Suppression of the Shear Viscosity as QCD Cools Into a Confining Phase

Yoshimasa Hidaka (RBRC)

Collaboration with R. D. Pisarski (BNL)

Based on arXiv:0803.0453[hep-ph]

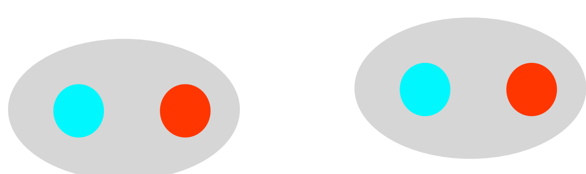
Introduction

Confinement-deconfinement Phase Transition ⟺ **Ionization of color charge**

Ionization parameter: Polyakov loop

$$\mathrm{trL} = \mathrm{tr}\ \mathrm{P} e^{ig \int d\tau A_0}$$

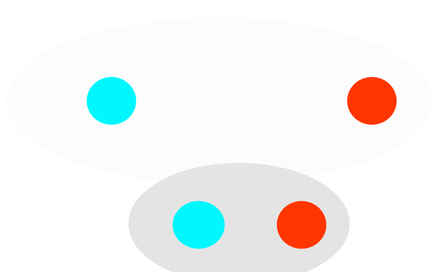
★ Confinement



No ionization

$$\left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle = 0$$

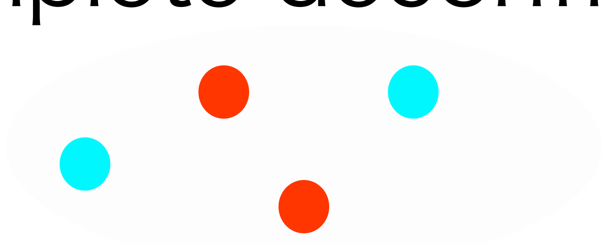
★ Partial deconfinement



Partial ionization

$$\left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle < 1$$

★ Complete deconfinement



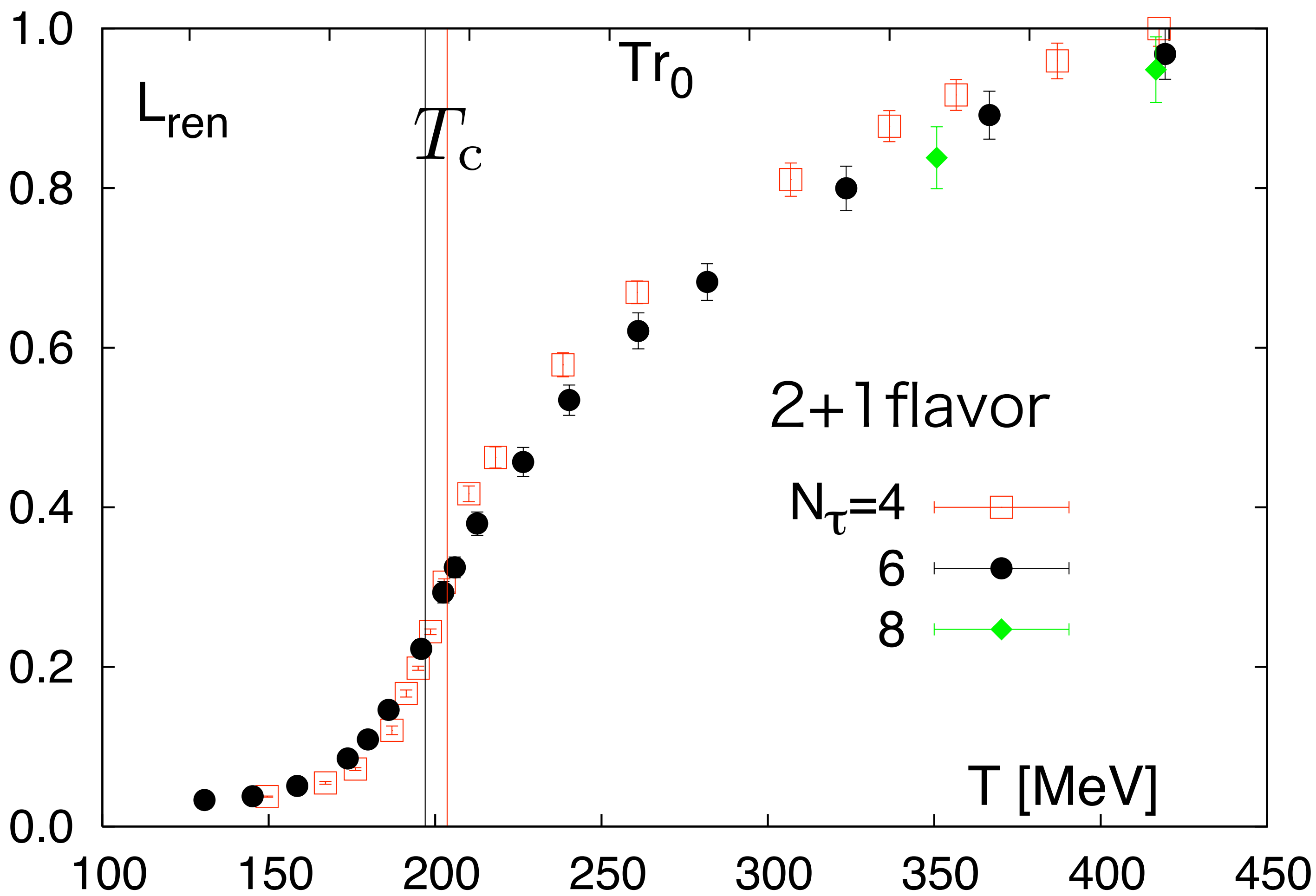
Complete ionization

$$\left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle \simeq 1$$

Semi-QGP

Confinement | **Semi-QGP** | perturbative-QGP

$\left\langle \frac{1}{N_c}\mathrm{tr}L \right\rangle \approx 0$ | $\left\langle \frac{1}{N_c}\mathrm{tr}L \right\rangle < 1$ | $\left\langle \frac{1}{N_c}\mathrm{tr}L \right\rangle \simeq 1$

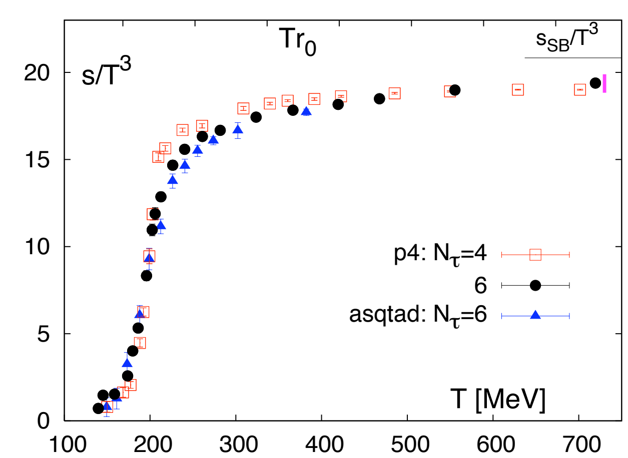


Ren'd Polyakov loop Cheng et. al.(2007)

Semi-QGP Window

$$0.8T_c - 2T_c$$

Expected temperature in RHIC experiment is in Semi-QGP!!



Cheng et. al.(2007)

Pressure, entropy, susceptibility, etc. drastically change in Semi-QGP.
How about transport coefficients?

Viscosities

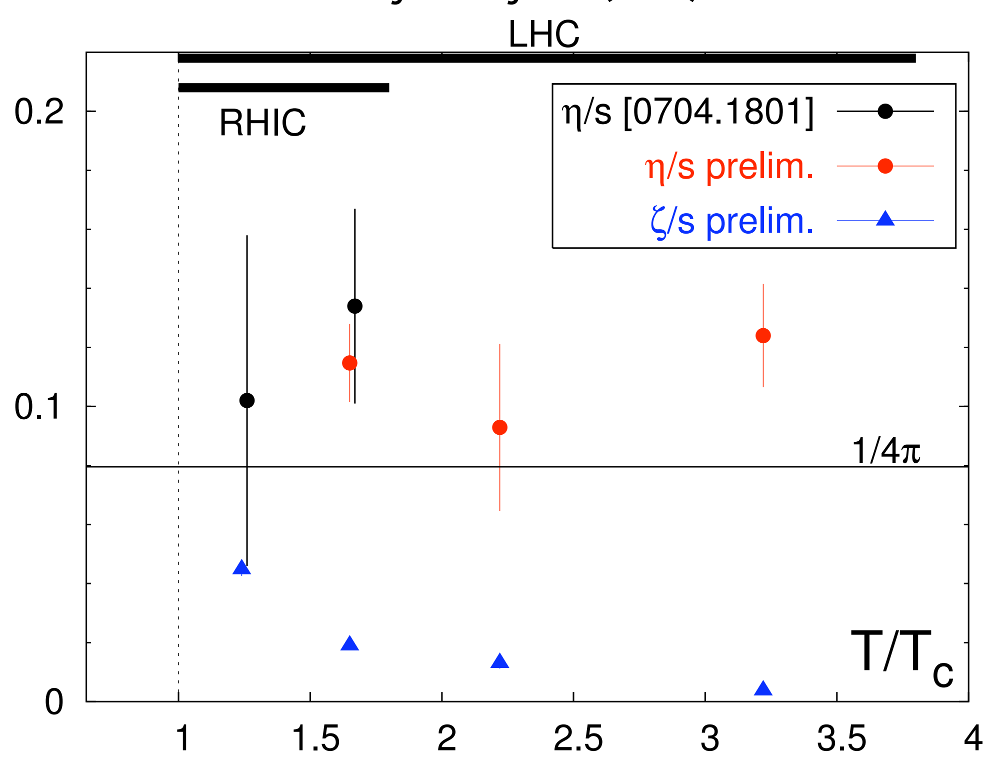
	shear viscosity	bulk viscosity
★ Perturbation theory $\alpha_s \ll 1$	$\eta \sim \frac{T^3}{\alpha_s^2 \log[1/\alpha_s]}$ Hosoya, Sakagami, Takao ('84);Hosoya, Kajantie ('85);Baym, Monien, Pethick, Ravenhall ('90), ('91); Arnold, Moore, Yaffe ('00), ('03)	$\zeta \sim \frac{\alpha_s^2 T^3}{\log[1/\alpha_s]}$ Arnold, Dogan, Moore ('06)
★ $\mathcal{N} = 4$ Super Yang-Mills $N_c \to \infty$ $\alpha_s N_c \to \infty$ Policastro, Son, Starinets ('01) Kovtun, Son, Starinets ('04)	$\frac{\eta}{s} = \frac{1}{4\pi}$	$\frac{\zeta}{s} = 0$ conformal

s: entropy density

★ Lattice

Karsch and Wyld('87);
Sakai and Nakamura('04);
Aarts, Allton, Justin Foley, Hands, Kim('07)
Meyer ('07)

Harvey Meyer ('07)



Bulk viscosity near Tc
Kharzeev, Tuchin (07); Karsch, Kharzeev, Tuchin(07)

AnomalousViscosity
Asakawa, Bass, Muller('06)

Hadron phase
Gavin('85) Prakash, Prakash, Venugopalan, Welke('93)
Davesne('96) Dobado, Santalla('01)
Dobado, Llanes-Estrada('03) Chen, Nakano ('06)
Itakura, Morimatsu,Otomo ('07)

Formulation of Semi-QGP

★ Decompose Polyakov loop to eigenvalue and gauge dependent part.

$$\mathrm{L} = \mathrm{P}\mathrm{e}^{ig\int d\tau A_0} = \Omega^\dagger \mathrm{e}^{iQ/T}\Omega \quad Q^a$$:diagonal and gauge invariant.

★ Integrating over A_μ expcept Q.

$$Z = \int \mathcal{D}A_\mu \exp(-S[A_\mu]) = \int \mathcal{D}Q \exp(-N_\mathrm{c}^2 S_\mathrm{eff}[Q])$$

★ Large-Nc approximation

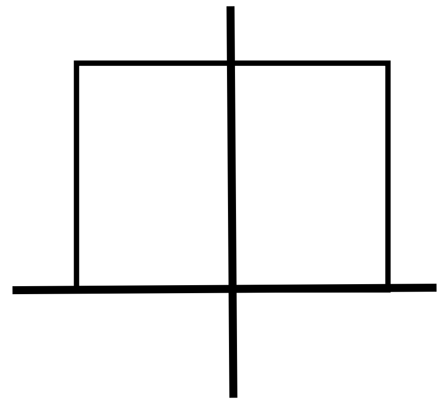
$$\frac{\delta}{\delta Q(x)} S_\mathrm{eff} = 0$$

⇒ eigenvalue-distribution function $\rho(\theta)$ with $\theta = Q/T$

$$\frac{1}{N_\mathrm{c}}\mathrm{trL}^n = \frac{1}{N_\mathrm{c}}\sum_a e^{in\theta^a} = \int da e^{i\theta(a)} = \int d\theta \rho(\theta) e^{in\theta}$$

Eigenvalue Distribution

Confinement phase

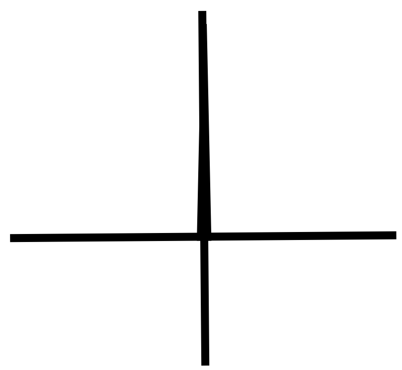


Distribution is constant: $\rho(\theta) = \frac{1}{2\pi}$

All Polyakov loops vanish:

$$\frac{1}{N_{\rm c}}{\rm trL}^n = \int d\theta \rho(\theta) {\rm e}^{in\theta} = 0$$

Complete ionization



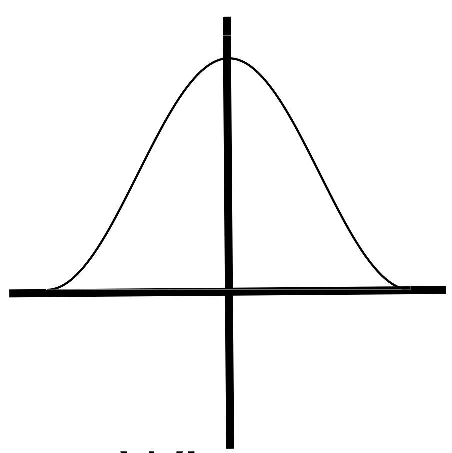
Distribution is the delta function:

$$\rho(\theta) = \delta(\theta)$$

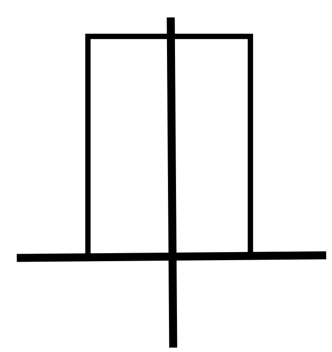
All Polyakov loops are unit:

$$\frac{1}{N_{\rm c}}{\rm trL}^n = 1$$

Semi-QGP



Gross-Witten model



Step function

We take some assumption of distribution, Gross-Witten model and Step function type.

Assumption

- ★ A_0 is decomposed to background and quantum field, $A_0 = Q/g + A_0^{\mathrm{qu}}$
- ★ Coupling is small $g \ll 1$
- ★ Background gauge field is large $Q \sim T$
- ★ Slowly changing $\partial Q/T \sim gT$

⇩ can use derivative expansion.

Analytical continuation

Q corresponds imaginary chemical potential,

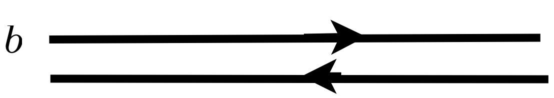
$$i\omega_n + iQ^a \rightarrow p_0 \pm i\epsilon$$

ω_n :Matsubara frequency

Propagator in the Background

We choose the basis of Lie algebra as eigenstates of the background field.

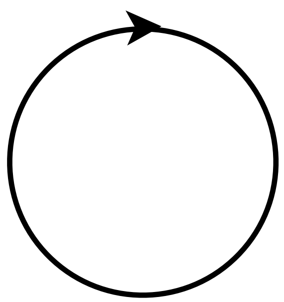
Quarks and gluons carry color "charge".

	Quarks	Gluons
★ Covariant derivative	$iD_0\psi^a = (k_0 + Q^a)\psi^a$	$iD_0 A_\mu^{ab} = (k_0 + Q^a - Q^b)A_\mu^{ab}$
★ Matsubara frequency	$k_0 = 2\pi(n + \frac{1}{2})T$	$k_0 = 2\pi n T$
★ Double line notation	Q^a 	$Q^a - Q^b$ 
★ Propagator	$\frac{1}{(\omega_n + Q^a)^2 + k^2 + m^2}$	$\frac{1}{(\omega + Q^a - Q^b)^2 + k^2}$
★ Distribution function	$\frac{1}{\exp(\omega - iQ^a) + 1}$	$\frac{1}{\exp(\omega - iQ^a + iQ^b) - 1}$

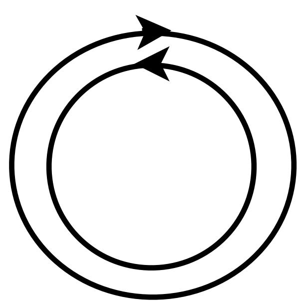
Expand the distribution function

$$\frac{1}{\mathrm{e}^{(E-iQ^a)/T}+1}=\sum_{n=1}^{\infty}(-)^{n+1}\mathrm{e}^{-n(E-iQ^a)/T}$$

Example: trace of the propagator



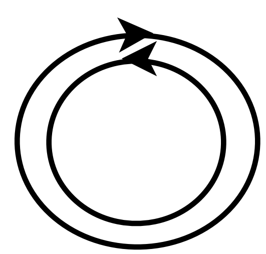
$$\sum_{a}\frac{1}{\mathrm{e}^{(E-iQ^a)/T}+1}=\sum_{n=1}^{\infty}(-)^{n+1}\mathrm{e}^{-nE/T}\mathrm{trL}^n$$

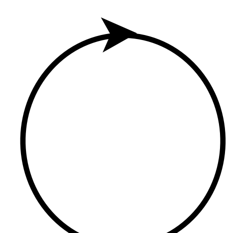


$$\sum_{a,b}\frac{1}{\mathrm{e}^{(E-i(Q^a-Q^b))/T}-1}=\sum_{n=1}^{\infty}\mathrm{e}^{-nE/T}|\mathrm{trL}^n|^2$$

Pressure(leading order)

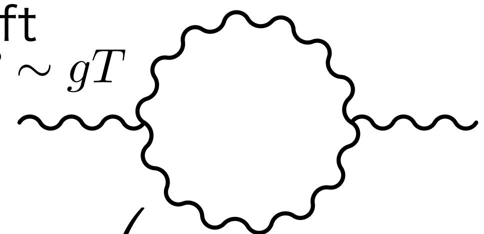
$$P=\frac{T^4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^4}\left(2\left(|\mathrm{trL}^n|^2-1\right)+4N_{\mathrm{f}}(-1)^{n+1}\mathrm{RetrL}^n\right)$$





Two point function

Hard Thermal Loop approximation

$\Pi^{\mu\nu}(P) =$  $+$ Tadpole, ghost diagrams

$$= (m_D^2)^{ab}\left(\delta^{\mu 0}\delta^{\nu 0} - \int \frac{d\Omega}{4\pi}\frac{(P)^0 \hat{K}^\mu \hat{K}^\nu}{P \cdot \hat{K}}\right) - i f^{ab}{}_c \langle (J^c)^0 \rangle \int \frac{d\Omega}{4\pi}\frac{\hat{K}^\mu \hat{K}^\nu}{P \cdot \hat{K}}$$

Ordinary HTL approx. term

The thermal mass is modified.

Debye mass $\quad [m_D^2(A_0^{\mathrm{cl}})]^{ab} = m_D^2 \times h^{ab}(A_0^{\mathrm{cl}})$ where $m_D^2 = \frac{1}{6} N_c g^2 T^2$

- ★ Completely deconfined phase $\quad h^{ab} = \delta^{ab}$
- ★ Semi-QGP phase $\quad h^{ab} < 1$
- ★ Confined pahse $\quad h^{ab} \sim 0$ because background is neutral.

Kinetic Theory

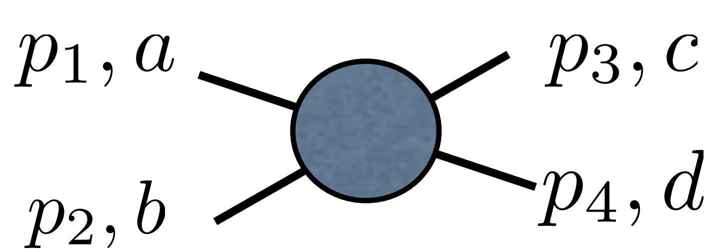
Boltzmann Equation

$$Df^a = -C^a[f] \qquad D = \frac{\partial}{\partial t} + \boldsymbol{v}_p \cdot \frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{F}_{\rm ext} \cdot \frac{\partial}{\partial \boldsymbol{p}}$$

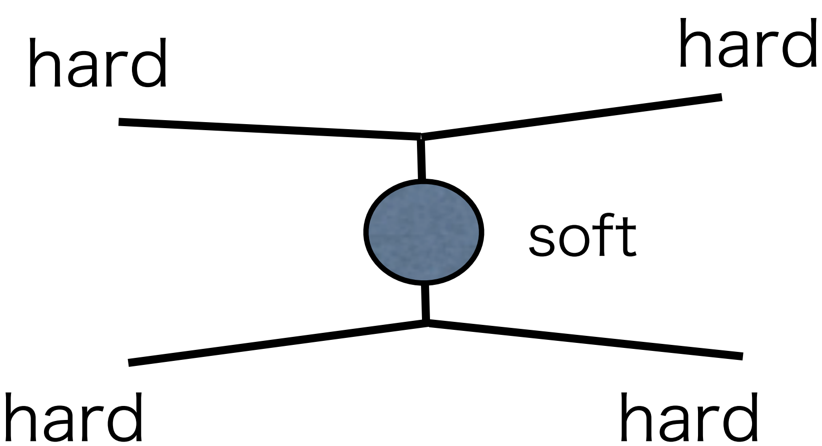
Collision term

$$C^a[f] = \frac{1}{2} \sum_{\rm color,spin,flavor} \int d\Pi (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathcal{M}|^2 f^a f^b (1 \pm f^c)(1 \pm f^d)$$

Two body scattering $\mathcal{M} =$



t-channel contributes to leading log



soft gluon exchange

$$|\mathcal{M}|^2 \sim \frac{1}{(q^2 + m_{\rm D}^2)^2}$$ (gluon exchange)

$m_{\rm D}$:color dependent Debye mass

Viscosities

★ Stress tensor

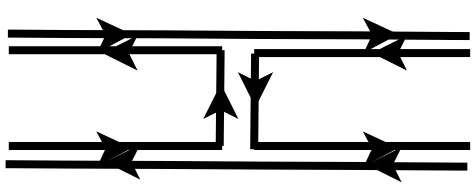
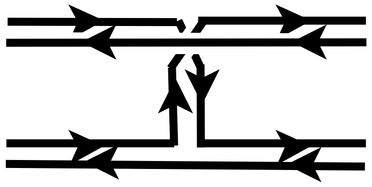
$$\langle T_{ij} \rangle = \delta_{ij}\langle \mathcal{P} \rangle - \eta\sqrt{6}X_{ij} - \xi\delta_{ij}\nabla^l u_l$$

$$X_{ij} = \frac{1}{\sqrt{6}}[\nabla_j u_i - \frac{2}{3}\delta_{ij}\nabla^l u_l]$$

In kinetic theory

$$\langle T_{\mu\nu}(x) \rangle = \sum_{\text{spin,flavor,color}} \int \frac{d^3p}{(2\pi)^3} \frac{p_\mu p_\nu}{2\epsilon} f^a(p,x)$$

★ Scattering amplitude

Pure glue $\mathcal{M} =$  $+$  $+ \cdots$

Viscosities(Cont.)

Arnold, Moore, Yaffe (01)

Linearized Boltzmann Equation

Assume that the system is near (global) equilibrium.

Expand the distribution function:

$$f^a = f_0^a + \frac{\partial f_0^a}{\partial \epsilon} X_{ij} I_{ij} \chi^a \qquad I_{ij} = \sqrt{\frac{3}{2}}(\hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij})$$

$$f_0^a = \frac{1}{\mathrm{e}^{(u_\mu(x)p^\mu(x) - iQ^a(x))/T(x)} \pm 1}$$ in (local) equilibrium

Linear equation is obtained as

$$S = C\chi$$

S and $C\chi$ correspond to Df and the collision term in Boltzmann equation, respectively.

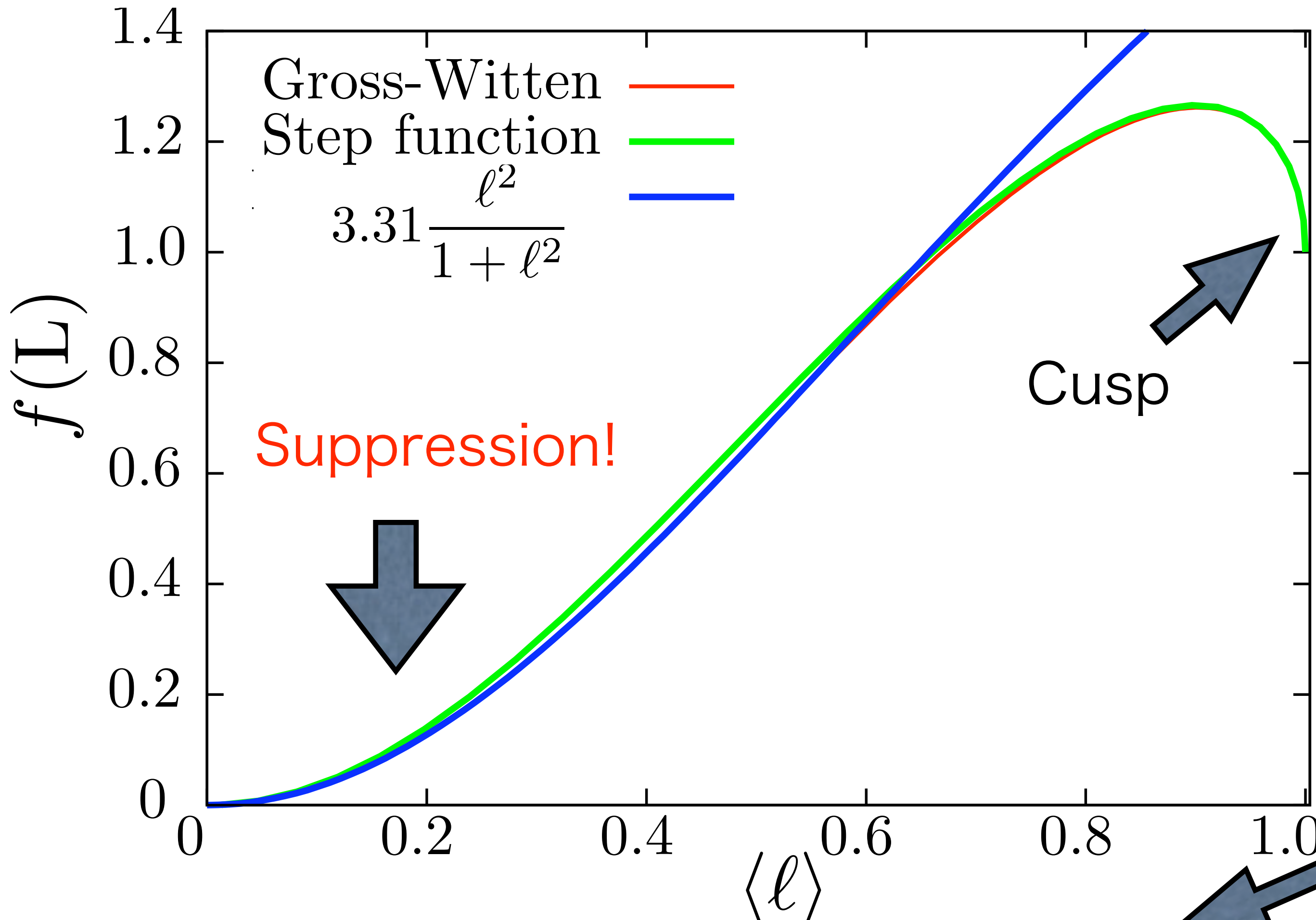
The solution is formally obtained: $\chi = C^{-1}S$

$$\Rightarrow \quad \eta = \frac{1}{15} S^t C^{-1} S$$

Result without quarks

Y.H., Pisarski('08)

Viscosity $\eta = \dfrac{c_\eta T^3}{g^4 \, \log(1/g)} f(\mathrm{L}) \qquad f(\mathrm{L}=1)=1$



Semi-QGP $\ell = \langle \frac{1}{N_c} \mathrm{tr}\ \mathrm{L} \rangle$

★ $\eta \sim |\ell|^2 T/\sigma$

Suppression! at small ℓ

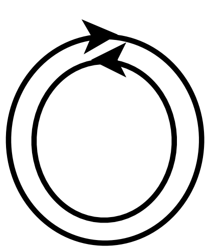
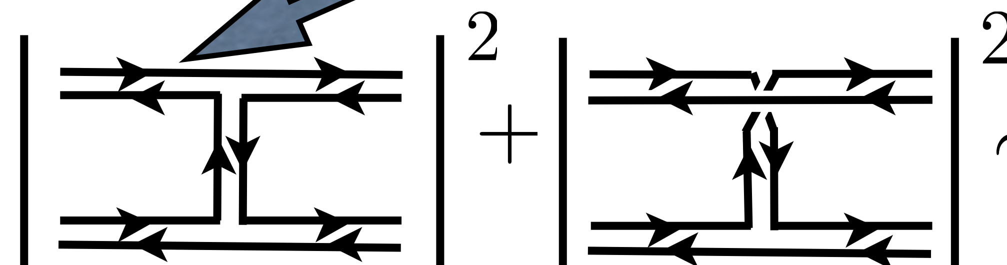
★ Classical dilute gas

$$\eta \sim T/\sigma$$

σ :cross section

cancel

$$\mathrm{e}^{iQ^a - iQ^b}\mathrm{e}^{iQ^b - iQ^c} = \mathrm{e}^{iQ^a - iQ^c}$$

$S \sim$  $\sim N_c^2 \ell^2 \quad C \sim \left| \right.$  $\left. \right|^2 \sim N_c^4(\ell^2 + \ell^4)$

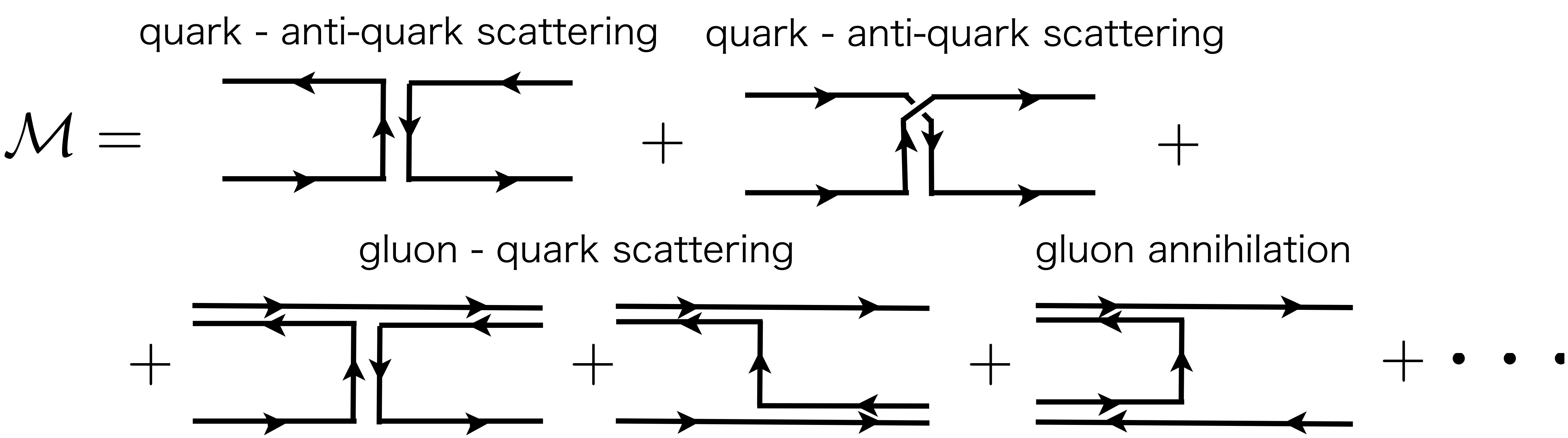
at small ℓ

$$\Rightarrow \eta = \frac{1}{15} S^t C^{-1} S \sim \frac{(\ell^2)^2}{\ell^2 + \ell^4} = \frac{\ell^2}{1+\ell^2}$$

With quarks

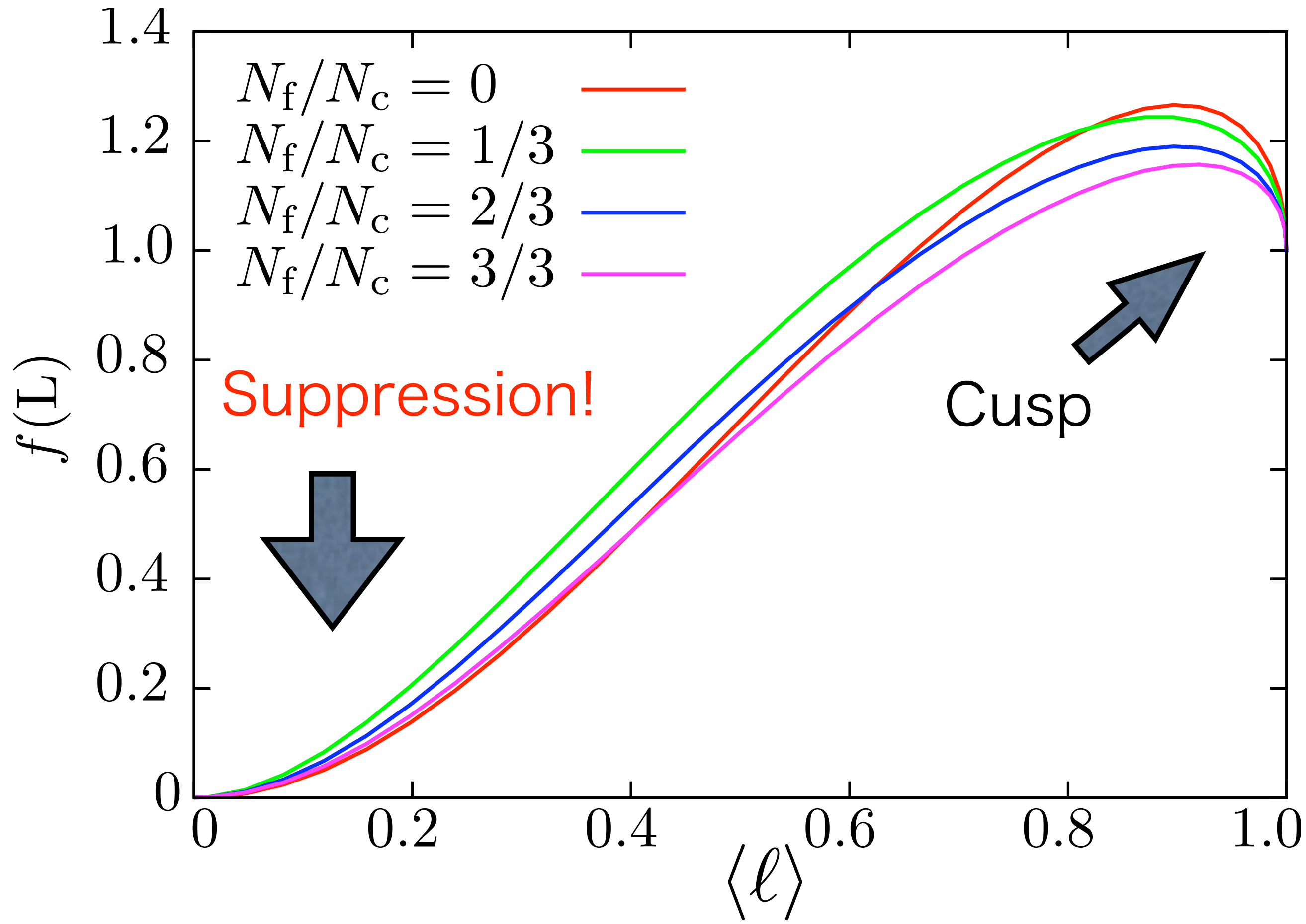
The scattering amplitude is more complicated.

Assume $N_{\rm f} \sim N_{\rm c} \gg 1$.



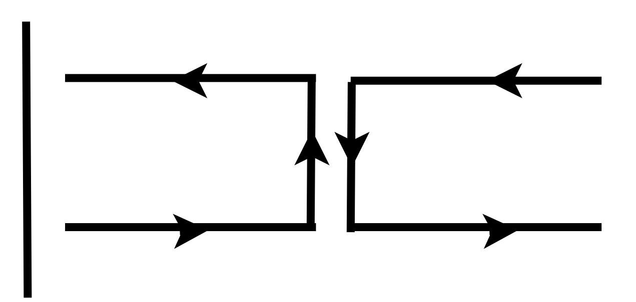
Results with quarks

Y.H., Pisarski('08)



$f(\mathrm{L}) \sim \ell^2$ at small ℓ

quark - anti-quark scattering

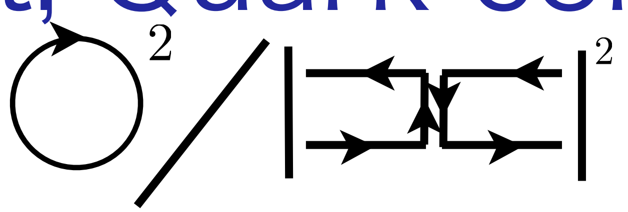


$^2 \sim 1$

no suppression at $\ell \ll 1$

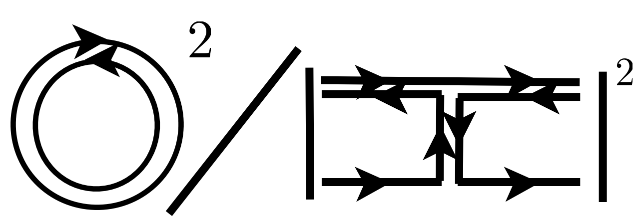
The results are similar to that without quarks,

but, Quark contribution dominates,



$\sim \ell^2/1 = \ell^2$

Gluon contribution is suppressed,

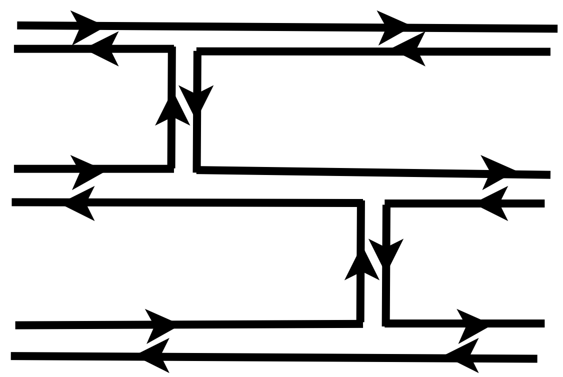


$\sim \ell^3$

Summary

- We find a strong suppression of the shear viscosity, $\sim \ell^2$, near Tc.

Higher order



$\sim \ell^2$

- Heavy ion collisions at RHIC have probed some region in the semi-QGP, where physical quantities such as real photon, dileptions emission and R_{AA} are at least partially suppressed by small Polyakov loops.

- Heavy ion collisions at the LHC may probe temperatures which are significantly higher, perhaps well into the complete QGP. If so, collisions at the LHC should be qualitatively different from those at RHIC.