Analog cosmology with spinor Bose-Einstein condensates

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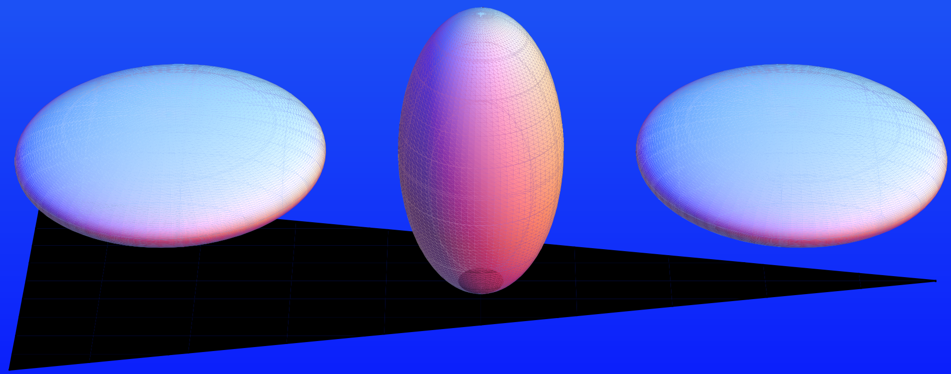
Summary

It is well known that both a particle with spin and a quantum field in a Bianchi type IX Universe are related to the quantum mechanical top. We will combine these insights to show that the dynamics of a cold gas of atoms with total momentum $F \neq 0$ may be used to explore the behavior of quantum matter near a cosmological singularity.



BECs

Because of the propensity of bosons to bunch together, at low enough temperature the lowest lying one-particle mode for a system of bosonic particles may acquire a macroscopic occupation number, that is, of the order of the total particle number. This is the phenomenon of Bose-Einstein condensation. The particles in the “condensate” are coherent, which leads to a number of observable phenomena such as superfluidity.

Bose-Einstein condensates are being actively investigated and have a number of applications, not least their role in a possible implementation of a quantum computer

To condense or not to condense

For a system of free bosons all particles fall to the condensate at zero temperature. If there is a repulsive interaction then a compromise is reached, and there is a nonzero number of noncondensate particles even at absolute zero.

For attractive interactions, in principle the condensed state is unstable. However, if the increase of density brings an increase in kinetic energy which outweighs the loss of potential energy, then a stable state may be found. This happens when the total particle number is less than a critical value.

Field theory of BEC

For a field-theoretic description of BEC we begin with a second - quantized field operator $\Psi(x,t)$ which removes an atom at the location x at times t. It obeys the canonical commutation relations

$$\left[\Psi(x,t), \Psi(y,t)\right] = 0 \tag{1}$$

$$\left[\Psi(x,t), \Psi^{\dagger}(y,t)\right] = \delta(x-y) \tag{2}$$

Field theory of BEC

The dynamics of this field is given by the Heisenberg equations of motion

$$-i\hbar\frac{\partial}{\partial t}\Psi = [\mathbf{H}, \Psi] \tag{3}$$

$$\mathbf{H} = \int d^d x \;\left\{\Psi^\dagger H \Psi + V_{int}\left[\Psi^\dagger, \Psi\right]\right\} \tag{4}$$

$$H\Psi = -\frac{\hbar^2}{2M}\nabla^2\Psi + V_{trap}\left(x\right)\Psi \tag{5}$$

Classical action

The Heisenberg equation of motion

$$i\hbar\frac{\partial}{\partial t}\Psi = H\Psi + \frac{\partial V_{int}}{\partial \Psi^\dagger} \tag{6}$$

is also the classical equation of motion derived from the action

$$S = \int d^{d+1}x \; i\hbar\Psi^* \frac{\partial}{\partial t}\Psi - \int dt \; \mathbf{H} \tag{7}$$

Particle number conservation

The theory is invariant under a global phase change of the field operator

$$\Psi \rightarrow e^{i\theta}\Psi, \qquad \Psi^\dagger \rightarrow e^{-i\theta}\Psi^\dagger \tag{8}$$

The constant of motion associated with this $U(1)$ invariance through Noether theorem is the total particle number.

Condensate wave function

There is a special one-particle state, with wave function ϕ_0, which, upon condensation, acquires a macroscopic occupation number N_0, comparable to the total number of particles N. We call this state the "condensate". We decompose Ψ into its component proportional to ϕ_0 and an orthogonal component $\Psi = \Phi + \chi$, where $\Phi = a_0\phi_0$. The operator a_0 is the destruction operator for the condensate.

Particle Number Conserving Formalism

The PNC approach is a systematic expansion in inverse powers of the total particle number. To lowest order ϕ_0 obeys the classical equation derived from the action, with the addition of a chemical potential term to enforce the normalization. This is the Gross-Pitaievskii equation. Higher quantum effects may be described by a suitable effective action

Analog Gravity with BECs

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- *focus on metrics*: if the condensate flow is inhomogeneous, long wavelength perturbations of the condensate behave as a relativistic scalar field in an effective metric

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- *focus on metrics*: if the condensate flow is inhomogeneous, long wavelength perturbations of the condensate behave as a relativistic scalar field in an effective metric
- *focus on mechanisms*: one could also mimic a specific mechanism without duplicating the whole metric. The best known example is using the *Bose-Nova* condensate collapse experiment to study cosmological particle creation.

Analog Cosmology

Our work is similar to the metric approach in that we shall use Bose-Einstein condensates to emulate a quantum field in a Bianchi type IX spacetime, but it is also close to the Bose-Nova work in that the necessary condensate configuration can be obtained in realistic experimental situations - indeed, several relevant experiments have already been performed: L. E. Sadler *et al.*, Nature 443, 312 (2006); T. Lahaye *et al.*, Nature 448, 672 (2007); M. Vengalattore *et al.* , quant-ph/0712.4182, etc.

The Bianchi type IX Universe

The Bianchi type IX (*Mixmaster*) Universe is the most general homogeneous but anisotropic model. The metric of the spatial sections can then be written in terms of Misner parameters as

$$ds_B^2 = e^{2\Omega}\left\{e^{-2\beta_+}\omega_1^2 + e^{\beta_+ + \sqrt{3}\beta_-}\omega_2^2 + e^{\beta_+ - \sqrt{3}\beta_-}\omega_3^2\right\} \tag{9}$$

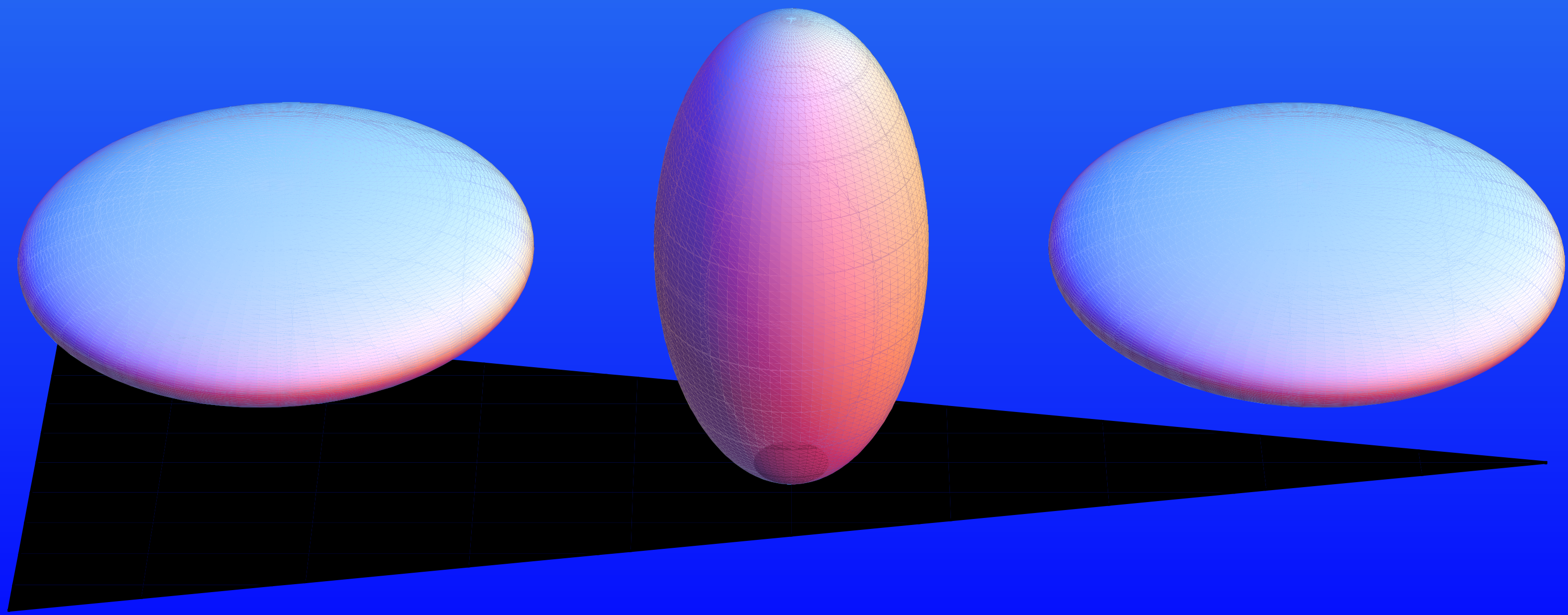
where the nonintegrable 1-forms ω_a satisfy

$$d\omega_a = -\frac{1}{2}\epsilon_{abc}\omega_b \wedge \omega_c \tag{10}$$

where ϵ_{abc} is the totally antisymmetric symbol.

Why Bianchi IX

It has been conjectured by Belinsky, Lifshitz and Khalatnikov that the Universe behaved locally like a vacuum Bianchi type IX Universe near the cosmological singularity. Moreover, the Mixmaster model is known to be chaotic



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- *Can a Bianchi IX Universe dissipate its anisotropy?*
- *Can it avoid recollapse?*
- *Can there be a smooth transition from a Bianchi IX to an inflationary evolution?*

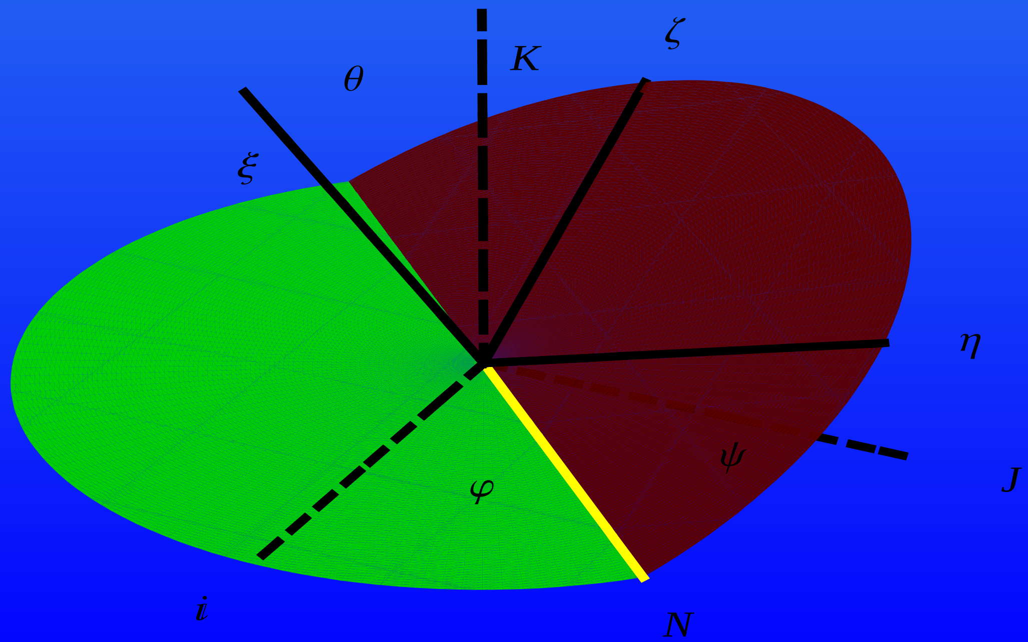
Spinor BECs

In a spinor BEC, the sum $\mathbf{F}$ of the electron spin $\mathbf{S}$ and the nuclear spin $\mathbf{I}$ is not zero in the ground state. Examples are rubidium ($\mathbf{F} = \mathbf{1}$), sodium ($\mathbf{F} = \mathbf{2}$) and chromium ($\mathbf{F} = \mathbf{3}$).

Spinor condensates show local point interactions, spin-exchange interactions and dipole-dipole interactions. The strenght of each of these can be controlled independently by tayloring the confining potential and/or applying external electromagnetic fields

From spinor BECs to Bianchi IX

The space of states of any spinning particle is a subspace of the Hilbert space of a quantum top (N. Rosen, Phys. Rev. 82, 621 (1951). Introduce Euler angles



From spinor BECs to Bianchi IX

The spin operators are identified with differential operators in the configuration space of the top

$$\begin{aligned} F_x &= \frac{\hbar}{i}\left\{\cos[\varphi]\frac{\partial}{\partial\theta} - \frac{\sin[\varphi]\cos[\theta]}{\sin[\theta]}\frac{\partial}{\partial\varphi} + \frac{\sin[\varphi]}{\sin[\theta]}\frac{\partial}{\partial\psi}\right\} \\ F_y &= \frac{\hbar}{i}\left\{\sin[\varphi]\frac{\partial}{\partial\theta} + \frac{\cos[\varphi]\cos[\theta]}{\sin[\theta]}\frac{\partial}{\partial\varphi} - \frac{\cos[\varphi]}{\sin[\theta]}\frac{\partial}{\partial\psi}\right\} \\ F_z &= \frac{\hbar}{i}\frac{\partial}{\partial\varphi} \end{aligned} \tag{11}$$

L. S. Schulman used this to develop a path integral for spin (Phys. Rev. 176, 1558 (1968))

The free spinor condensate

We are almost there. Because of the hyperfine splitting, the Hamiltonian H for a spinor condensate contains a term $(A/2)\mathbf{F^2}$ (for rubidium, $A = 2\,10^{-6}$eV). After second quantization, this term becomes ($\chi^a = (\theta, \varphi, \psi)$)

$$H_0 = \frac{A}{2} \int d^3\chi \, \sqrt{g_0} \;\; g_0^{ab} \frac{\partial \Psi^\dagger}{\partial \chi^a} \frac{\partial \Psi}{\partial \chi^b} \tag{12}$$

where g_{0ab} is the metric of the sphere, given by

$$ds_0^2 = d\theta^2 + d\varphi^2 + d\psi^2 + 2\cos\theta \; d\varphi d\psi \tag{13}$$

Turn on B

Under the influence of an external magnetic field B, there appear linear and quadratic Zeeman shifts. The former can be elliminated by going to a rotating frame, and the latter adds to the second quantized Hamiltonian a term

$$H_Z = \frac{\mu_B^2 B^2}{8A} \int d^3\chi \, \sqrt{g_0} \, \frac{\partial \Psi^\dagger}{\partial \varphi} \frac{\partial \Psi}{\partial \varphi} \tag{14}$$

Turn on B

Repeating the above procedure, we see that the metric of the sphere has been deformed into

$$\begin{aligned} ds_B^2 &= \left[1+\mathcal{B}^2\right] d\theta^2 + d\varphi^2 + \left[1+\mathcal{B}^2 \sin^2\theta\right] d\psi^2 \\ &+ 2\cos\theta \, d\varphi d\psi \end{aligned} \quad (15)$$

where

$$\mathcal{B} = \frac{\mu_B B}{2A} \quad (16)$$

Bianchi IX at last

To see that this is indeed a Bianchi type IX metric, identify

$$\begin{aligned}\omega_1 &= d\varphi + \cos\theta\; d\psi \\ \omega_2 &= \cos\varphi\, d\theta + \sin\varphi \sin\theta\; d\psi \\ \omega_3 &= -\sin\varphi\, d\theta + \cos\varphi \sin\theta\; d\psi \end{aligned} \tag{17}$$

Bianchi IX at last

The metric can then be written in terms of Misner parameters as

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where $\beta_- = 0$, $\beta_+ = \Omega$ and

$$\Omega = \frac{1}{3}\ln\left[1 + \mathcal{B}^2\right] \tag{19}$$

Final remarks

- We have shown that a spinor BEC has a natural representation as a field on a Bianchi type IX Universe whose scale factor and shape depend upon the external magnetic field. This opens up opportunities for both cosmology and BEC physics

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- *For cosmology*: build a bounce in the laboratory!
- *For BEC*: bring the sustantial literature on quantum fields on homogeneous spaces to bear on the computation of quantum corrections to the Gross-Pitaievskii equation.
- *More next time!*