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Frontiers in nuclear physics, KITP, Santa Barbara
August 22 - November 4, 2016

Nuclear Effective Field Theories - the crux of the matter

„There are no right answers to wrong questions“

Ursula K. Le Guin

Special thanks to my collaborator on these topics Jambul Gegelia.

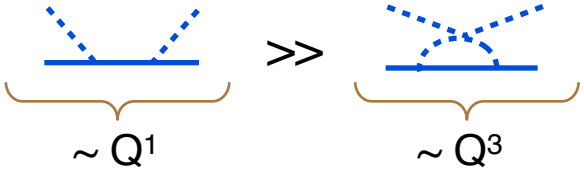
Thanks to Mike for illuminating discussions.

Nuclear EFT: The crux of the matter

What is power counting?

PC = counting powers of soft scales for **renormalized** contributions to observables!

E.g., in ChPT:



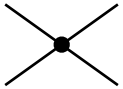
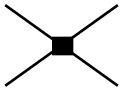
However, evaluating the loop integral with a cutoff $\Lambda \sim 4\pi F_\pi$ one finds:

Only **after** subtracting the UV divergences $\sim \Lambda^2$ and $\sim \log(\Lambda/\mu)$, the result is in agreement with PC

What is W.'s approach?

Let's start with a simple case of pionless EFT…

Nuclear EFT: The crux of the matter

W.: define the potential V =  +  + …, introduce an UV cutoff $\Lambda \sim M_\pi$, solve the Schrödinger eq. and tune $C_0(\Lambda)$ and $C_2(\Lambda)$ to a and r.

For a sharp cutoff, one finds at NLO:

$$mC_0 = \frac{6\pi^2\left(\beta - 6\sqrt{3}\sqrt{\alpha(\pi - 2a\Lambda)^2}\right)}{5\alpha\Lambda}\,, \qquad mC_2 = \frac{6\pi^2\left(\sqrt{3}\sqrt{\alpha(\pi - 2a\Lambda)^2} - \alpha\right)}{\alpha\Lambda^3}$$

where I have introduced:

$$\alpha \equiv 16a^2\Lambda^2 - \pi a\Lambda\left(a\Lambda^2 r + 12\right) + 3\pi^2\,,$$
$$\beta \equiv 64a^2\Lambda^2 - \pi a\Lambda\left(3a\Lambda^2 r + 62\right) + 18\pi^2$$

(Implicitly) renormalized expression for the inverse amplitude:

$$-\frac{4\pi}{m}\mathrm{Re}(T^{-1}) = -\frac{1}{a} + \frac{1}{2}rk^2 + \frac{\pi\left(8 - 3a\Lambda^2 r(\pi\Lambda r - 8)\right) - 64a\Lambda}{12\pi\Lambda^3(\pi - 2a\Lambda)}k^4 + \mathcal{O}(k^6)$$

$\longrightarrow$ well-defined and correct (modulo higher-order terms) result for $\Lambda \sim r^{-1} \sim M_\pi$; things may (and generally will!) go wrong for $\Lambda \gg r^{-1}$ (complex C_i, Wigner bound, peratization…)

Nuclear EFT: The crux of the matter

Well… But in QCD, QED, ChPT, …, one normally takes the limit $\Lambda \to \infty$. Why?

Because it simplifies calculations (symmetries…) and eliminates finite-Λ artifacts (in an EFT, it is sufficient to take $\Lambda \sim \Lambda_b$ …)

Is it justified to take this limit?

Yes, but only after absorbing all positive powers (and Log's) of Λ into c.t.. This is not always possible: in χEFT with nonperturbative V_π, infinitely many c.t.'s are needed; it is unknown how to implement subtractive renormalization in practice for S=1 channels…

Which power counting does W. approach correspond to?

The answer depends on the **choice of renormalization conditions**!

Subtractive renormalization:

1. Define: $I_n \equiv -\int \frac{d^3l}{(2\pi)^3} l^{n-3}\theta(\Lambda - l) = \left(I_n + \frac{\mu_n^n}{2n\pi^2}\right) - \frac{\mu_n^n}{2n\pi^2} \equiv \Delta_n(\mu_n) + I_n^{\rm r}(\mu_n)$
2. Replace: $I_n \to I_n^{\rm r}(\mu_n), \quad C_i \to C_i^{\rm r}(\{\mu_n\})$
3. (optionally) Take the limit: $\Lambda \to \infty$

Nuclear EFT: The crux of the matter

Some possible choices of renormalization conditions:

- $Q \ll \mu_i \sim M_\pi$ ⟵ most general (NDA for $C_i^{\rm r}$ compatible with arbitrary fine tuning) but least useful (cancellations…)
- $\mu_{3,5,\ldots} \sim Q \ll M_\pi$ For $\mu_{3,5,\ldots} = 0$: $mC_0^{\rm r}(\mu_1) = \dfrac{4\pi^2 a}{\pi - 2a\mu_1}$, $mC_2^{\rm r}(\mu_1) = \dfrac{\pi^3 a^2 r}{(\pi - 2a\mu_1)^2}$

  and $\mathrm{Re}(T^{-1}) = \left(\dfrac{1}{C_0^{\rm r}} + \dfrac{\mu_1 m}{2\pi^2}\right) - \dfrac{2k^2}{C_0^{\rm r2}} C_2^{\rm r} + \dfrac{4k^4}{C_0^{\rm r3}} C_2^{\rm r2} - \dfrac{8k^6}{C_0^{\rm r4}} C_2^{\rm r3} + \mathcal{O}\left(C_0^{\rm r4}\right)$

Specifying the scale μ_1:

(1): $Q \ll \mu_1 \sim M_\pi$ ⟵ NDA for LECs describes both the natural ($a \sim M_\pi^{-1}$) and unnatural ($a \sim Q^{-1} \gg M_\pi^{-1}$) cases, i.e. in both cases: $mC_n^{\rm r}(\mu_1) \sim M_\pi^{-n-1}$

Not the most economic scheme for $a \sim M_\pi^{-1}$ …

Expansion of V in k ≠ expansion of T for $a \sim Q^{-1}$, but still a self-consistent & systematic approach!

(2): $\mu_1 \sim Q \ll M_\pi$ ⟵ NDA for LECs describes the natural case; KSW scaling of LECs for the unnatural case

Nuclear EFT: The crux of the matter

Notice: while both approaches are equivalent in the π-less case, they yield different predictions when generalized to χEFT (nonperturbative vs perturbative V_π)

What does the RG analysis of Mike tell us?

It determines the scaling of $C_i^{\rm r}$ for $\mu_i \sim Q \ll M_\pi$ for a system near the trivial or non-trivial fixed point (renormalization conditions as in **(2)**, i.e. KSW)

Do the findings of the RG analysis imply that the NDA scaling of $C_i^{\rm r}$ is wrong?

No. (Different choices of renorm. conditions possible, e.g. **(1)**)

Can one use implicit renormalization without specifying renorm. conditions (W.)?

Yes. Trivially, there is a 1:1 mapping between π-less EFT and ERE…

Does W.'s approach correspond to an expansion around a trivial fixed point?

No. It is designed to match the ERE regardless of the size of a.

Is the RG analysis more predictive? E.g., it seems to suggest that the range correction is perturbatively small…

No. One has to specify the starting conditions for the RG equation - that's where one needs data. A stronger fine tuning cannot be excluded a priori.

Nuclear EFT: The crux of the matter

π-less case is trivial. How to generalize to an EFT with long-range interactions?

- If $V_{long}(r)$ is known exactly, generalization is straightforward.
- In reality, $V_{long}(r)$ is only known within the chiral expansion.
  Common approach: terms in (bare) $V = V_{cont} + V_{long}$ are organized according to the standard χ counting (i.e. NDA). This corresponds to the minimal set of contact terms and is the correct assignment in a weakly-interacting case.

What expansion of the amplitude does this correspond to?

- ERE for $V_{long}(r) = 0$, MERE for the case of an exactly known $V_{long}(r)$
- Generally, no rigorous answer is known (to me). But can be tested by looking at the convergence pattern of (implicitly) renormalized predicted observables.

Can RG analysis help one to determine the expansion?

Perhaps (…even though I am not convinced it can in practice…)

Are there alternative suggestions to organize the expansion of V?

Yes, typically suggesting to promote some terms in V_{cont} to lower orders.

Can these approaches be tested?

Yes, e.g. by comparing predictions for few-N observables (at higher orders).

Nuclear EFT: The crux of the matter

Have such tests been performed?

No, the only approach which has so far been pushed to high orders (N^4LO) is based on the standard NDA assignment (W.)…

Do these calculations give any indication of the need to depart from NDA (W.)?

No. In fact, the opposite is true…

1) Chiral expansion of observable seems, in general, to agree reasonably well with the NDA pattern.

Expansion parameter for an observable X(p): $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

Λ_b (estimated from error plots): $\Lambda_b \sim 600$ MeV [for not too soft cutoffs]

Chiral expansion of the np total cross section [R = 0.9 fm]

$E_{lab} = 50$ MeV [p = 153 MeV]: $\sigma_{tot} = \overbrace{183.6}^{Q^0} - \overbrace{17.1}^{Q^2} + \overbrace{0.5}^{Q^3} - \overbrace{0.2}^{Q^4} + \overbrace{0.7}^{Q^5} = 167.5$ mb

Q = 153/ 600 ~ 0.26 → expect: ~ 12, ~ 3, ~ 0.8, ~ 0.2

$E_{lab} = 96$ MeV [p = 212 MeV]: $\sigma_{tot} = 84.8 - \underbrace{9.7}_{\sim 11} + \underbrace{3.2}_{\sim 4} - \underbrace{0.8}_{\sim 1.3} + \underbrace{0.5}_{\sim 0.5} = 78.0$ mb

Q = 212 / 600 ~ 0.35 → expect: ~ 11, ~ 4, ~ 1.3, ~ 0.5

$E_{lab} = 200$ MeV [p = 307 MeV]: $\sigma_{tot} = 34.9 + \underbrace{1.0}_{\sim 9} + \underbrace{6.7}_{\sim 5} + \underbrace{0.6}_{\sim 2.4} - \underbrace{0.5}_{\sim 1.2} = 42.7$ mb

Q = 307 / 600 ~ 0.5 → expect: ~ 9, ~ 5, ~ 2.4, ~ 1.2

Nuclear EFT: The crux of the matter

2) Description of experimental data:

Energy bin	LO	NLO	N^2LO	N^3LO	N^4LO
neutron-proton data					
0 – 100 MeV	130.11	3.79	1.46	1.08	1.08
0 – 200 MeV	104.71	19.88	3.21	1.14	1.09
0 – 300 MeV	111.24	52.03	8.78	1.51	1.15
proton-proton data					
0 – 100 MeV	2046.58	33.68	6.67	0.86	0.84
0 – 200 MeV	1649.58	115.60	81.11	1.95	1.34
0 – 300 MeV	1301.41	104.38	84.24	2.73	1.46
	2 LECs	+ 7 + 2 IB LECs		+ 15 LECs	+ 1 IB LEC

no new LECs → (NLO → N^2LO)

+1 LEC ($^1S_0$) → (N^3LO → N^4LO)

no new LECs → (proton-proton data, N^3LO → N^4LO)

Clear evidence of the (parameter-free) chiral 2π-exchange!

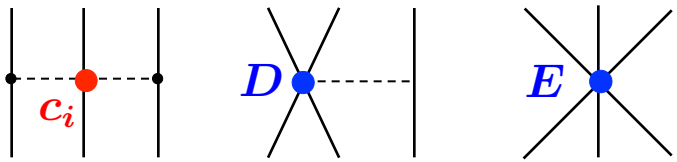
If short-range interactions had to be promoted relative to V_{long}, one wouldn't expect a substantial reduction in χ^2 at N^2LO and N^4LO…

3) Similarly, alternative schemes suggest that the contributions to e.g. the $^1S_0$ and $^3S_1$ - $^3D_1$ channels are enhanced relative to NDA. However, we find these channels to be nearly perfectly described at N^3LO and N^4LO in W.'s scheme. The error (χ^2) is dominated by different waves…

Nuclear EFT: The crux of the matter

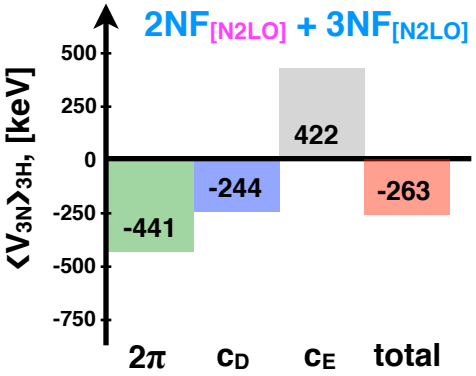
4) The size of the 3N force

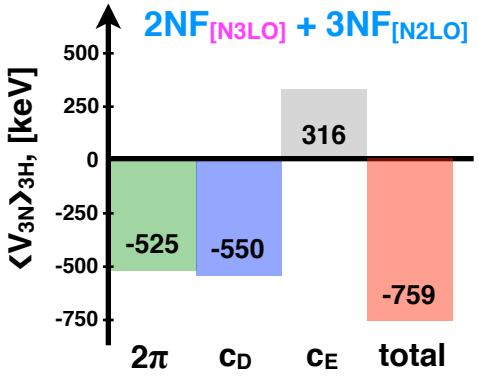
In W.'s scheme, all 3 diagrams contribute at N^2LO (Q^3). On the other hand, the RG analysis of Mike suggests:

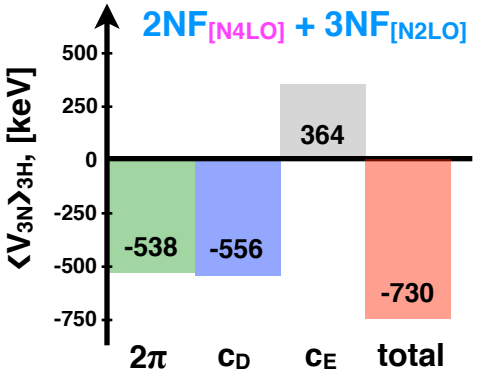


$$V_{2\pi} \sim Q^3 \,, \quad V_D \sim Q^{5/4} \,, \quad V_E \sim Q^{(>3)}$$

Contributions to the ^3H binding energy [R = 0.9 fm]







Nuclear EFT: The crux of the matter

How are the truncation errors estimated?

A difficult task… Proposed a simple approach based on the calculated predictions for an observable at low orders in the chiral expansion [EE, Krebs, Meißner, EPJA 51 (2015) 53].

Let $X(p)$ be some observable with p denoting the corresponding momentum scale and $X^{(n)}(p), \;\; n = 0, 2, 3, 4, \ldots$ a prediction at order Q^n in the chiral expansion:

$$X^{(n)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(n)}$$

calculated in the chiral expansion

Expect based on NDA: $\Delta X^{(n)} \sim \mathcal{O}\left(Q^n \, X^{(0)}\right)$ with $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \; \frac{p}{\Lambda_b}\right)$

Estimation of the theoretical uncertainty $\delta X^{(n)}$ (size of neglected higher-order terms):

$$\begin{aligned}
\delta X^{(0)} &= Q^2 \, |X^{(0)}| \,, \\
\delta X^{(2)} &= \max\left(Q^3 \, |X^{(0)}|, \;\; Q \, |\Delta X^{(2)}|\right), \\
\delta X^{(3)} &= \max\left(Q^4 \, |X^{(0)}|, \;\; Q^2 \, |\Delta X^{(2)}|, \;\; Q \, |\Delta X^{(3)}|\right), \\
&\ldots \\
+ \text{ demand: } \delta X^{(n)} &\geq \max\left(\Delta X^{(>n)}\right)
\end{aligned}$$

Easy to implement, works remarkably well for all observables considered so far (validation)!

Nuclear EFT: The crux of the matter

Can one do better, e.g. a more elaborate RG-based error analysis?

Maybe. Would like to see convincing evidence…

Summary

The take-away message

Don't take it too seriously when you hear next time „It's well known that W. is wrong." :-)
Ask for details, arguments and definitions!

Future perspectives for W. approach

Consistency checks (e.m. and weak reactions), 3N force, precision few-N physics, …

Comparing with other power-counting schemes

Looking forward to see results from the competition.
There are plenty of possibilities to test the (predictive power of the) theory!