# Anisotropic QCD plasmas

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# Anisotropic plasmas

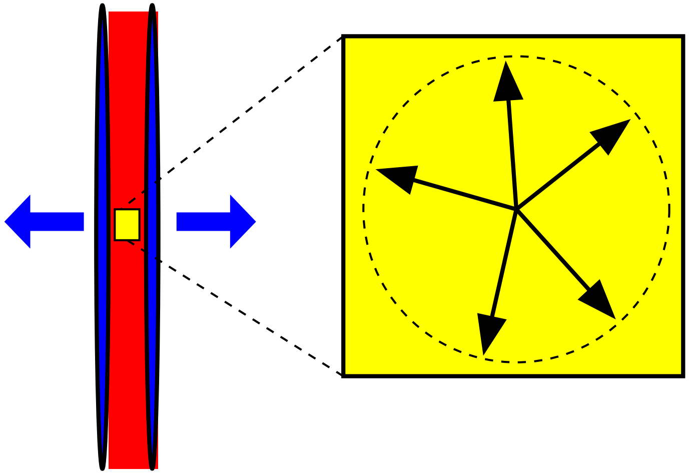
Two causes of anisotropy

1. Production in very narrow region, $\gamma \sim 1000$ for 1TeV/A

2. weak coupling

experimental signal $\Rightarrow$ wQCD

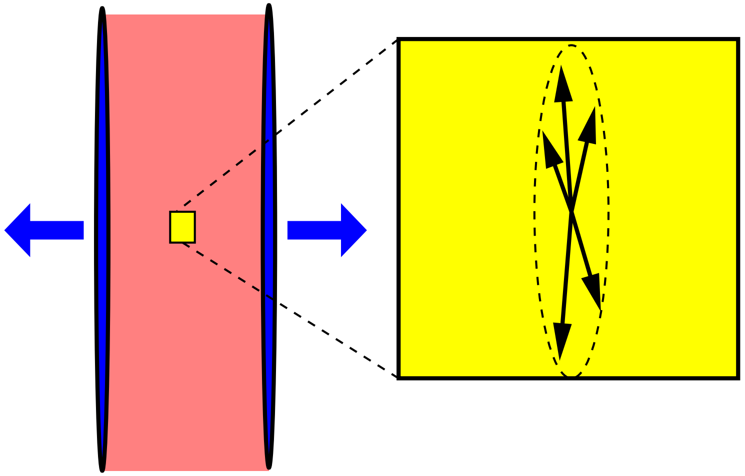
# Momentum anisotropy



$\tau \lesssim Q^{-1}$: production of "hard" gluons with

$$p \sim Q$$

(isotropic momentum distribution)



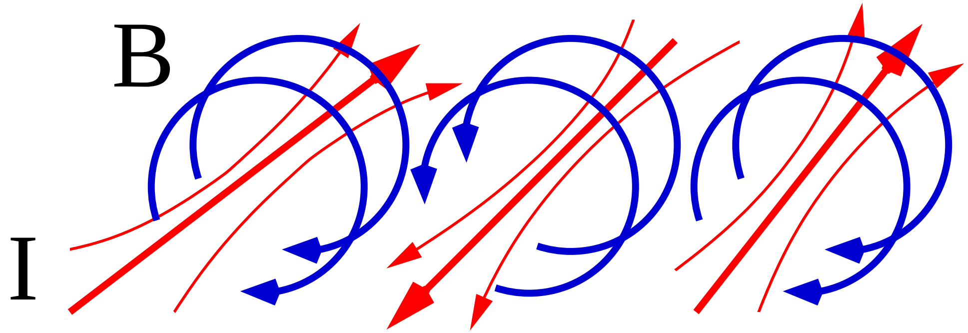
free expansion in $z$-direction $\longrightarrow$

$$p_z \ll p_\perp$$

while $p_z \sim p_\perp$ in equilibrium

# Plasma instabilities

anisotropic plasmas are unstable [Weibel,...], small fluctuations grow exponentially




instability for weak field initial conditions

probably no true instability

momenta mostly $\perp$ beam direction

they help thermalization through isotropization

# Which modes are unstable

characteristic momentum of anisotropic particles $Q$

unstable modes $$k^2 \sim m^2 \, , \qquad m^2 \sim N g^2 f Q^2$$

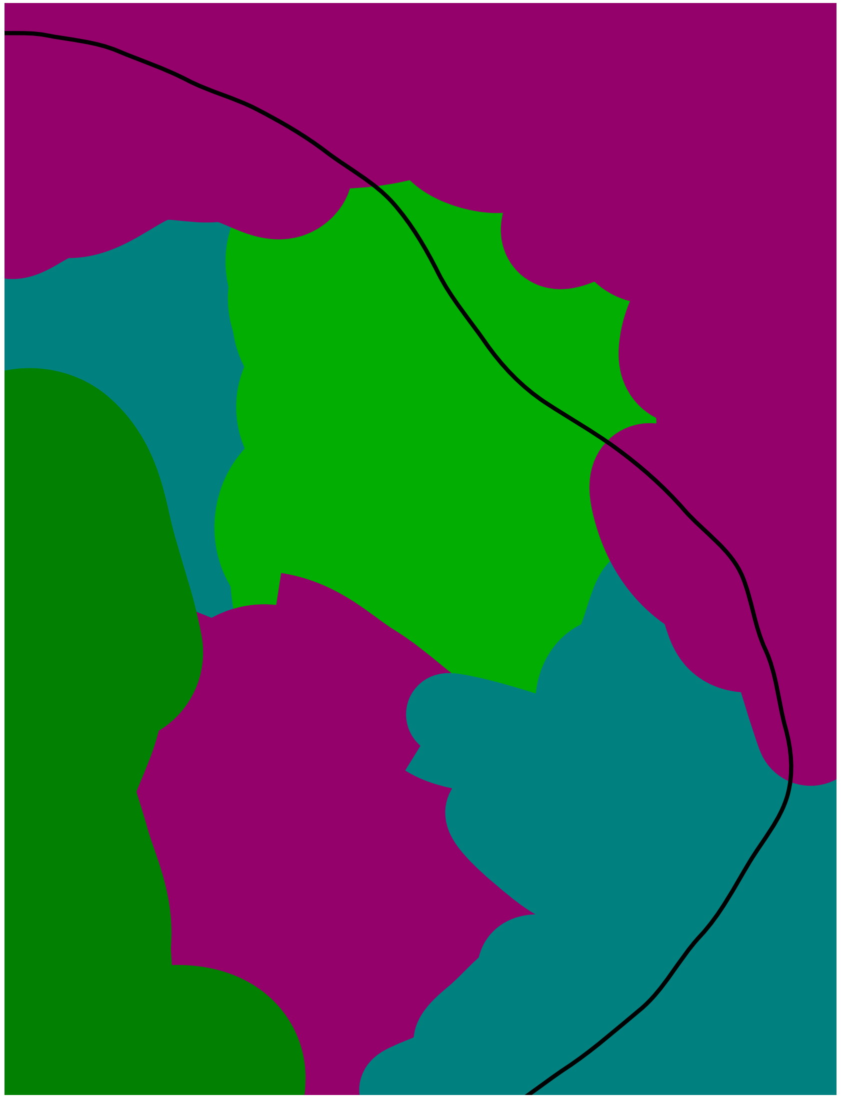
$$m \ll Q \qquad \text{for} \qquad f \ll \frac{1}{g^2 N}$$

unstable modes are soft

strong anisotropy:

$\exists$ unstable modes with $k \gg m$

# Effect of unstable modes on hard gluons



randomly oriented domains of long wavelength gluon fields

hard gluon momenta perform random walk

$\Rightarrow p_z$-broadening, isotropization

more efficient than elastic scattering

# What we would like to know

**how fast does the system isotropize?**

How large do the unstable modes grow?

1) until non-linearities kick in? $(gA \sim k)$
not sufficient for fast isotropization

2) until rapid isotropization?

# What we would like to know

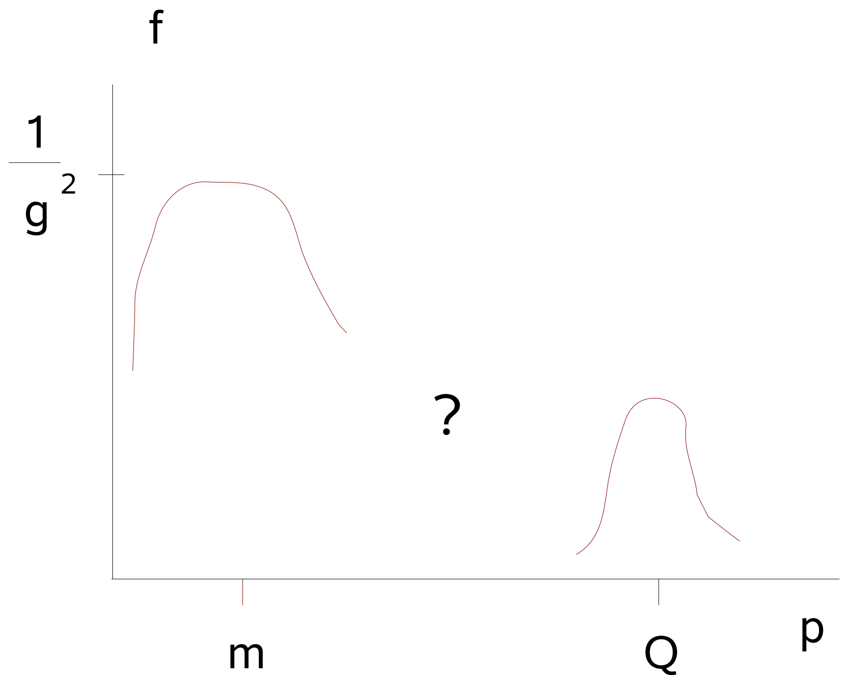
what is $f(\boldsymbol{p})$ between $m$ and $Q$?

cascading from $m$ to higher $p$

similar to Kolmogorov wave turbulence

lattice results indicate $f \propto 1/p^2$ [Arnold,Moore]

Boltzmann equation calculation gives $1/p$ [Mueller,Shoshi,Wong]




what happens for strong anisotropy?

# Numerical simulations of QCD plasma instabilities

classical field approximation for soft gluon fields

eikonal approximation for hard gluons, "hard loop approximation"

neglecting

- expansion
- back-reaction on hard gluon momentum distribution

# Simulation of strongly anisotropic plasmas

system with "hard" classical particles + soft classical fields

neglect bending of hard particle momenta "hard loop approximation"

$$(D_\mu F^{\mu\nu})^a = g \int \frac{d^3p}{(2\pi)^3} v^\nu f^a,$$

$$(v \cdot D f)^a + g v^\mu F^a_{\mu i} \frac{\partial \bar{f}}{\partial p^i} = 0$$

we use the $W$ field method, SU(2) gauge group

$$W^a(x, \boldsymbol{v}) \equiv 4\pi g \int_0^\infty \frac{dp p^2}{(2\pi)^3} f^a(x, \boldsymbol{v}p)$$

# Simulation of strongly anisotropic plasmas

we expand $W$, $\bar{f}$ in spherical harmonics

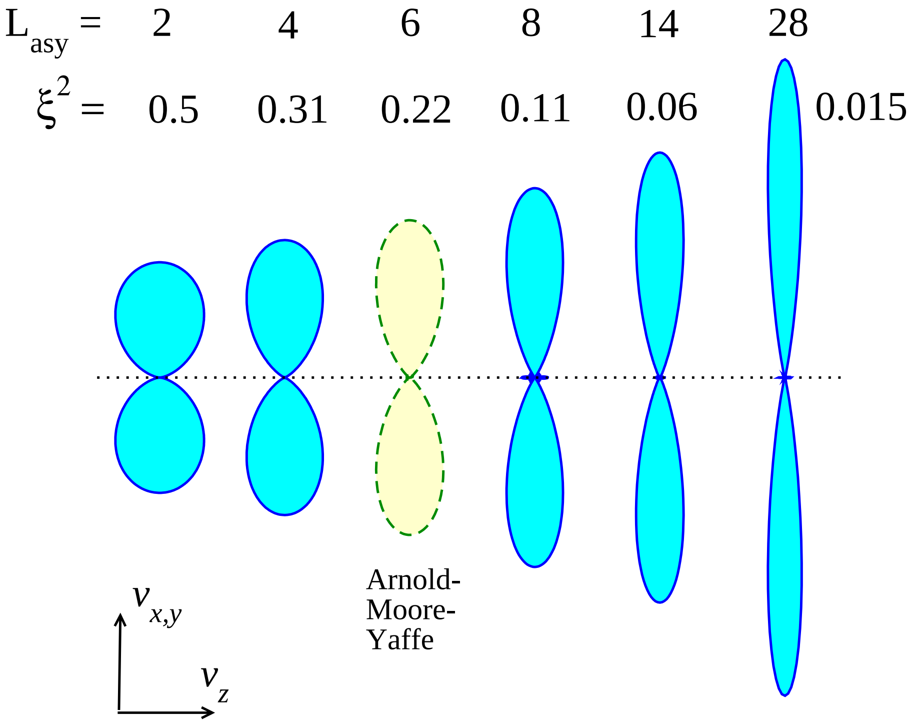
$$\bar{f}(p\boldsymbol{v}) = \sum_{l}^{L_{\text{asym}}} \bar{f}_l(p) Y_{l0}(\boldsymbol{v})$$

$$W(x,\boldsymbol{v}) = \sum_{l,m}^{L_{\max}} W_{lm}(x) Y_{lm}(\boldsymbol{v})$$

$3+1$ dim theory with many fields

# Simulation of strongly anisotropic plasmas

for each $L_{\mathrm{asym}}$ we try to maximally localize the distribution along the $xy$-plane




propellor shaped distribution

asymmetry parameter

$\xi^2 \equiv \frac{v_z^2}{\boldsymbol{v}^2}$

$\xi = 0.5, \ldots, 0.015$

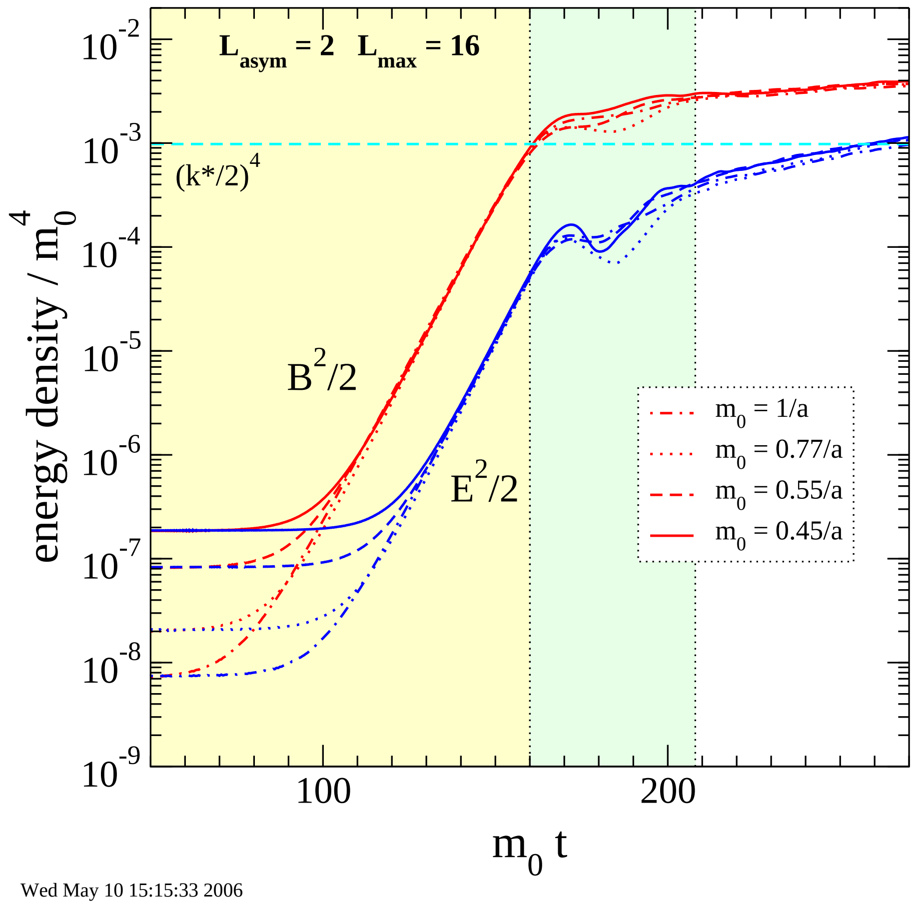
one needs $L_{\max} > L_{\mathrm{asym}}$

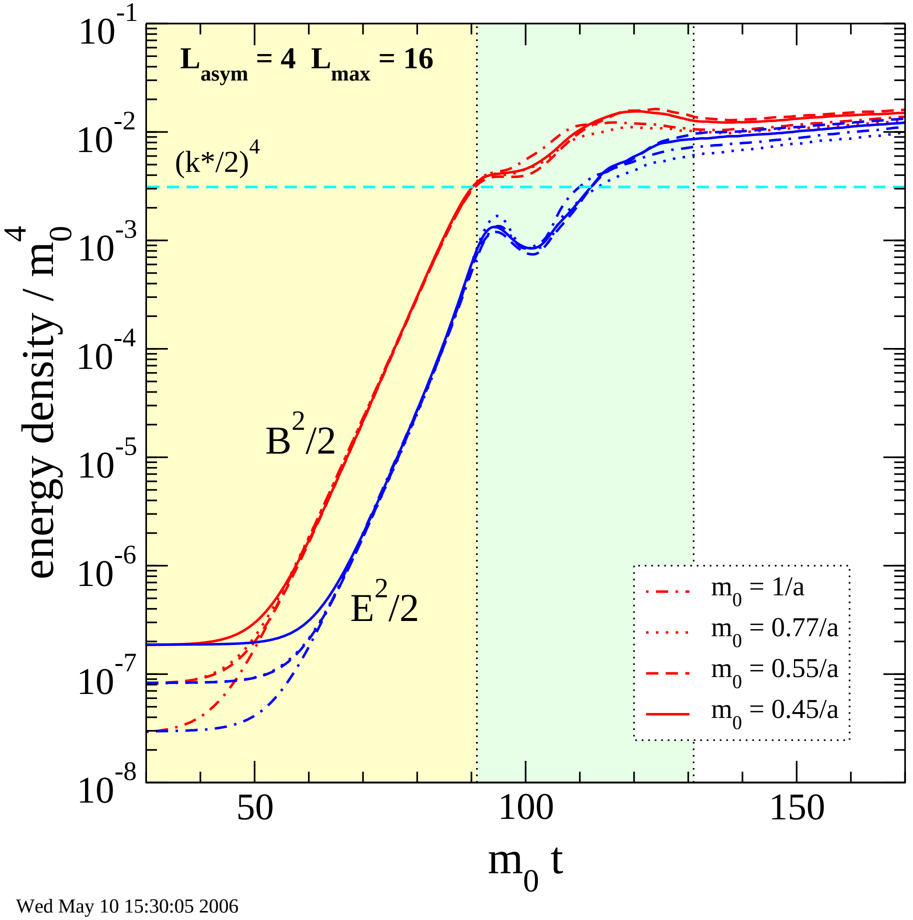
# Results: energy growth for weak anisotropy

little growth beyond weak field regime

lattice UV modes far from saturated

consistend with Arnold, Moore, Yaffe ($L_{\rm asym} = 6$)

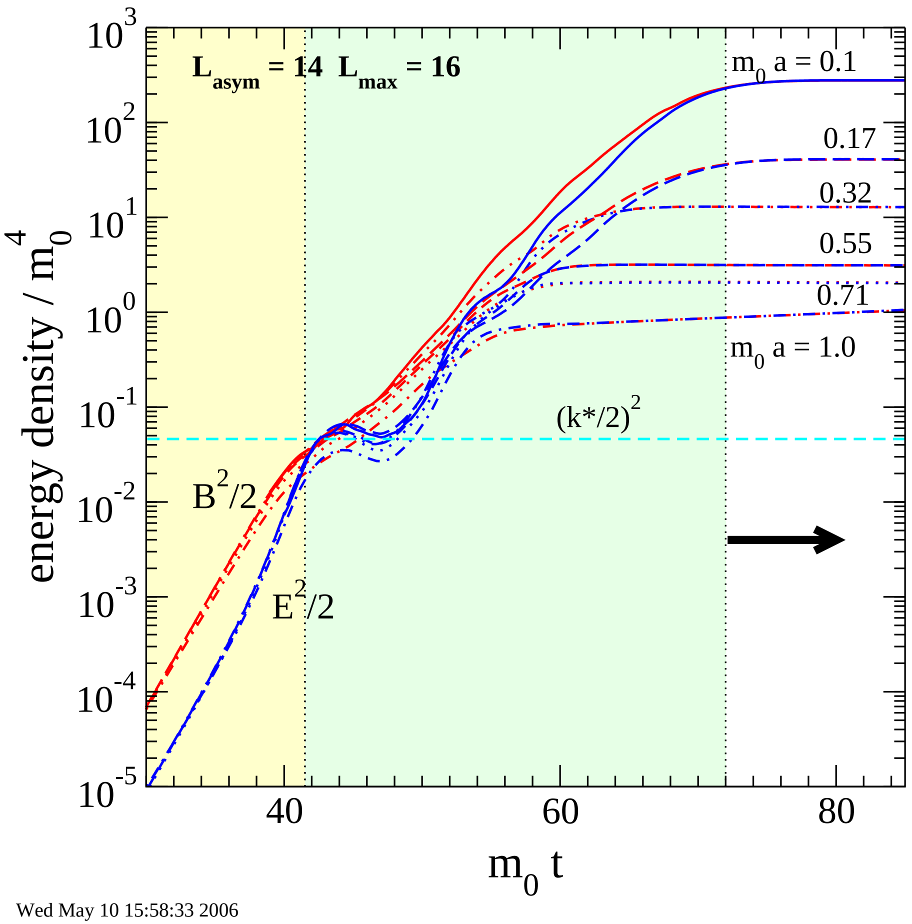


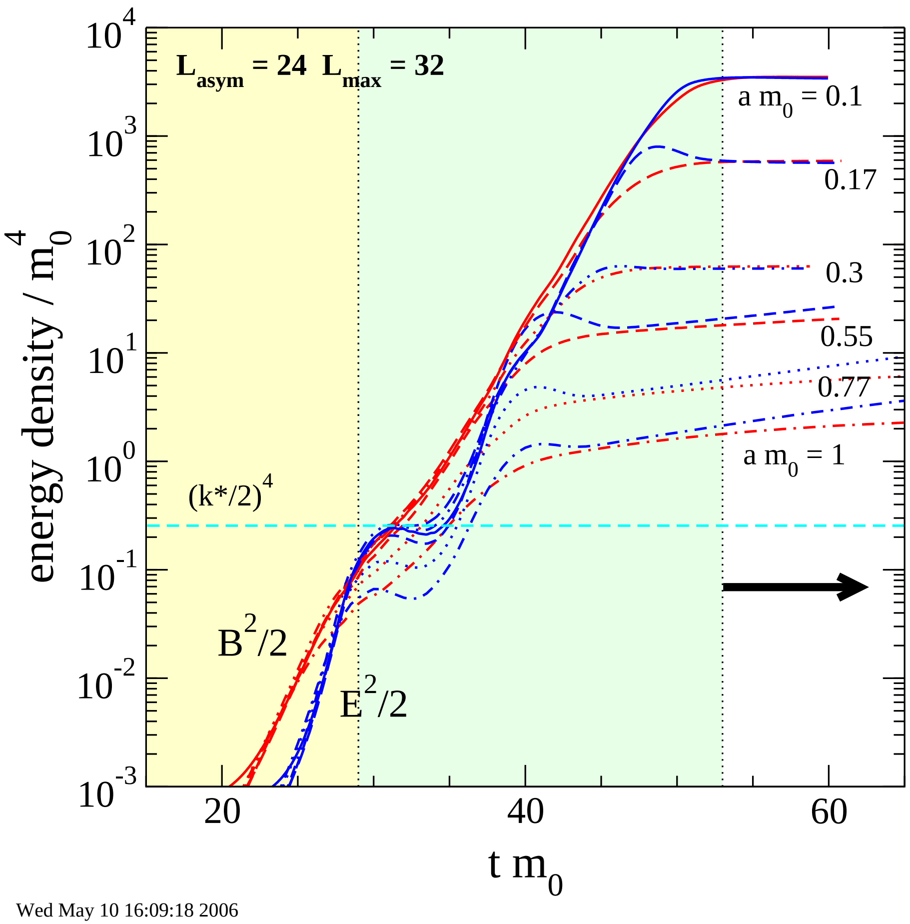



# Results: growth of energy for strong anisotropy

quasi-exponential growth beyond weak field regime

how far does it continue when $a \longrightarrow 0$?



# What is going on?

unstable modes for $|\boldsymbol{k}|$ up to

$$k_{\rm max} \sim \frac{m_0}{\xi} \gg m_0$$

large $\xi$ enhances energy density at non-abelian saturation

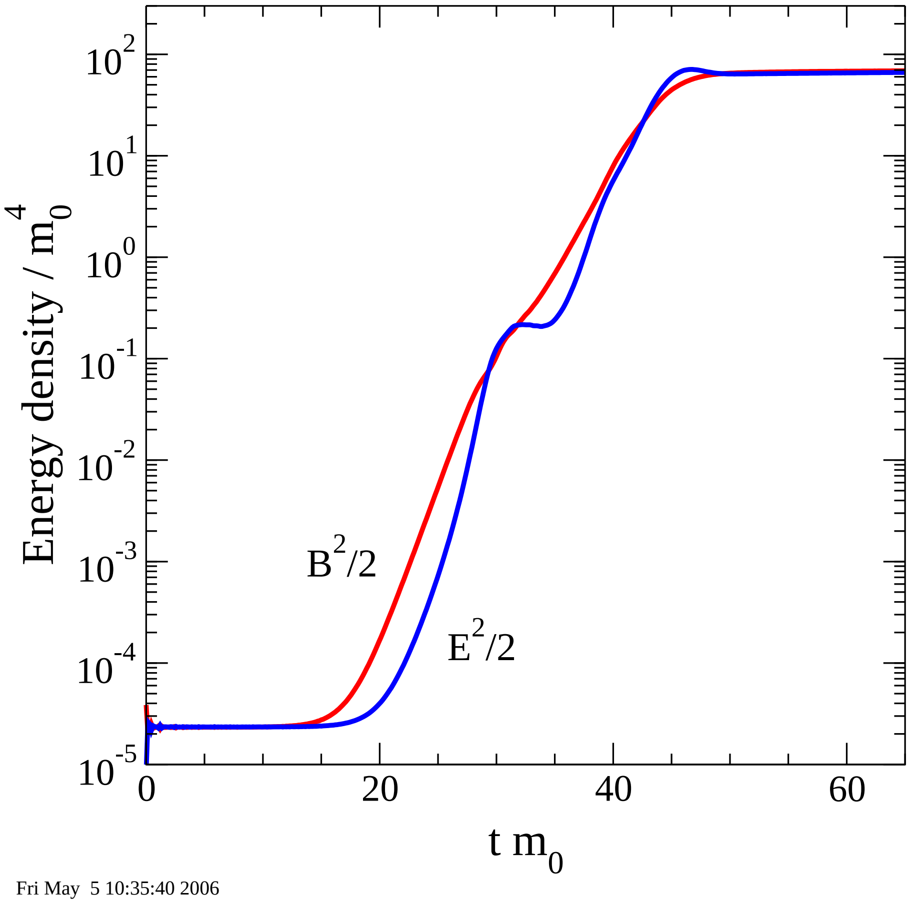
$$\varepsilon \sim \frac{k_\perp^2}{g^2} \int^{k_{\rm max}} dk^z k^z \sim \frac{m_0^4}{g^2 \xi^2}$$

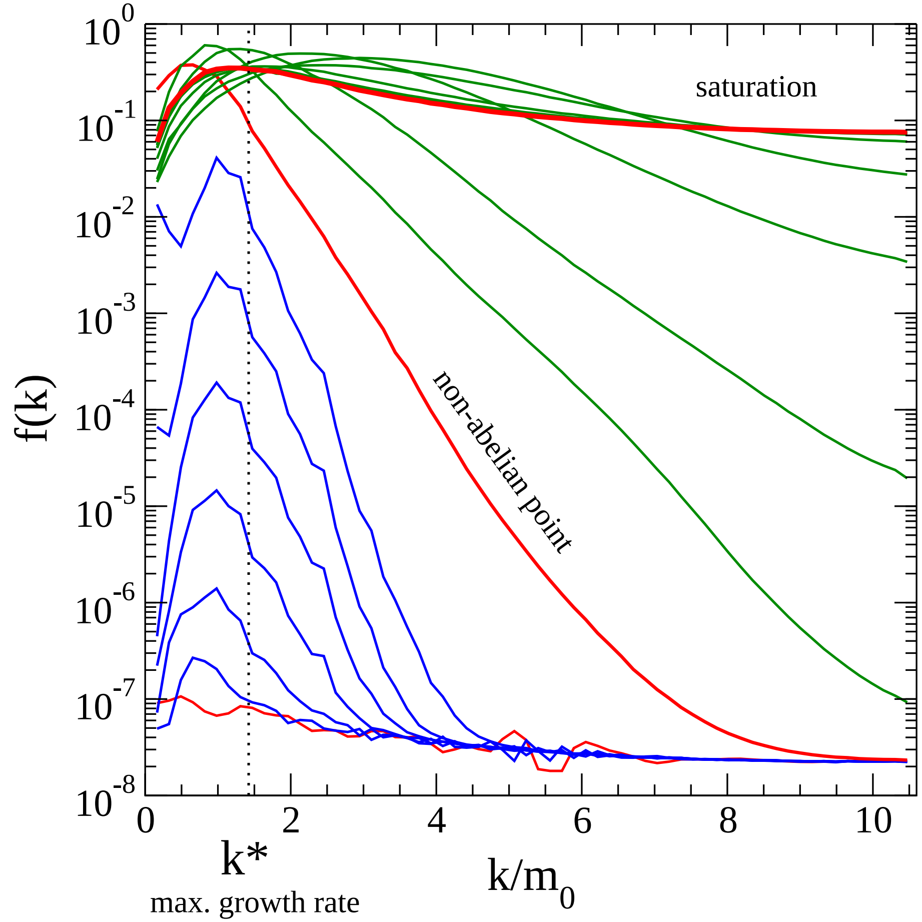
enhancement factor $= 67 \; (L_{\rm asym} = 28)$

cannot account for 3 orders of magnitudes

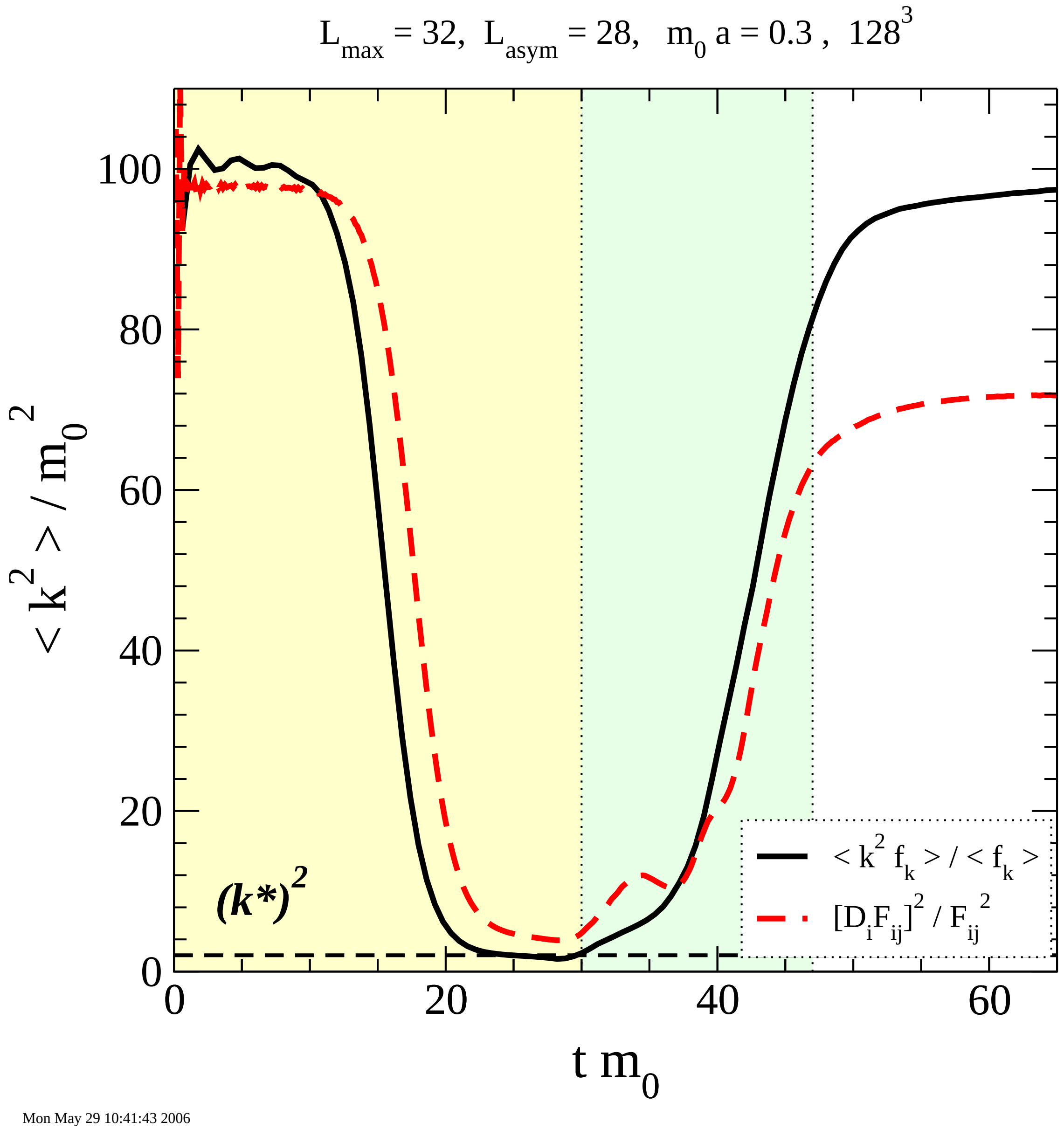
# Coulomb gauge spectrum

gauge fixed to Coulomb gauge, $f_{\text{soft}}(\boldsymbol{k}) \propto kA(\boldsymbol{k})$



# Average momentum

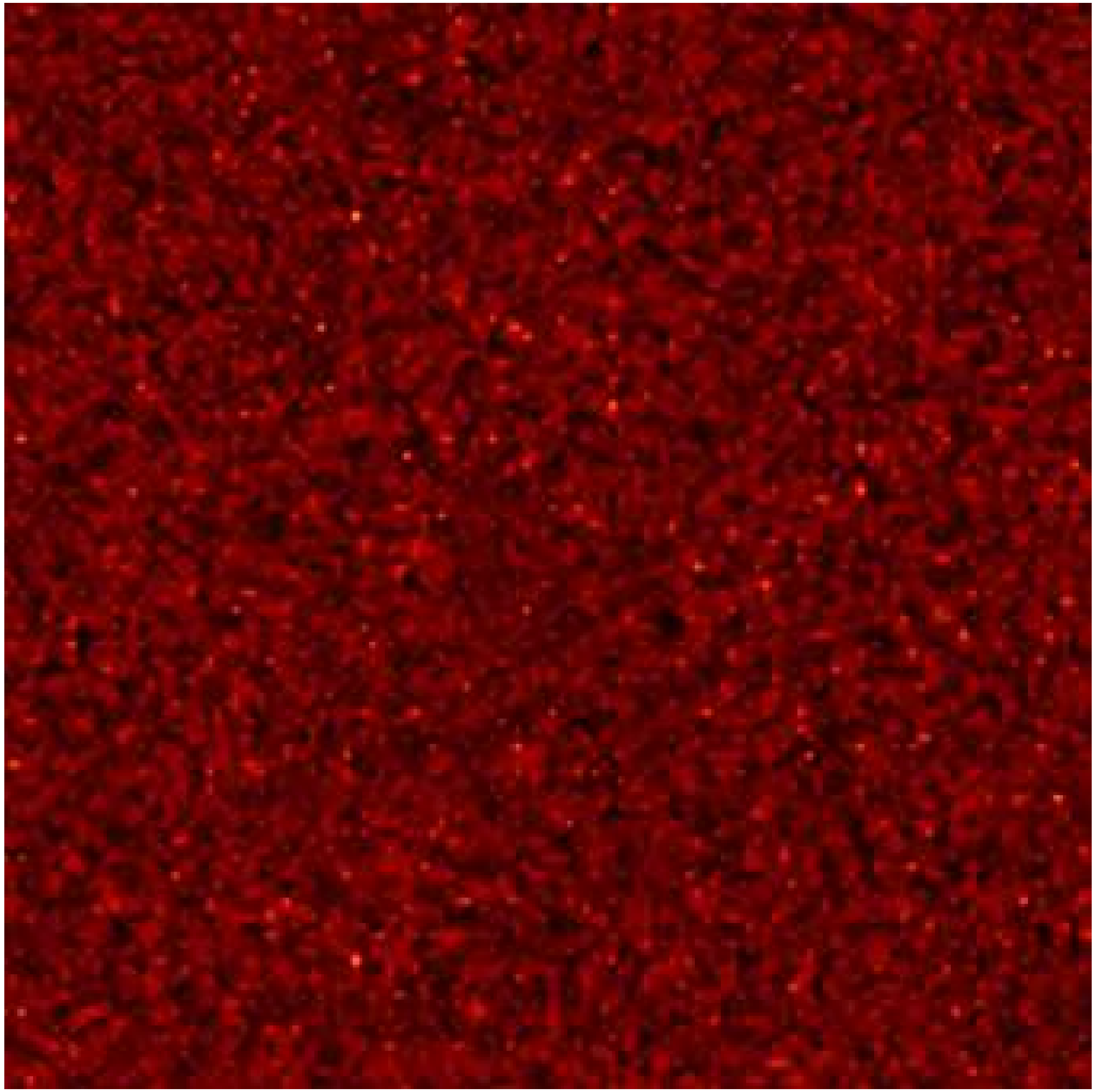



gauge invariant definition of average momentum

$$\langle \boldsymbol{k}^2 \rangle_{\mathrm{GI}} \equiv \frac{\int \mathrm{tr}(\boldsymbol{D} \times \boldsymbol{B})^2 d^3x}{\int \mathrm{tr}\boldsymbol{B}^2 d^3x}$$

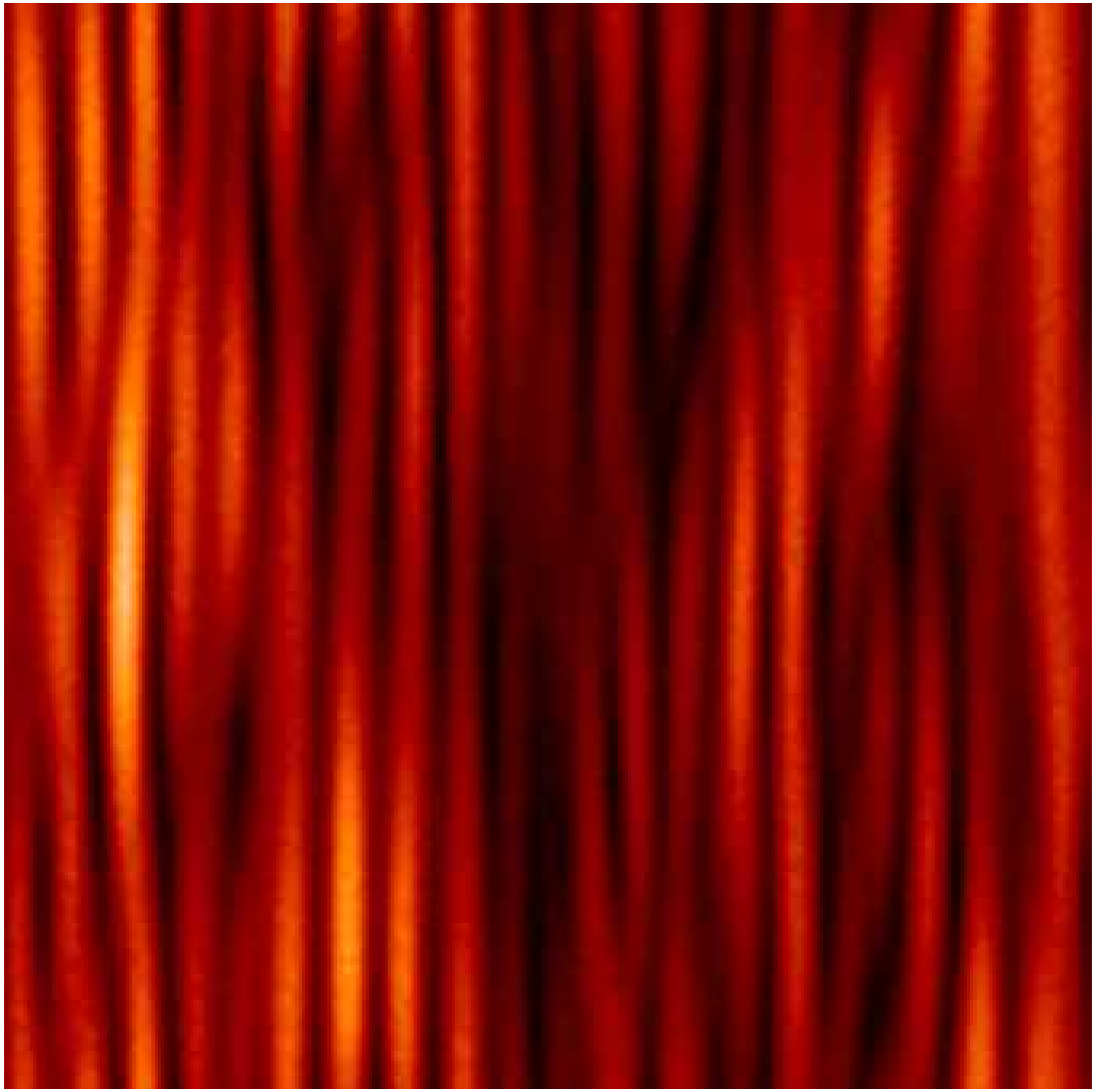
consistent with $\langle \boldsymbol{k}^2 \rangle$ from $f_{\mathrm{Coulomb}}$

# Energy density in position space



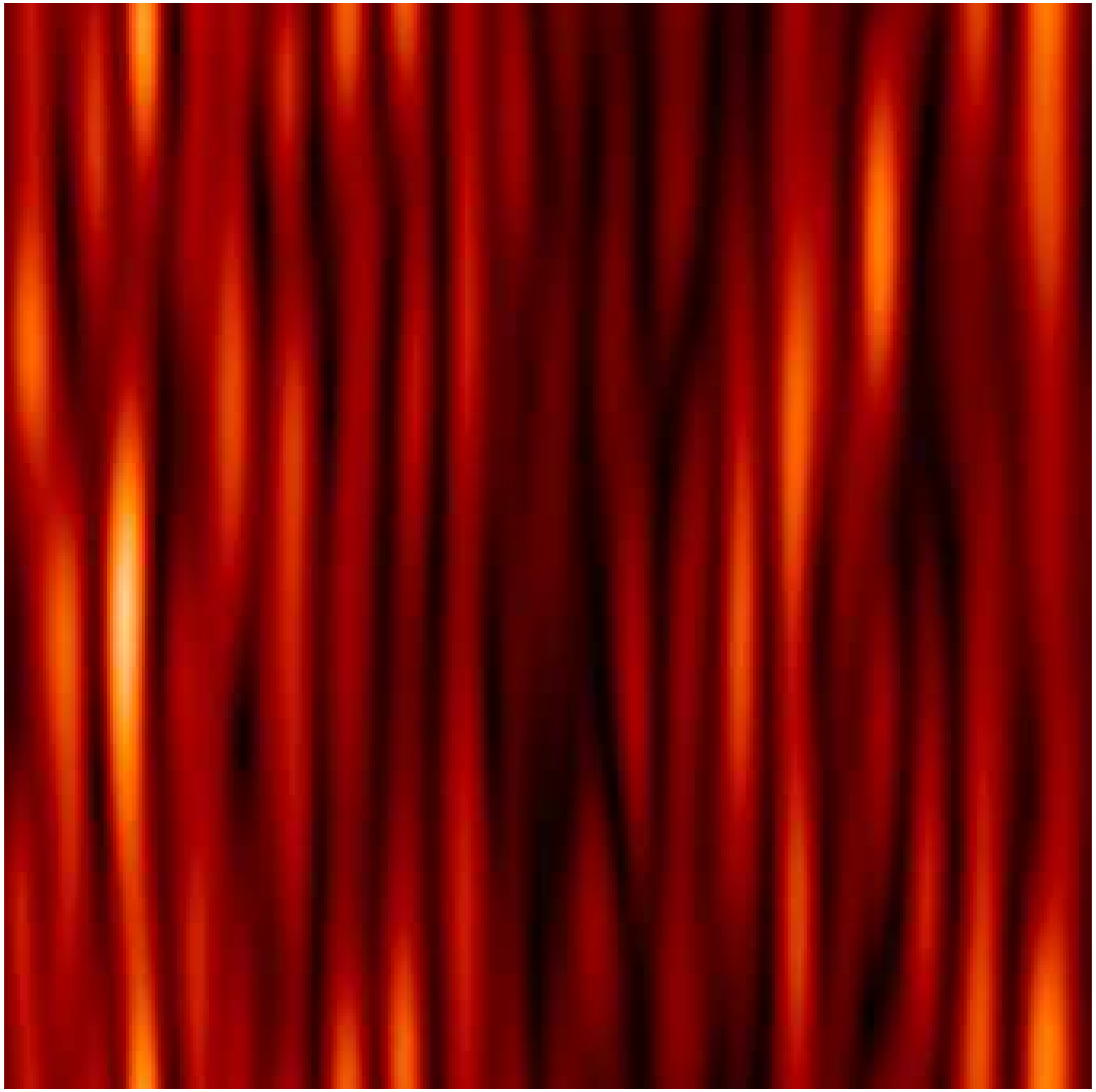
$t = 12/m_0$

# Energy density in position space



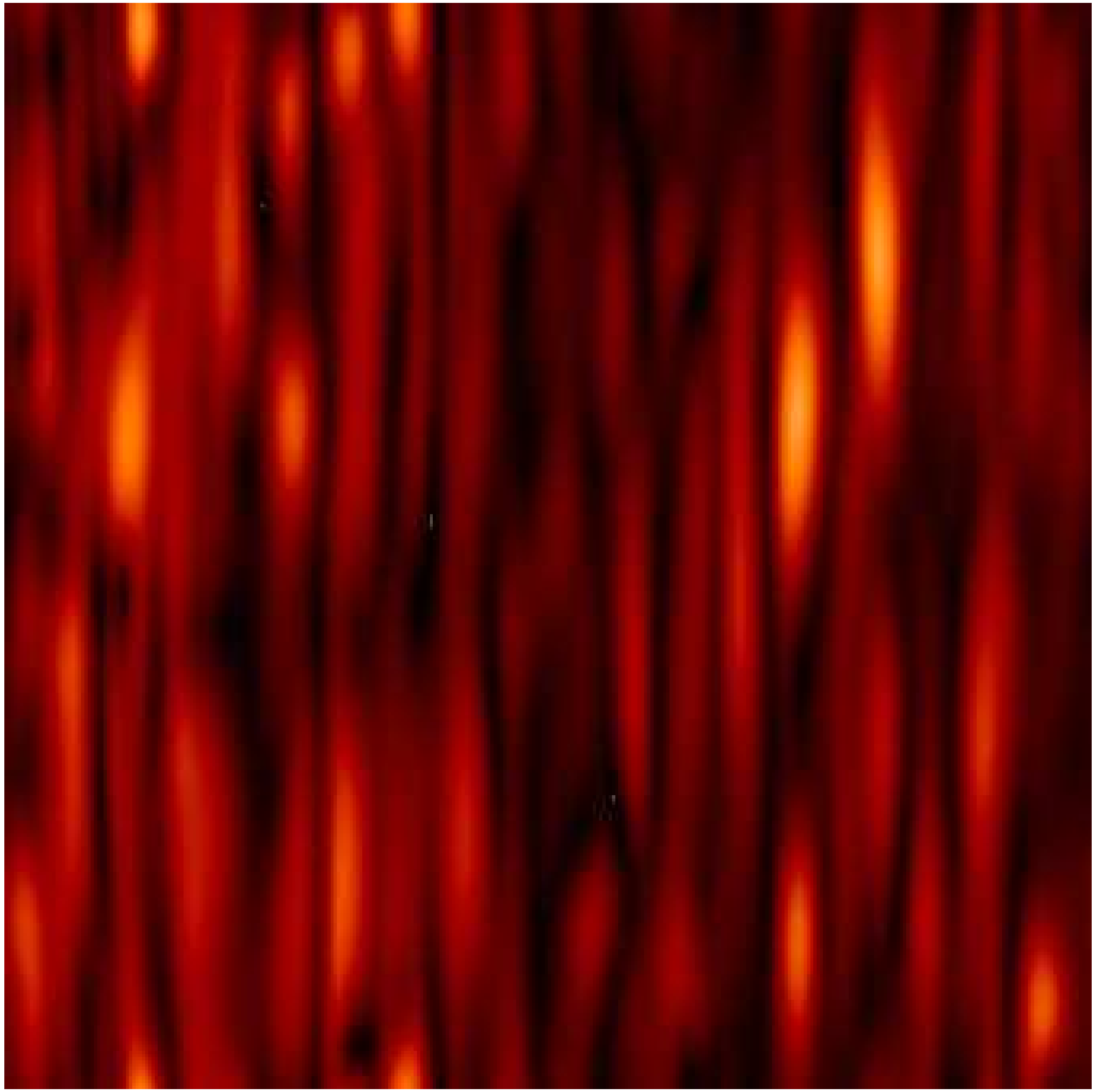
$t = 24/m_0$

# Energy density in position space



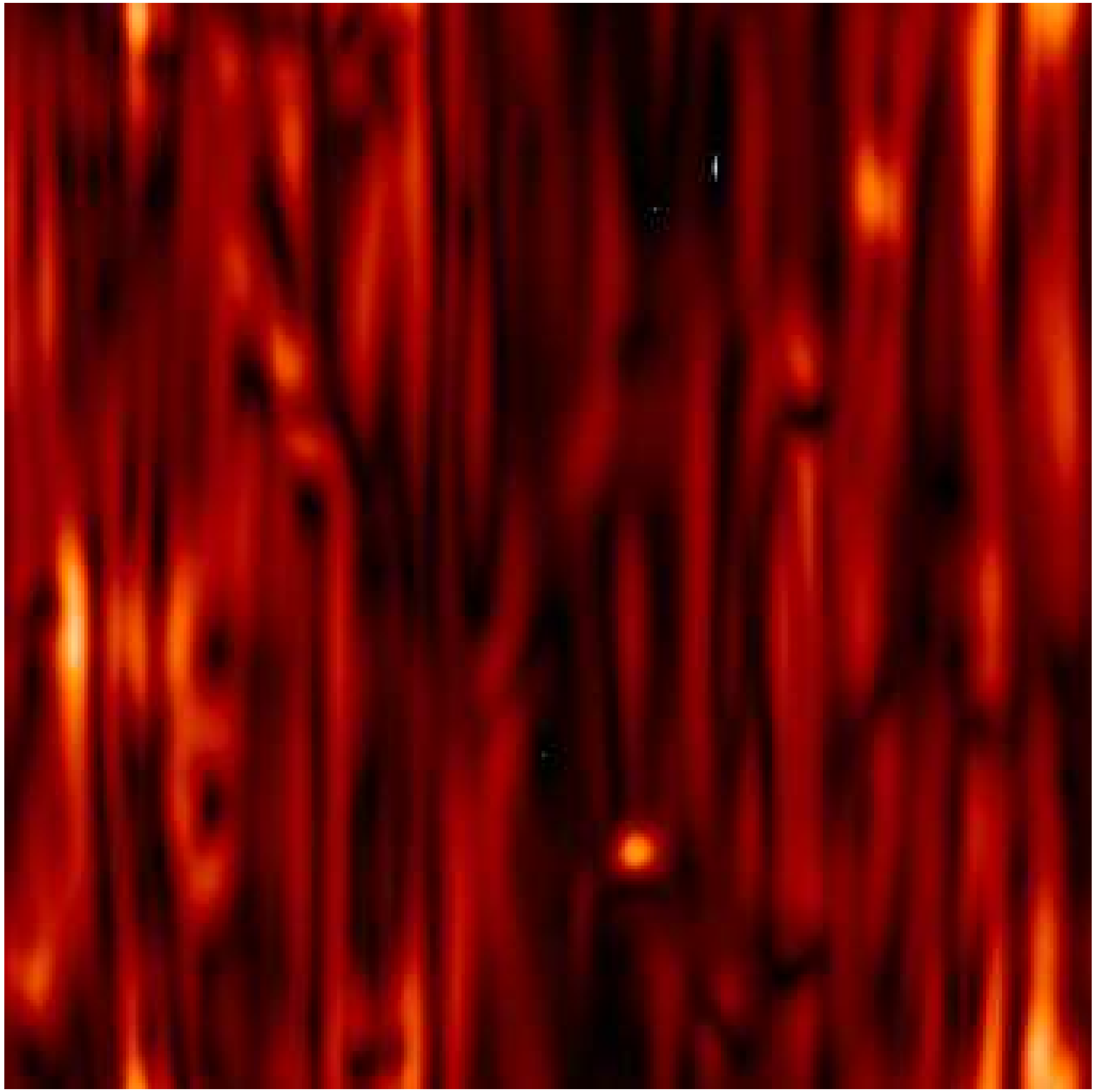
$t = 28/m_0$

# Energy density in position space



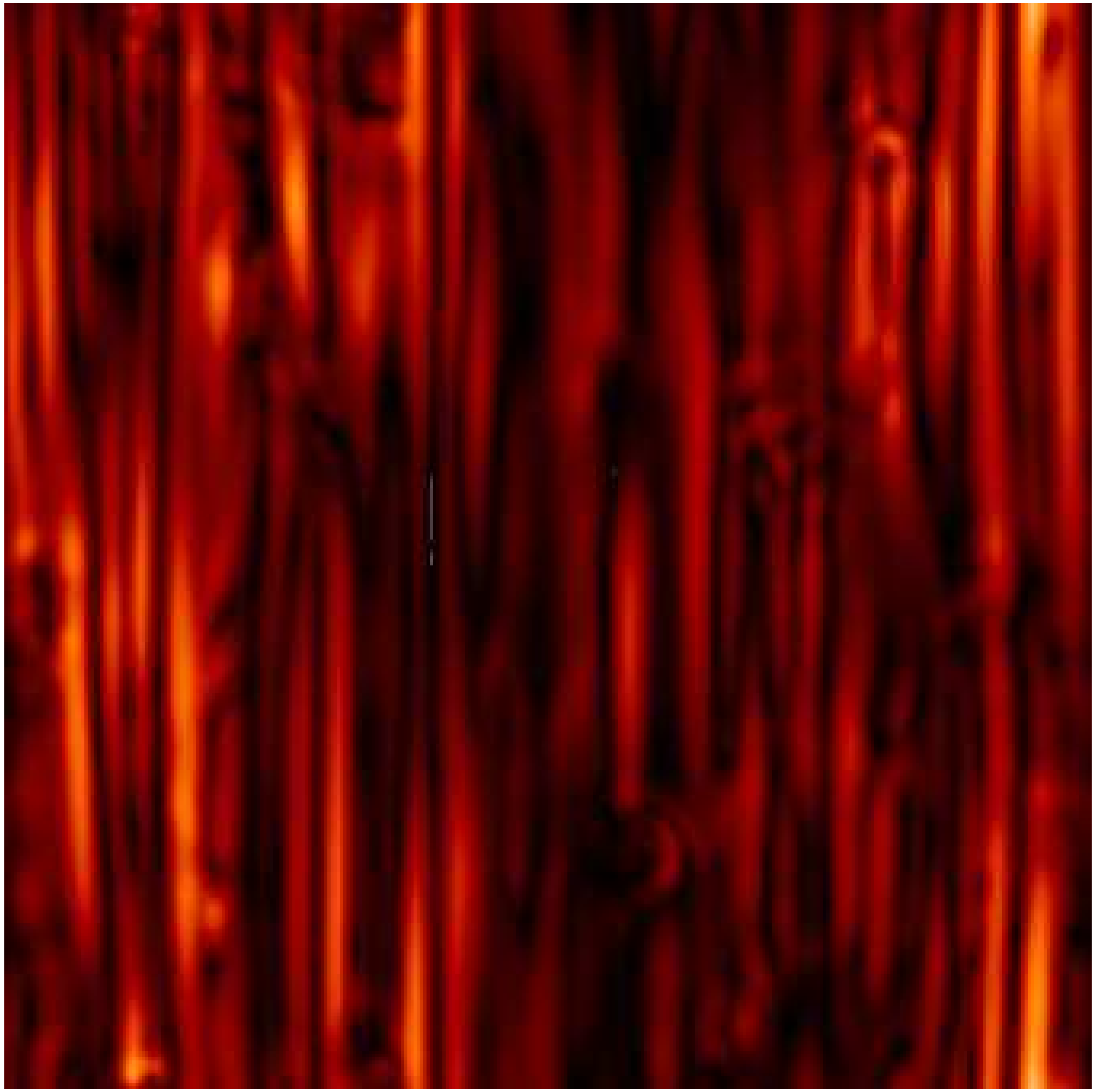
$t = 29/m_0$

# Energy density in position space



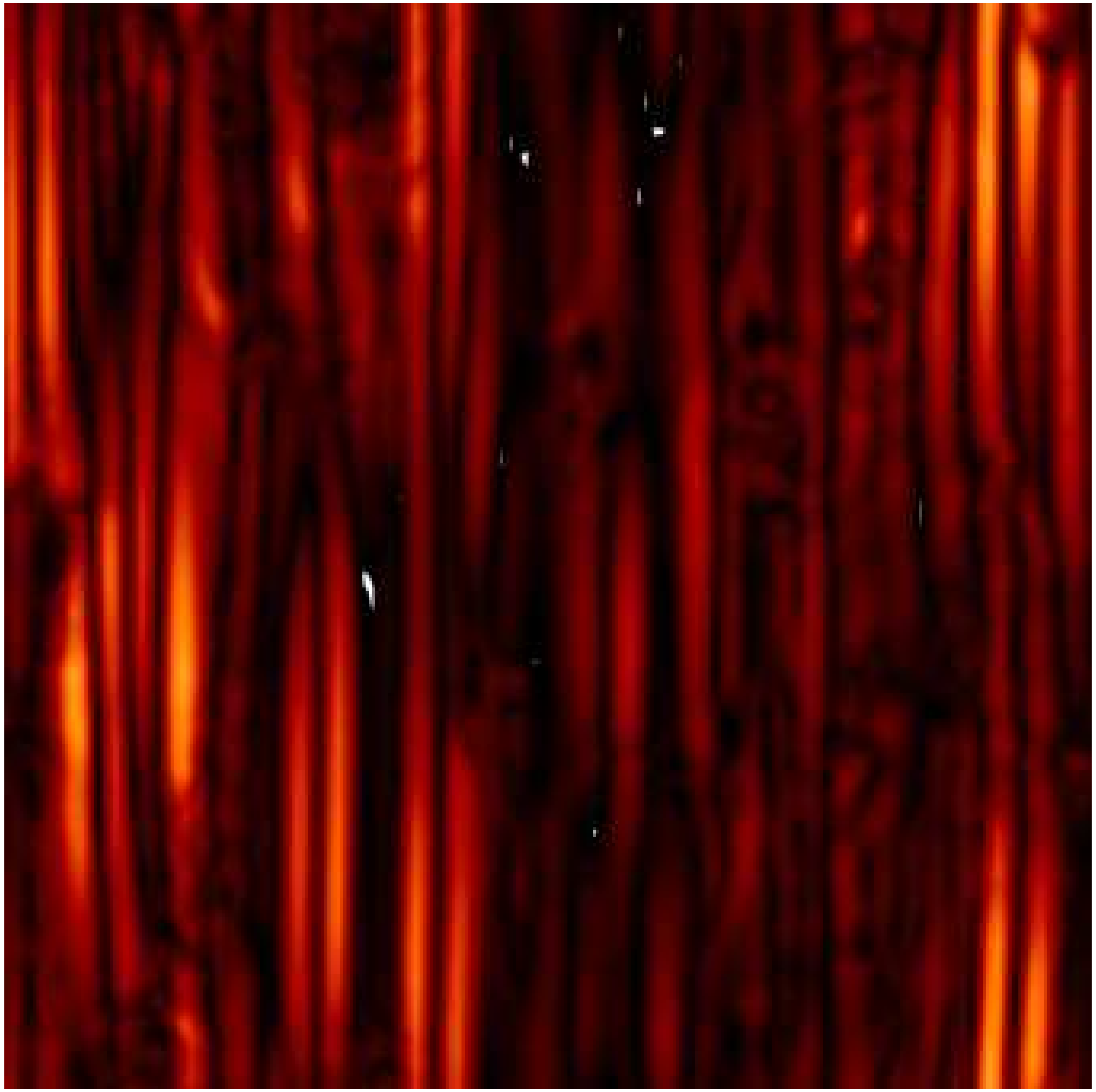
$t = 31/m_0$

# Energy density in position space



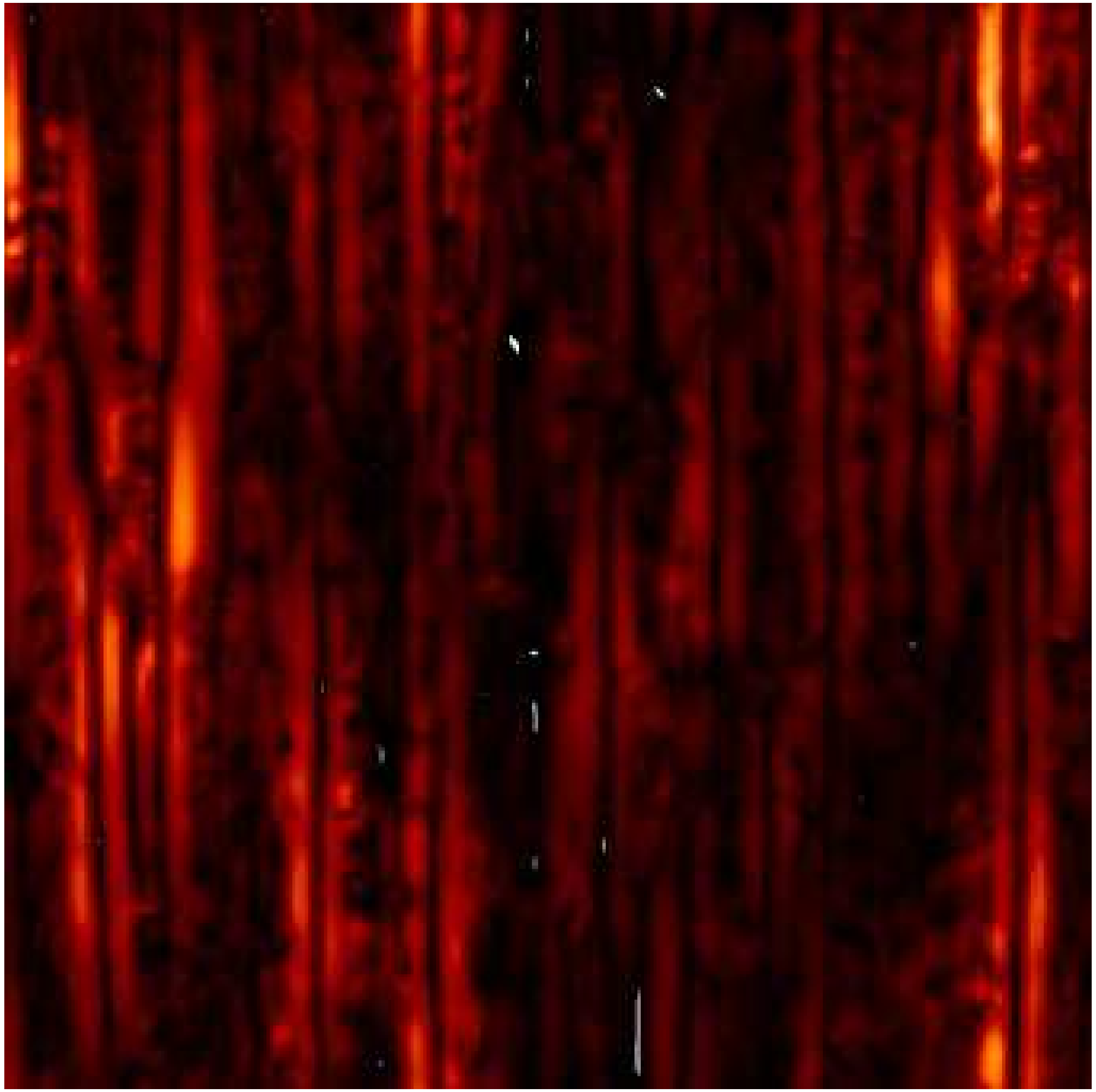
$t = 34/m_0$

# Energy density in position space



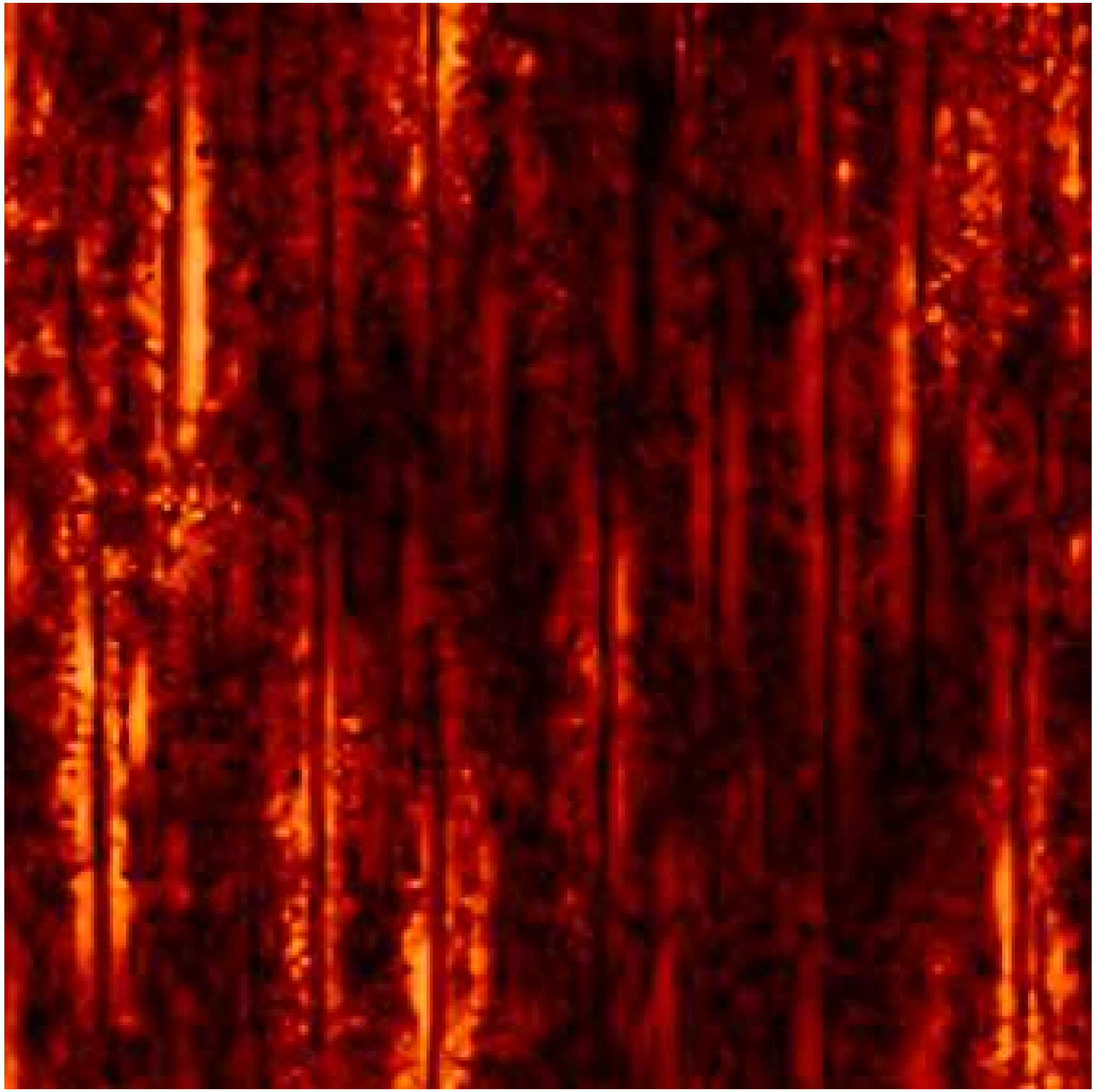
$t = 36/m_0$

# Energy density in position space



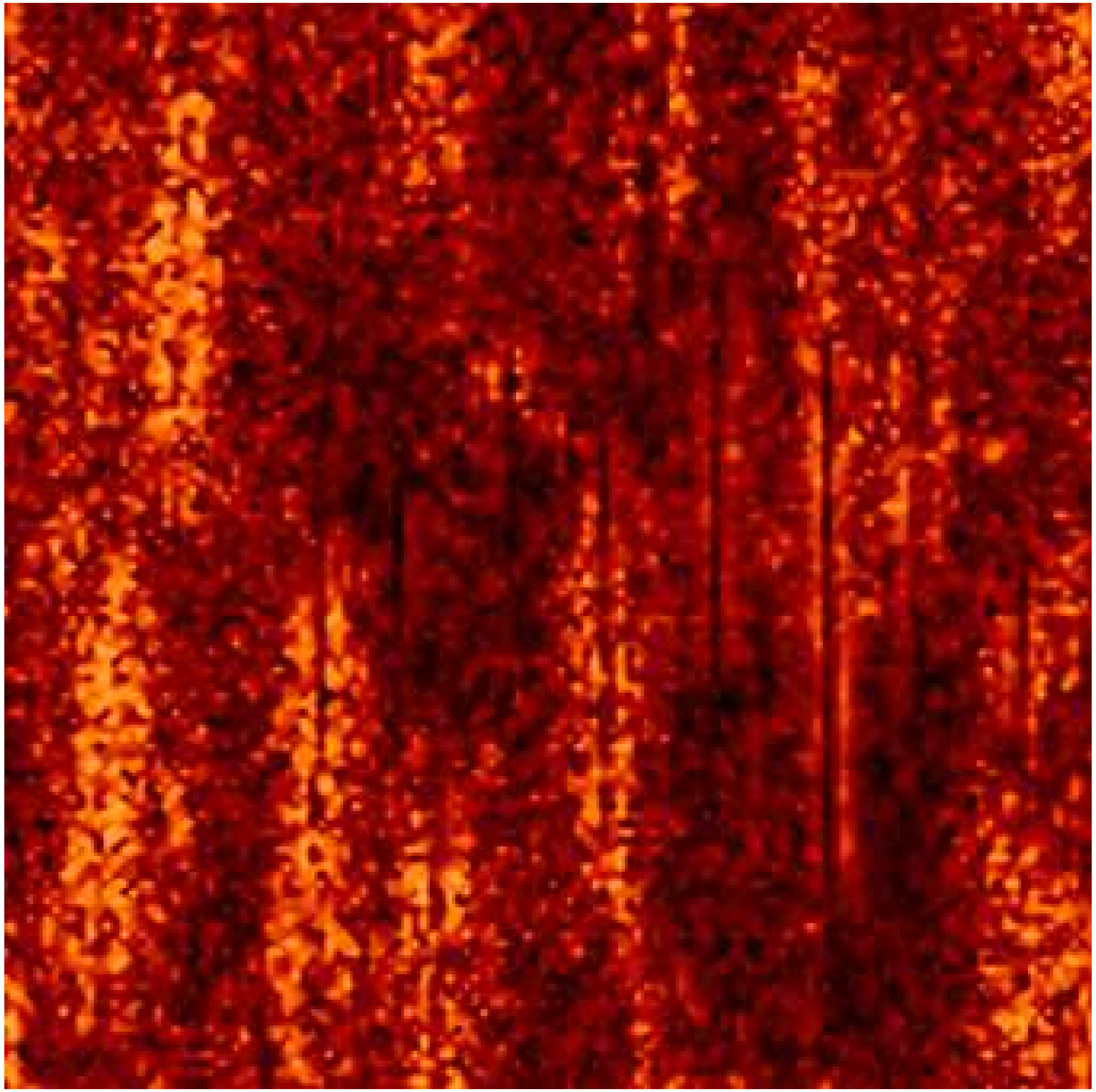
$t = 39/m_0$

# Energy density in position space



$t = 42/m_0$

# Energy density in position space



$t = 45/m_0$

# Summary

- expanding systems of gluons produced in HIC is anisotropic
- anistropy causes plasma instabilities in weak coupling
- unstable modes grow to large field amplitudes with $f \gtrsim 1/g^2$
- they affect the isotropization
- growth of instabilities beyond naive saturation bound for strong anisotropy
- energy growth in very thin sheets $\perp$ beam axis

# Open questions

- what is the mechanism for saturation?
- at which amplitude does saturation occur for strong anisotropy?
- what is the spectrum of the UV cascade?
- is there a universal parametric result for the thermalization time and temperature?