Euclidean correlators and spectral functions in lattice QCD

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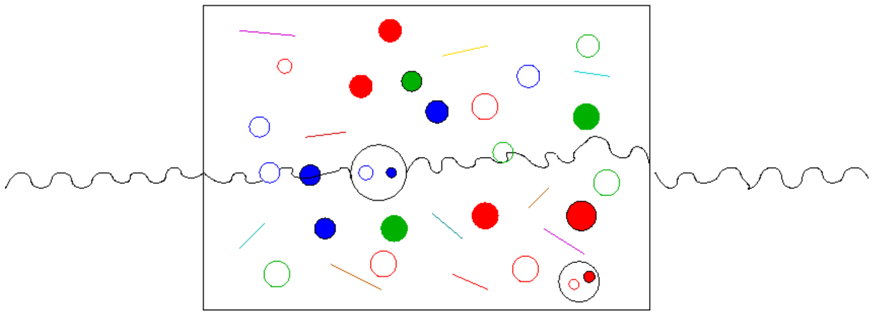
- Euclidean correlators, spectral functions and Maximum Entropy Method
- Quarkkonium correlators an spectral functions in lattice QCD and potential models
- Light meson correlators and spectral functions

Meson correlators and spectral functions

Spectral (dynamic structure) function



$$\frac{D^>(\omega)-D^<(\omega)}{2\pi}=\frac{1}{\pi}\mathrm{Im}\,D_R(\omega)=\sigma(\omega)$$

Example : virtual photon

$$R(\omega)=\frac{\sigma_{e^+e^-\to hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}=\sigma(\omega)/\omega^2$$

$$\frac{dW}{d\omega d^3p}=\frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{\omega^2(e^{\omega/T}-1)}\sigma(\omega,\vec{p},T)$$

What are the excitations (dof) of the system ?

$$G(\tau,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(\tau,\vec{x})\,J_H^+(0,0)\right\rangle,\quad J_H(\tau,\vec{x})=\overline{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

$$\Gamma_H=1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\cdot\gamma_\mu$$

quenched approximation is used !

$$G(\tau,T)=D^>(-i\tau)$$

Imaginary time   Real time

$$G(\tau,T)=\int_0^\infty d\omega\,\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T)))}{\sinh(\omega/(2T))}$$

Reconstruction of the spectral functions : MEM

$$G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \cdot K(\omega, T)$$

$\mathcal{O}(10)$ data and $\mathcal{O}(100)$ degrees of freedom to reconstruct

Bayesian techniques: find $\sigma(\omega, T)$ which maximizes $P[\sigma|DH]$

data ⇧ ⇧ Prior knowledge

H :

$\sigma(\omega, T) > 0$ ➡ Maximum Entropy Method (MEM)

Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma|DH] = \exp(-\frac{1}{2}\chi^2 + \alpha S)$$

Likelihood function ⇧ Shannon-Janes entropy: ⇧

$$S = \int_0^\infty d\omega[\sigma(\omega) - m(\omega) - \sigma(\omega) \ln \frac{\sigma(\omega)}{m(\omega)}]$$

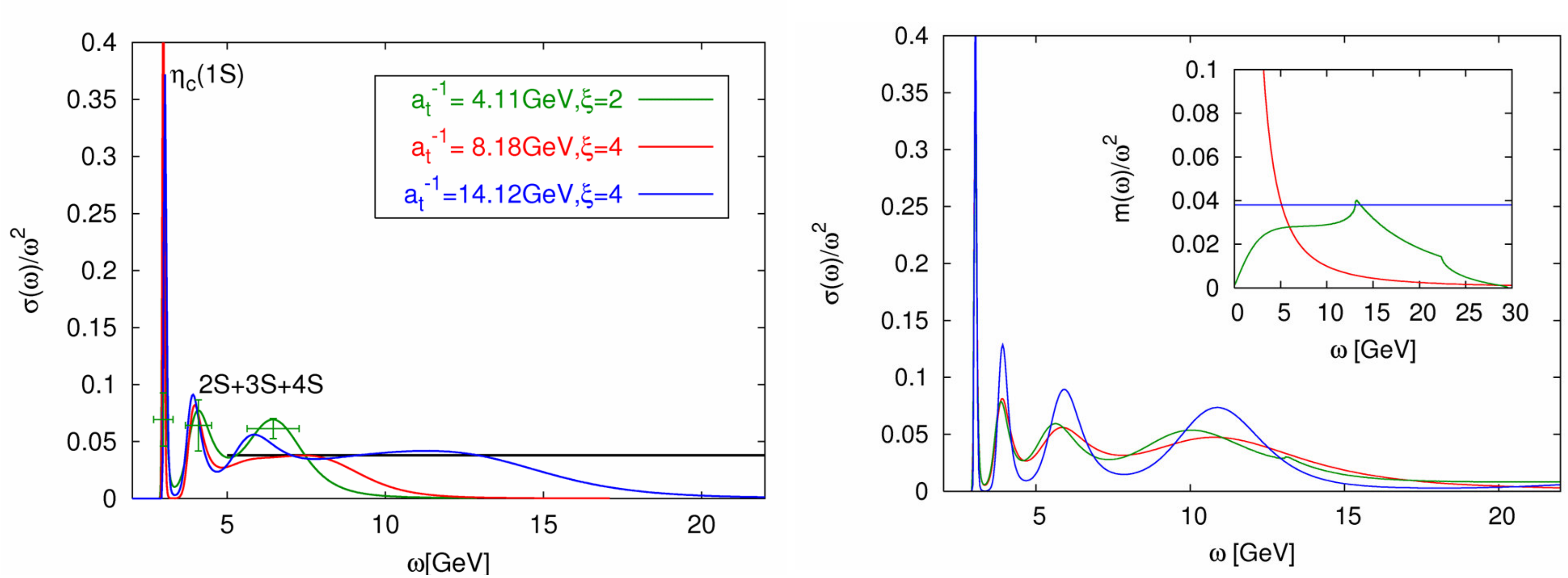
$m(\omega)$ - default model $m(\omega \gg \Lambda_{QCD}) = m_0 \omega^2$ -perturbation theory

Charmonia spectral functions at T=0

Anisotropic lattices: $16^3 \times 64, \xi = 2$ $16^3 \times 96, \xi = 4$, $24^3 \times 160, \xi = 4$
$L_s = 1.35 - 1.54$fm, #configs=500-930;
Wilson gauge action and Fermilab heavy quark action

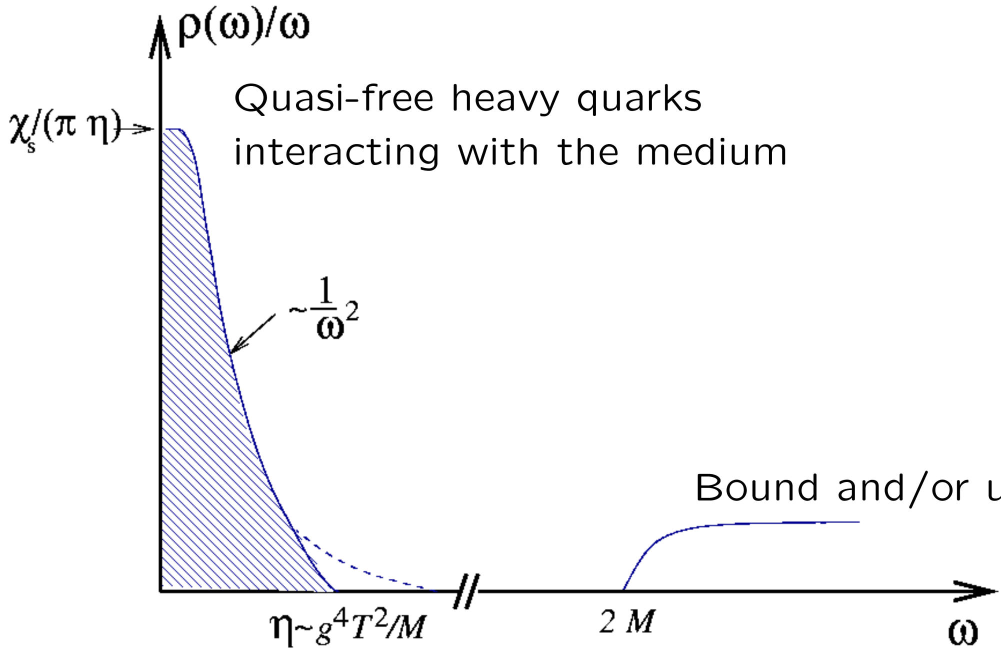
Jakovác, P.P., Petrov, Velytsky, PRD 75 (07) 014506

Pseudo-scalar (PS) $\rightarrow$ $^1S_0$-states



For $\omega > 5$ GeV the spectral function is sensitive to lattice cut-off ;
Strong default model dependence in the continuum region

Quarkonium spectral functions at T>0



interactions :

$$\delta(\omega) \to \frac{1}{\pi} \cdot \frac{\eta}{\omega^2 + \eta^2} \qquad \eta = \frac{T}{M}\frac{1}{D} \ll T$$

P.P., Teaney, PRD 73 (2006) 014508
see also Aarts, Martinez-Resco

E.g. free theory :

$$\sigma_V^{ii}(\omega) = \theta(\omega^2 - 4M^2)\frac{1}{4\pi^2}\omega^2\sqrt{1 - \frac{4M^2}{\omega^2}} + \chi(T)\left(\frac{T}{M}\right)\omega\delta(\omega)$$

$$\sigma^{\text{high}}(\omega, T) \quad + \quad \sigma^{\text{low}}(\omega, T)$$

$$\chi(T) = \frac{6}{\pi^2}\int_0^\infty dp p^2 \left(-\frac{\partial n_F}{\partial E_p}\right), \quad E_p^2 = p^2 + M^2, \quad n_F = 1/(\exp(E_p/T)+1)$$

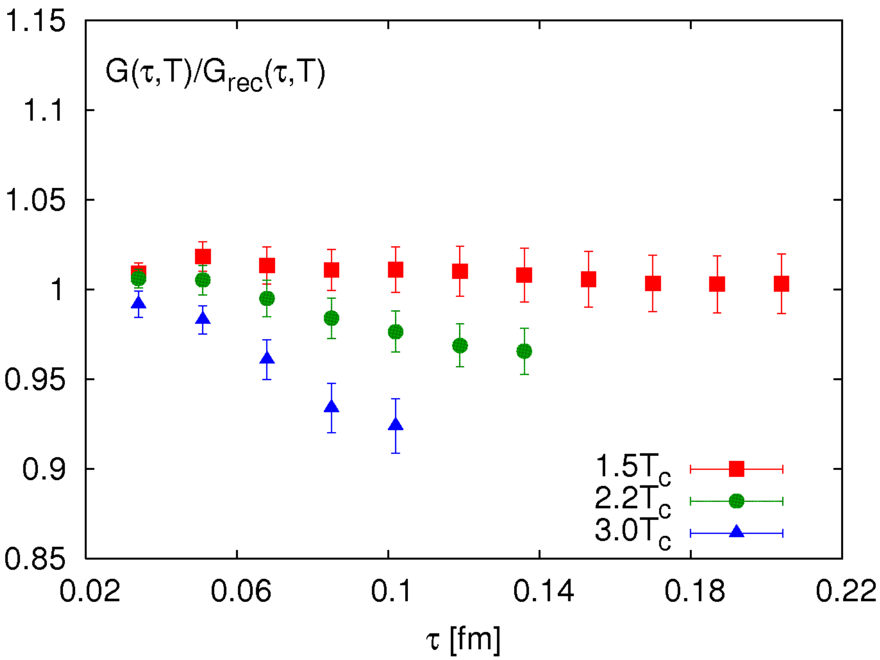
quark number susceptibility

Temperature dependence of quarkonium

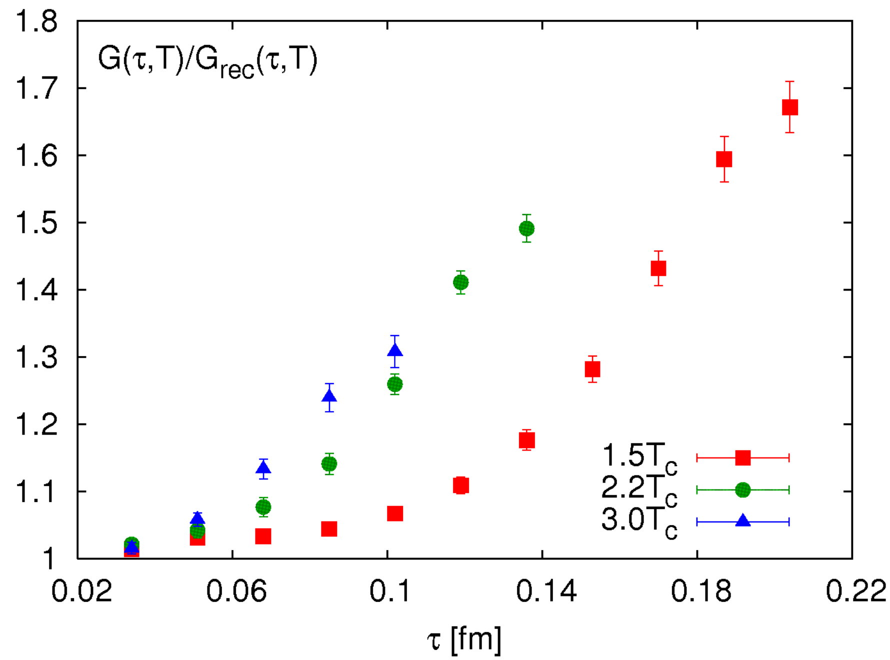
$$G_{rec}(\tau,T) = \int_0^\infty d\omega\sigma(\omega,T^*)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))},\quad T^*\ll T_c$$

study the T-dependence of : $G(\tau,T)/G_{rec}(\tau,T)$

Pseudo-scalar



Scalar



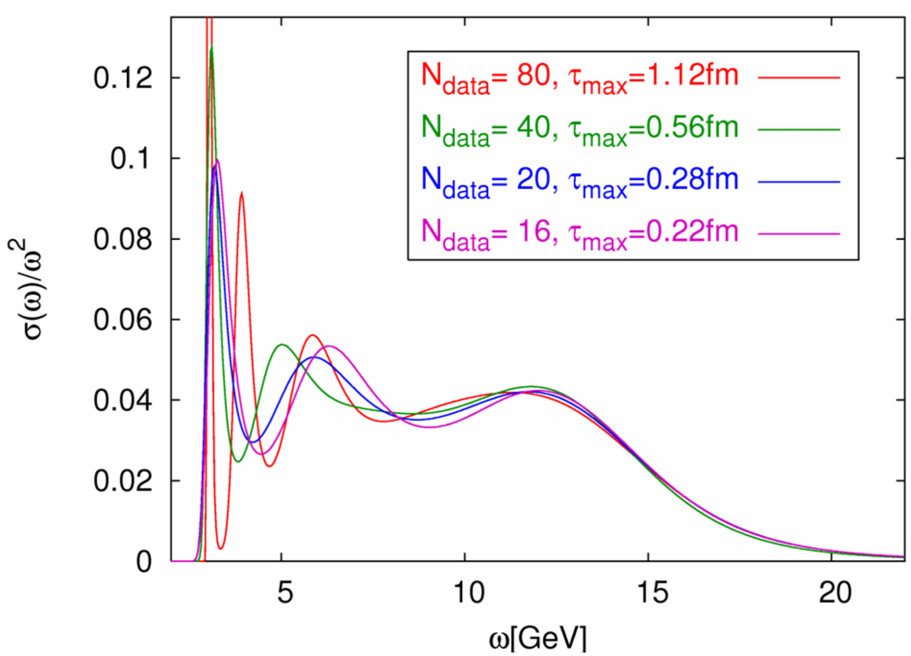
Datta, Karsch, P.P , Wetzorke, PRD 69 (2004) 094507

Charmonia spectral functions in PS channel at T>0

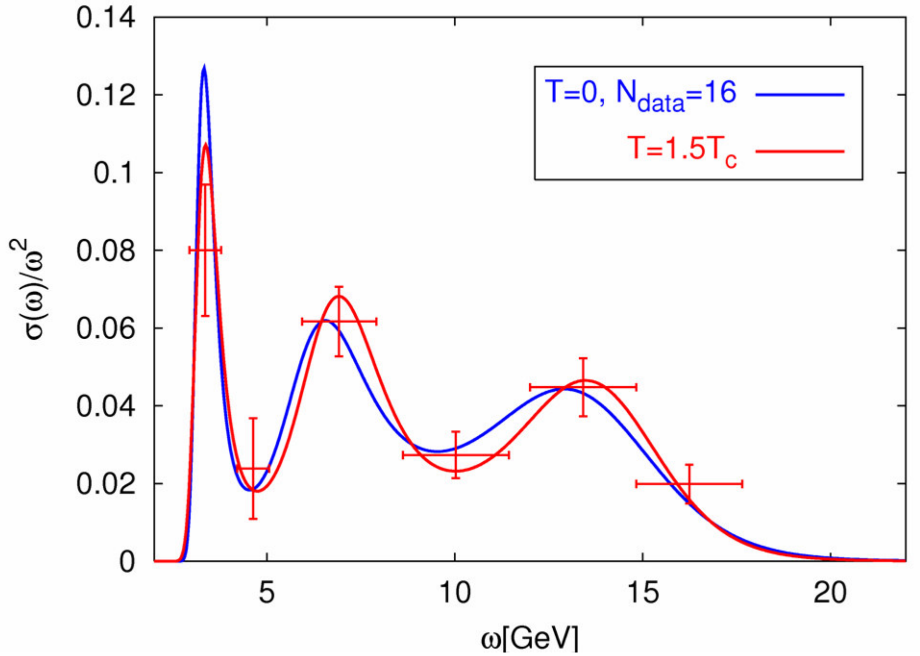
$$T = 1/(N_t a) \quad \leftrightarrow \quad \tau_{max} = 1/(2T)$$

$$\text{PS, } 24^3 \times N_t,\ a_t^{-1} = 14.12 \text{ GeV},\ \xi = 4,\ \#conf \simeq 2000$$

$T = 0, \quad N_t = 160$



$T = 1.5T_c, \quad N_t = 32$



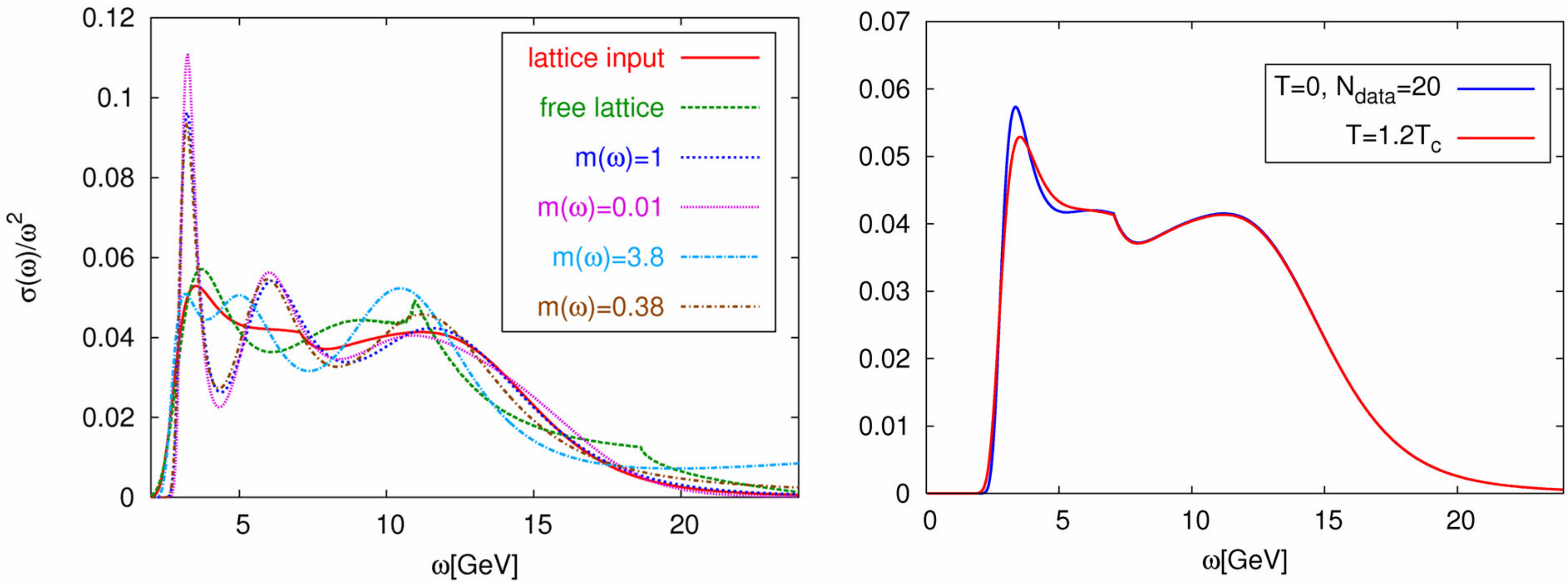
ground state peak is shifted, excited states are not resolved when $\tau_{max}, \quad N_{data}$ become small

no temperature dependence in the PS spectral functions within errors

Jakovác, P.P., Petrov, Velytsky, PRD 75 (07) 014506

Charmonia spectral functions at T>0 (cont'd)

PS, $24^3 \times 40$, $a_t^{-1} = 14.12$ GeV, $\xi = 4$, $T = 1.2T_c$

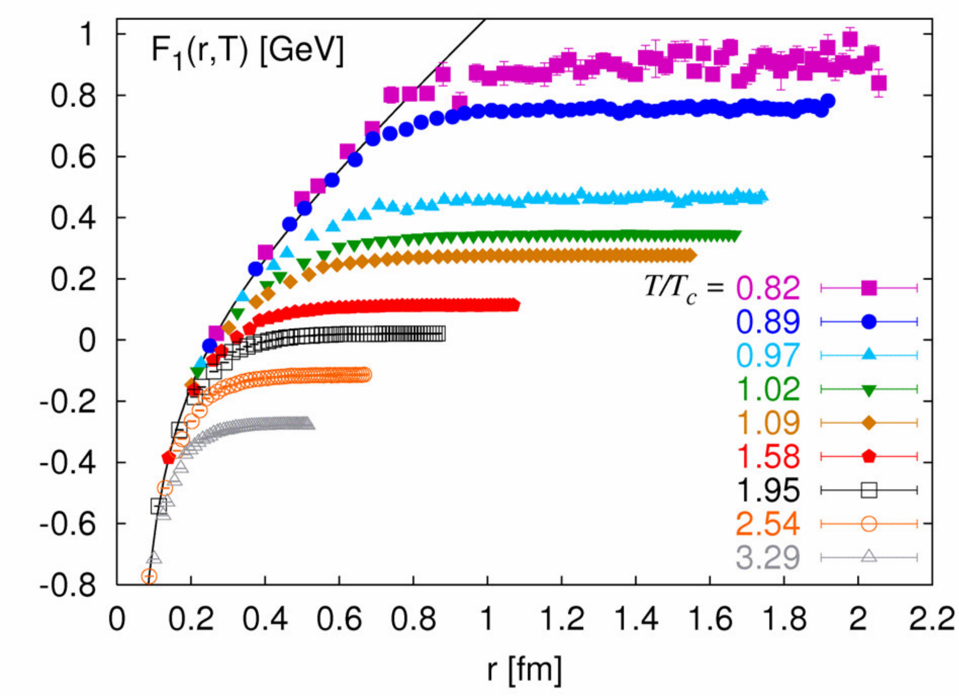


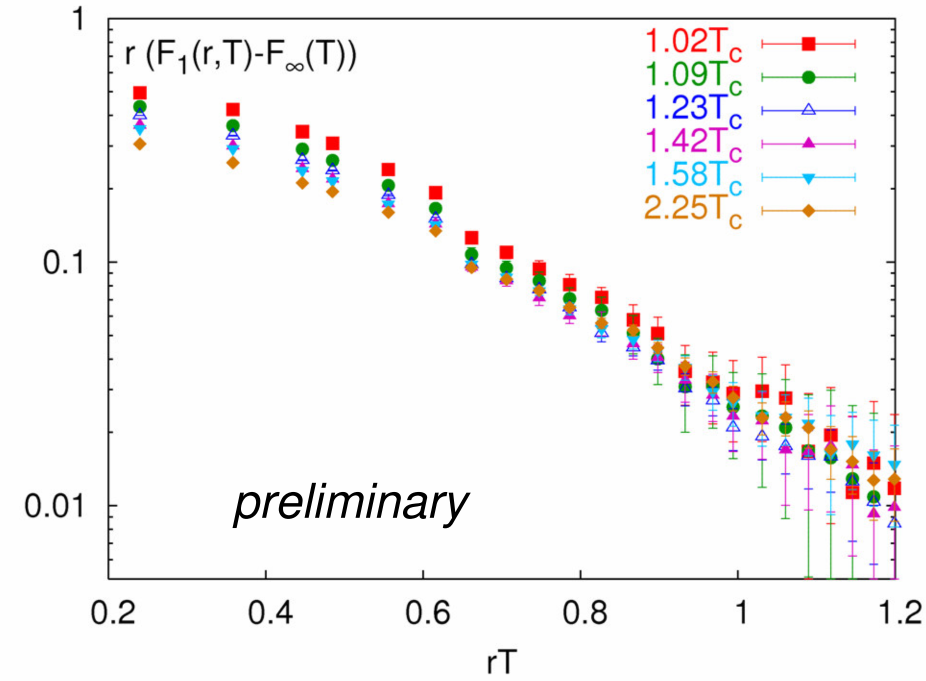
there is a strong dependence on the default model $m(\omega)$ at finite temperature or for small τ_{max}

$\Downarrow$

MEM is not a reliable for reconstructing spectral functions at finite temperature

Screening of static color charges in QCD





$F_1(r,T)$ is temperature independent for

$r < \frac{0.4\text{fm}}{T/T_c}$

$\Downarrow$

r is the dominant scale:

$\alpha_{eff}(r, T > 1.1T_c) < 0.6$

$$F_1(r \gg 1/T) = -\frac{4}{3}\alpha_s(T)\frac{e^{-m_D r}}{r} + F_\infty$$

$F_1(r,T)$ scales with T and is exponentially screened for $r > 0.8/T$

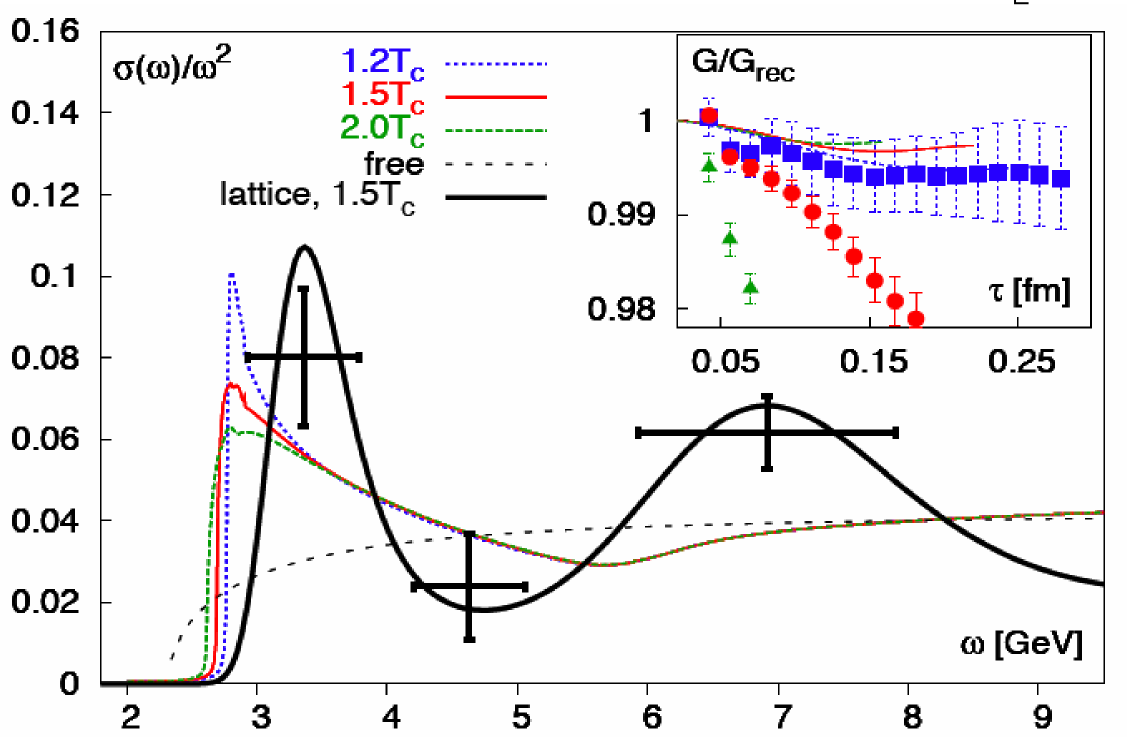
$\Downarrow$

T is the dominant scale:

Can 1S charmonia state survive deconfinement ?

What about color screening ?

$$\left[-\frac{1}{m}\nabla^2 + V(\vec{r}) + E\right] G^{NR}(\vec{r},\vec{r}\,',E) = \delta^3(\vec{r}-\vec{r}\,')$$



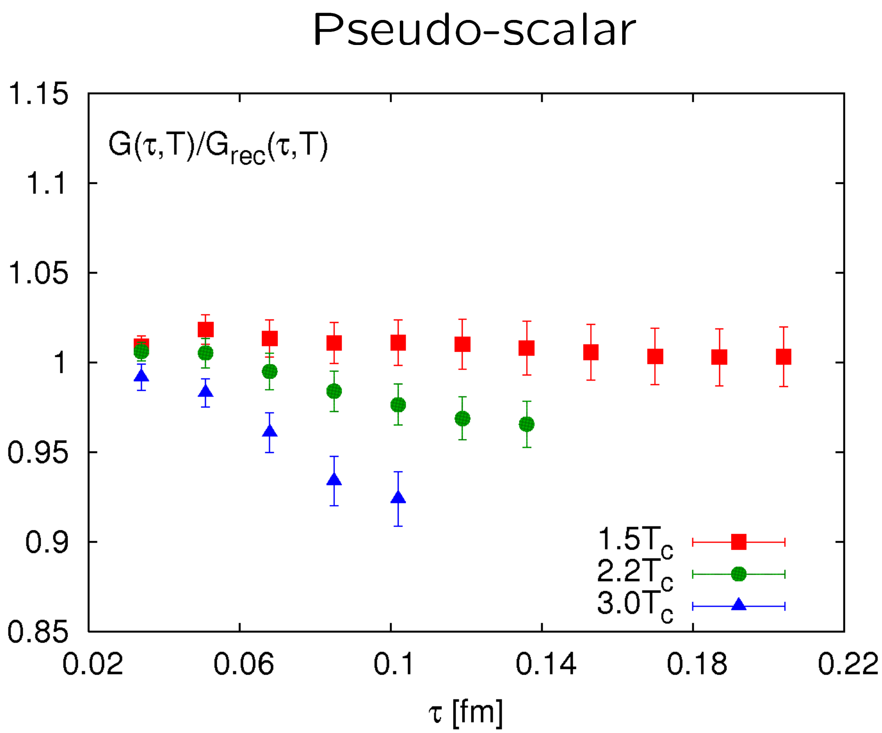
$$\sigma(E) = \frac{2N_c}{\pi} \mathrm{Im}\, G^{NR}(\vec{r},\vec{r}\,',E)_{\vec{r}=\vec{r}\,'=0}$$

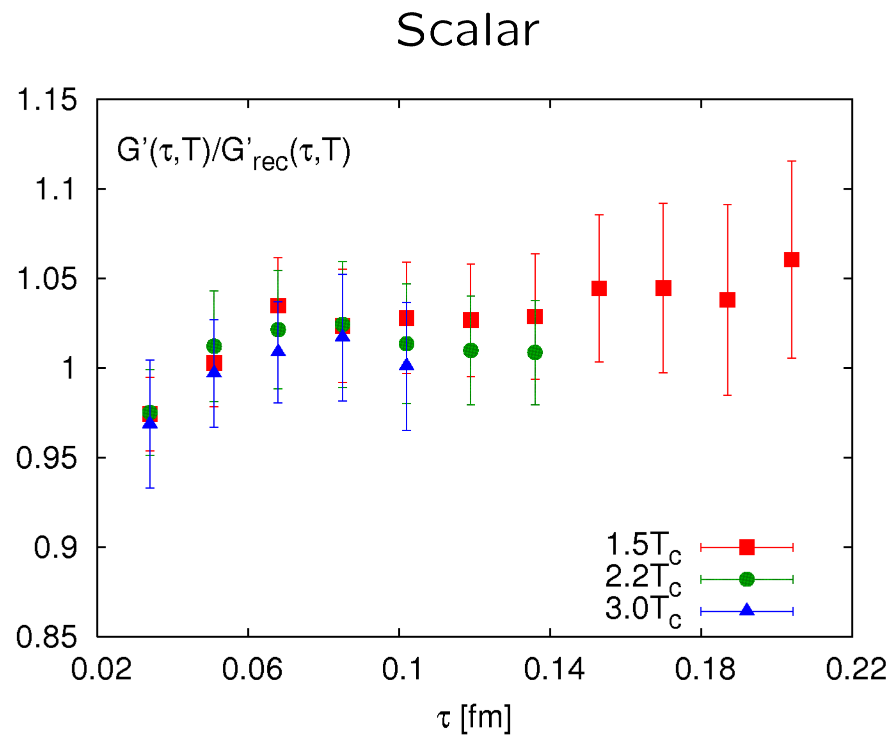
Mócsy, P.P, PRD 77 (2008) 014501
PRL 99 (2007) 211602

similar conclusions:
Burnier, Laine, Vepsalainen
JHEP 0801 (2008) 04,
Laine et al,
JHEP 0703 (2007) 054

- resonance-like structures disappear already by 1.2Tc
- strong threshold enhancement
- contradicts lattice results ? No !

Temperature dependence of quarkonium





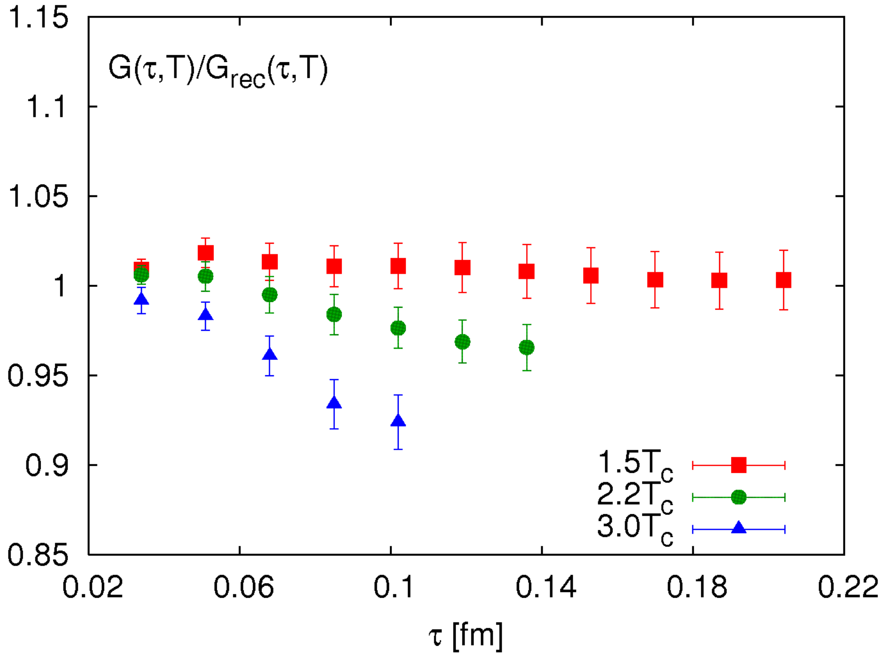
zero mode contribution is not present in the time derivative of the correlator
Umeda, PRD 75 (2007) 094502

No change in the derivative of the scalar quarkonium correlator up to $3T_c$! Almost the entire temperature dependence of the scalar correlators is given by the zero mode contribution !

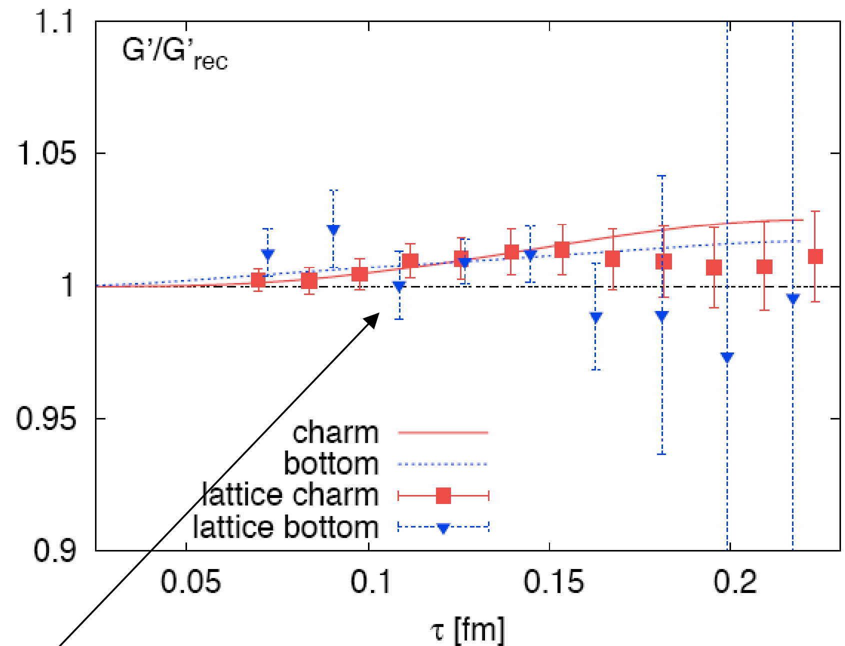
In agreement with potential models Mócsy, P.P, PRD 77 (2008) 014501

Temperature dependence of quarkonium

Pseudo-scalar



Scalar



zero mode contribution is not present in the time derivative of the correlator

Umeda, PRD 75 (2007) 094502

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In agreement with potential models   Mócsy, P.P, PRD 77 (2008) 014501

Quarkonium correlators in Euclidean time

$$G_i(\tau,T)=\int_0^\infty d\omega \sigma_i(\omega,T)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

$i = vc, sc, ax$

$$G_i(\tau,T)=G_i^{\text{low}}(\tau,T)+G_i^{\text{high}}(\tau,T)$$

The high energy part can be estimated from the low T spectral functions: $G^{\text{high}}(\tau,T)\simeq G_{rec}(\tau,T)$

Since $\eta=\frac{T}{M}\frac{1}{D}\ll T \Rightarrow \sigma_i^{\text{low}}(\omega)\simeq \chi^i(T)\omega\delta(\omega)$

$G^{\text{low}}(\tau)\simeq \chi^i(T)T$ - zero mode contribution

$\chi^i(T)$ can be calculated for free quarks:

$$\chi^i(T)=\frac{6}{\pi^2}\int_0^\infty dp p^2\left(a_i+b_i\frac{M^2}{E_p^2}+c_i\frac{p^2}{E_p^2}\right)\left(-\frac{\partial n_F}{\partial E_p}\right)$$

$a_{sc}=0,\ a_{ax}=1,\ a_{vc}=0;$
$b_{sc}=1,\ b_{ax}=2,\ b_{vc}=0;$
$c_{sc}=0,\ c_{ax}=0,\ c_{vc}=1;$

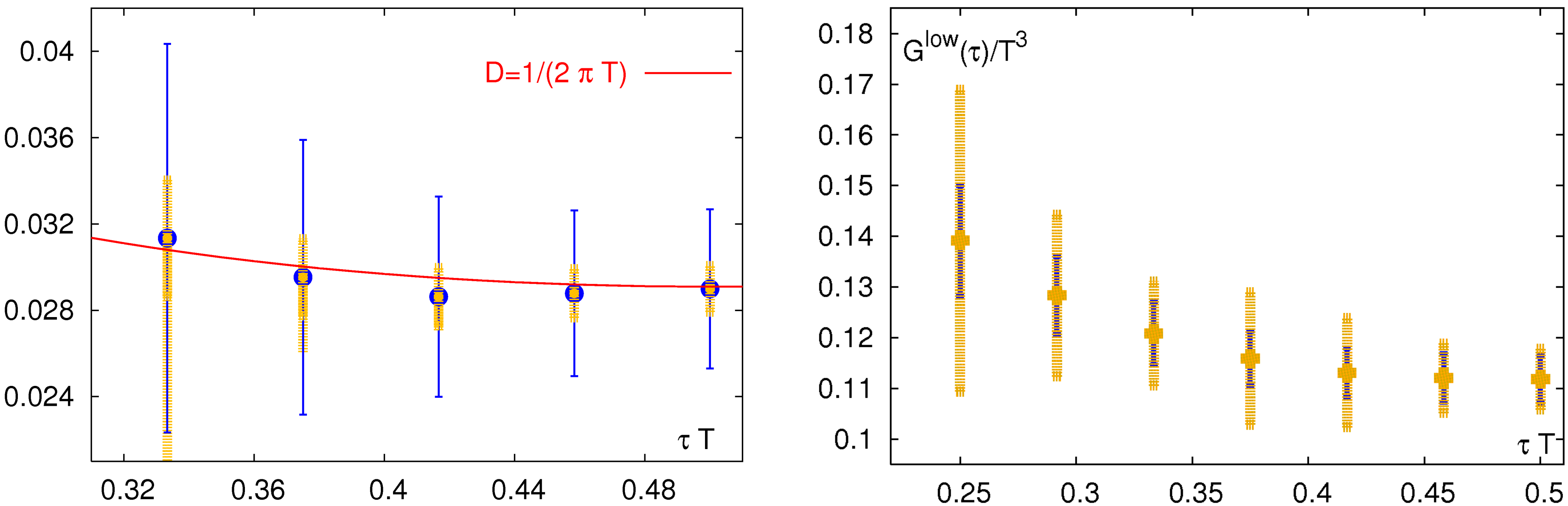
$E_p^2=p^2+M^2,\quad n_F=1/(\exp(E_p/T)+1)$

for temporal component of the vector correlator $a_0=-1,\ b_0=c_0=0$
and $G_{00}=-T\chi(T)$ (exact)

Estimating the zero mode contribution

$$G^{\mathrm{low}}(\tau,T) = G(\tau,T) - G_{rec}(\tau,T)$$

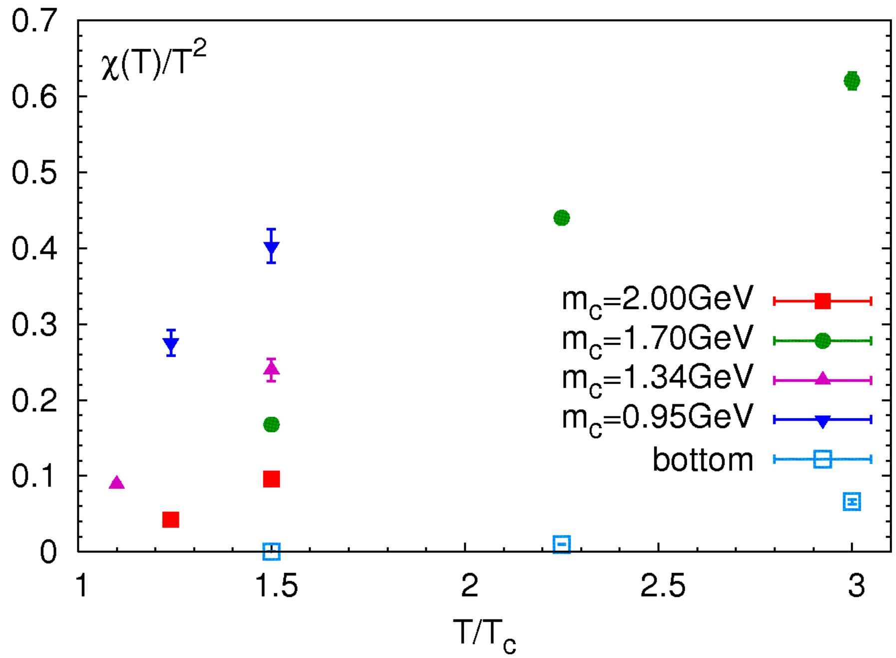
vector $1.5T_c$ axial-vector

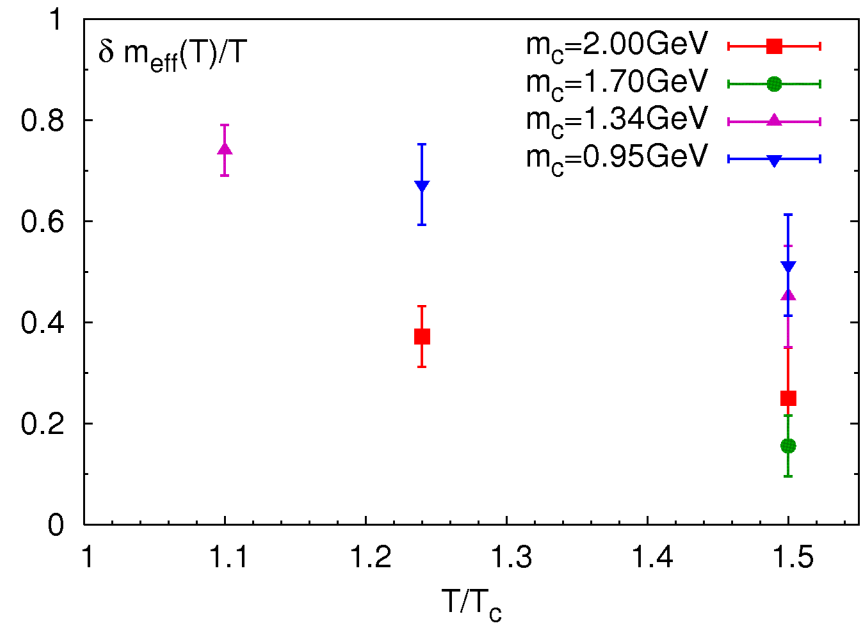


The curvature of $G_i^{\mathrm{low}}(\tau,T)$ is governed by heavy quark diffusion
No diffusion ($D=\infty \leftrightarrow \eta=0$): $G_i^{\mathrm{low}} = const = T\chi_i(T)$

$G_i^{\mathrm{low}}(\tau,T)$ is τ-idependent within errors and
$\chi_i(T) \simeq G_i^{\mathrm{low}}(\tau = 1/(2T), T)$

Temporal component of the vector correlator





Fit $\chi(T)$ using quasi-particle model with T-dependent effective quark mass

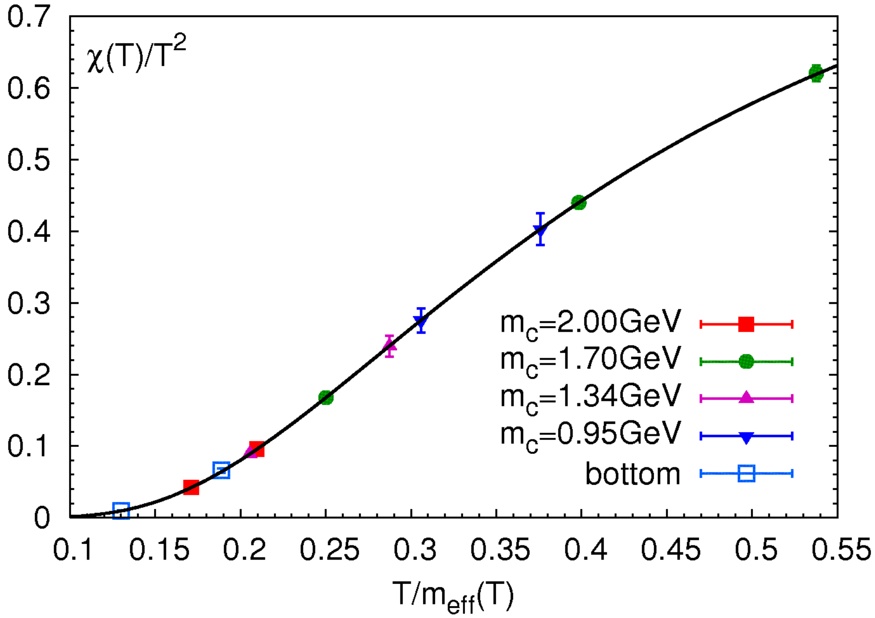
$$\chi(T) = \frac{6}{\pi^2}\int_0^\infty dp p^2 \left(-\frac{\partial n_F}{\partial E_p}\right), \quad E_p^2 = p^2 + m_{eff}^2(T)$$

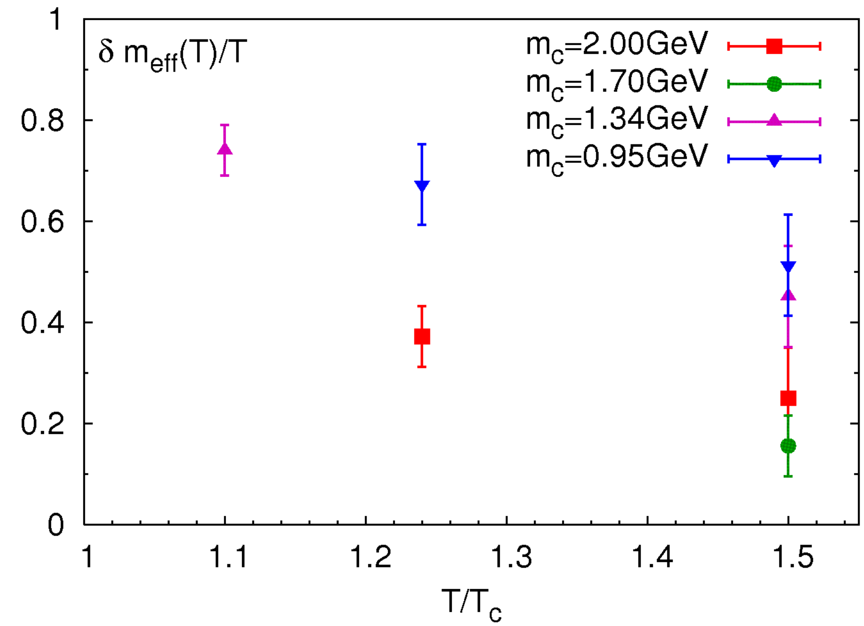
$$\delta m_{eff}(T) = m_{eff}(T) - m_c$$

negligable for $T > 1.5 T_c$

decreases with increasing m_c

Temporal component of the vector correlator





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$$\delta m_{eff}(T) = m_{eff}(T) - m_c$$

negligable for $T > 1.5T_c$

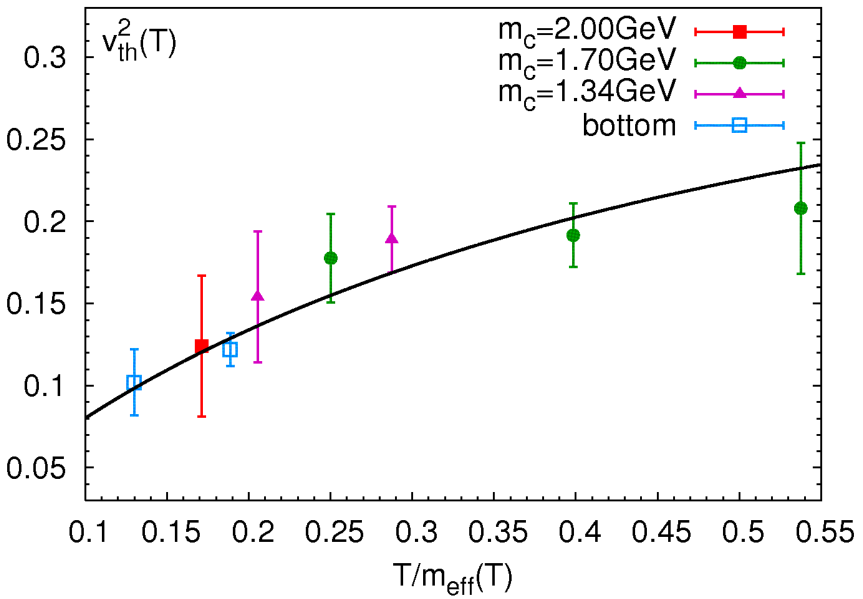
decreases with increasing m_c

negligable for bottom quarks

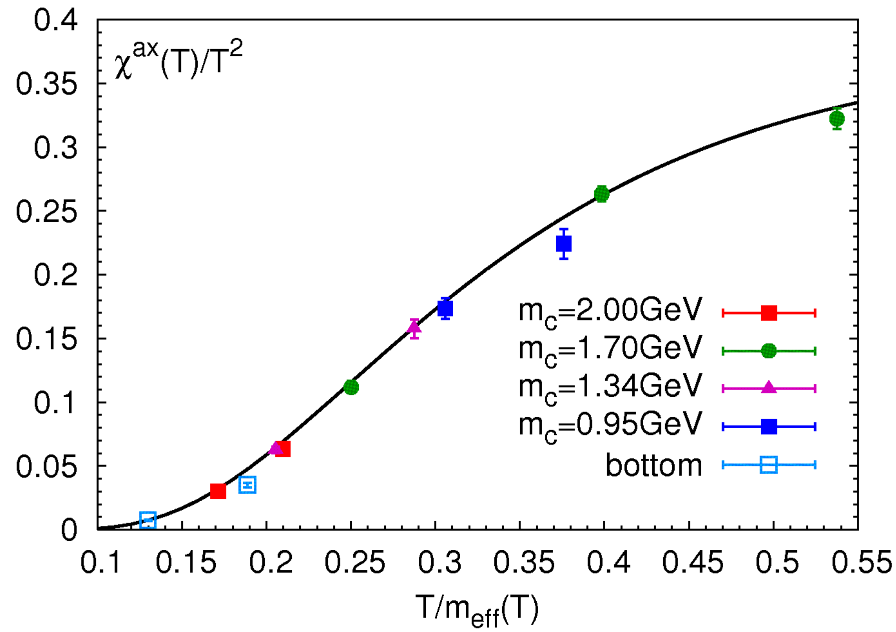
Zero mode contribution in the vector and axial-vector correlator

Vector

$$G^{\text{low}}_{Vii}/G_{00} \simeq \frac{\int d^3p \frac{p_i^2}{E_p^2} e^{-E_p/T}}{\int d^3p e^{-E_p/T}} \simeq v_{th}^2$$



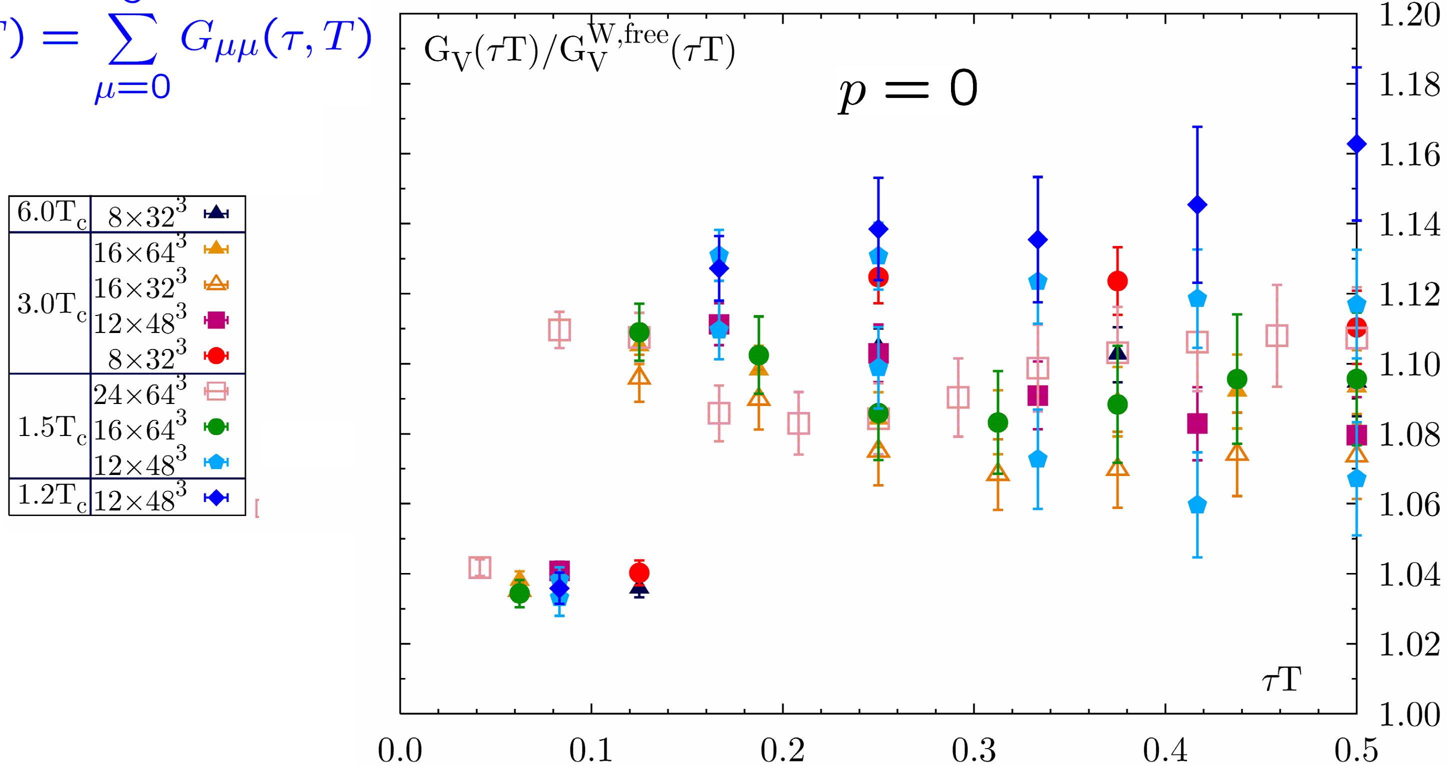
Axial-vector



The zero mode contribution is function of T/m_{eff} only and well described by the free gas limit

Vector correlators in the light quark sector

$$G_V(\tau,T) = \sum_{\mu=0}^{3} G_{\mu\mu}(\tau,T)$$

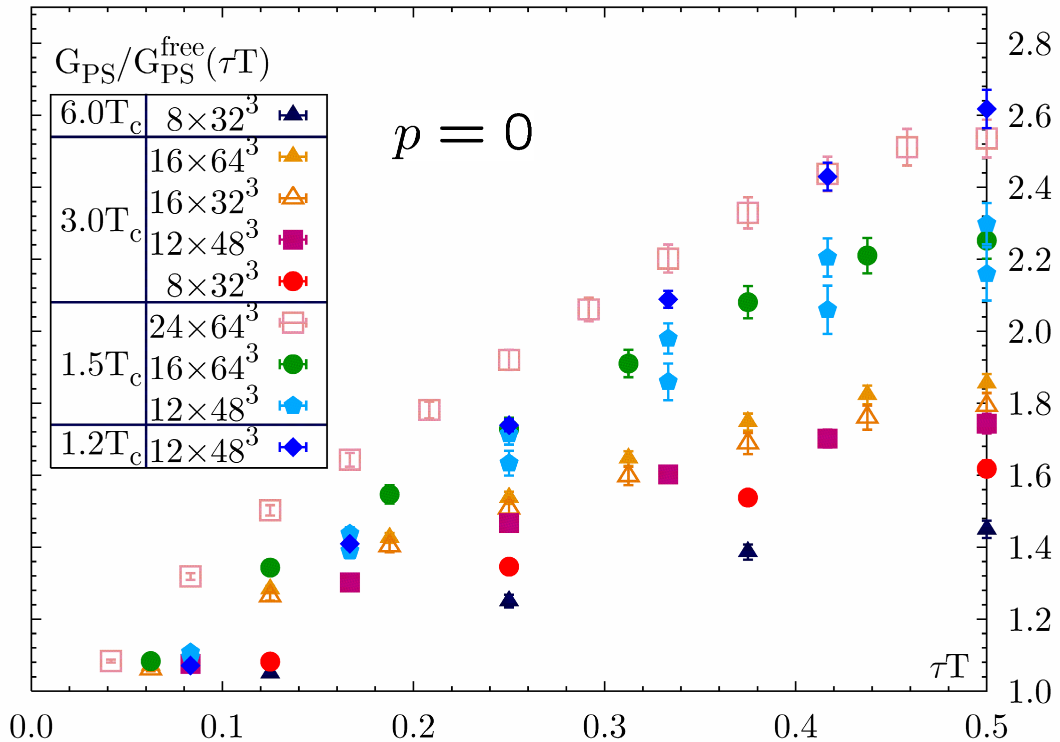


Lattice spacing dependence is small !

$$G(\tau = \frac{1}{2T}, p, T) = \int_0^\infty d\omega \frac{\sigma(\omega, p)}{\sinh(\omega/(2T))}$$

➡ constraints on the spectral functions at small energies

Pseudo-scalar correlators in the light quark sector



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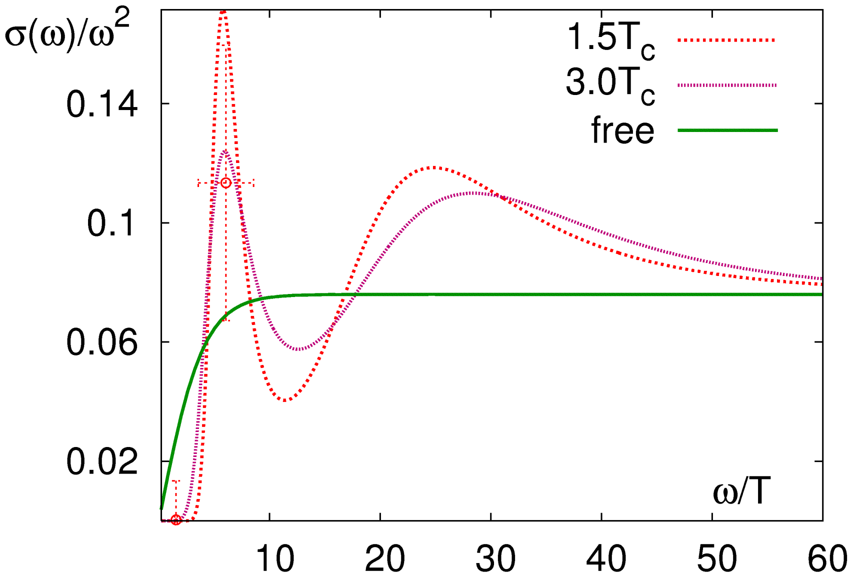
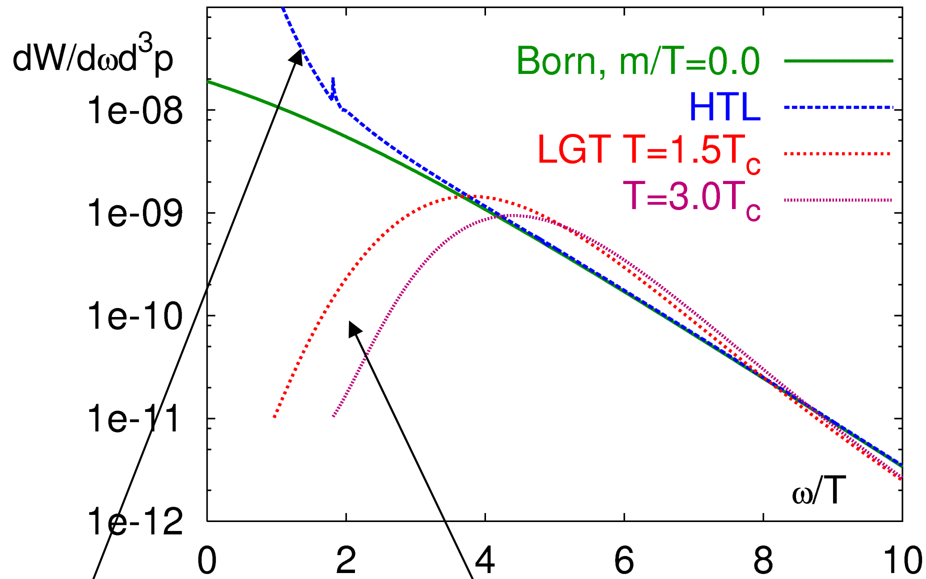
➡ constraints on the spectral functions at small energies

Spectral function and thermal dilepton rate

$$p = 0$$





Karsch, Laermann, Petreczky,
Stickan, Wetzorke, PLB 530 (02) 147

suppression of low mass dileptons

in sharp contradiction with perturbative expectations
Braaten, Pisarski, Yuan, PRL 64 (90) 2242
Moore, Robert, hep-ph/0607172

Conclusions

- Euclidean correlators contain information about different parts of the spectral functions but it is very difficult to resolve the details with MEM

- Broadening-melting of heavy quark bound states is not reflected in large change in the Euclidean correlators, lattice data are consistent with potential model predictions (melting of quarkonia states)

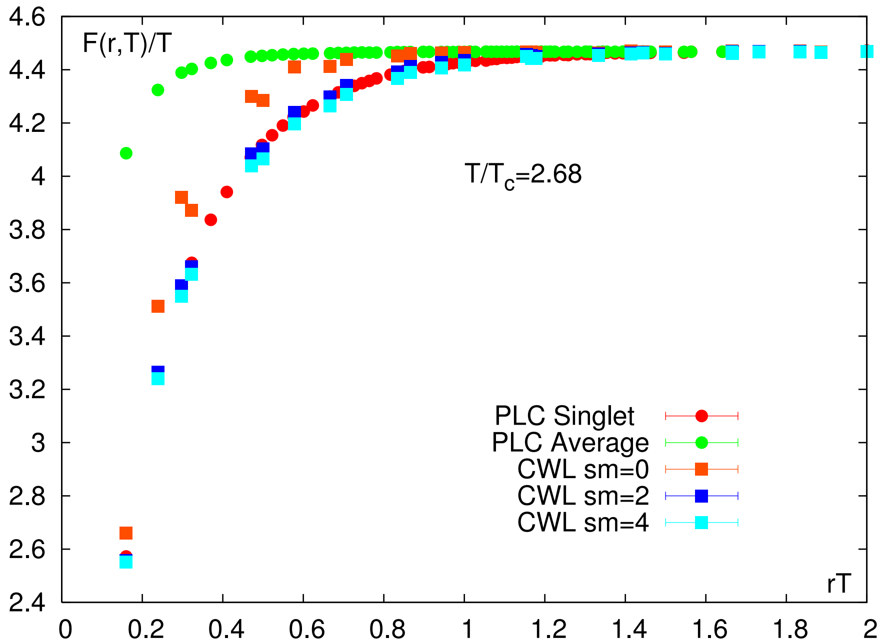
- The zero mode contribution can be identified in the lattice correlators and is the dominant source of the T-dependence of the Euclidean correlators

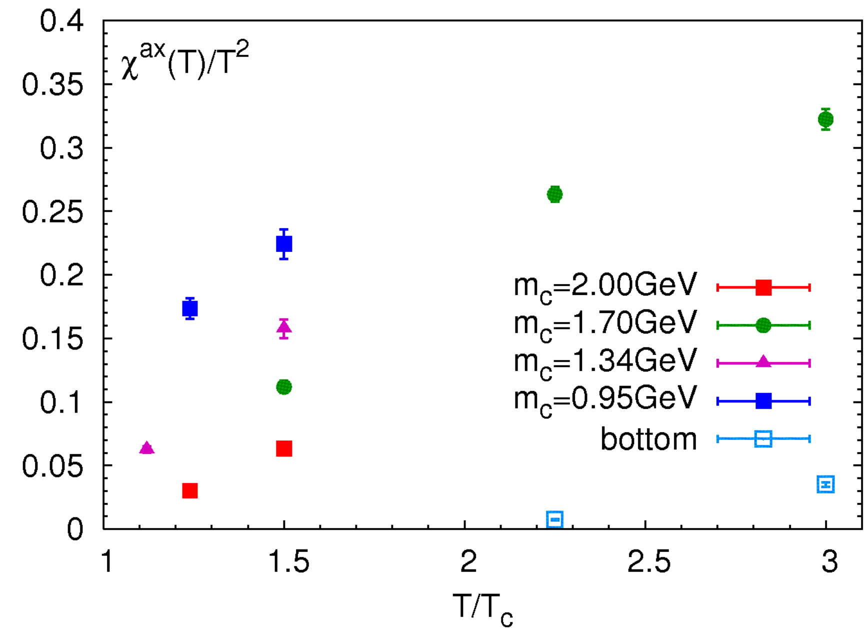
- The zero mode contribution depends on the effective mass of the heavy quarks in units of the temperature and is well described by the free gas limit

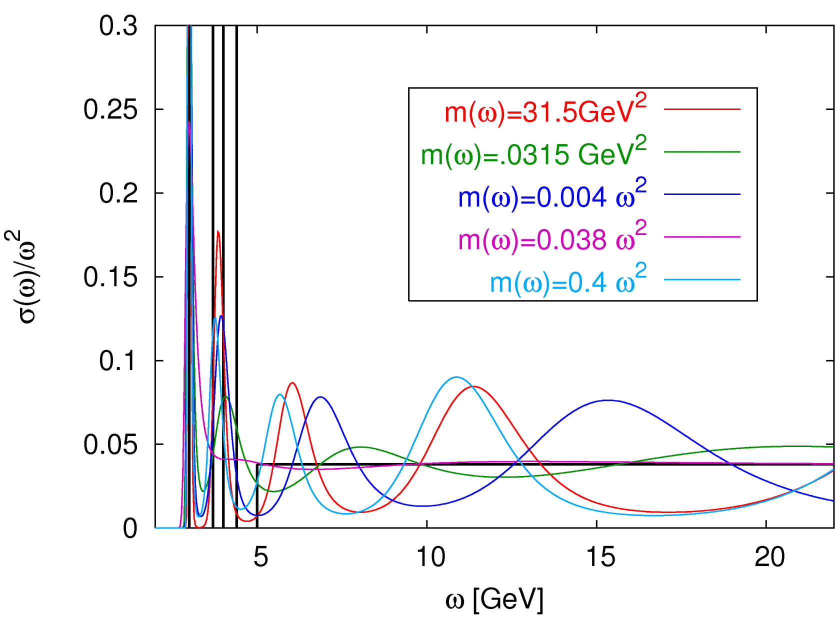
- In the light quark sector the vector correlation functions are close to the free case, while the vector correlation functions shows significant deviation from the free case

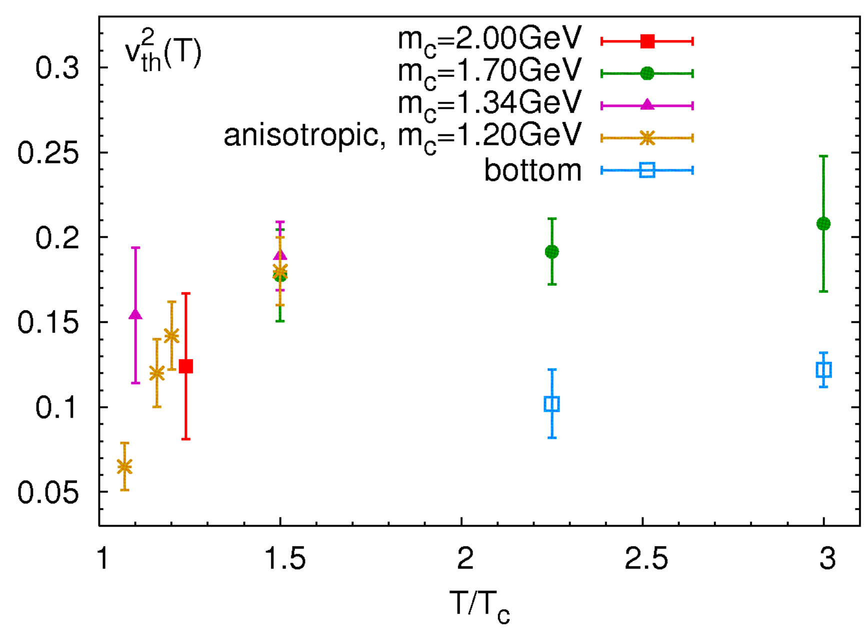
- The vector spectral functions extracted from MEM show significant suppression compared to the free spectral functions (problems with MEM ? strong residual interactions between quarks ?)

Back-up slide



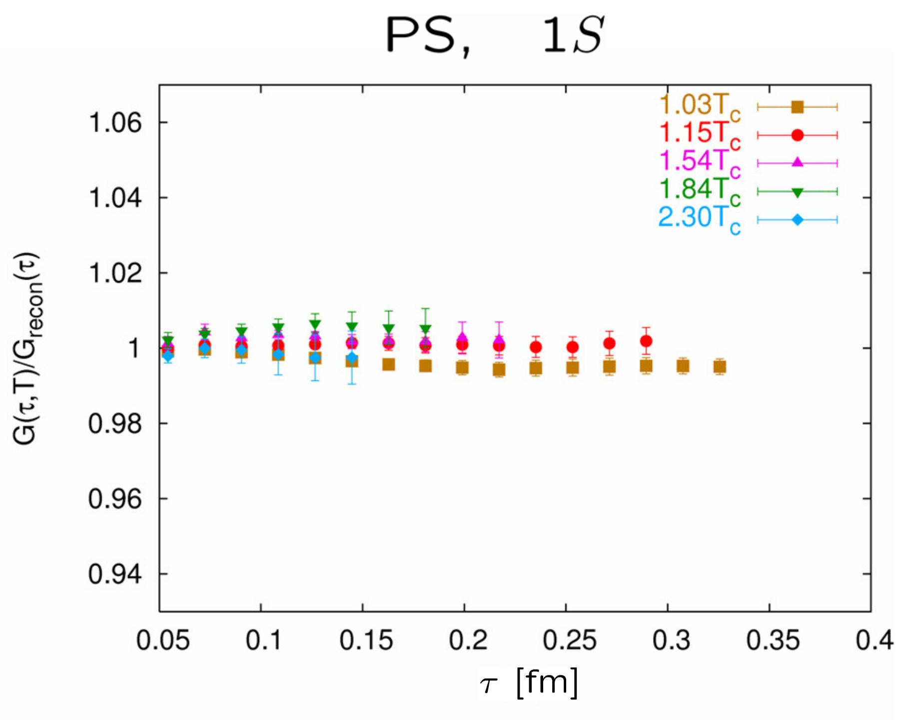


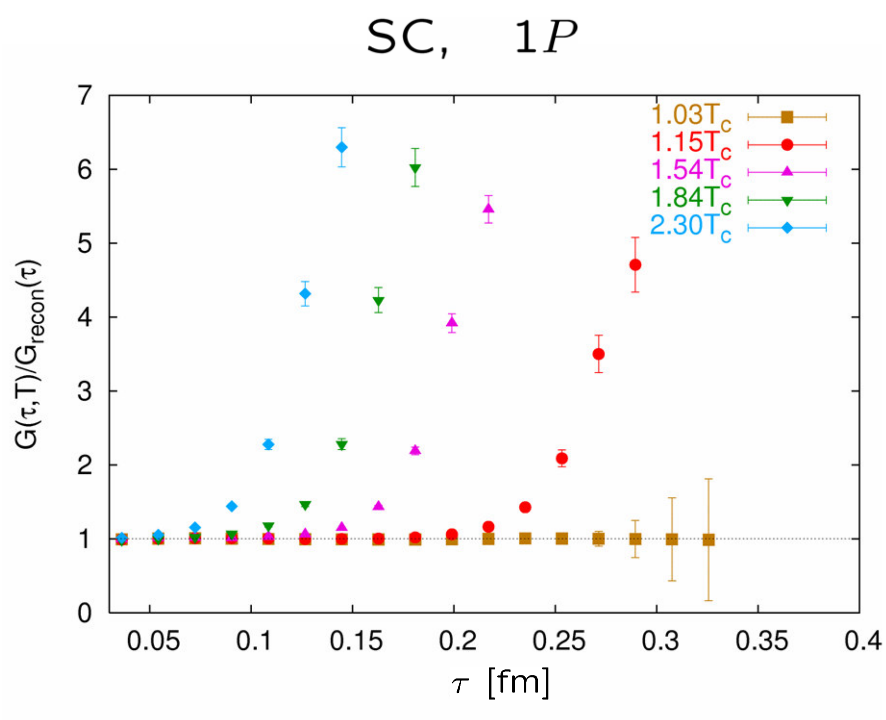




Bottomonium correlators

Jakovác, P.P., Petrov, Velytsky, PRD 75 (07) 014506





T-dependence of the bottomonium correlators is similar to the charmonium ones, but $\langle r^2 \rangle_{\chi_b} \simeq \langle r^2 \rangle_{\eta_c}$

Does the increase in the scalar correlator mean χ_b dissolution ?