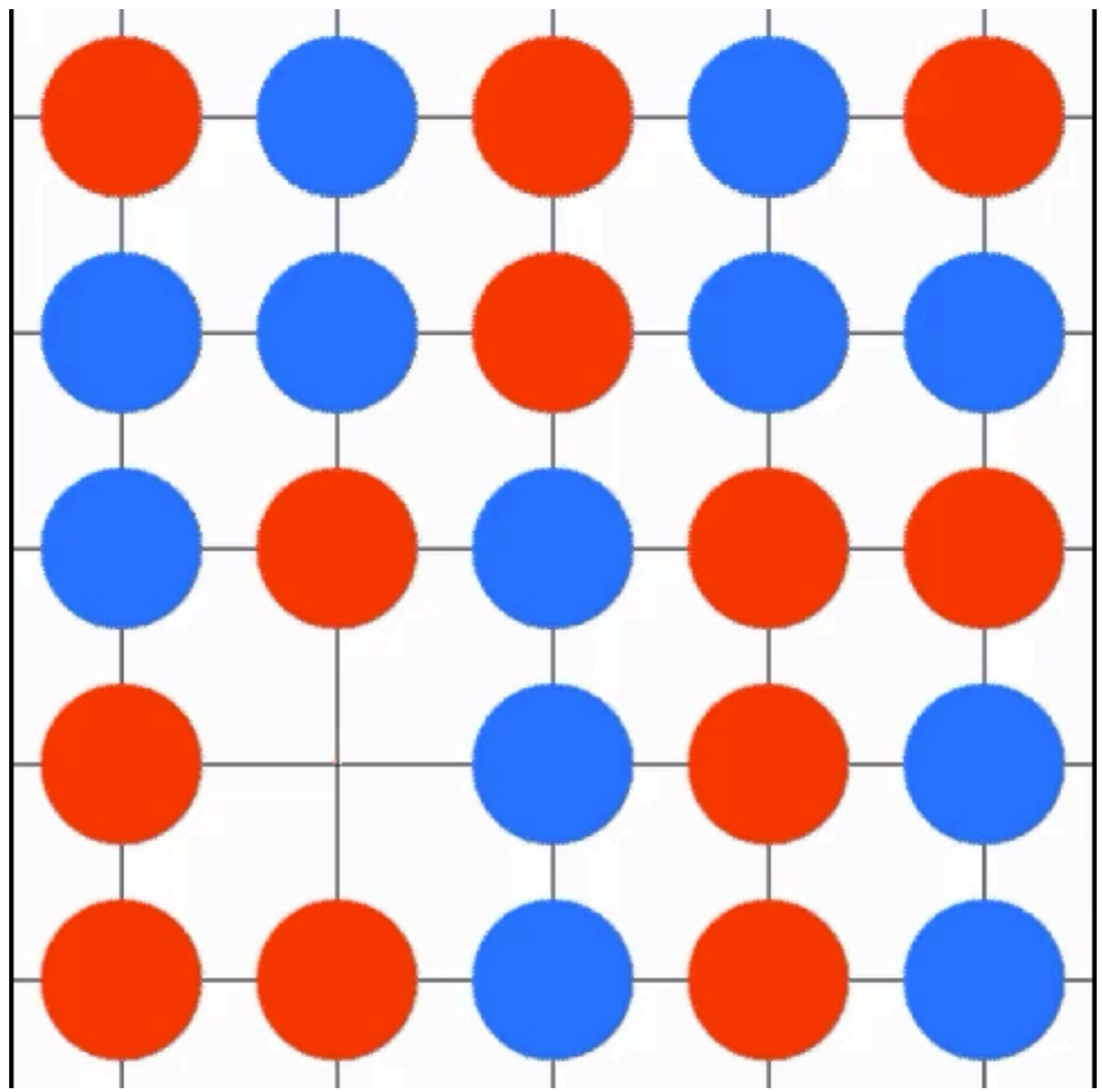
metal



b
U > W, ⟨n⟩ < 1
ω
0
−π
0
π
k
d
0
1
2
Mott insulator
2
a
b
U/W
1
Non-Fermi liquid
c
0
0
1
x = 1 − ⟨n⟩

what does the HK model leave out??

$$[H_t, H_U] \neq 0$$



dynamical spectral weight transfer

Cooper Instability

$$H = H_{\mathrm{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

$$\langle n_{k\sigma}\rangle = 0$$

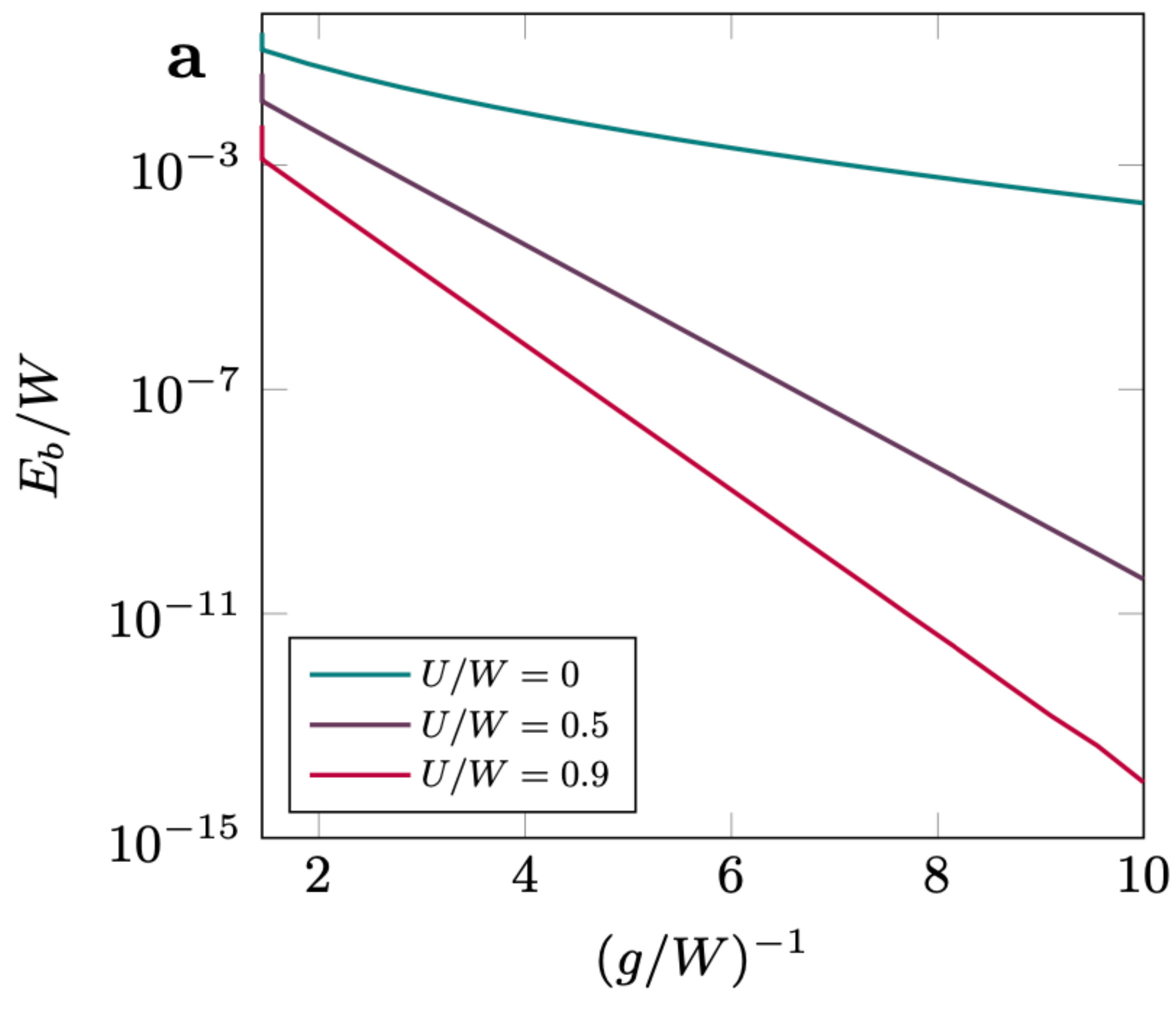
$$E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle\psi|\mathrm{H}|\psi\rangle \lessgtr 0$$

$$1 = -\frac{g}{L^d}\sum_{k\in\Omega_0} \frac{\langle 1 - n_{k\uparrow} + n_{-k\downarrow}\rangle}{E - 2\xi_k - U\langle n_{k\downarrow} + n_{-k\uparrow}\rangle}$$

$$1 = -g\int_{\mu}^{W/2} d\epsilon \frac{\rho(\epsilon)}{E - 2\epsilon + 2\mu},$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



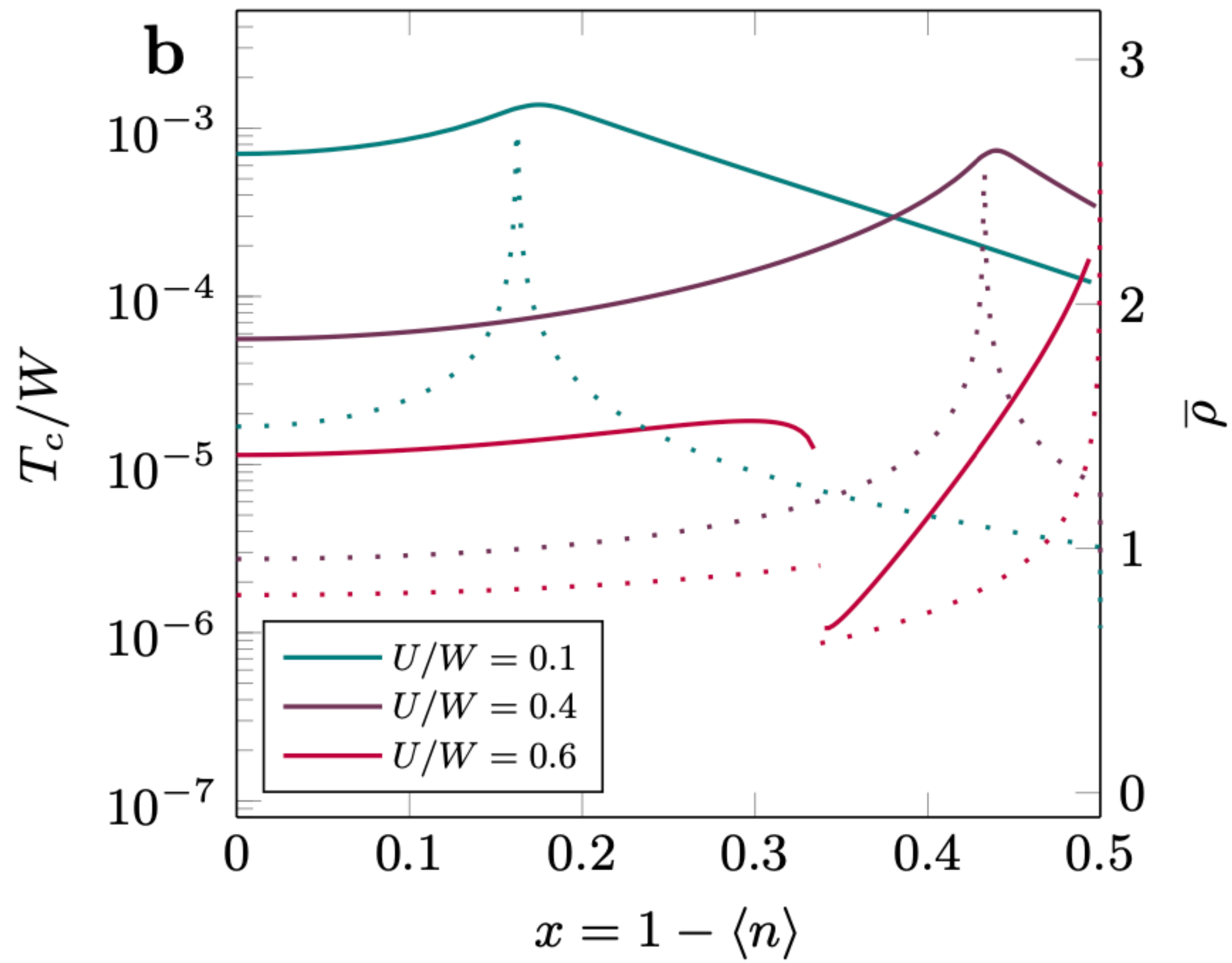
Pair Susceptibility

$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

$$= \frac{\chi_0}{1-g\chi_0}$$

$$\sum_k G_{-k\downarrow}(\tau)G_{k\uparrow}(\tau)$$

solve for T_c



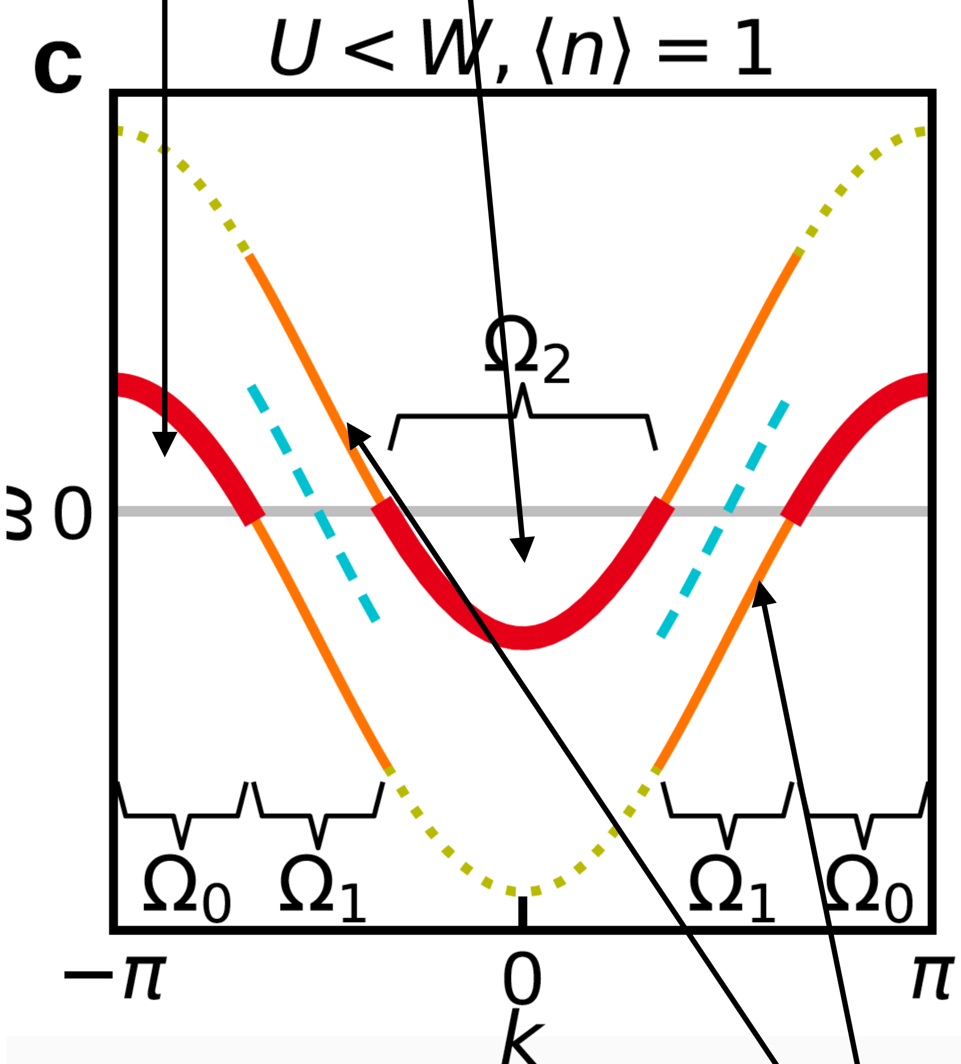
$$T_c = (W - U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

variational wave function

$$|\psi_{\mathrm{BCS}}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle$$

$$|\psi_{\mathrm{BCS}}\rangle = \prod_{k>0} (u_k^2 + v_k^2 b_k^\dagger b_{-k}^\dagger + u_k v_k (b_k^\dagger + b_{-k}^\dagger))|0\rangle$$

$$|\psi\rangle = \prod_{k>0}\left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}}(b_k^\dagger + b_{-k}^\dagger)\right)|0\rangle$$



HK
generalization

$$|\psi\rangle = \prod_{k>0}\left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}}(b_k^\dagger + b_{-k}^\dagger)\right)|0\rangle$$

$\boxed{\text{three variational parameters}}$

$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

$\boxed{\text{gap equation}}$

$$1 = \frac{g}{W} \sinh^{-1}\left(\frac{W-U}{2\Delta}\right) + \frac{g}{W} \sinh^{-1}\left(\frac{U}{2\Delta}\right)$$

$$\Big\downarrow \; \Delta \ll U, W$$

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

gap/T_c ratio

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

non-BCS superconductivity

Bogoliubov excitations

$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$

$$\cancel{\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\overline{\sigma}}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\overline{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\overline{\sigma}}$$

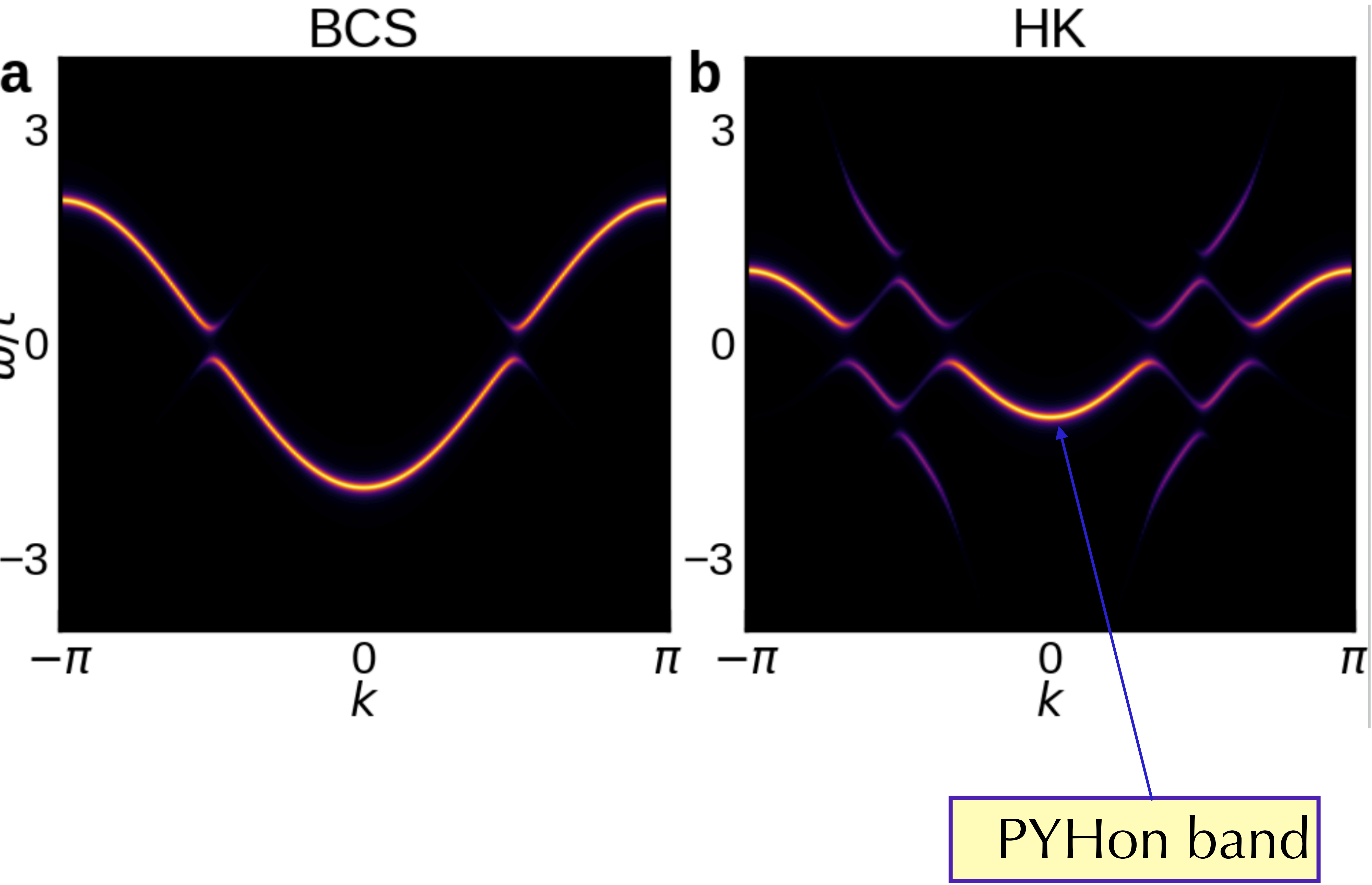
Excitation spectrum

$$\gamma_{k\sigma}^{u/l}|\psi\rangle = 0$$

$$\langle\psi|\gamma_{k\sigma}^{u/l} H \gamma_{k\sigma}^{u/l})^{\dagger}|\psi\rangle = \langle\psi|H|\psi\rangle + E_k^{u/l}$$

$$E_k^{u/l} = \sqrt{{\xi_k^{u/l}}^2 + \Delta^2}$$

superconductivity affects both bands!



BCS
HK
a
b
3
0
−3
ω/t
$-\pi$
0
π
k
PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

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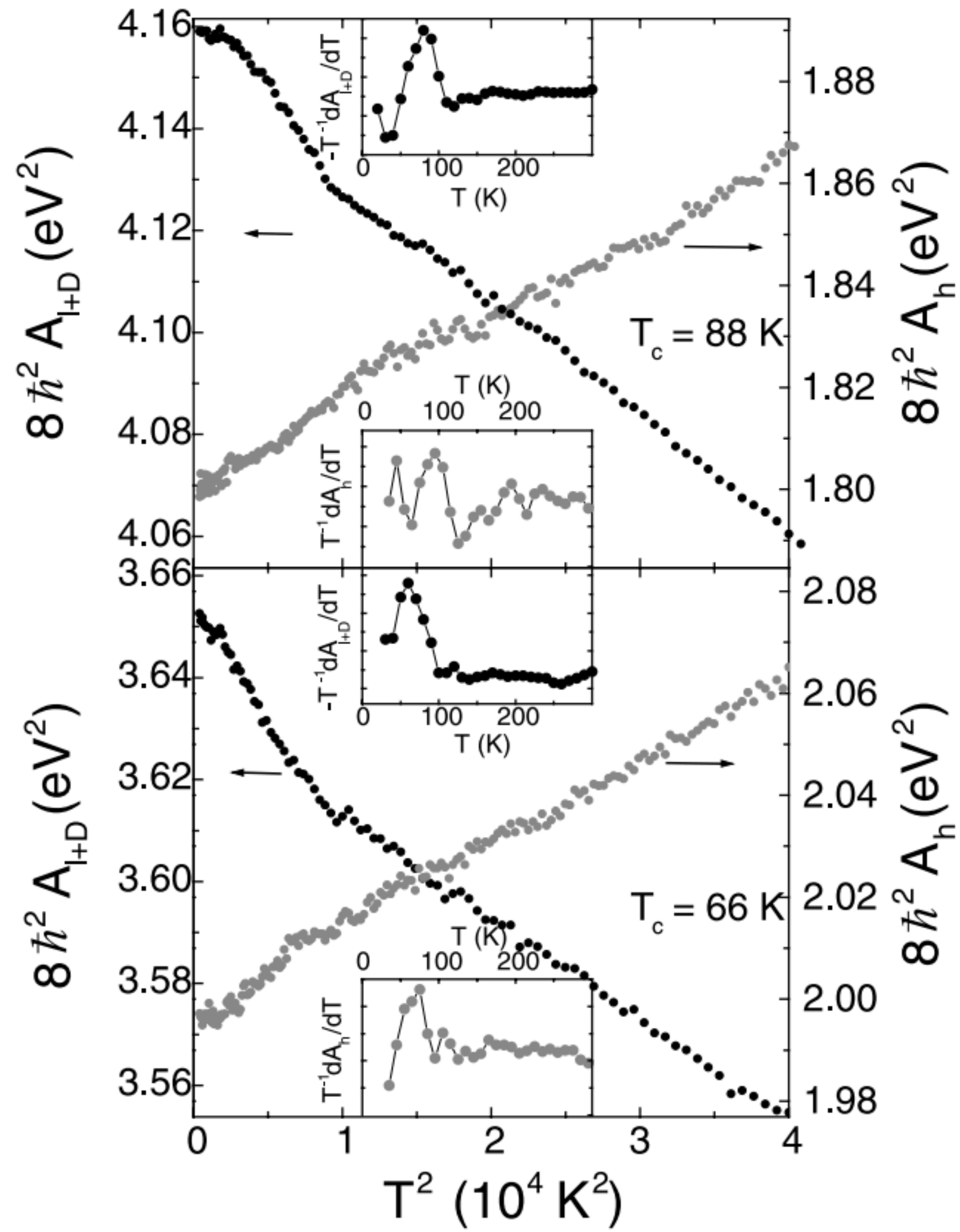
$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$\frac{\Delta A_l}{A_l} \propto 3\%$$

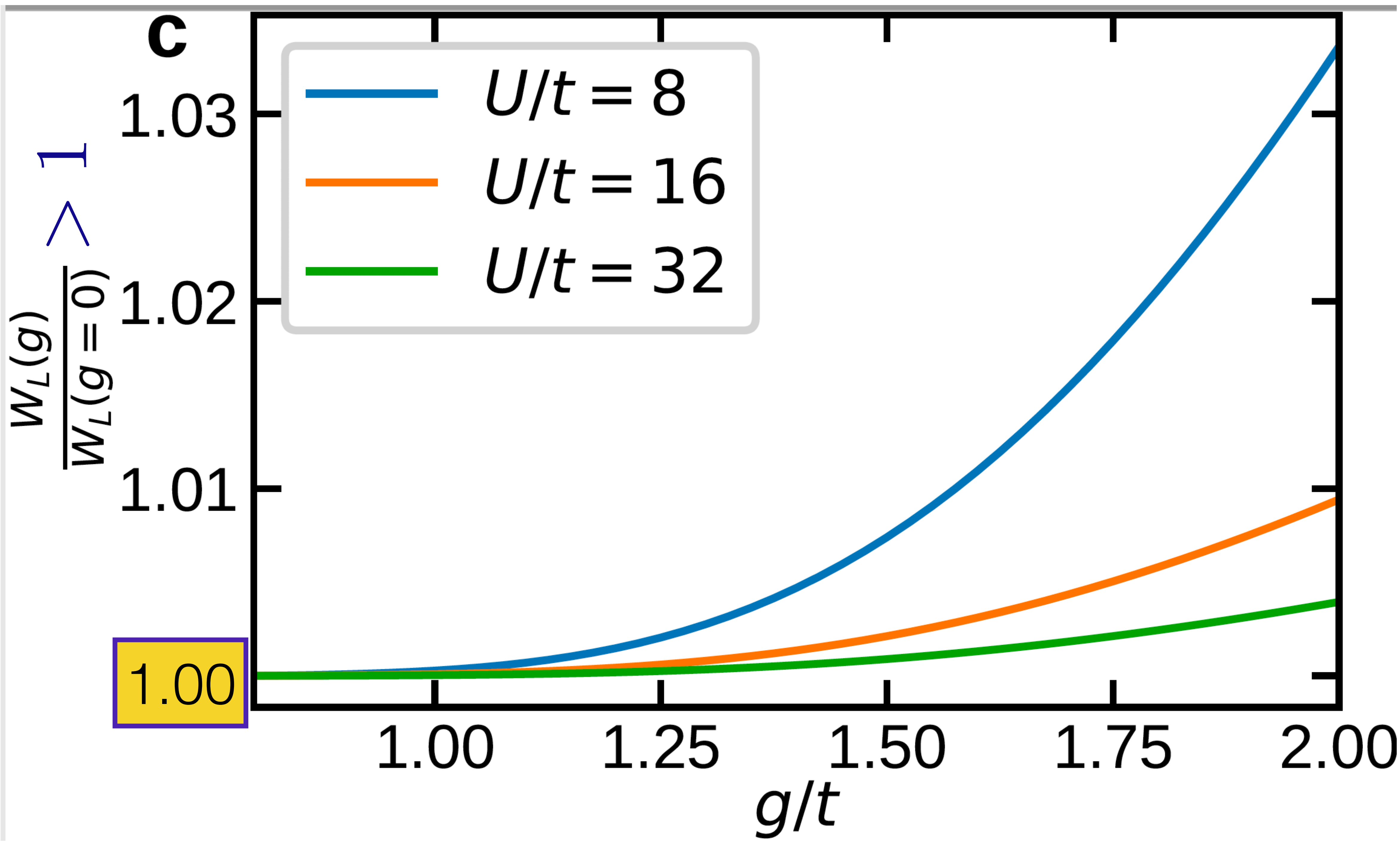


condensation energy

Optical data are reported on a spectral weight transfer over a broad frequency range of $Bi_2Sr_2CaCu_2O_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least 2×10^4 cm^{-1}, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing

condensation energy:HK model



$\Delta W_L \propto O(2-3\%)$ as in experiments why?

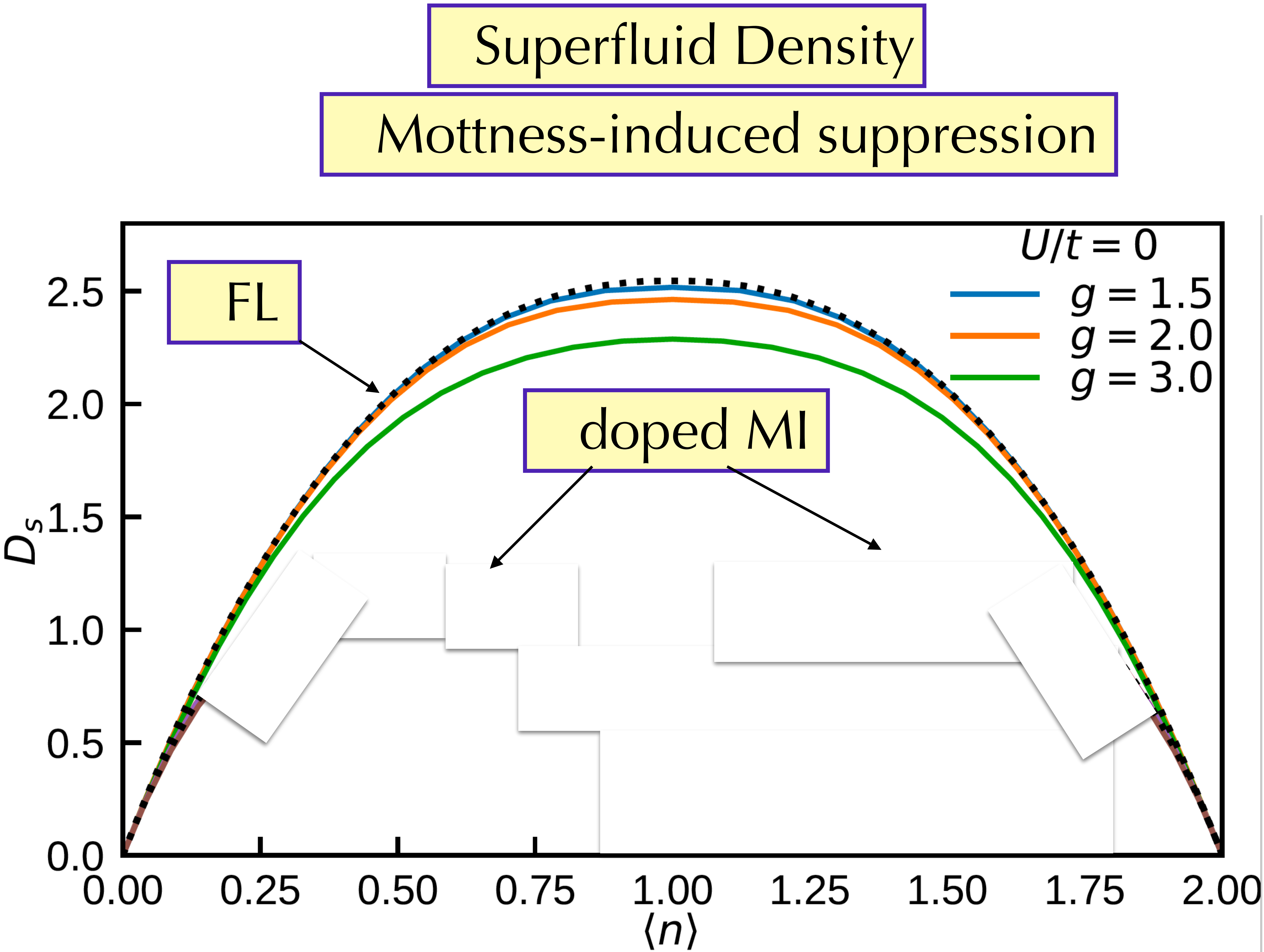
$$H = H_{\mathrm{HK}} + H_p$$

$$[H_{\mathrm{HK}}, H_p] \neq 0$$

$$\Big\downarrow$$

dynamical
spectral weight
transfer

is this the general mechanism of the color change?



Superfluid Density
Mottness-induced suppression
FL
doped MI
$U/t=0$
$g=1.5$
$g=2.0$
$g=3.0$
D_s
$\langle n \rangle$
0.0
0.5
1.0
1.5
2.0
2.5
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Mott gap

$$[H_t, H_V] = 0$$

Mottness in momentum space

$$n_{k\uparrow} n_{k'\downarrow}$$

$$\gamma^{l}_{k\sigma} \propto \sqrt{2} x_k \zeta^{\dagger}_{k\sigma} - \sigma z_k \zeta_{-k\bar{\sigma}}$$

$$\gamma^{u}_{k\sigma} \propto z_k \eta^{\dagger}_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\bar{\sigma}}$$

PYHons

$$[H_{\mathrm{HK}}, H_p] \neq 0$$

$$\frac{\Delta}{T_c} \propto \infty$$

non-BCS superconductivity

$$\frac{W_L(g)}{W_L(g=0)} > 1$$