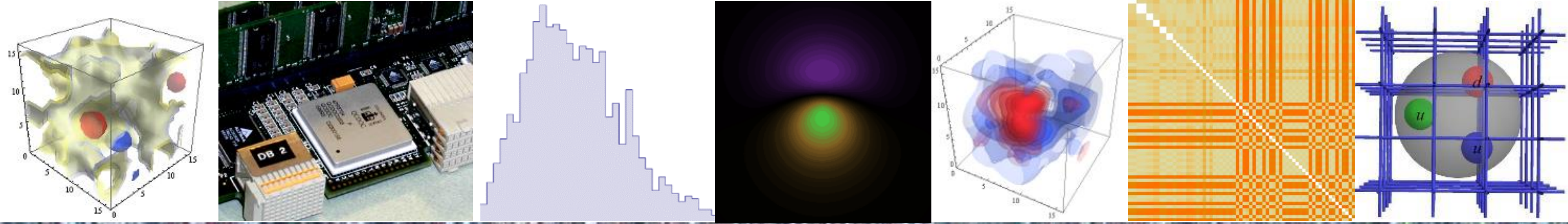


PRECISION NEUTRON-DECAY MATRIX ELEMENTS

HUEY-WEN LIN

PNDME

Precision Neutron-Decay Matrix Elements

`https://sites.google.com/site/pndmelqcd/`

Tanmoy Bhattacharya

Rajan Gupta

HWL

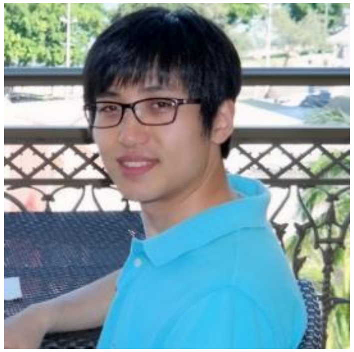
+

Vincenzo Cirigliano

Saul Cohen

Anosh Joseph

Yong-Chull Jang



Boram Yoon

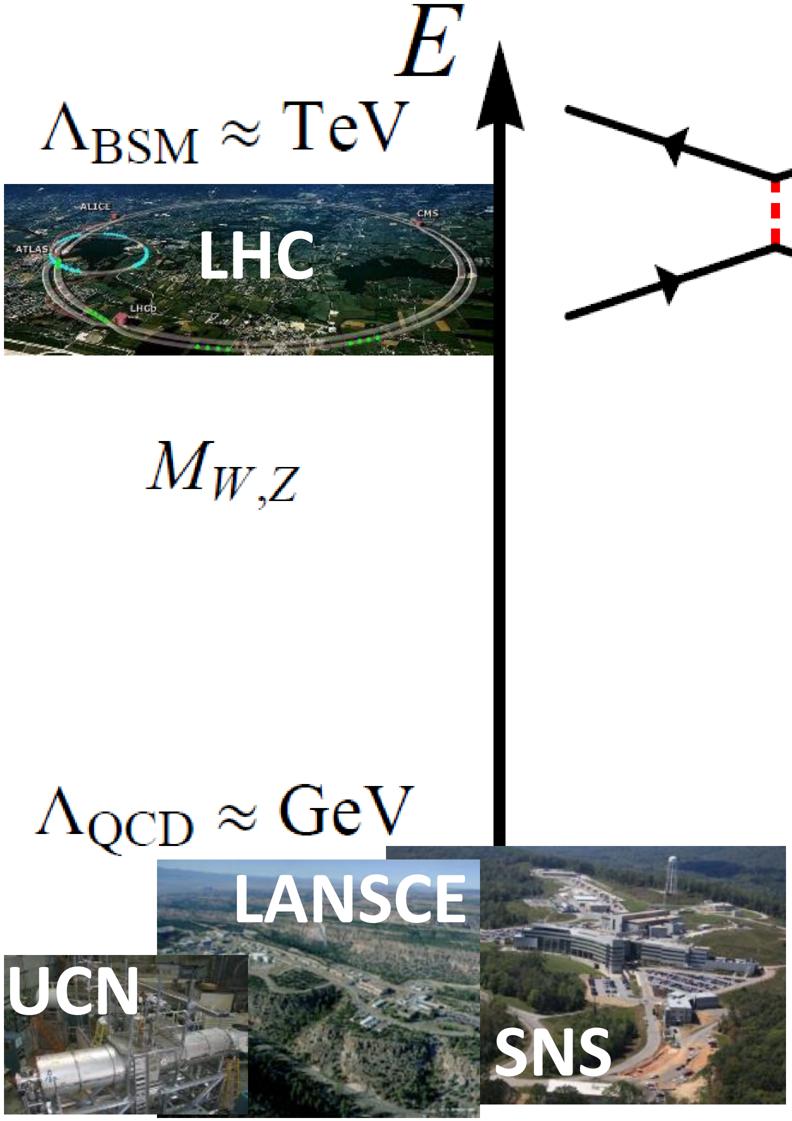
A Tale of Two Scales

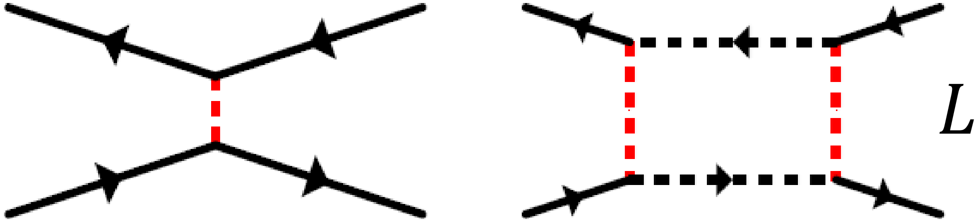
- LHC strikes out onto the high-energy frontier (13 TeV)
  - Direct production of Higgs and BSM particles
  - Parton distribution functions for SM background
- Many experiments refine low-energy measurements
  - Discern small discrepancies from the Standard Model

    Muon $g-2$, Q_{weak}, CKM matrix…
  - Probe small signals that are suppressed in the SM

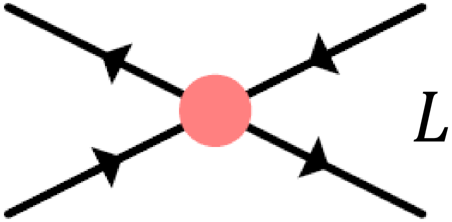
    dark matter, nEDM, $0\nu\beta\beta$, **neutron β decay**…

New Physics in TeV Scale

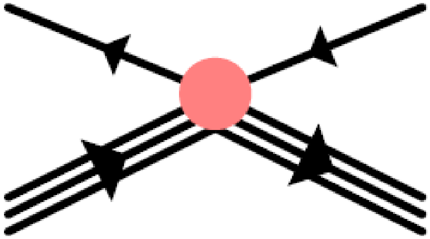




$$L_{\mathrm{SM}} + L_{\mathrm{BSM}}$$



$$L_{\mathrm{SM}} + \sum_{i=S,T} \varepsilon_i^{\mathrm{BSM}} \hat{O}_i^{\mathrm{BSM}}$$



$$g_S = \langle n|\bar{u}d|p\rangle$$

$$g_T = \langle n|\bar{u}\sigma_{\mu\nu}d|p\rangle$$

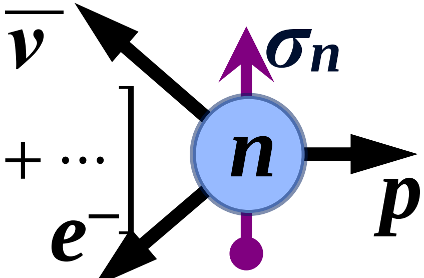
New Interactions

§ Neutron beta decay could be related to new interactions:

$$H_{\mathrm{eff}} = G_F \left(J_{V-A}^{\mathrm{lept}} \times J_{V-A}^{\mathrm{quark}} + \sum_i \varepsilon_i^{\mathrm{BSM}} \hat{O}_i^{\mathrm{lept}} \times \hat{O}_i^{\mathrm{quark}} \right)$$

- ε_S and ε_T are related to the masses of the new TeV-scale particles
- Parameters sensitive to new physics

$$d\Gamma \propto F(E_e)\left[1 + A\frac{\vec{\sigma_n}\cdot\vec{p_e}}{E_e} + b\frac{m_e}{E_e} + \left(B_0 + B_1\frac{m_e}{E_e}\right)\frac{\vec{\sigma_n}\cdot\vec{p_\nu}}{E_\nu} + \cdots\right]$$



Fierz interference term:
Deviations from the leading-order e^- spectrum

Energy-dependent part of the **neutrino asymmetry parameter** with neutron spin

$$\boxed{\{b,B\}_{\mathrm{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})}$$

Precision LQCD input
($m_\pi \approx 140$ MeV, $a \to 0$)

$$\boldsymbol{\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}}$$

New Interactions

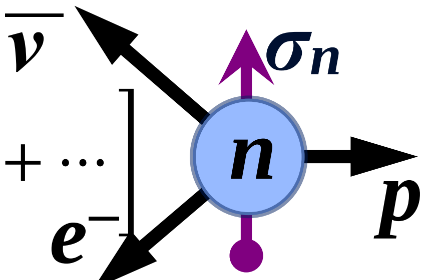
§ Neutron beta decay could be related to new interactions:

$$H_{\text{eff}} = G_F \left(J^{\text{lept}}_{V-A} \times J^{\text{quark}}_{V-A} + \sum_i \varepsilon_i^{\text{BSM}}\, \hat{O}_i^{\text{lept}} \times \hat{O}_i^{\text{quark}} \right)$$

- ε_S and ε_T -scale particles
- Paramete

$$d\Gamma \propto F(E_e)$$

$$\vec{\nu} + \cdots$$



Fierz inter

Deviations

leading-or

t of the

arameter

Ongoing and Future Experiments	Expected Precision
UCNb & UCNB at LANL	10^{-3} to 10^{-4}
Nab at ORNL	10^{-3}
FRMII in Munich, …	
CENPA ^{6}He(b_{GT})	10^{-3}

$$\boxed{\{b,B\}_{\text{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})}$$

Precision LQCD input
(m_π≈140 MeV, a→0)

$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Outline

- Lattice Nucleon Structure 101
- Precision nucleon couplings
  - Isovector $g_{S,T}$ and the impact on new-physics searches
  - Lattice axial charge calculations

Introducing the Lattice

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

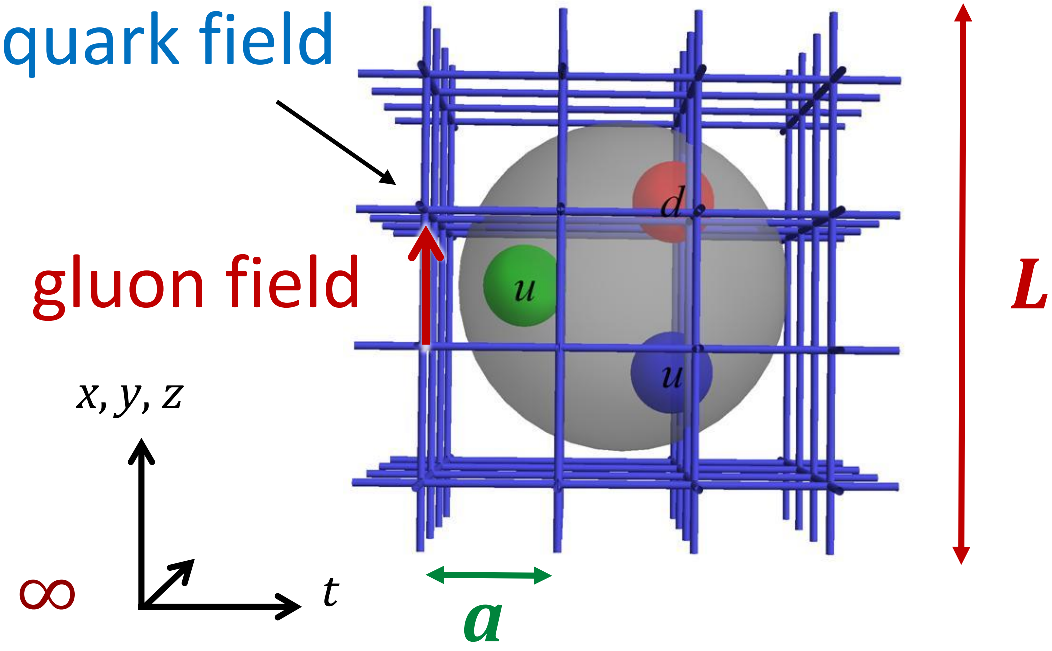
$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O\left(\bar{\psi},\psi,A\right)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume

§ Recover physical limit

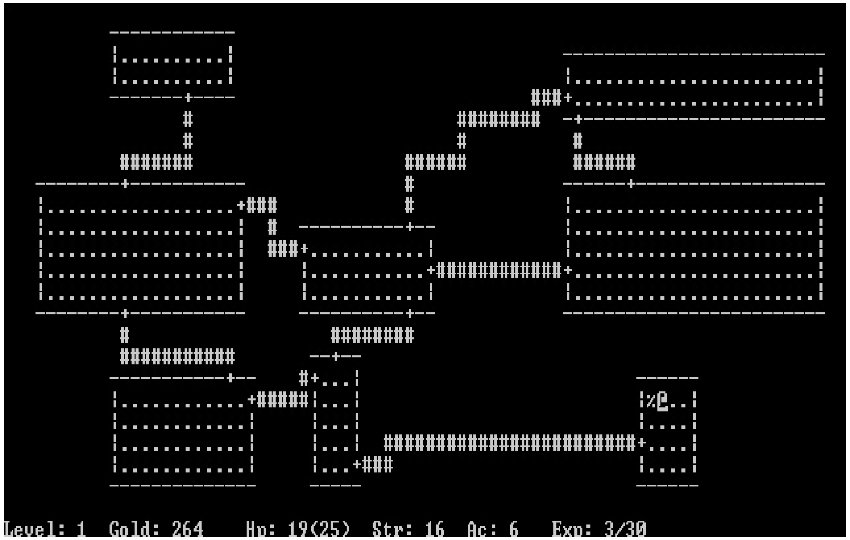
$$m_\pi \to m_\pi^{\text{phys}},\ \boldsymbol{a} \to \mathbf{0},\ \boldsymbol{L} \to \infty$$



Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources

1980s



```
Level: 1  Gold: 264   Hp: 19(25)  Str: 16  Ac: 6   Exp: 3/30
```

Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

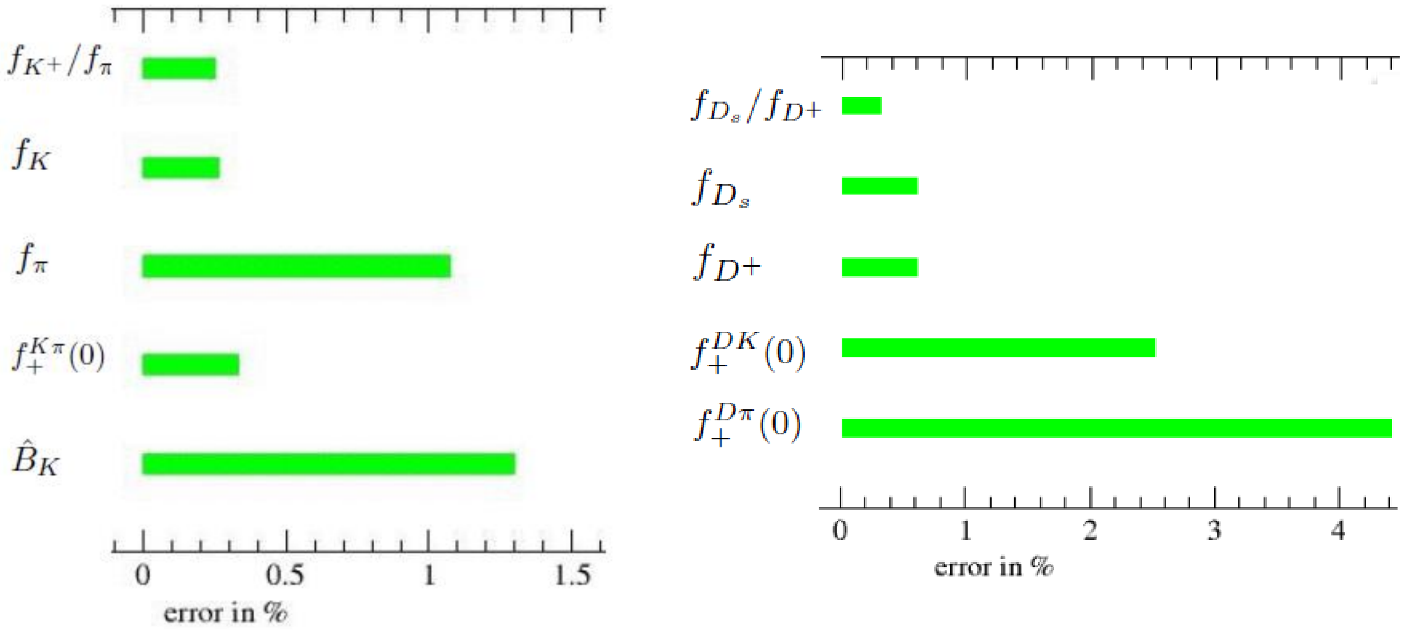
Successful Examples

- Lattice flavor physics provides precise inputs from the SM

  A. El-Khadra, Sep. 2015, INT workshop "QCD for New Physics at the Precision Frontier"

  - Very precise results in many meson systems

errors (in %) **(preliminary) FLAG-3 averages**



- We are beginning to do precision calculations in nucleons

The Trouble with Nucleons

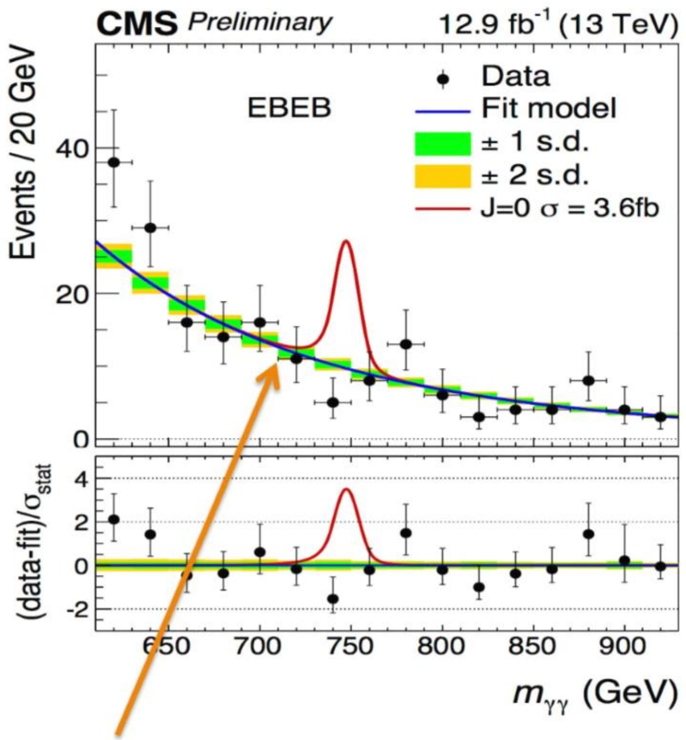
Nucleons are more complicated than mesons because…

- Noise issue
  - Signal diminishes at large t_E relative to noise
  - Get worse when quark mass decreases
- Excited-state contamination
  - Nearby excited state: Roper(1440)
- Hard to extrapolate in pion mass
  - Δ resonance nearby; multiple expansions, poor convergence…
  - Less an issue in the physical pion-mass era
- Requires larger volume and higher statistics
  - Ensembles are not always generated with nucleons in mind
  - **High-statistics**: large measurement and long trajectory

The Trouble with Nucleons

Nucleons are more complicated than mesons because…

- Noise issue
  - Signal diminishe
- Excited-state cor
  - Nearby excited s
- Hard to extrapol
  - Δ resonance near r convergence…
  - Less an issue in t
- Requires larger v s
  - Ensembles are n leons in mind
  - **High-statistics:** amples



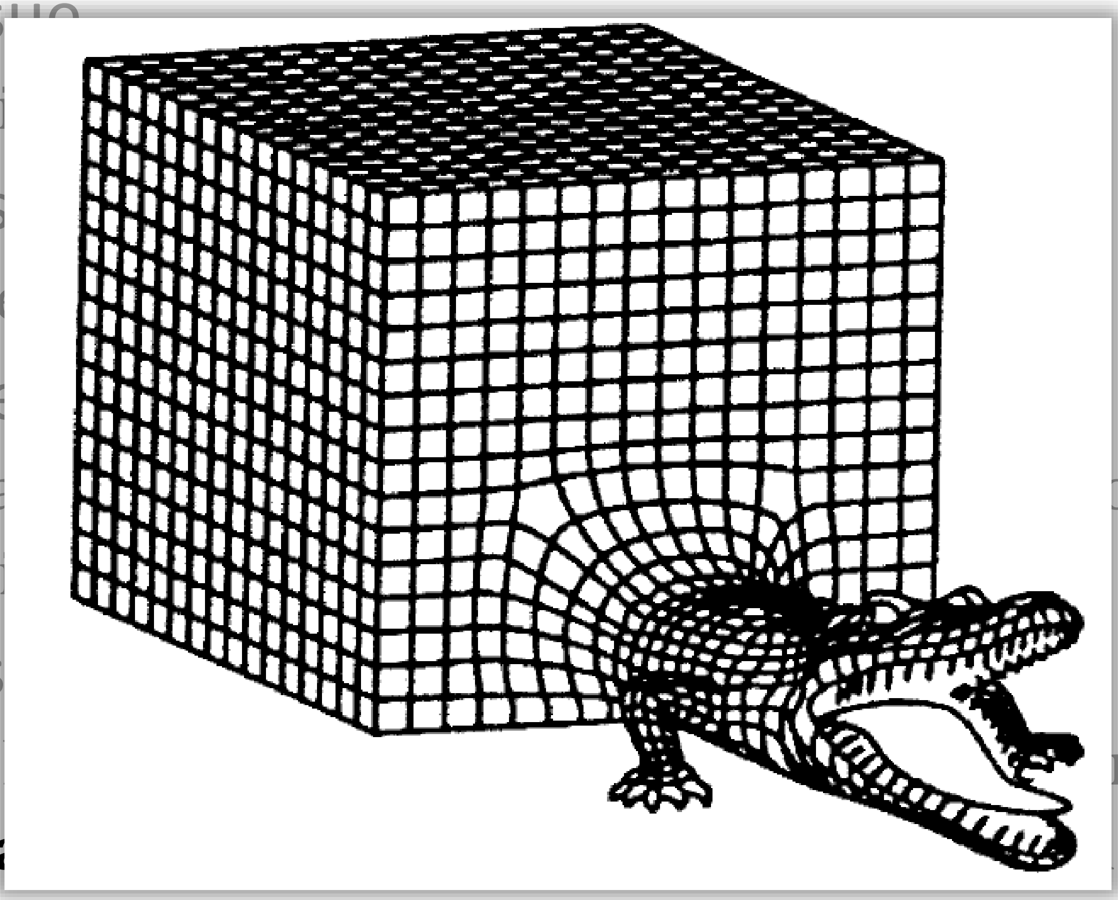
A signal with cross-section as the largest excess in 2015+8TeV would look like this

The disappearance of X(750)

The Trouble with Nucleons

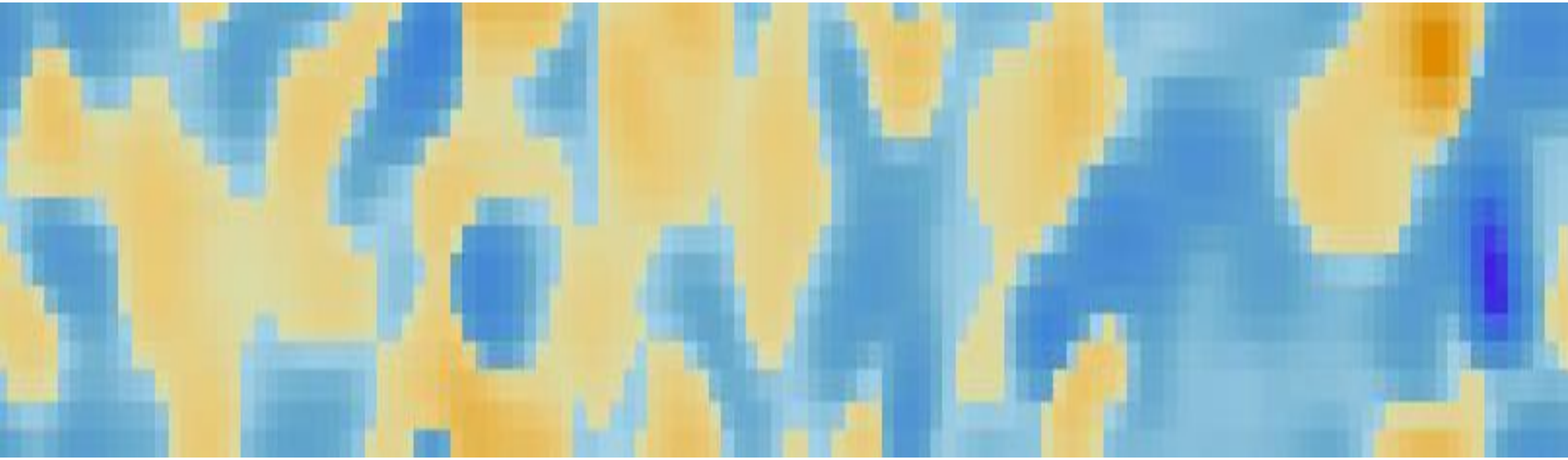
Nucleons are more complicated than mesons because…

- Noise iss
  - Signal di
- Excited-s
  - Nearby
- Hard to e
  - Δ resona onvergence…
  - Less an
- Requires
  - Ensemb ns in mind
  - **High-sta ples**



PROCEED WITH CAUTION

Nucleon Matrix Elements

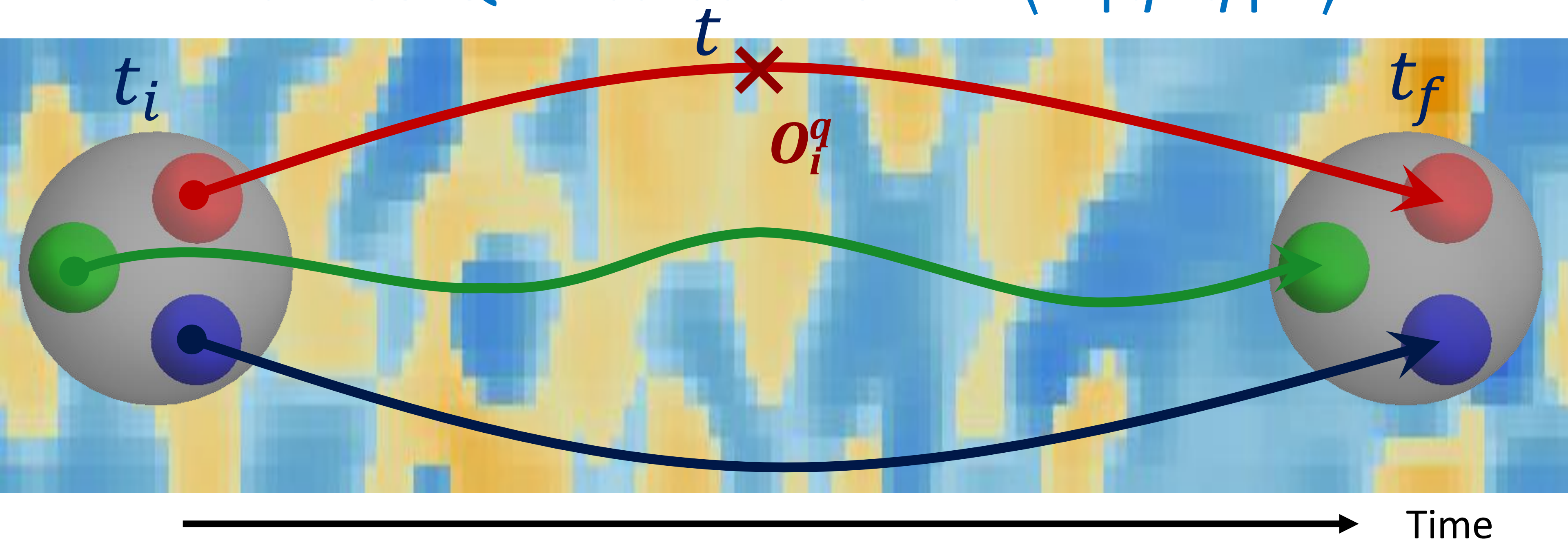


§ Pick a QCD vacuum

- Gauge/fermion actions, flavour (2, 2+1, 2+1+1), m_π, a, L, …

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

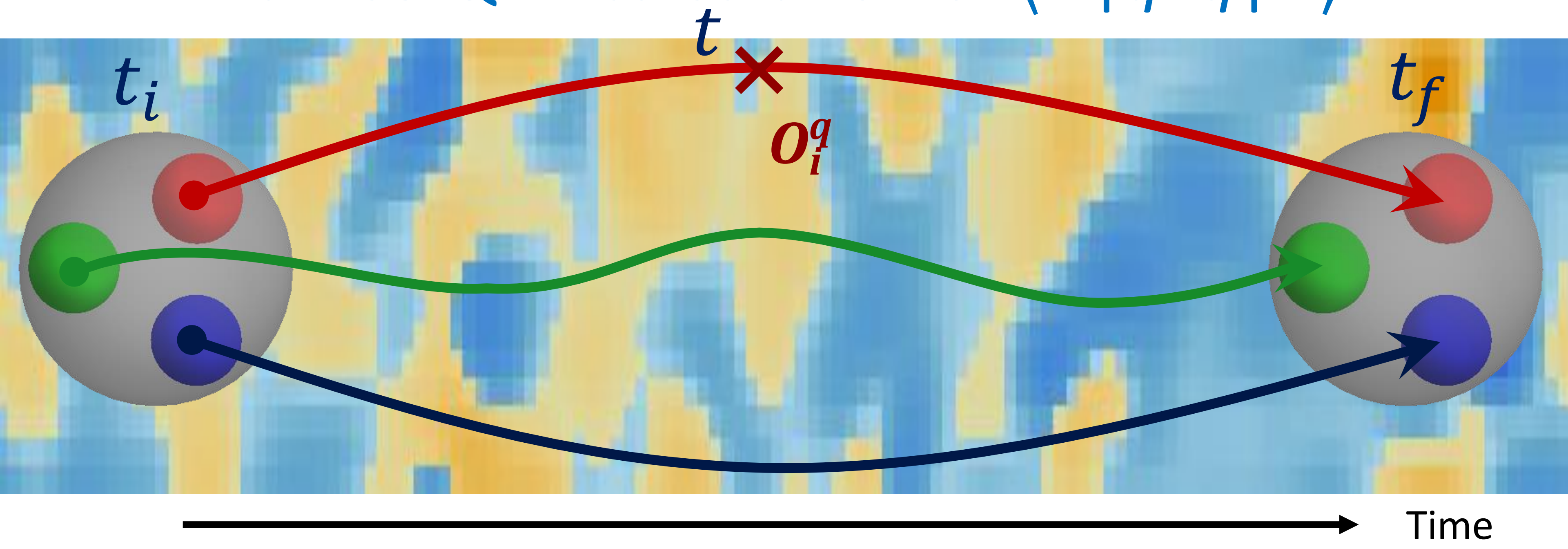


- Construct correlators (hadronic observables)
  - Requires “quark propagator”

    Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



§ Analysis (extract couplings)

$$C^{3\text{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

$$C^{2\text{pt}}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)} + \ldots$$

Precision Nucleon Couplings

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

a (fm)	*V*	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	24k
0.12	$32^3 \times 64$	4.38	220	8,10,12	7.6k
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	64.6k
0.09	$32^3 \times 96$	4.51	310	10,12,14	7.0k
0.09	$48^3 \times 96$	4.79	220	10,12,14	7.1k
0.09	$64^3 \times 96$	3.90	130	10,12,14	56.5k
0.06	$48^3 \times 144$	4.52	310	16,20,22,24	64.0k
0.06	$64^3 \times 144$	4.41	220	16,20,22,24	41.6k
0.06	$96^3 \times 192$	3.80	130		

Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME) $a = 0.12$ fm, 310-MeV pion

  - Move the **excited-state systematic** into the statistical error

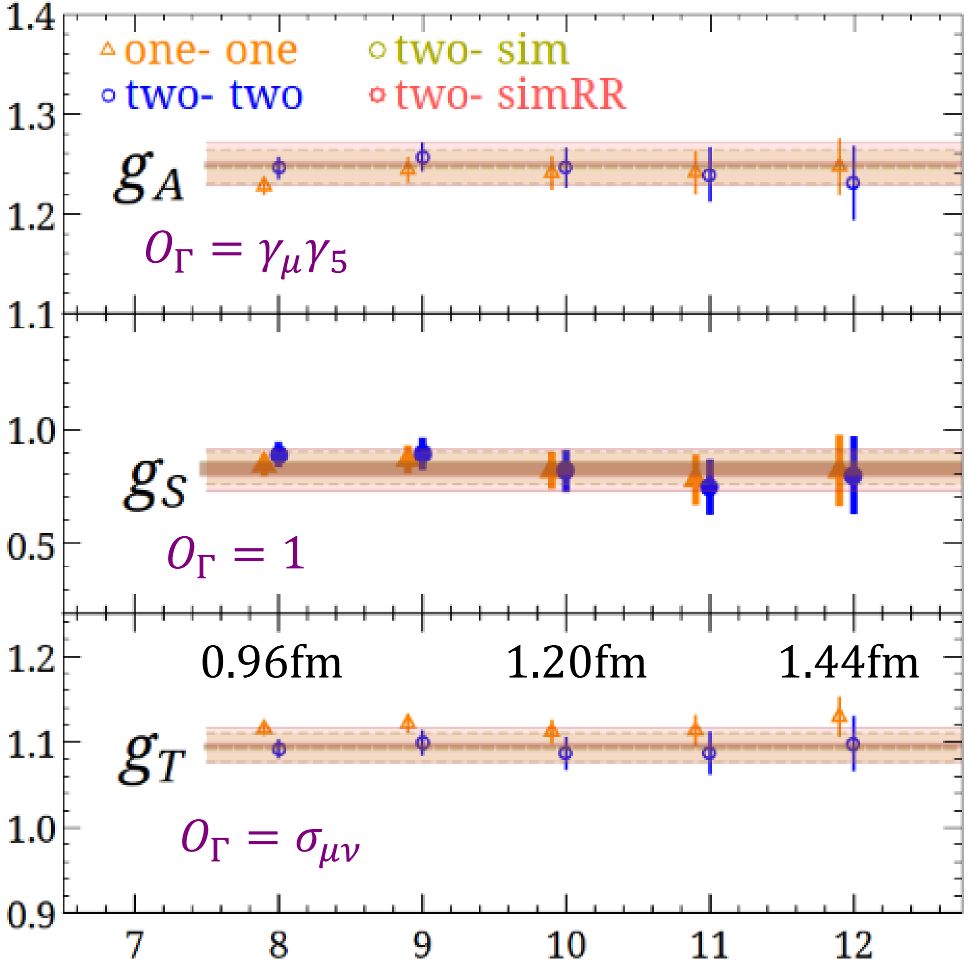
$$C^{3\mathrm{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$

$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0| \quad 1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$

$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e \quad (t-t_i)e^{-M_0(t_f-t)}$$

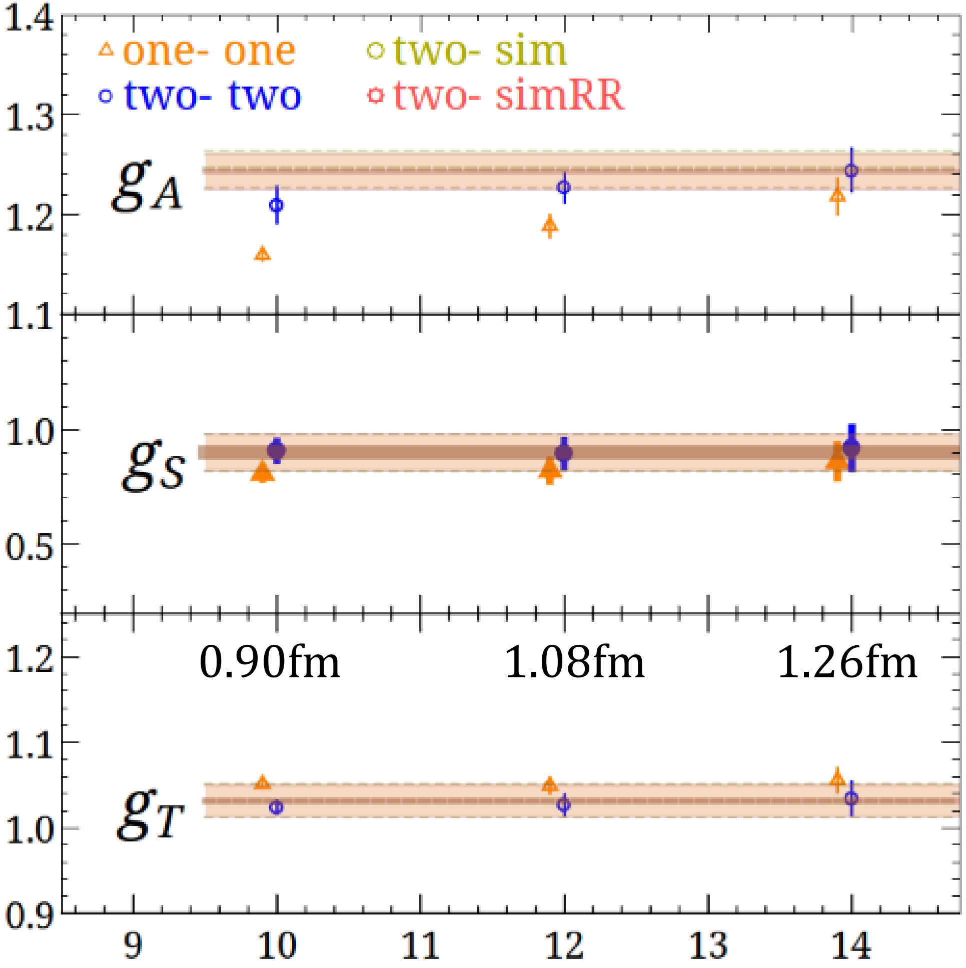
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e \quad t_i)$$

  - No obvious contamination between 0.96 and 1.44 fm separation



Systematic Control

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME) $a = 0.09$ **fm**, 310-MeV pion



- Move the **excited-state systematic** into the statistical error

$$C^{3\text{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$

$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0| \qquad 1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$

$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e \qquad (t-t_i)\, e^{-M_0(t_f-t)}$$

$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e \qquad t_i)$$

- Much stronger effect at finer lattice spacing!
- Needs to be studied case by case

Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

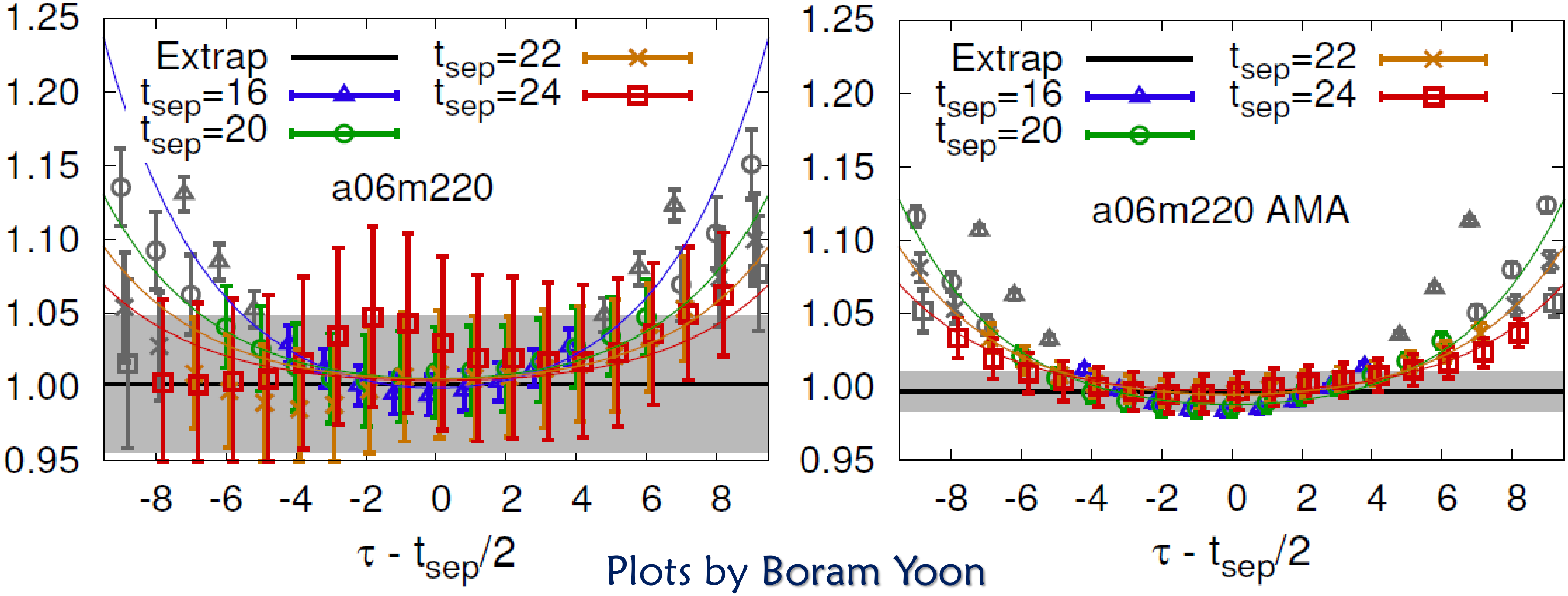
Statistical effect

$a = 0.06$ fm, 220-MeV pion

g_T^{bare}

2.6k

41.6k



Plots by **Boram Yoon**

Systematic Control

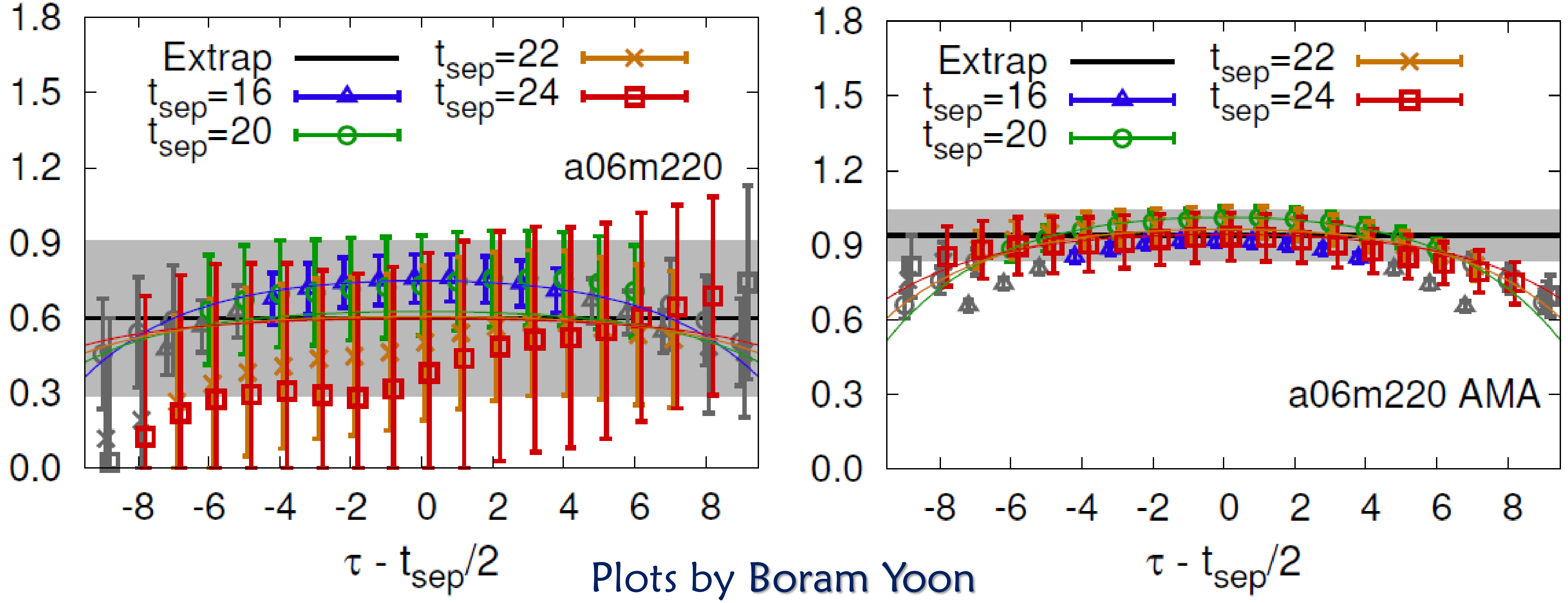
- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)
  - Statistical effect

$a = 0.06$ fm, 220-MeV pion

g_S^{bare}

2.6k

41.6k

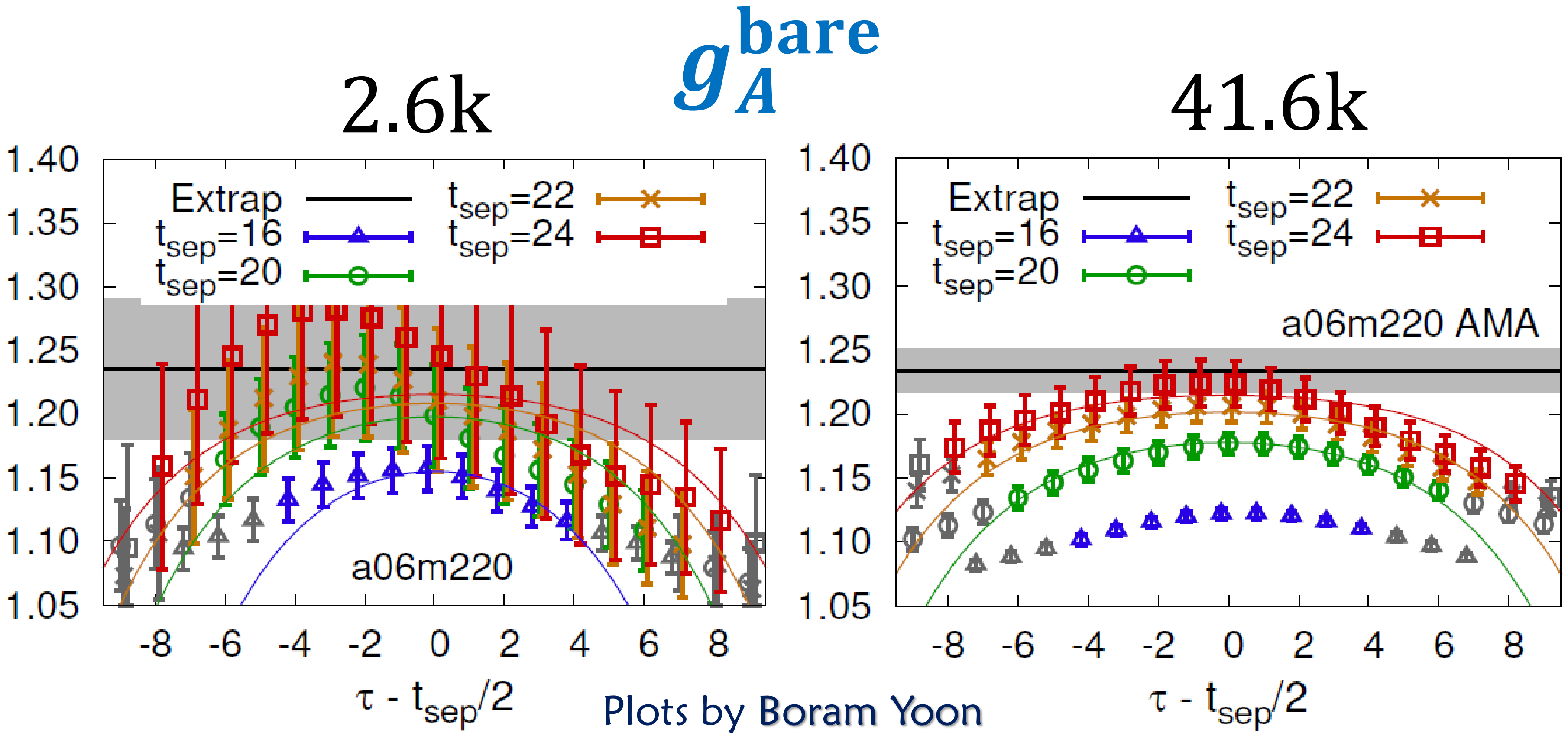


Plots by **Boram Yoon**

Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

Statistical effect (worst case) **a = 0.06 fm**, 220-MeV pion



g_A^{bare}

2.6k

41.6k

Plots by Boram Yoon

Systematic Control

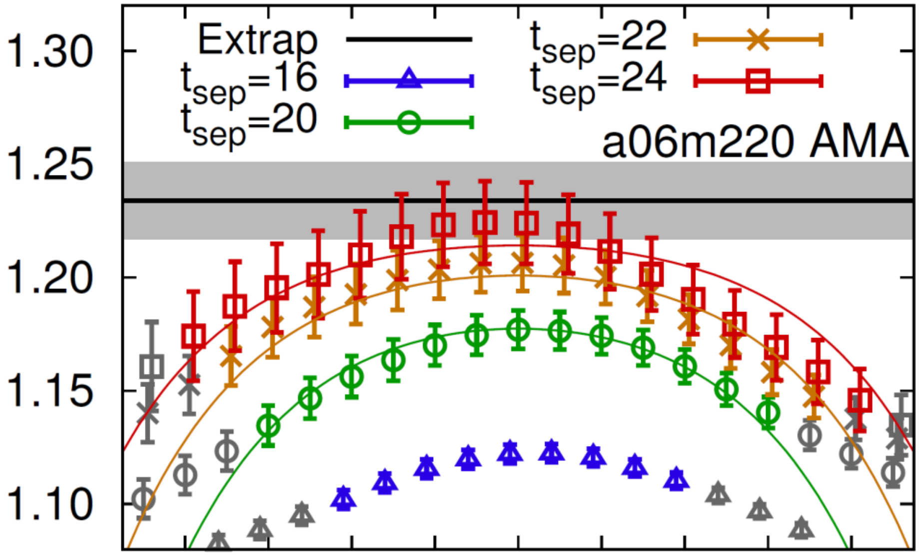
- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

➳ Robustness of the 2-state fit

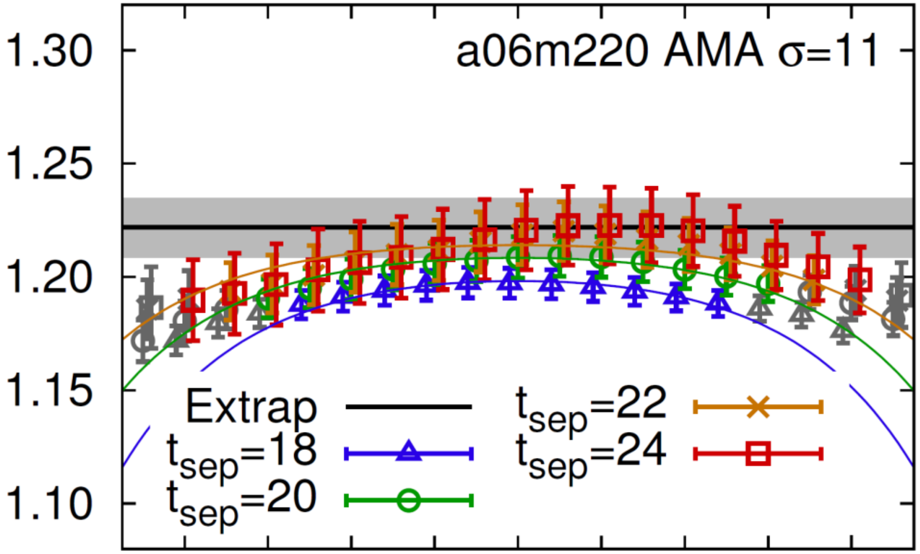
$a = 0.06$ fm, 220-MeV pion

g_A^{bare}

2.6k



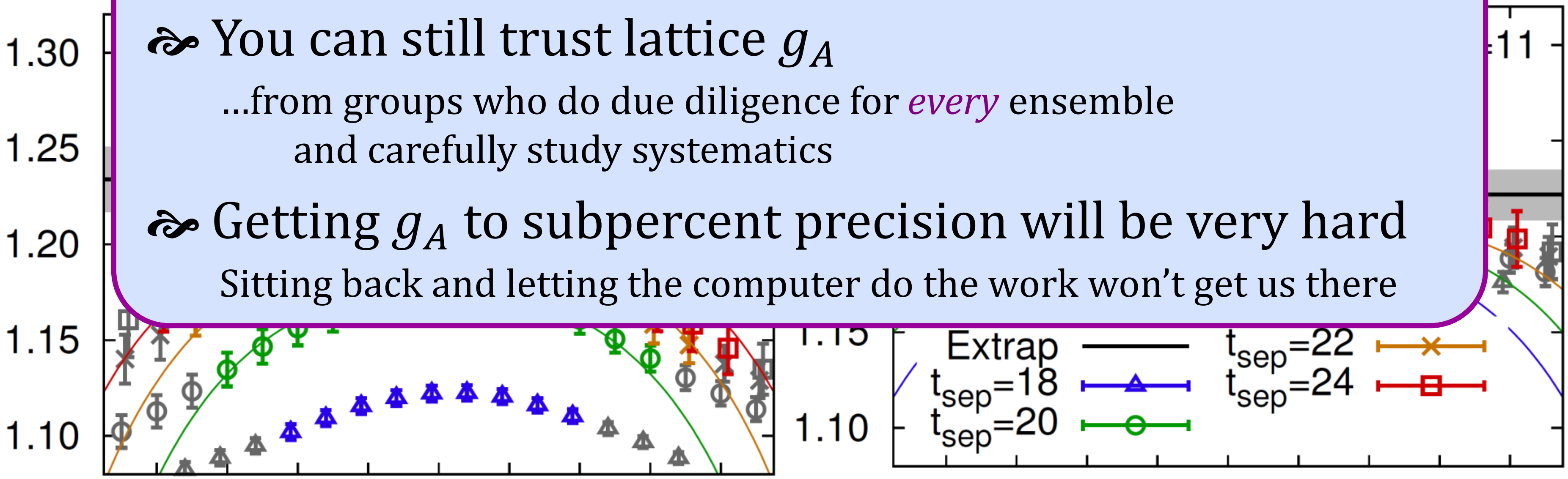
41.6k



Plots by **Boram Yoon**

Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)
  - R



My Two Cents

- g_A is *not* a gold-plated quantity

  Early impressions that g_A would be easy underestimated systematics

- You can still trust lattice g_A

  ...from groups who do due diligence for *every* ensemble
  and carefully study systematics

- Getting g_A to subpercent precision will be very hard

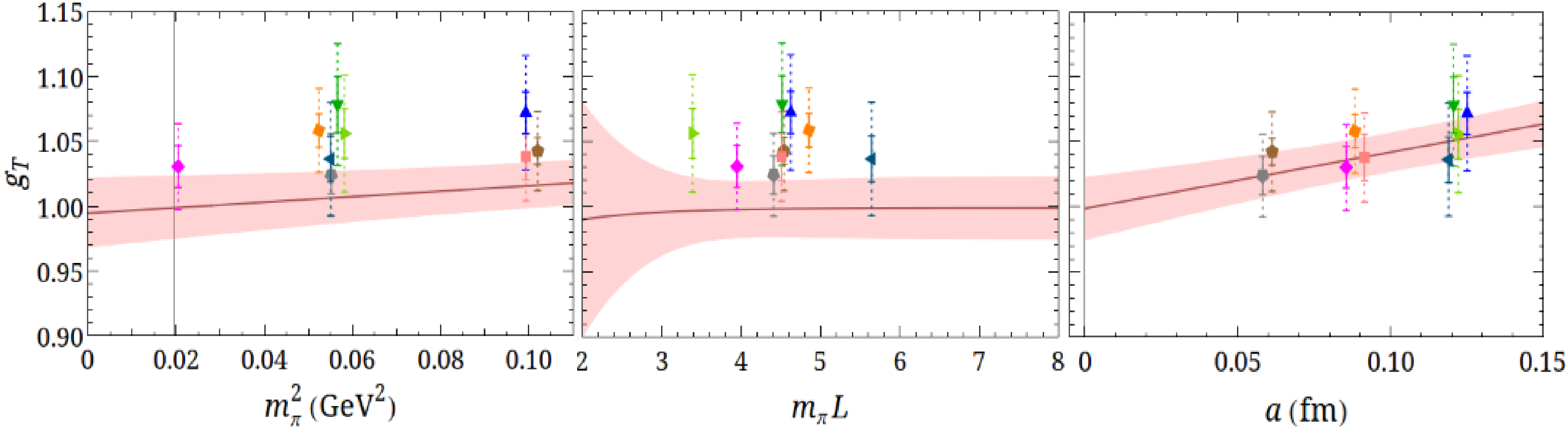
  Sitting back and letting the computer do the work won't get us there

Plots by **Boram Yoon**

Precision Nucleon Couplings

- § Much effort has been devoted to controlling **systematics**
- § A state-of-the art calculation (PNDME)
  - Extrapolate to the physical limit

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

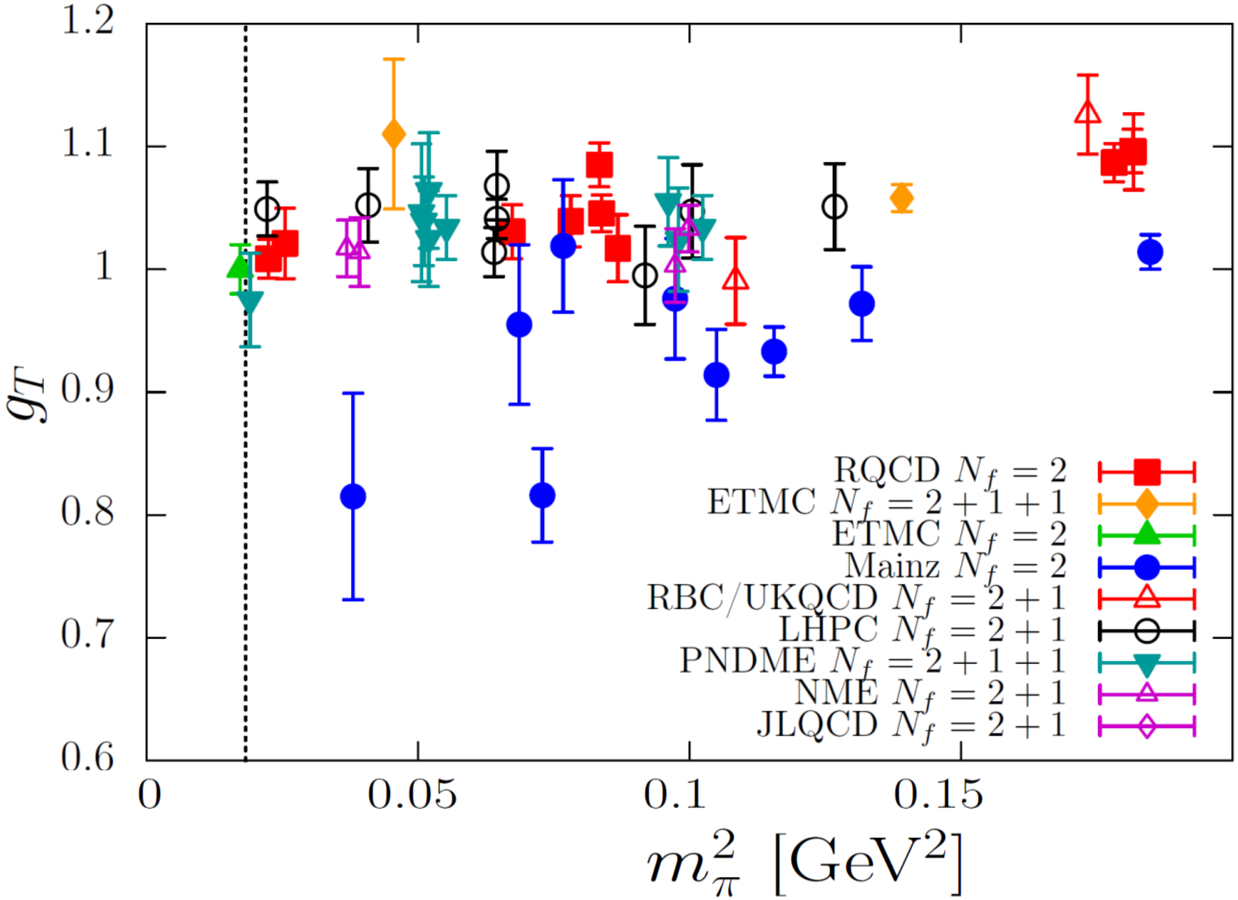


First extrapolation to the physical limit
of a nucleon matrix element!

Precision Nucleon Couplings

§ Usually more than one LQCD calculation

- For example, tensor charge
- Lattice results should agree in the continuum limit



Sara Collins,
Lattice 2016

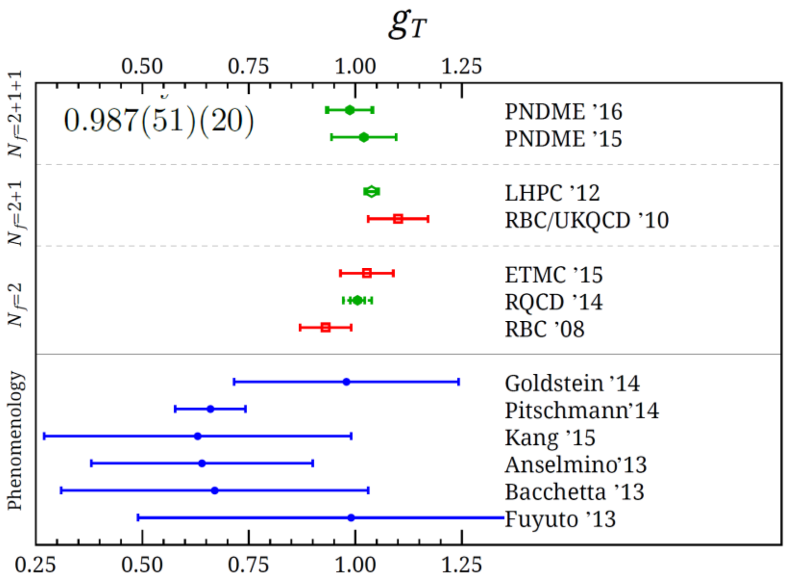
Precision Nucleon Couplings

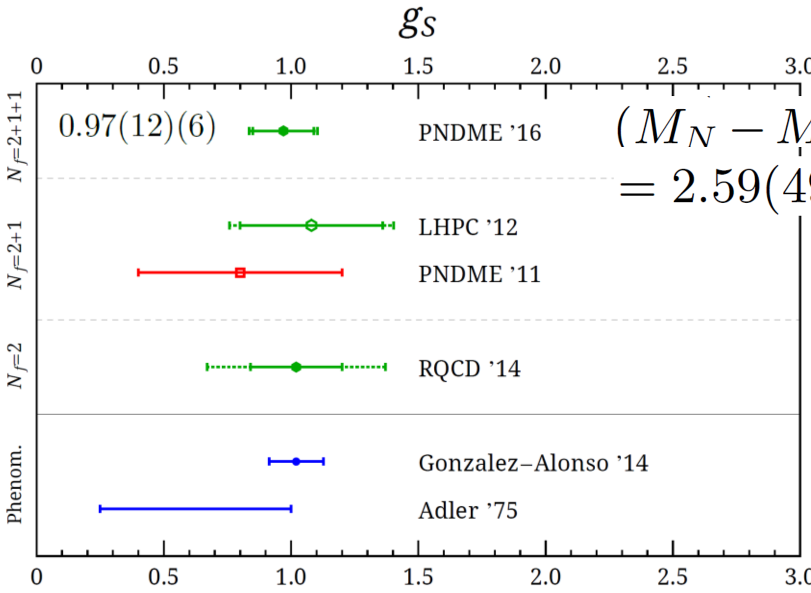
FLAG rating system

PNDME, 1506.06411; 1606.07049

New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7) [d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46) [f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6) [g]



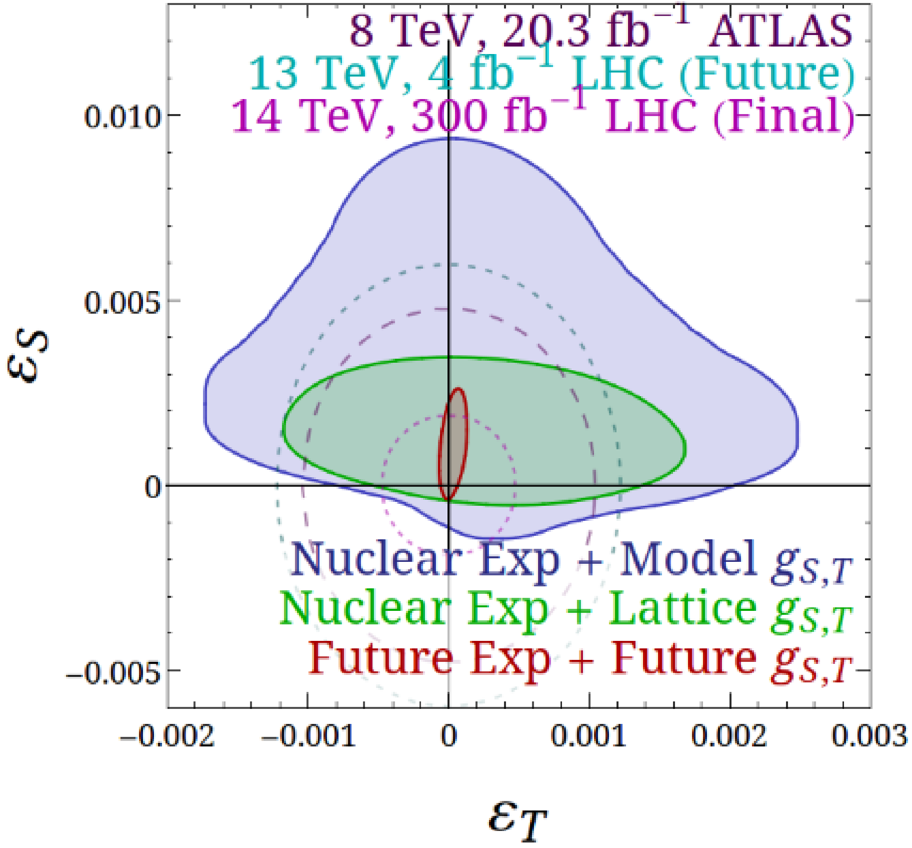


$$(M_N - M_P)^{\rm QCD} = 2.59(49)\ \text{MeV}$$

Beta Decays & BSM

- Given precision $g_{S,T}$ and $O_{\rm BSM}$, predict new-physics scales

Low-Energy Expt → $O_{\rm BSM} = f_O(\varepsilon_{S,T}\, g_{S,T})$ ← Precision LQCD input ($m_\pi \to 140$ MeV, $a \to 0$)



$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp't

$$|B_1 - b|_{\rm BSM} < 10^{-3}$$
$$|b|_{\rm BSM} < 10^{-3}$$

CENPA: ^{6}He($b_{\rm GT}$) at 10^{-3}

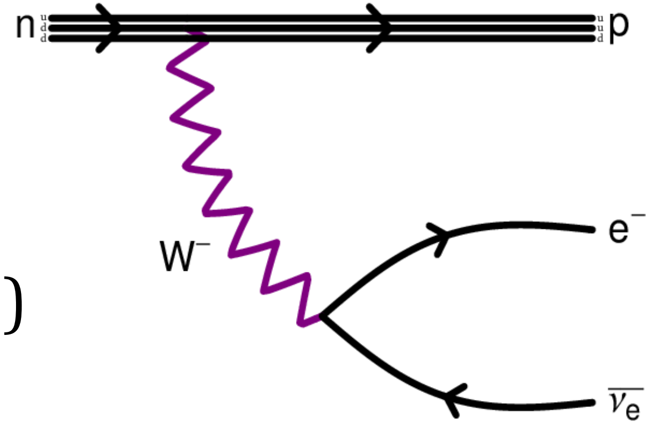
PNDME, PRD85 054512 (2012); 1306.5435; 1606.07049

$$\Lambda_S > 7 \text{ TeV}$$
$$\Lambda_T > 13 \text{ TeV}$$

Nucleon Axial Charge

- A fundamental measure of nucleon structure
- Axial-vector–current matrix element

$$g_A = G_A^{u-d}(Q_2 = 0)$$

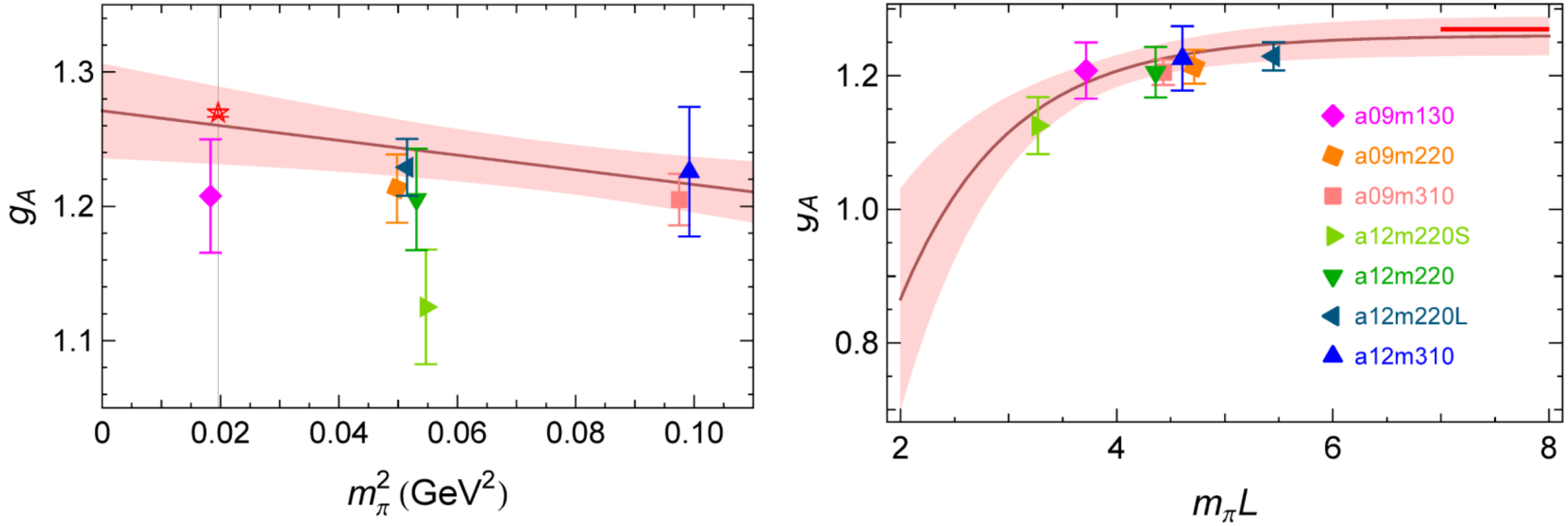


- Important to many nuclear processes
  - The rate of *pp* fusion (as in Sun-like stars)
  - *n*-lifetime when combining with V_{ud}
  - New-physics searches such as right-handed neutrinos
  - $0\nu\beta\beta$ searches, "quenching" g_A^4

- In lattice QCD: A benchmark for nucleon structure

Nucleon Axial Charge

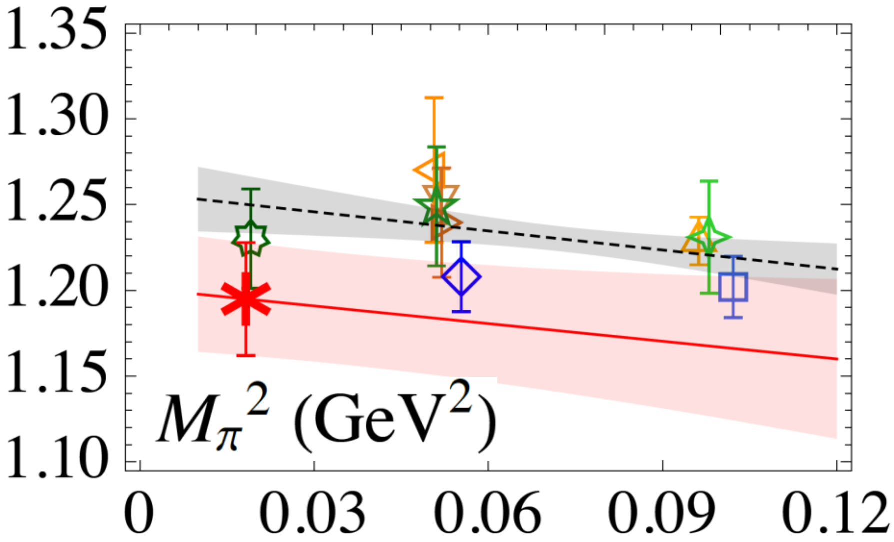
§ Finite-volume/statistical effects

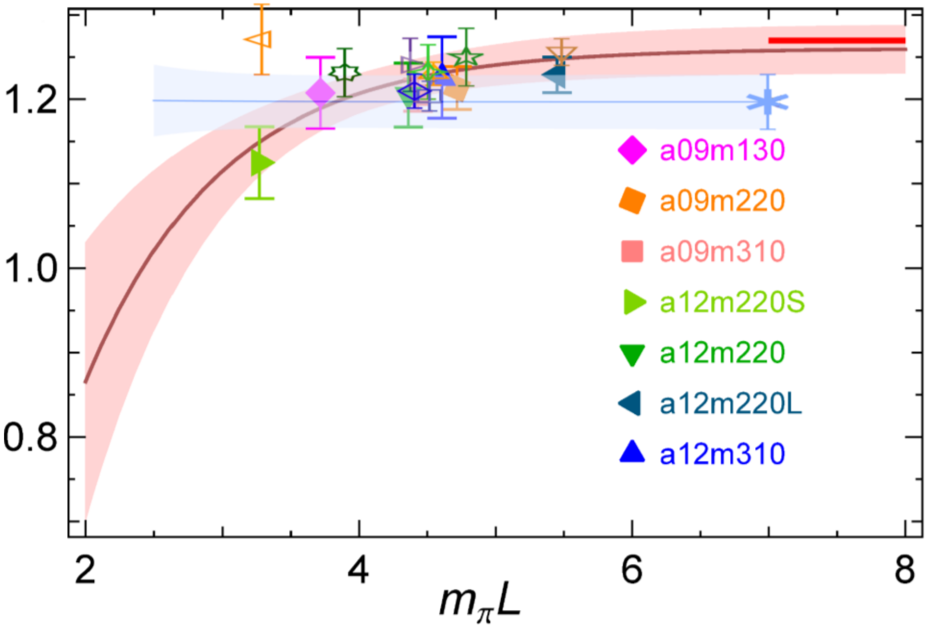


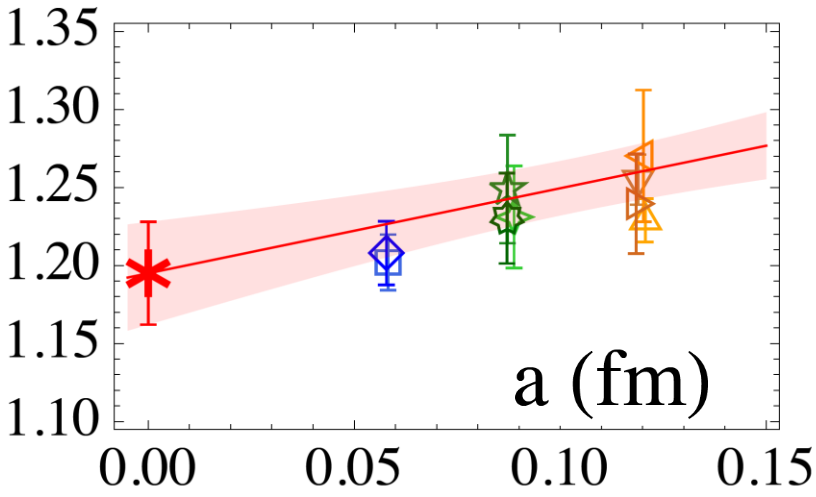
2013 Results

Nucleon Axial Charge

§ Finite-volume/statistical effects



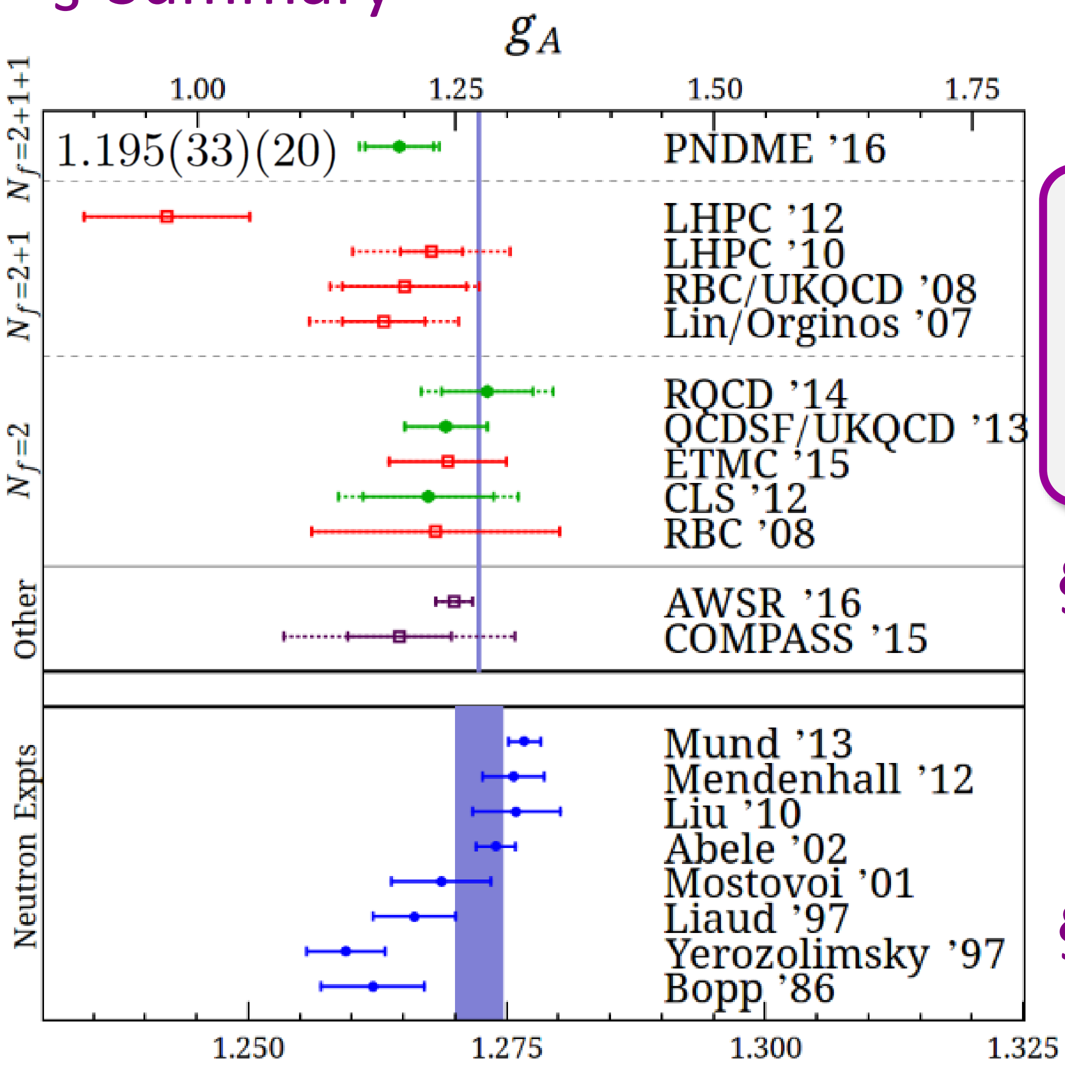




2016 Results

Nucleon Axial Charge

§ Summary



§ Implications?

- 2σ might go away with greater statistics

Lattice 2016 Prelim.

- RBC* 2+1f 1.15(4)
- PACS* 2+1f 1.8(4)

§ New physics?

- $\lambda = g_A/\ g_V\ f_{NP}$

$$A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}$$

§ Stay tuned…

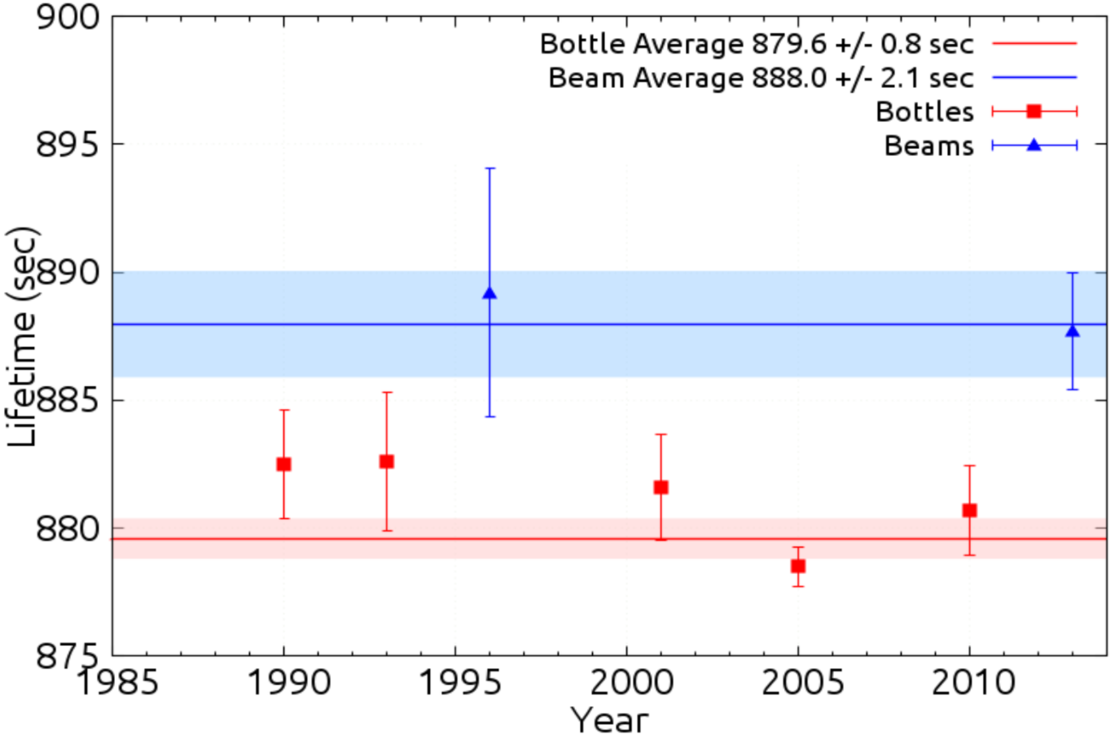
n-Lifetime Discrepancy

§ Neutron lifetime discrepancy?

- $\tau_n = 980(50)$ s Using lattice g_A and V_{ud}

The Situation…Today

Slide by Geoff Greene



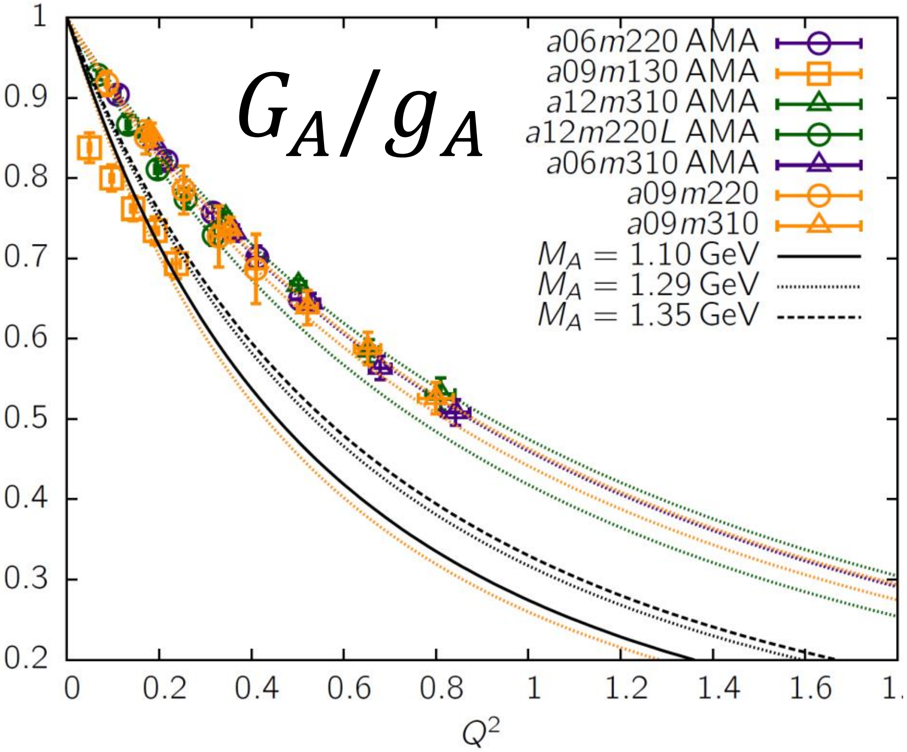
A. Yue, et al, Phys Rev Lett, 111, 222501 (2013)

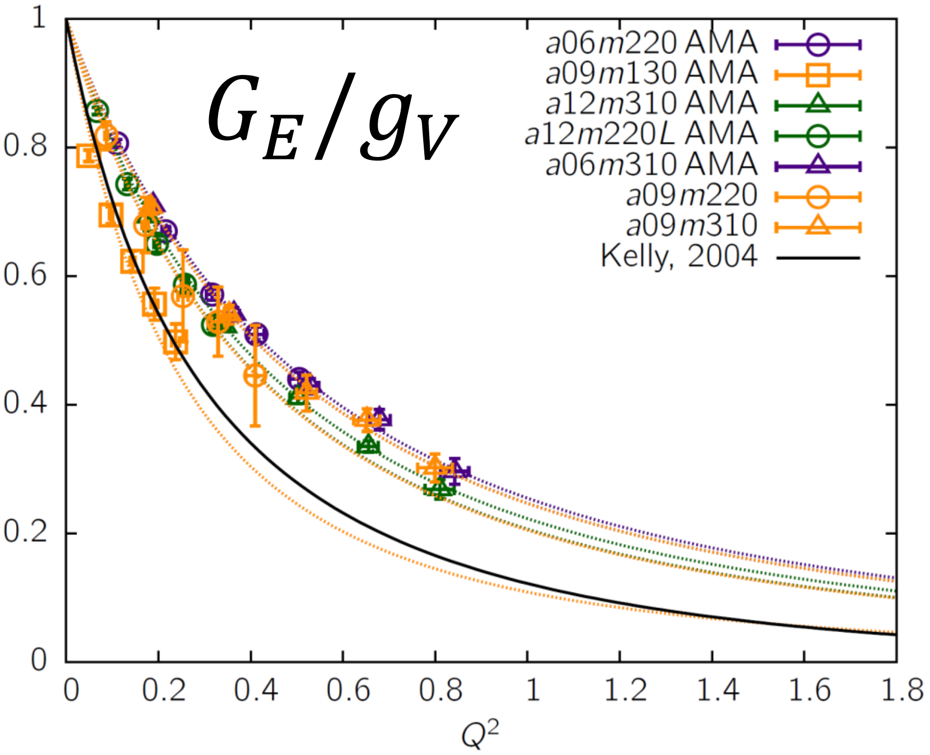
plot courtesy K. Grammer

Others Results

§ Isovector form factors

Plots by **Yong-Chull Jang**





§ Flavor-dependent couplings, 1^{ST} moments on PDFs, …

- qEDM by Cirigliano (this afternoon)

Precision Nucleon Couplings

§ Improvement within the next year

a (fm)	V	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	~30k
0.12	$32^3 \times 64$	4.38	220	8,10,12	~30k
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	64.6k
0.09	$32^3 \times 96$	4.51	310	10,12,14,16	Planned
0.09	$48^3 \times 96$	4.79	220	10,12,14,16	Planned
0.09	$64^3 \times 96$	3.90	130	10,12,14	56.5k
0.06	$48^3 \times 144$	4.52	310	16,20,22,24	64.0k
0.06	$64^3 \times 144$	4.41	220	16,20,22,24	41.6k
0.06	$96^3 \times 192$	3.80	130	16,20,22,24	~20k

Summary

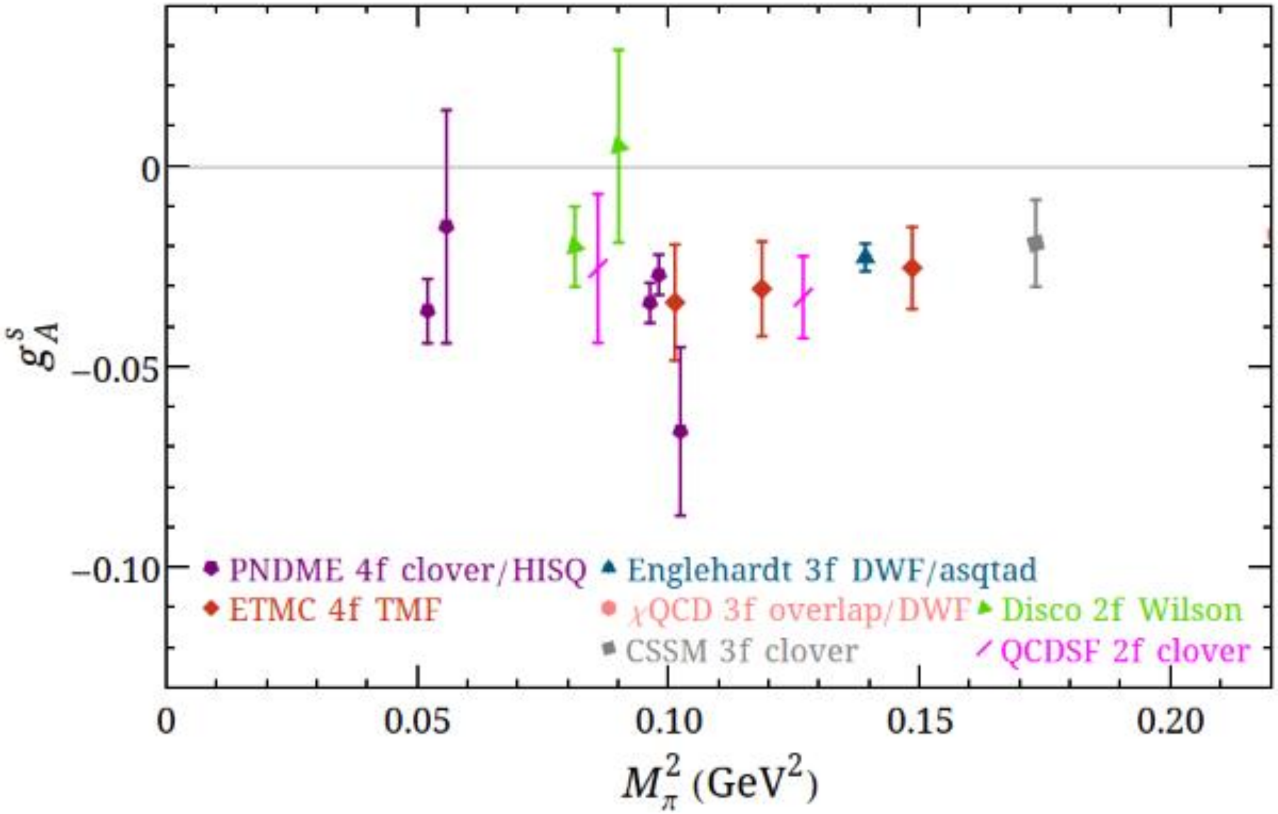
- Exciting era using LQCD for precision SM nucleon inputs
  - Increased computational resources and improved algorithms
  - Many near-physical ensemble calculations ongoing…
- Precision frontier enables us to probe BSM physics
  - Probes high-energy (TeV) physics at low energy (GeV)
  - Combined effort from experiment and theory sides to set bounds on new-physics scenarios, constrain BSM models
- LQCD is necessary when experiment is less known (e.g. g_T, g_s^A) or impossible to measure (e.g. g_S)
- More precise measurement is needed for g_A

Backup Slides

Spin-Dependent Cross Section

§ Still poorly known experimentally

❧ Good chance to have LQCD to improve the numbers

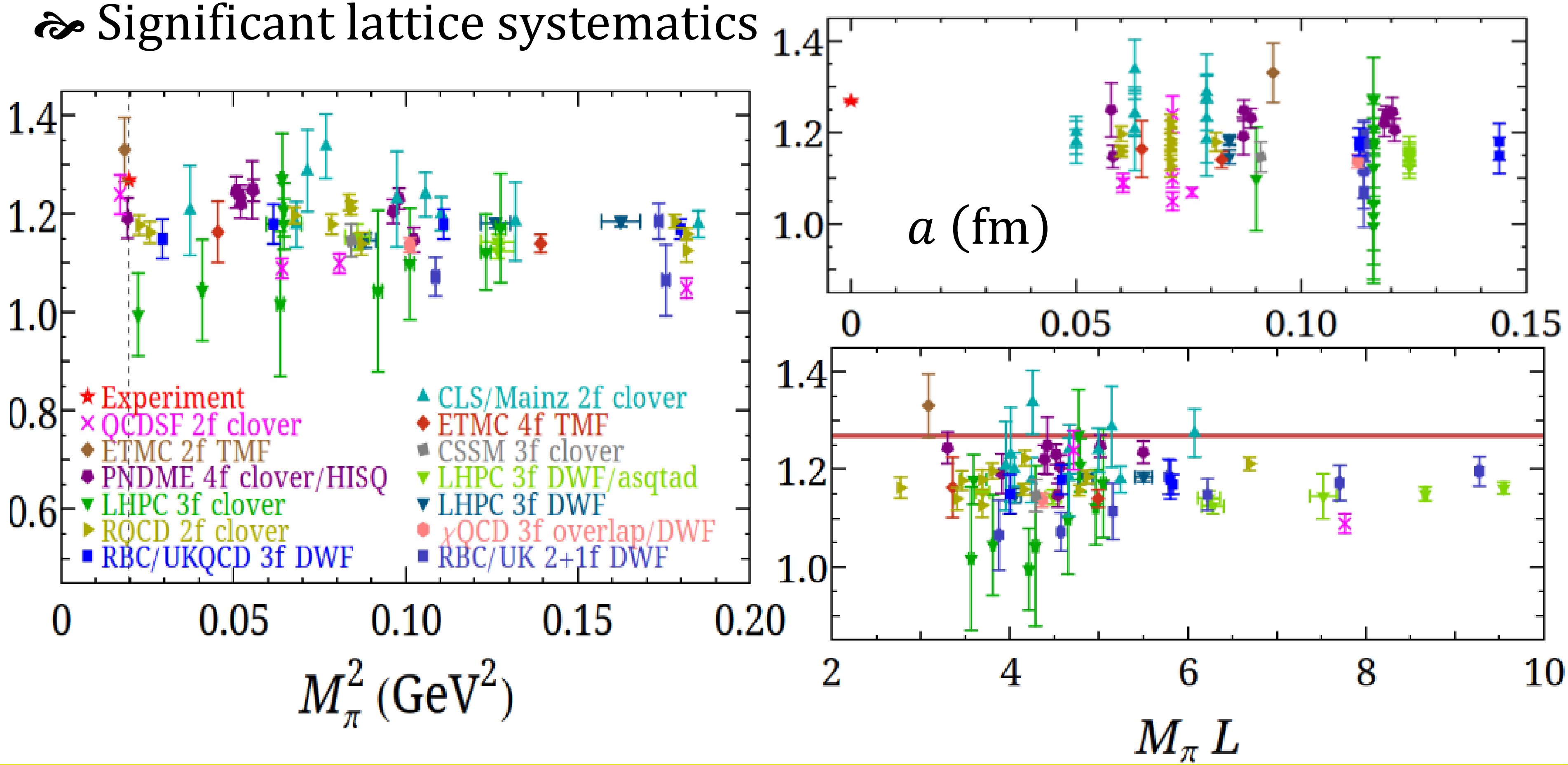


❧ Current LQCD data suggests about −0.04 contribution

Progress

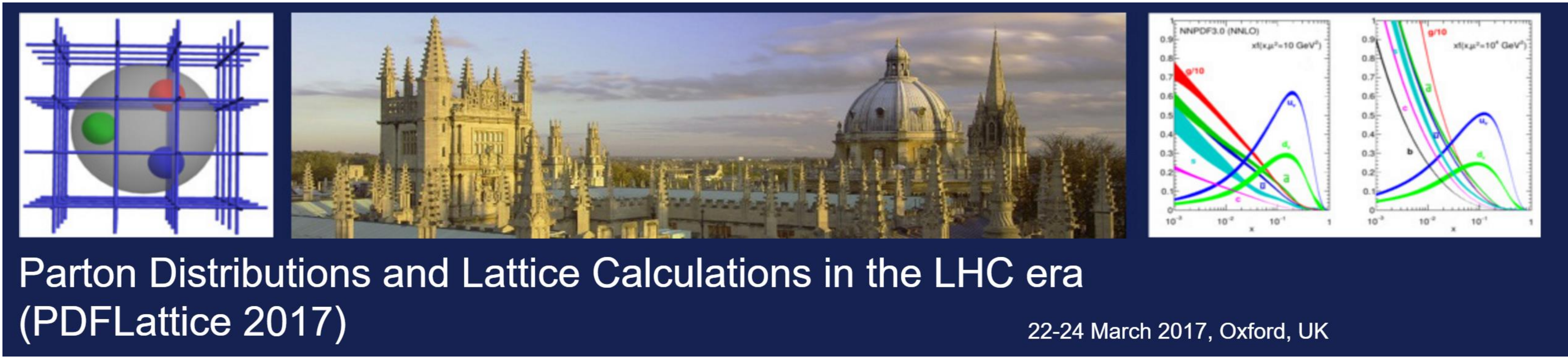
§ Axial charge

- Most well-known coupling with subpercent errors
- Important to the rate of *pp* fusion, *n*-lifetime, ...
- Significant lattice systematics



Future Prospects

- A first joint workshop with global-fitting community to address key LQCD inputs
  - http://www.physics.ox.ac.uk/confs/PDFlattice2017



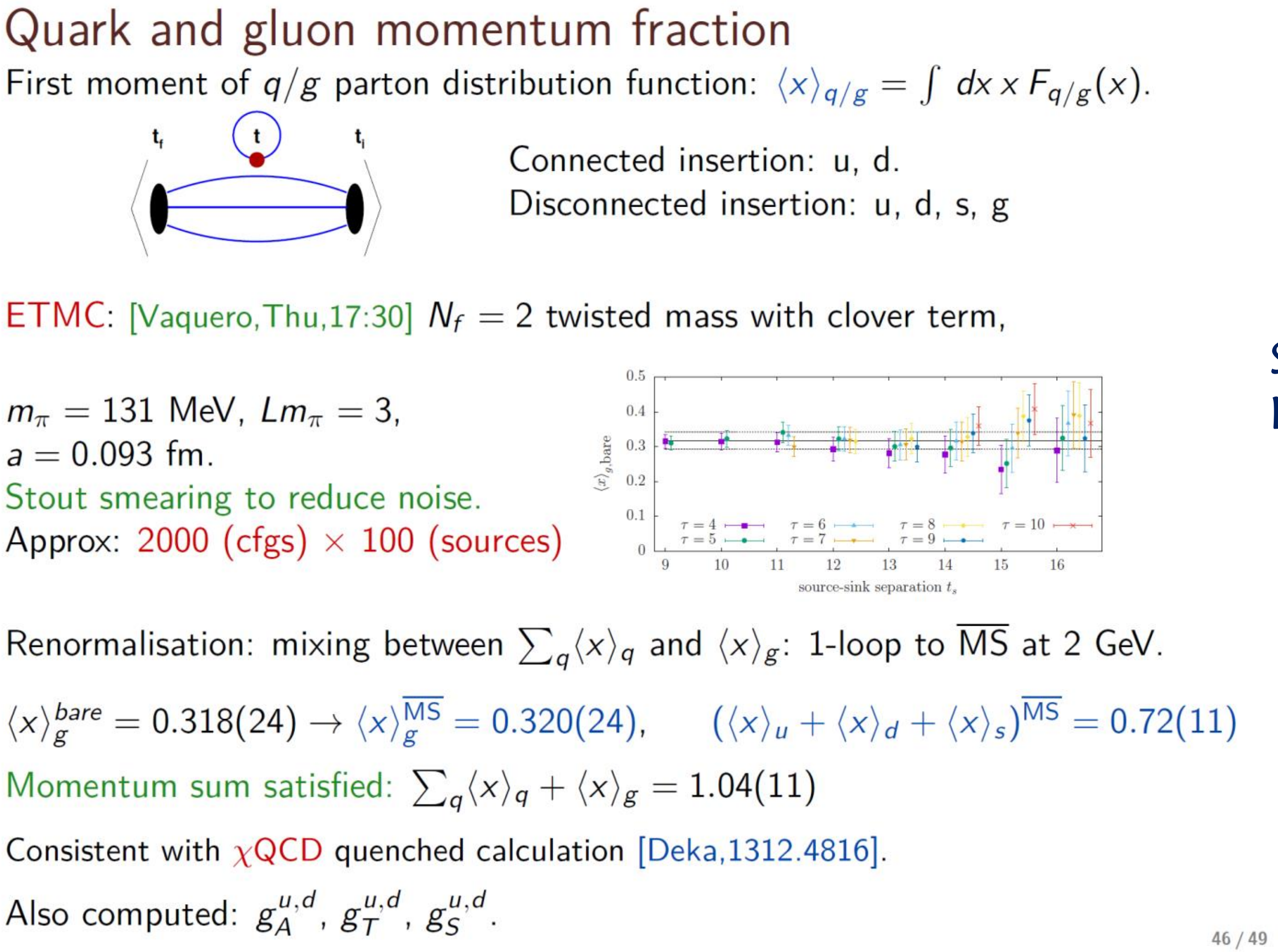
Parton Distributions and Lattice Calculations in the LHC era
(PDFLattice 2017)

22-24 March 2017, Oxford, UK

"The goal of this workshop is to ***bring together the global PDF analysis and lattice-QCD communities*** *to explore ways to improve current PDF determinations. In particular, we plan to* ***set precision goals for lattice-QCD*** *calculations so that these calculations, together with experimental input, can achieve more reliable determinations of PDFs. In addition we will discuss what impact such improved determinations of PDFs will have on future new-physics searches."*

Gluon

§ Updates from Lattice 2016



Quark and gluon momentum fraction

First moment of q/g parton distribution function: $\langle x \rangle_{q/g} = \int dx\, x\, F_{q/g}(x)$.

Connected insertion: u, d.
Disconnected insertion: u, d, s, g

ETMC: [Vaquero,Thu,17:30] $N_f = 2$ twisted mass with clover term,

$m_\pi = 131$ MeV, $Lm_\pi = 3$,
$a = 0.093$ fm.
Stout smearing to reduce noise.
Approx: 2000 (cfgs) × 100 (sources)

Renormalisation: mixing between $\sum_q \langle x \rangle_q$ and $\langle x \rangle_g$: 1-loop to $\overline{\text{MS}}$ at 2 GeV.

$\langle x \rangle_g^{bare} = 0.318(24) \rightarrow \langle x \rangle_g^{\overline{\text{MS}}} = 0.320(24)$, $\quad (\langle x \rangle_u + \langle x \rangle_d + \langle x \rangle_s)^{\overline{\text{MS}}} = 0.72(11)$

Momentum sum satisfied: $\sum_q \langle x \rangle_q + \langle x \rangle_g = 1.04(11)$

Consistent with χQCD quenched calculation [Deka,1312.4816].

Also computed: $g_A^{u,d}$, $g_T^{u,d}$, $g_S^{u,d}$.

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Sara Collins,
Lattice 2016