EFT meets GR

Or who is afraid of Mr. Feynman

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NRGR

The problem of motion

- The size scale (finite extension) $k \sim 1/r_s$
- The orbit scale (Internal problem) $k \sim 1/r$
- The radiation scale (External problem) $k \sim v/r$

EFT for extended non-spinning objects

I. Rothstein & W. Goldberger Phys Rev D73, 104029 (2006)
For a review see W. Goldberger, hep-ph/0701129 (2007)

Main Idea: Insertion of non minimal terms in the action to handle divergences and thus account for the finite size of the constituents (decoupling). Unknown parameters describing the internal structure are fixed by *matching* observables in the one-particle sector such as scattering amplitudes.

$$S_{pp} = -\sum_a \int d\tau_a \left(m_a + c_R^{(a)} R(x_a) + c_V^{(a)} R_{\mu\nu} v_a^\mu v_a^\nu + \ldots \right)$$

Spin give us more degrees of freedom to play with.
However, for spinless particle the first contribution starts at v^10

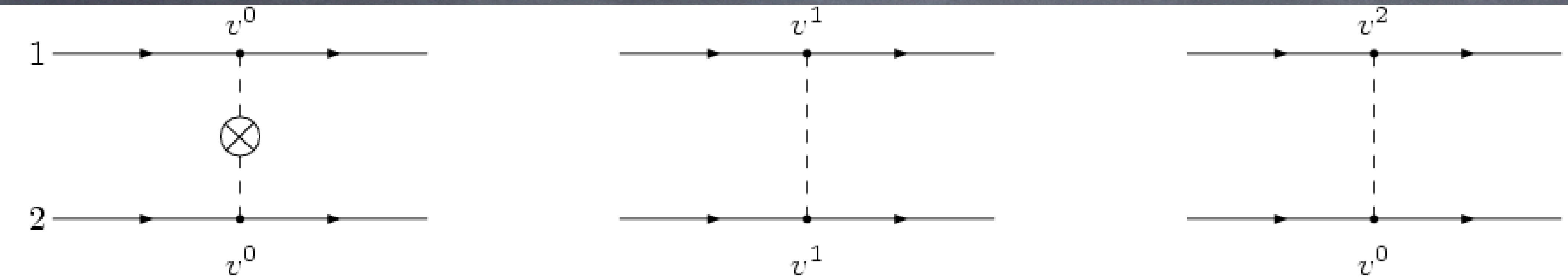
$$B_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\alpha\beta\sigma} C^{\alpha\beta}{}_{\nu\rho} u^\rho u^\sigma$$

$$E_{\mu\nu} = C_{\mu\beta\nu\rho} u^\beta u^\rho$$

$$\int d\tau B^{\mu\nu} B_{\mu\nu} \qquad \int d\tau E^{\mu\nu} E_{\mu\nu}$$

NLO (1PN) EIH action

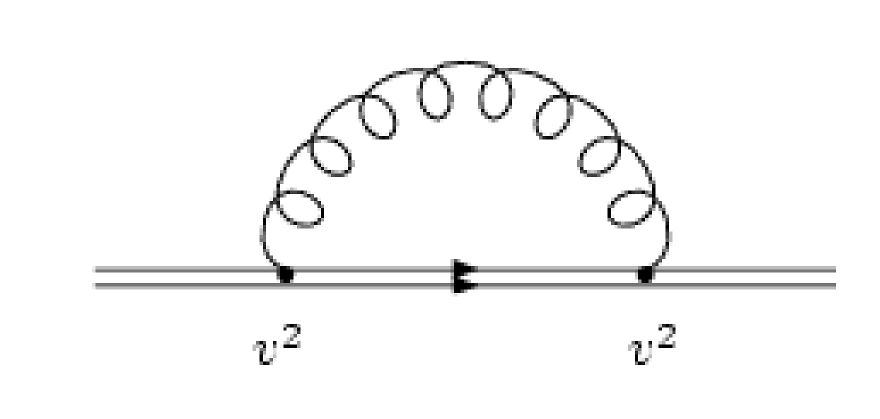
[For a toy model, see RAP & R. Sturani gr-qc/070106 (2007)]



$$G(p) \propto \frac{1}{p_0^2 - \vec{p}^2} \approx -\frac{1}{\vec{p}^2} - \frac{p_0^2}{\vec{p}^4} + \ldots$$

$$L_{EIH} = \frac{1}{8}\sum_a m_a \mathbf{v}_a^4 + \frac{G_N m_1 m_2}{2|\mathbf{x}_1 - \mathbf{x}_2|}\left[3(\mathbf{v}_1^2 + \mathbf{v}_2^2) - 7(\mathbf{v}_1 \cdot \mathbf{v}_2) - \frac{(\mathbf{v}_1 \cdot \mathbf{x}_{12})(\mathbf{v}_2 \cdot \mathbf{x}_{12})}{|\mathbf{x}_1 - \mathbf{x}_2|^2}\right] - \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_1 - \mathbf{x}_2|^2}$$

Radiation: Quadrupole Formula



$$Q_{ij} = \sum_a m_a \left(\mathbf{x}_{ai}\mathbf{x}_{aj} - \frac{1}{3}\delta_{ij}\mathbf{x}_a^2 \right)$$

$$\frac{1}{T} Im S_{eff}[x_a] = \frac{1}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}$$

$$Im S_{eff} = -\frac{1}{80 m_{Pl}^2}\int_{\mathbf{k}} \frac{1}{2|\mathbf{k}|}\mathbf{k}^4 |Q_{ij}(|\mathbf{k}|)|^2$$

$$\rightarrow \frac{dE}{dt} = -\frac{G_N}{5}\langle \frac{d^3 Q_{ij}}{dt^3}\frac{d^3 Q_{ij}}{dt^3}\rangle$$

Spinning Up

RAP, Phys Rev D73, 104031 (2006), gr-qc/0701106 (2007)

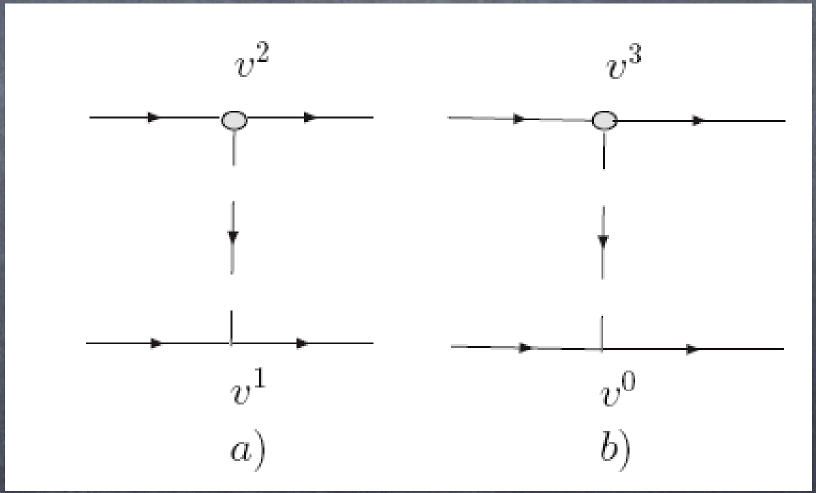
$$\mathcal{R} = -\sum_q \left(\int m_q \sqrt{u_q^2} d\lambda_q + \int \frac{1}{2} S_{Lq}^{ab} \omega_{ab\mu} u_q^\mu - \frac{1}{2m_q} R_{deab}(x_q) S_{Lq}^{cd} S_{Lq}^{ab} u_q^e u_c^q \, d\lambda_q \right)$$

$$\frac{\delta \mathcal{R}}{\delta x^\mu} = 0, \quad \frac{dS_L^{ab}}{d\tau} = \{S_L^{ab}, \mathcal{R}\}$$

Papapetrou eqs in the covariant SSC

To obtain PN corrections one calculates R perturbatively.

Example: Spin-Orbit EOM



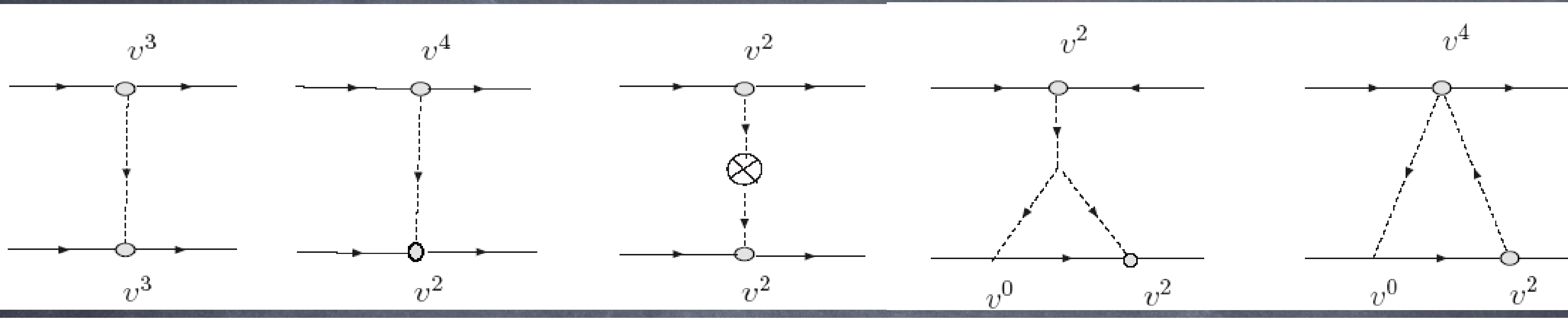
$$V_{1.5PN}^{so} = \frac{G_N m_2}{r^2} n^j \left(S_1^{j0} + S_1^{jk}(v_1^k - 2v_2^k) \right) + 1 \leftrightarrow 2$$

New results at 3PN for spinning binaries

RAP, Phys Rev D73, 104031 (2006)
RAP & I. Rothstein, Phys Rev Lett97, 021101 (2006)
RAP, gr-qc/0701106 (2007)

one graviton exchange

Non-linear terms



$$
\begin{aligned}
V_{3PN}^{ss} = & -\frac{G_N}{2r^3}\left[\vec{S}_1\cdot\vec{S}_2\left(\frac{3}{2}\vec{v}_1\cdot\vec{v}_2 - 3\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n} - \left(\vec{v}_1^2+\vec{v}_2^2\right)\right) - \vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{v}_2 - \frac{3}{2}\vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_1 + \vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_2\right.\\
& + \vec{S}_2\cdot\vec{v}_1\vec{S}_1\cdot\vec{v}_1 + 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\left(\vec{v}_1\cdot\vec{v}_2 + 5\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}\right) - 3\vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{n}\vec{v}_2\cdot\vec{n} - 3\vec{S}_2\cdot\vec{v}_2\vec{S}_1\cdot\vec{n}\vec{v}_1\cdot\vec{n}\\
& + 3(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n} + 3(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_1\times\vec{S}_2)\cdot\vec{n} - \frac{3}{2}(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n}\\
& \left. - 6(\vec{v}_1\times\vec{S}_2)\cdot\vec{n}(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}\right] + \frac{3G_N^2(m_1+m_2)}{r^4}\left(\vec{S}_1\cdot\vec{S}_2 - 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\right),
\end{aligned}
$$

Includes first non-linear corrections to the spin-spin interaction in the NW SSC

The spin(1)spin(2) Hamiltonian up to 3PN

RAP & I. Rothstein, Phys Rev Lett97, 021101 (2006)
RAP & I. Rothstein, arXiv:0712.2032 (2007)

Steinhoff et al. recently computed the next to leading order spin(1)-spin(2) Hamiltonian in arXiv:0712.1716, and claimed our previous result 'cannot be regarded as correct'

However, for comparison one must include subleading spin-orbit effects which contribute to the spin(1)spin(2) Hamiltonian once written in terms of the PN frame. These are of the type spin(1)spin(2)-orbit in the NW SSC.

$$S_1^{i0} = \frac{1}{2} S_1^{ij} v_1^j + \frac{1}{2} S_1^{ij} e_0^j(\vec{x}_1) + \ldots = \frac{1}{2} (\vec{v}_1 \times \vec{S}_1)^i + \frac{G_N}{2r^2} \left((\vec{n} \times \vec{S}_2) \times \vec{S}_1 \right)^i + \ldots$$

Adds a term in the spin(1)spin(2) potential

$$\frac{G_N^2}{2r^2} \left(m_2 n^i S_1^{ij} e_0^j(\vec{x}_1) - m_1 n^i S_2^{ij} e_0^j(\vec{x}_2) \right) = \frac{G_N^2 M}{2r^4} \left((\vec{S}_1 \times \vec{n}) \cdot (\vec{n} \times \vec{S}_2) \right)$$

And one gets up to 3pn

$$
\begin{aligned}
V^{s1s2} = & -\frac{G_N}{2r^3}\left[\vec{S}_1\cdot\vec{S}_2\left(\frac{3}{2}\vec{v}_1\cdot\vec{v}_2 - 3\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n} - \left(\vec{v}_1^2+\vec{v}_2^2\right)\right) - \vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{v}_2 - \frac{3}{2}\vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_1 + \vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_2\right.\\
& + \vec{S}_2\cdot\vec{v}_1\vec{S}_1\cdot\vec{v}_1 + 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\left(\vec{v}_1\cdot\vec{v}_2 + 5\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}\right) - 3\vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{n}\vec{v}_2\cdot\vec{n} - 3\vec{S}_2\cdot\vec{v}_2\vec{S}_1\cdot\vec{n}\vec{v}_1\cdot\vec{n}\\
& + 3(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n} + 3(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_1\times\vec{S}_2)\cdot\vec{n} - \frac{3}{2}(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n}\\
& \left. - 6(\vec{v}_1\times\vec{S}_2)\cdot\vec{n}(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}\right] + \frac{G_N^2 M}{2r^4}\left(5\vec{S}_1\cdot\vec{S}_2 - 17\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\right) - \frac{G_N}{r^3}\left(\vec{S}_1\cdot\vec{S}_2 - 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\right),
\end{aligned}
$$

And the Hamiltonian of Steinhoff et al. is equivalent to ours after a canonical transformation, contrary to their original claim.

The canonical transformation is generated by:

$$
g = \frac{G_N}{2r^2}\left[\frac{1}{m_2}\vec{S}_1\cdot\vec{p}_2\vec{S}_2\cdot\vec{n} - \frac{1}{m_1}\vec{S}_2\cdot\vec{p}_1\vec{S}_1\cdot\vec{n}\right] + \frac{G_N}{2r^2}\left[\frac{1}{m_1}\vec{S}_1\cdot\vec{S}_2\vec{p}_1\cdot\vec{n} - \frac{1}{m_2}\vec{S}_2\cdot\vec{S}_1\vec{p}_2\cdot\vec{n}\right]
$$

Divergences with spin insertion & finite size effects

Spin insertions they start out formally at 3PN, and 5PN for maximally rotating bodies through higher dimensional operators such as

$$D^2 R_{\mu\nu\alpha\beta} S^{\mu\rho} S^{\alpha}_{\rho} u^{\nu} u^{\beta}$$

$$c_{D^2}(\partial^4 H_{\mathbf{k}} d^3\mathbf{k}) S^2 d\tau \sim \sqrt{L} v^{6+2s}$$

$$\mu \frac{dC_D^2}{d\mu} \sim \frac{m}{m_{Pl}^4}$$

Self induced effects do not get renormalized and show up at leading order

$$R_{\mu\nu\alpha\beta} S^{\mu\sigma} S^{\alpha}_{\sigma} u^{\nu} u^{\beta} \rightarrow V_{S^2O} = C_{RS^2} \frac{1}{r^3}\left(3(\vec{S}\cdot\vec{S})^2 - \vec{S}\cdot\vec{S}\right)$$

$$C_{RS^2} \sim \frac{m}{r_s^2 m_{Pl}^4} \sim \frac{1}{m}$$

Absorption

I. Rothstein & W. Goldberger PRD73 104030 (2006)
RAP, arxiv:0710.5150 (2007)

New degrees of freedom (Q's) in the worldline
(stretched horizon?)

$$S_{int} = -\int d\tau Q^E_{ab} E^{ab} - \int d\tau Q^B_{ab} B^{ab} + \cdots,$$

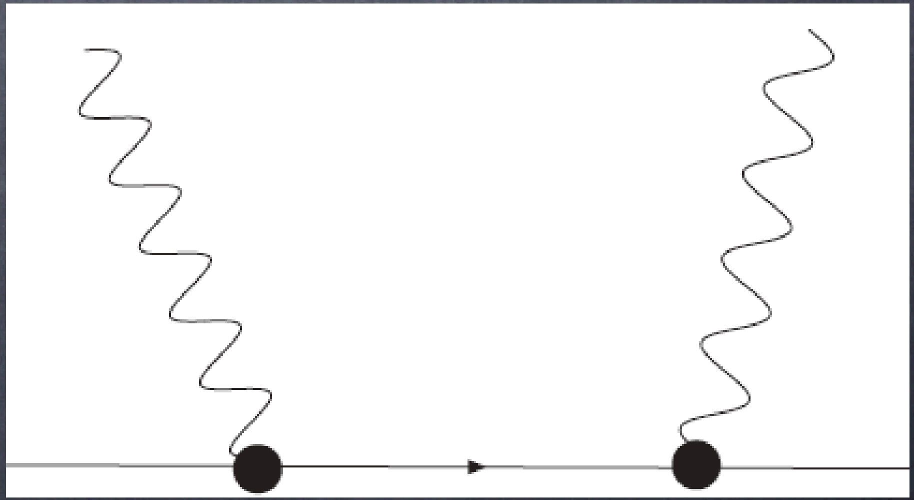
$$\int dx^0 e^{-i\omega x^0} \langle T Q^{E(B)}_{ij}(0) Q^{E(B)}_{kl}(x^0)\rangle_{spin} = -\frac{i}{2} S_{ijkl} F_s(\omega)$$

$$S_{ijkl} = [\delta_{ik}S_{jl} + \delta_{il}S_{jk} + \delta_{jl}S_{ik} + \delta_{jk}S_{il}]\,(1 + \alpha \mathbf{S}^2 + \ldots) + \ldots$$

Matching with graviton abs
cross section for Kerr BH

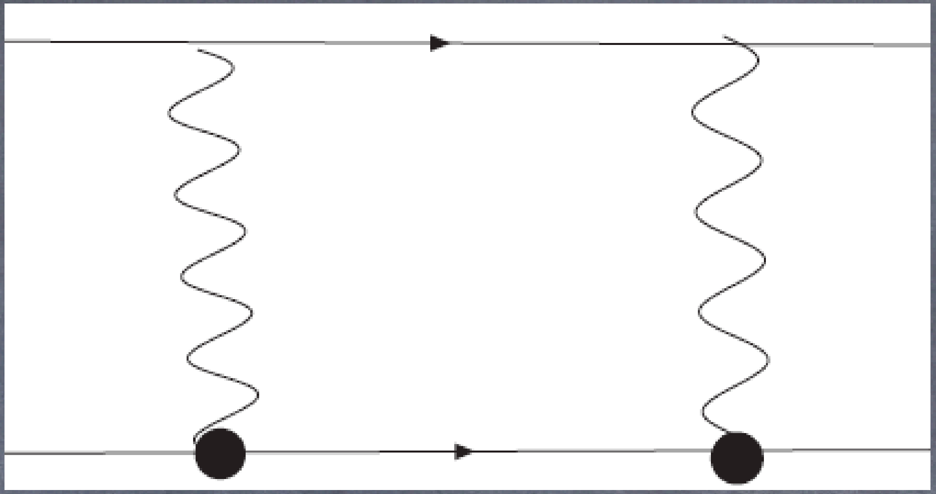
$$\sigma^{full}_{abs}(\omega) = \sigma^{EFT}_{abs}(\omega)$$

$$F_s(\omega) = \frac{4}{45} G_N^3 m^3$$



For the spinning binary

Enhanced for spin by 3 powers of v



$$P_{abs}^{spin} = -\epsilon \frac{8}{5} \frac{G_N^6 m_2^5 m_1^2}{r^7} (a_* + 3a_*^3)$$

Test particle in a spacetime background

$$P_{abs} = \frac{8}{45} a_* (1 + 3a_*^2) G_N^4 m^5 \left(\frac{d}{dt} E_{ij} E_{il} + \frac{d}{dt} B_{ij} B_{jl} \right) s_{jl}$$

Conclusions

- The problem of motion reduced to a tower of EFTs.
- World-line operators encoding finite size structure. Tidal effects start at (formally 3PN) 5PN for maximally rotating compact objects. Self induced effects show up already at 2PN.
- Systematic method to calculate to all orders in the PN expansion. Textbook renormalization. No ambiguities. Divergences absorbed into short distance parameters. Matching.
- Already new results at 3PN for spinning bodies, including finite size effects.
- Absorption -> new wordline degrees of freedom + matching.
- Self-force can be also tackled by EFT techniques. Applicable to other kinematical scenarios (large-small mass ratio).
- Work in progress = Templates

Other applications:

Caged BHs:

Yi-Zen Chu, W. Goldberger, I. Rothstein JHEP 0603:013 (2006)
B. Kol, M. Smolkin, arXiv0712.2822 (2007)

Non-Relativistic Gravitation

B. Kol, M. Smolkin, arXiv07124116 (2007)

Self-force

C. Galley, B. L. Hu, arXiv0801.900(2008)