Spatio-temporal chaos in rotating Rayleigh-Bénard convection

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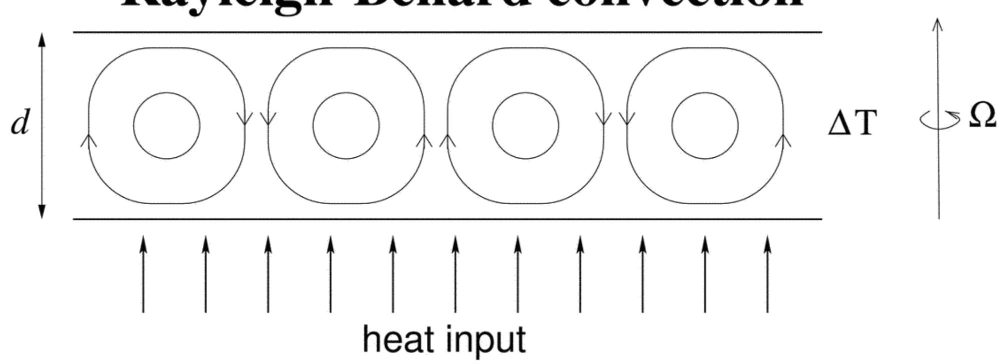
Rayleigh-Bénard convection



Convection occurs for $\Delta T > \Delta T_c$

Reduced temperature difference $\varepsilon = \frac{\Delta T}{\Delta T_c} - 1$

Reduced rotation frequency $\Omega = \frac{2\pi f d^2}{\nu}$

Vertical thermal diffusion time $\tau_v = \frac{d^2}{\kappa}$

Movie of shadowgraph visualization of Küppers-Lortz dynamics

$\varepsilon = 0.14$
$\Omega = 16.2$

Model equations

$$\tau \dot{A} = \varepsilon A + \xi^2 \nabla^2 A + N.L.$$

Scaling laws near onset

$$\tau \propto \varepsilon^{-1} \qquad \xi \propto \varepsilon^{-1/2}$$

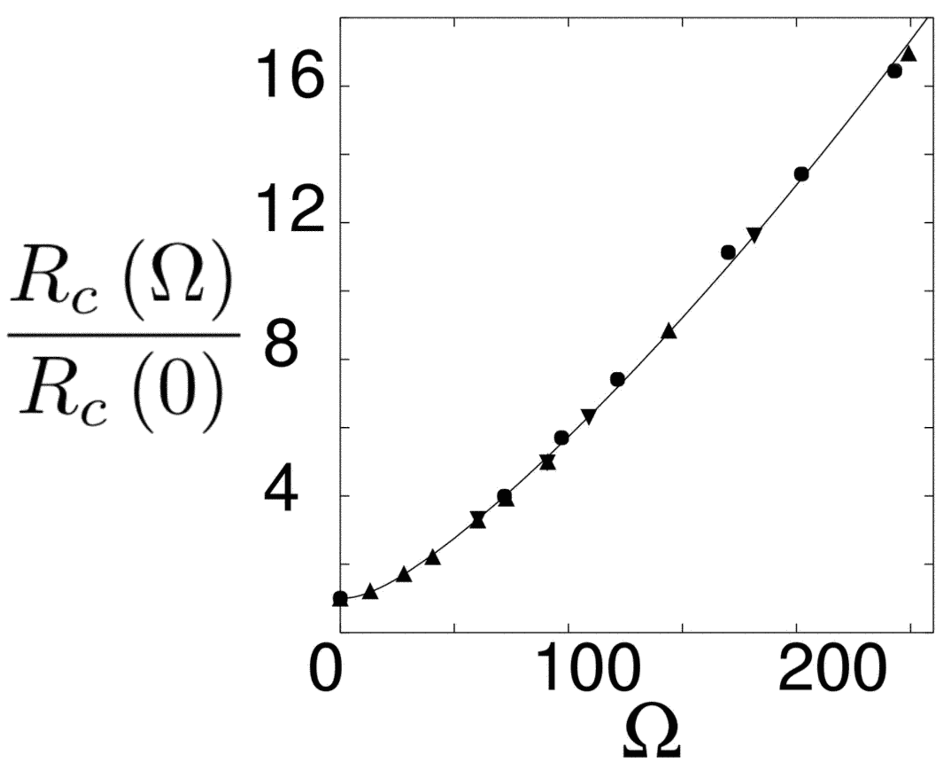
Y. Tu and M.C. Cross, Phys. Rev. Lett. **69** 2515 (1992)

Rayleigh-Bénard convection

Rayleigh number $$R = \frac{\alpha g d^3 \Delta T}{\kappa \nu}$$

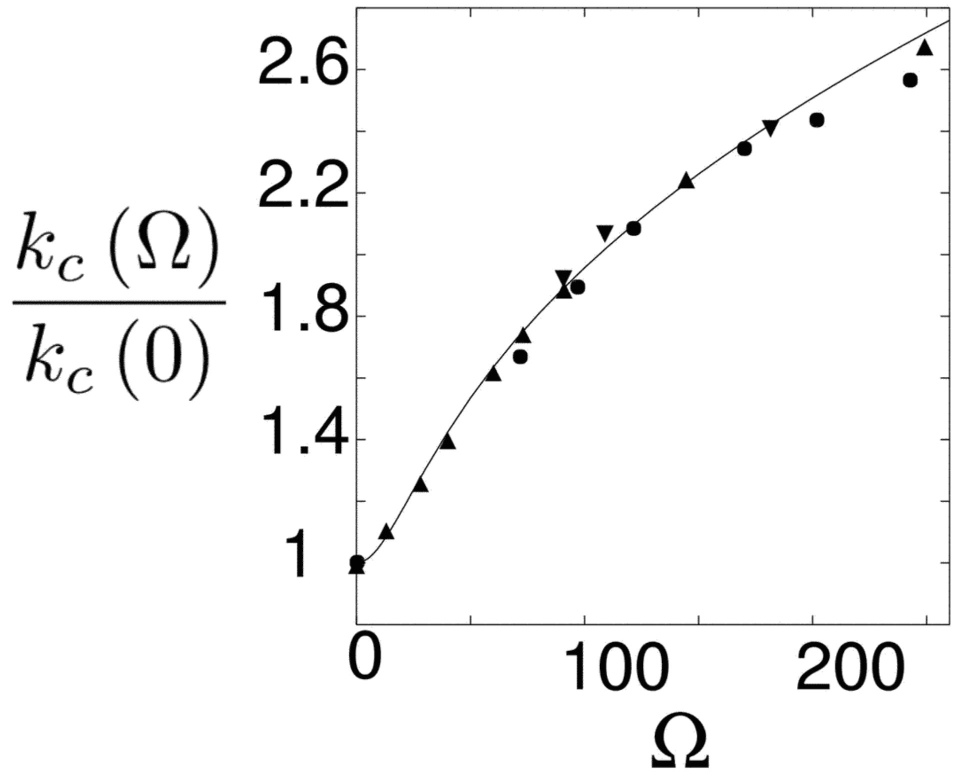
Wave-number $$k = \frac{2\pi d}{\lambda}$$

Onset of convection (from linear stability)



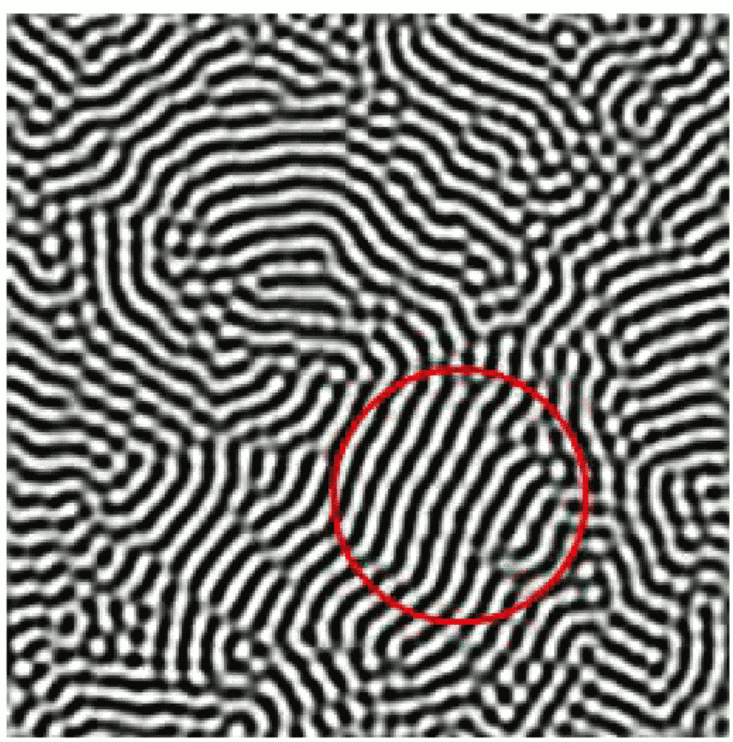
E. Bodenschatz, W. Pesch, and G. Ahlers, Annu. Rev. Fluid Mech. **32**, 709 (2000)

Wave-number at onset (from linear stability)

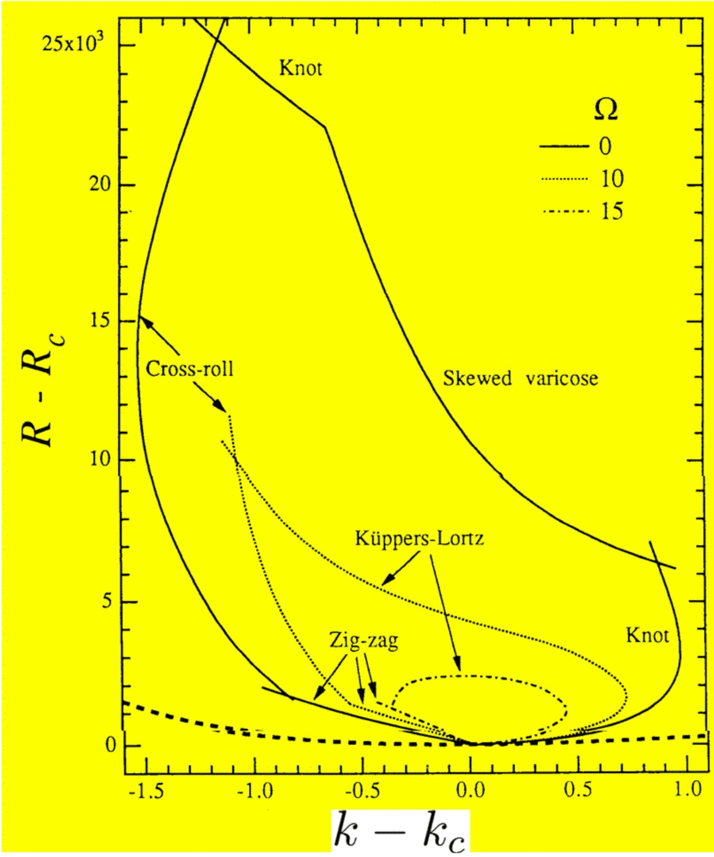


E. Bodenschatz, W. Pesch, and G. Ahlers, Annu. Rev. Fluid Mech. **32**, 709 (2000)

Küppers-Lortz bulk instability



Instability regimes - Busse Balloon



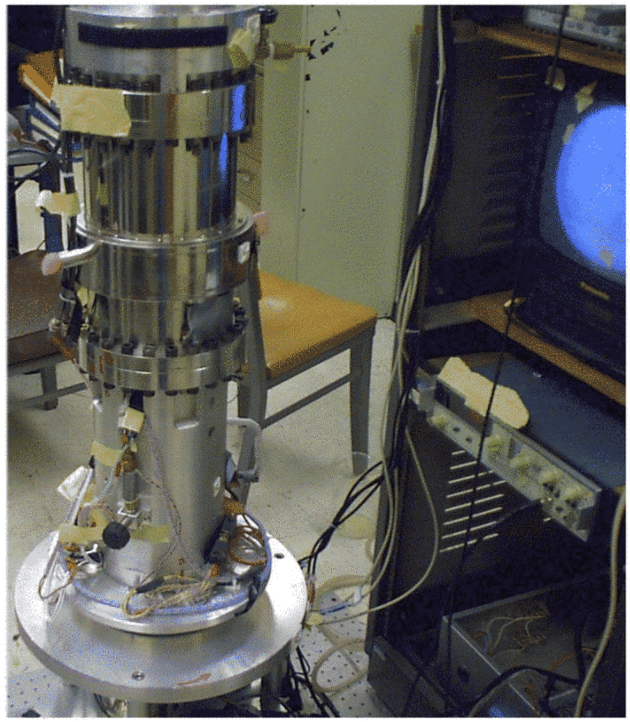
The Küppers-Lortz unstable region shrinks to zero at Ω_c. This results in the straight rolls constantly switching because the modes that grow due to instabilities are themselves unstable to the same instabilities.

From: F. Zhong, R. Ecke, and V. Steinberg, Physica D **51** 596 (1991)

Domain wall motion

E. Bodenschatz, D.S. Cannell, J. R. de Bruyn, R. Ecke, Y. Hu, K. Lerman, and G. Ahlers, Physica D **61**, 77 (1992)

Photos of the rotating gas convection apparatus



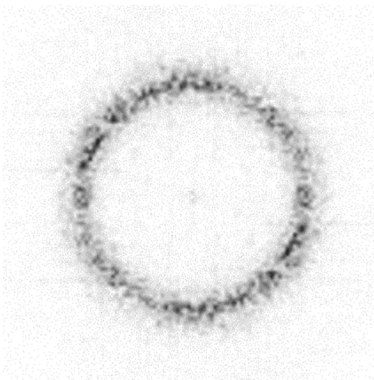
Fourier analysis

Realspace image

Fourier analysis

Realspace image

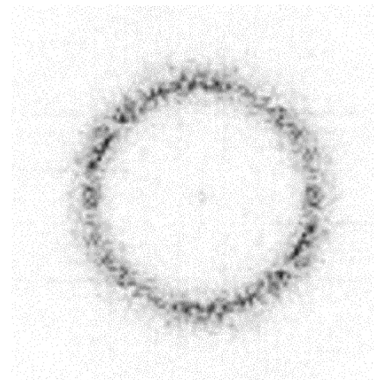
$|FT|^2$



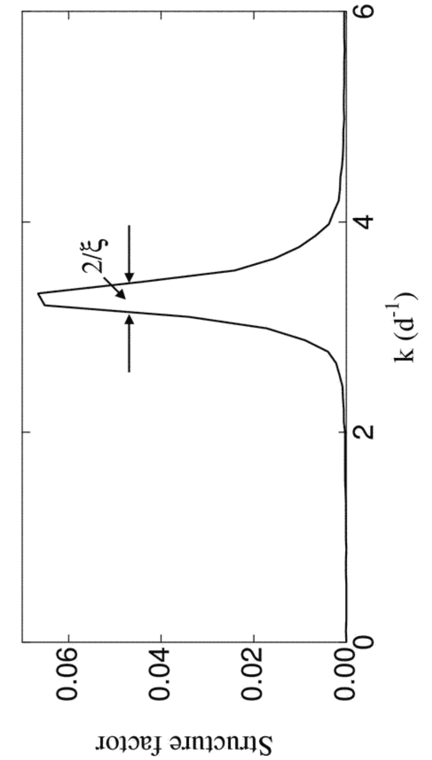
Fourier analysis

Realspace image

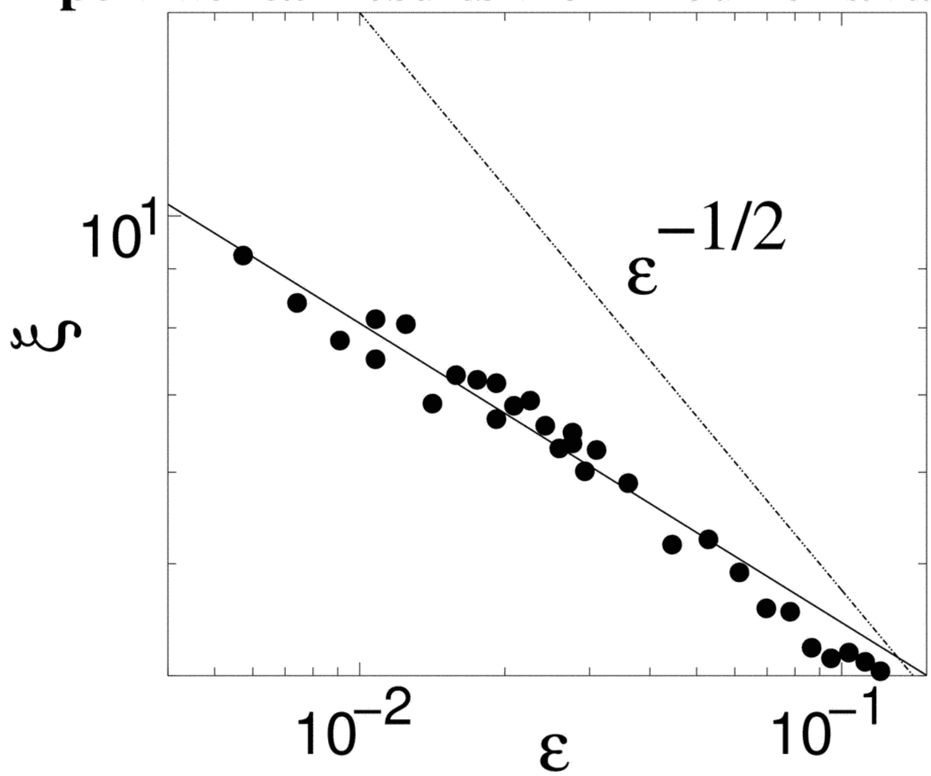
$|FT|^2$



Azimuthal average of $|FT|^2$



Experimental results from Fourier analysis



Y. Hu, R. Ecke, G. Ahlers, Phys. Rev. Lett. **74**, 5040 (1995)

Küppers-Lortz Domains

$\varepsilon = 0.125$
$\Omega = 16.2$

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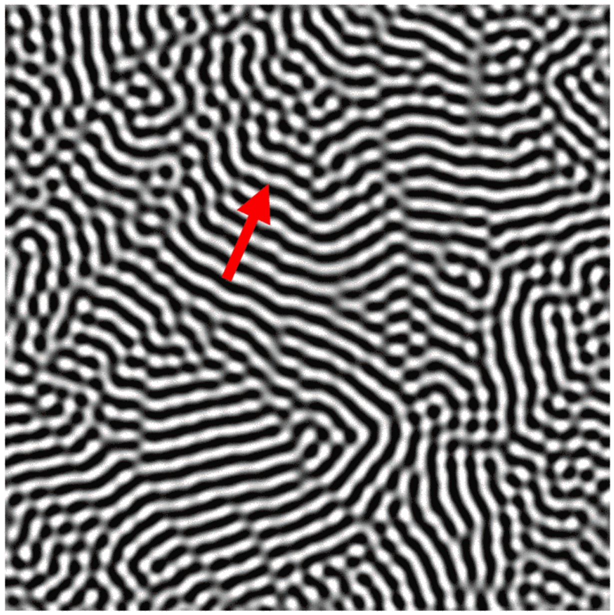
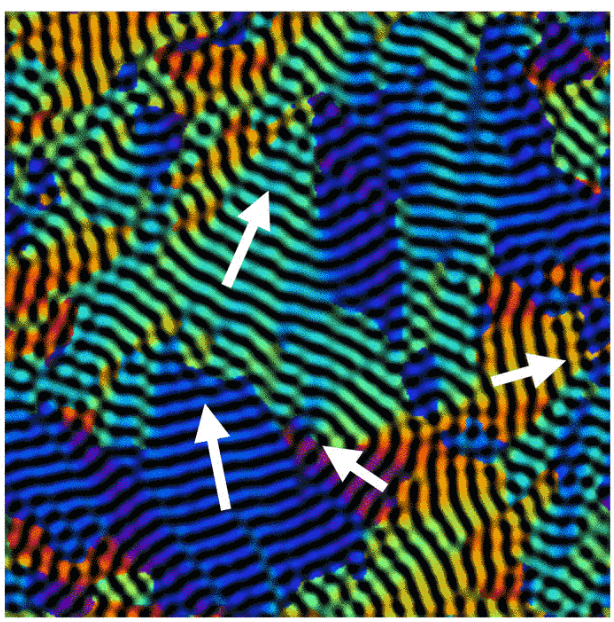
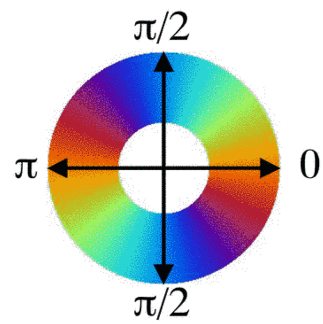


Küppers-Lortz Domains with local wave-director field

$\varepsilon = 0.125$
$\Omega = 16.2$





M.C. Cross, D. Meiron, and Y. Tu, Chaos **4**(4), 607 (1994).
D. Egolf, I. Melnikov, E. Bodenschatz, Phys. Rev. Lett. **80**, 3228 (1998).

Autocorrelation function of the angular component of the local wave-director field

$$C\left(\delta \mathbf{x}\right) = \int \cos 2\left[\theta\left(\mathbf{x}\right) - \theta\left(\mathbf{x} + \delta \mathbf{x}\right)\right] d\mathbf{x}$$

Autocorrelation function of the angular component of the local wave-director field

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•Use a cosine function so that the correlation varies smoothly from -1 to 1.
•Compare angles by looking at their difference.

$\cos\left(2 \times 0\right) = 1$ $\cos\left(2 \times \pi\right) = 1$

$\cos\left(2 \times \frac{\pi}{2}\right) = -1$

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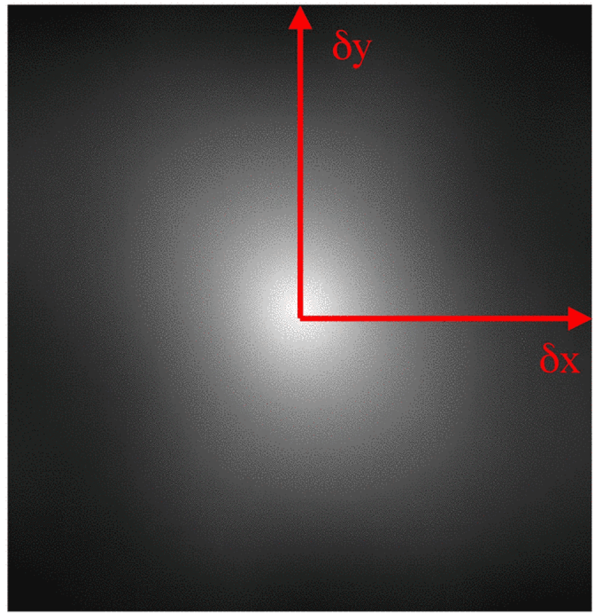
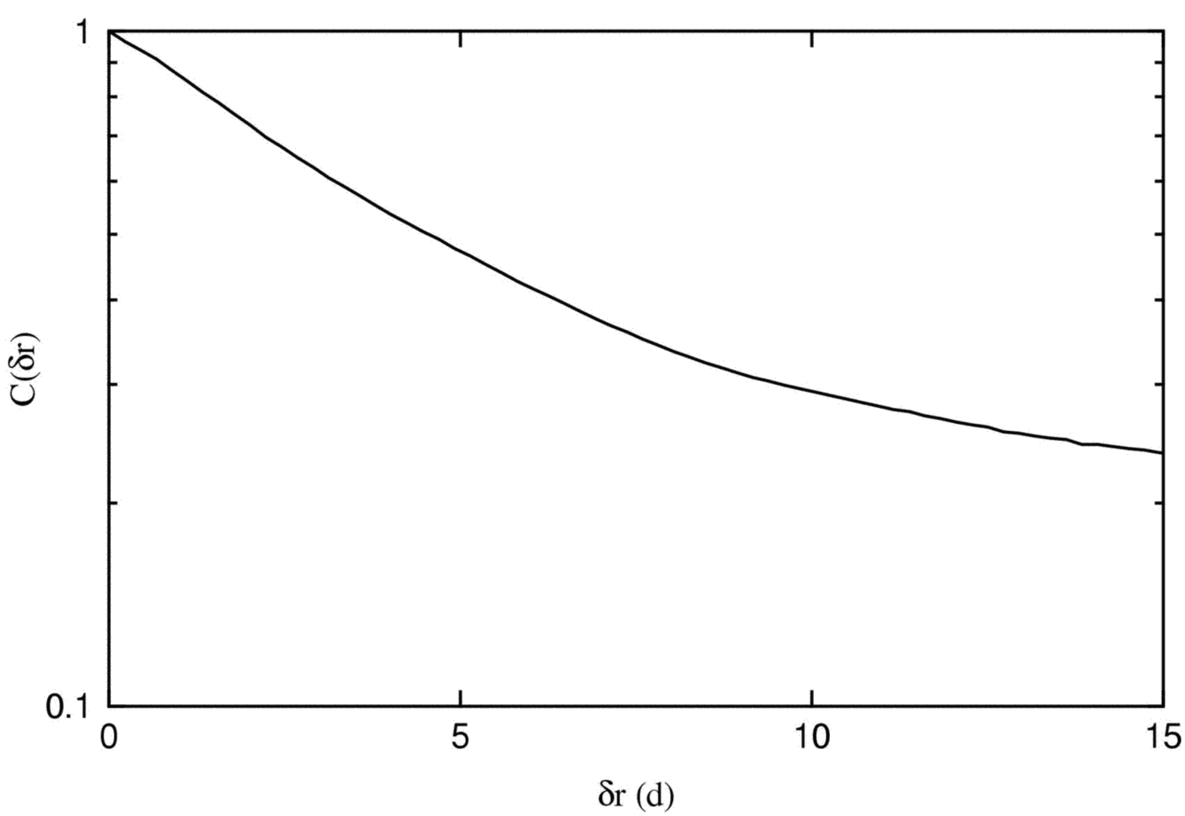
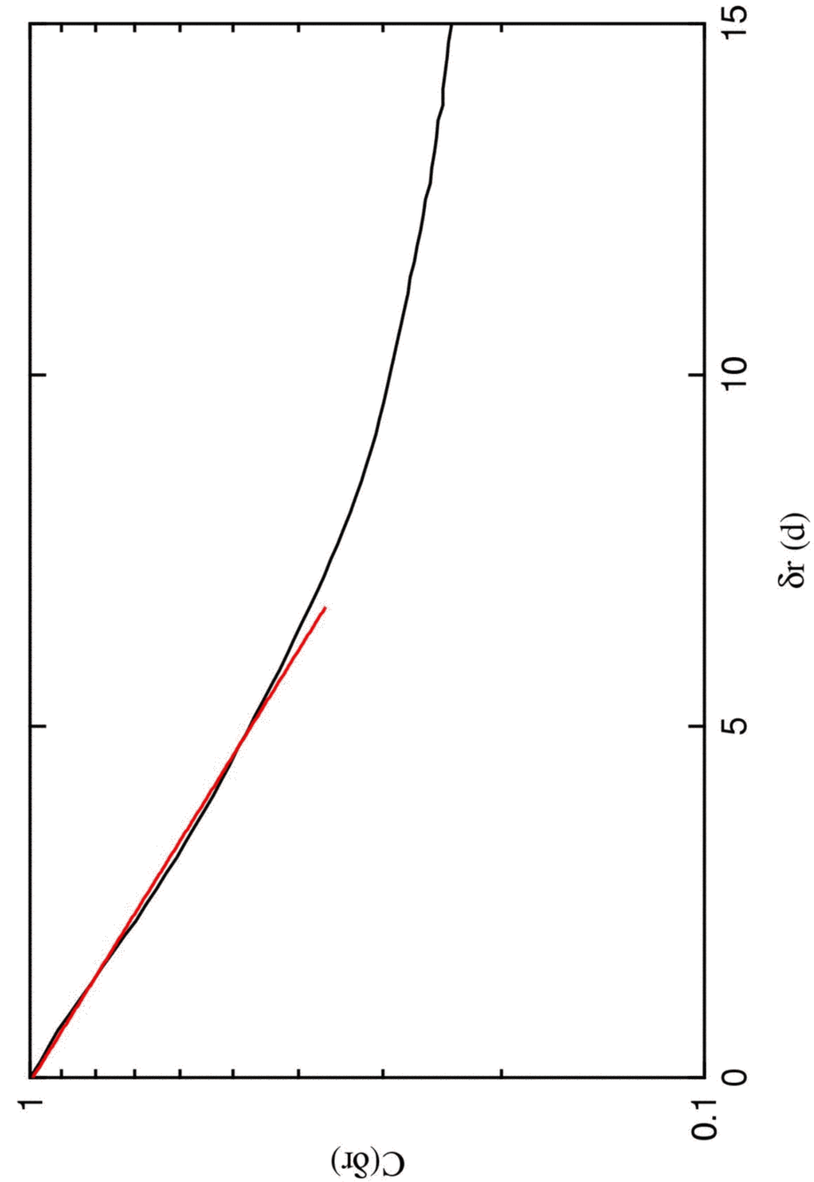
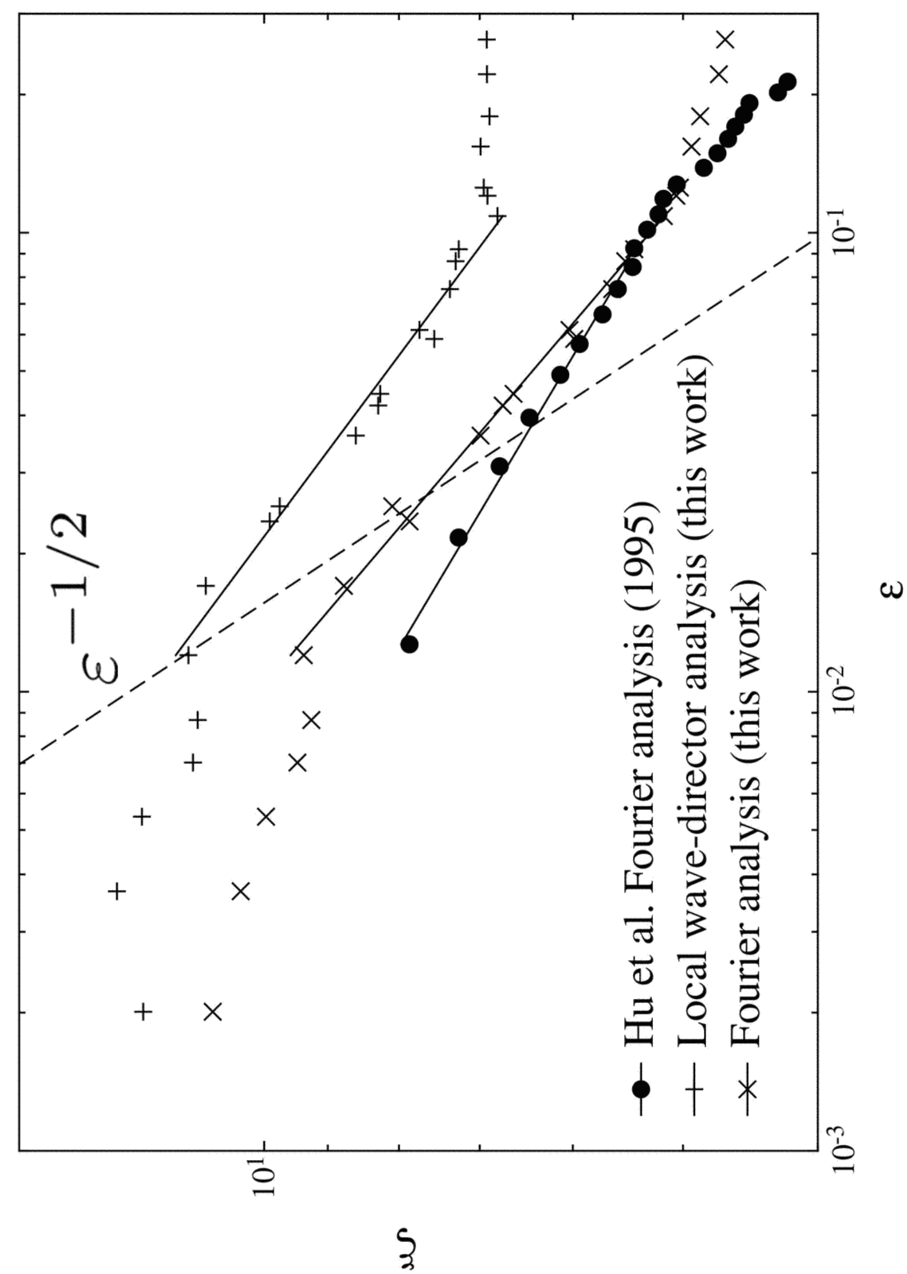


Azimuthal average of autocorrelation



Azimuthal average of autocorrelation





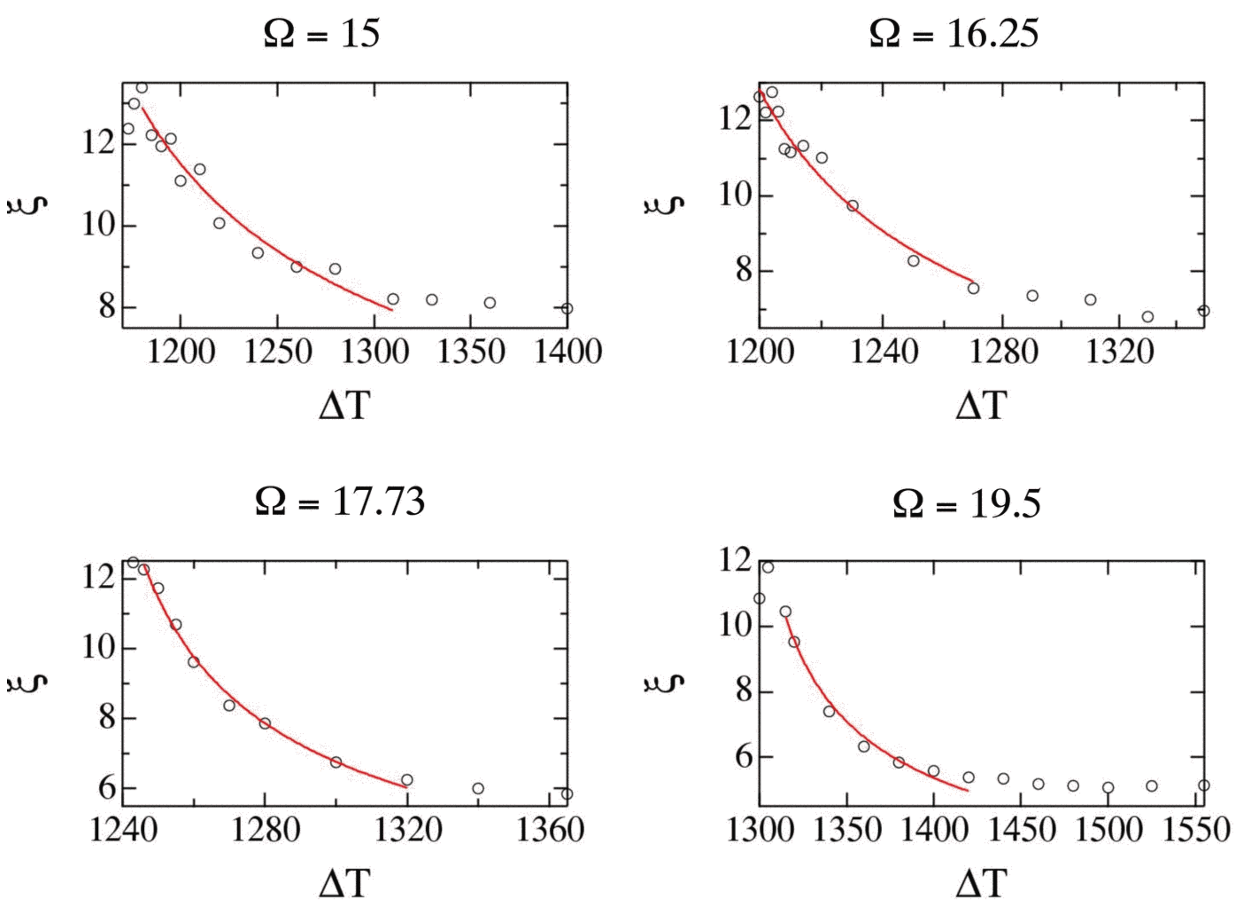
Theoretical prediction revisited

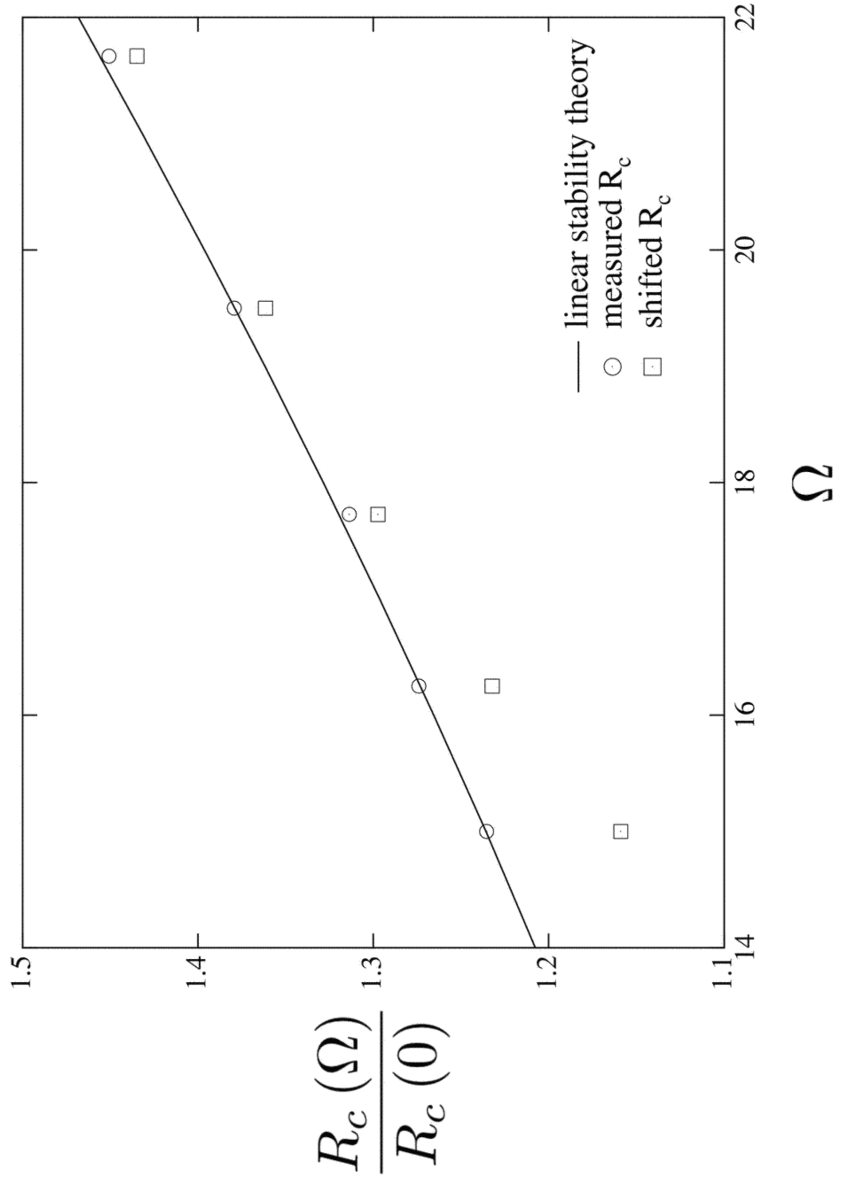
$$\xi \propto \varepsilon^{-1/2}$$

$$\varepsilon \equiv \frac{\Delta T}{\Delta T_c} - 1$$

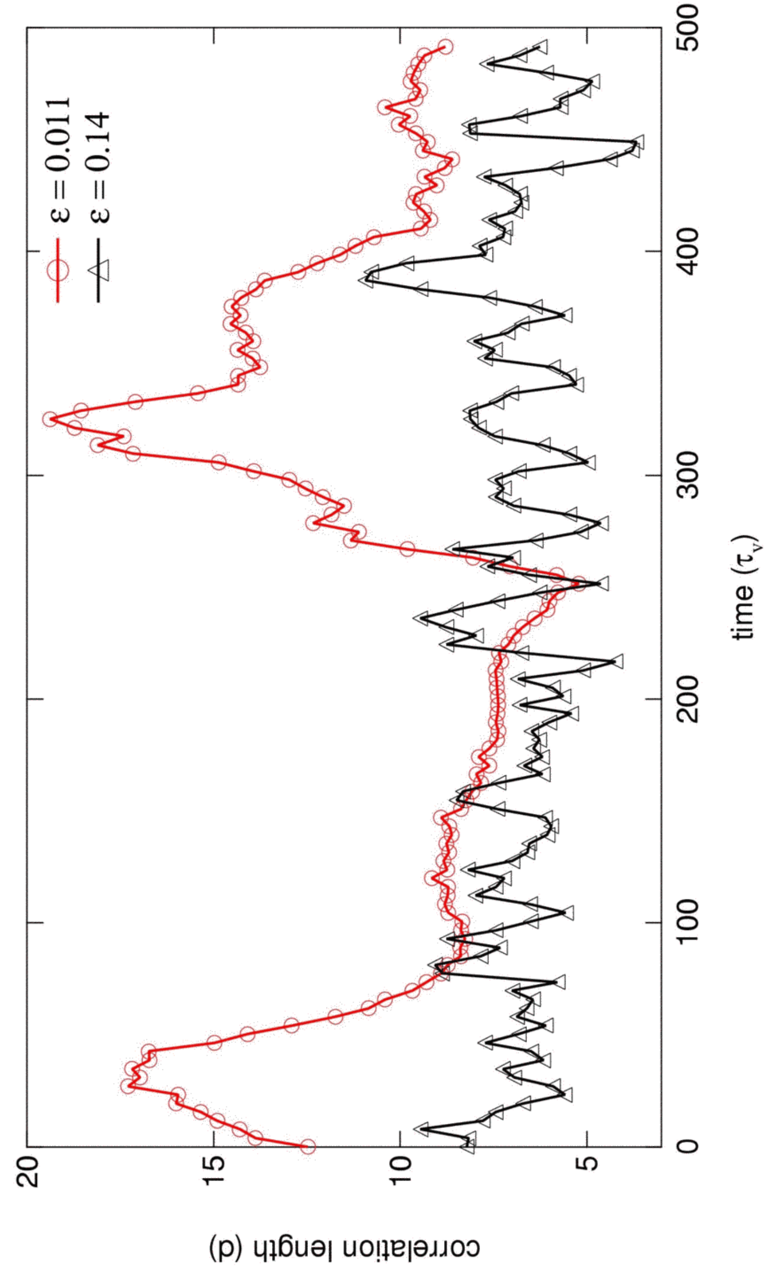
Fitting equation with adjustable onset temperature

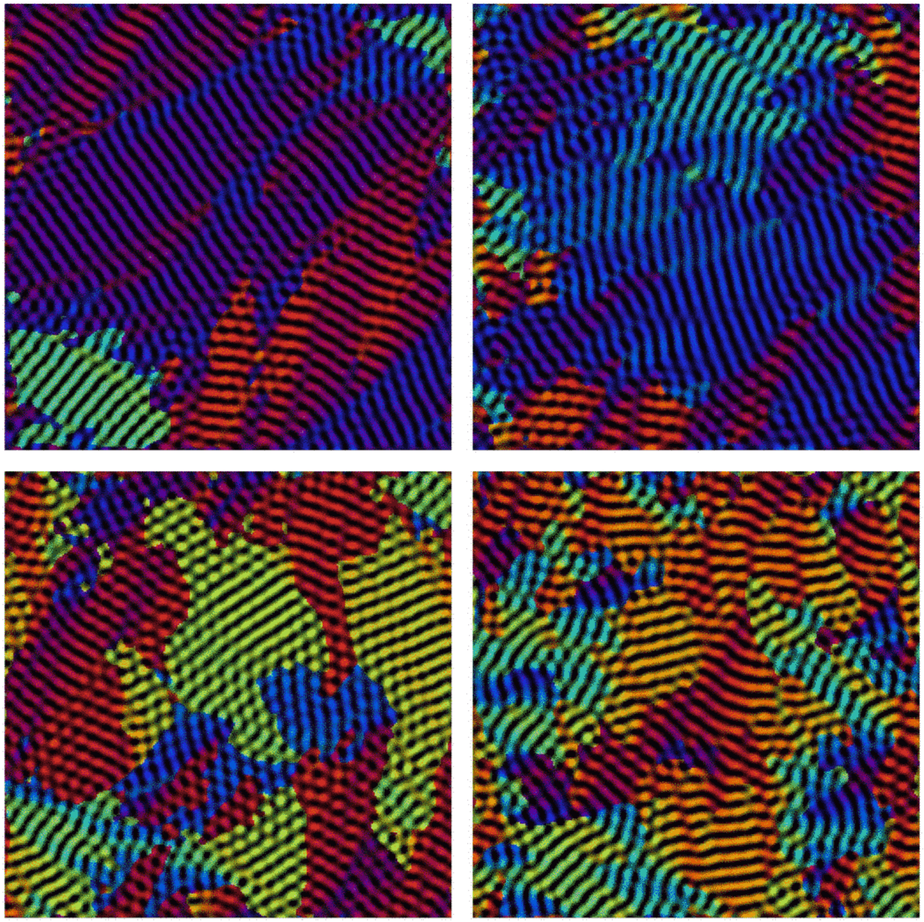
$$\xi \propto \left(\frac{\Delta T}{\Delta T_c} - 1 \right)^{-1/2}$$





Time dependence of exponential fit parameter

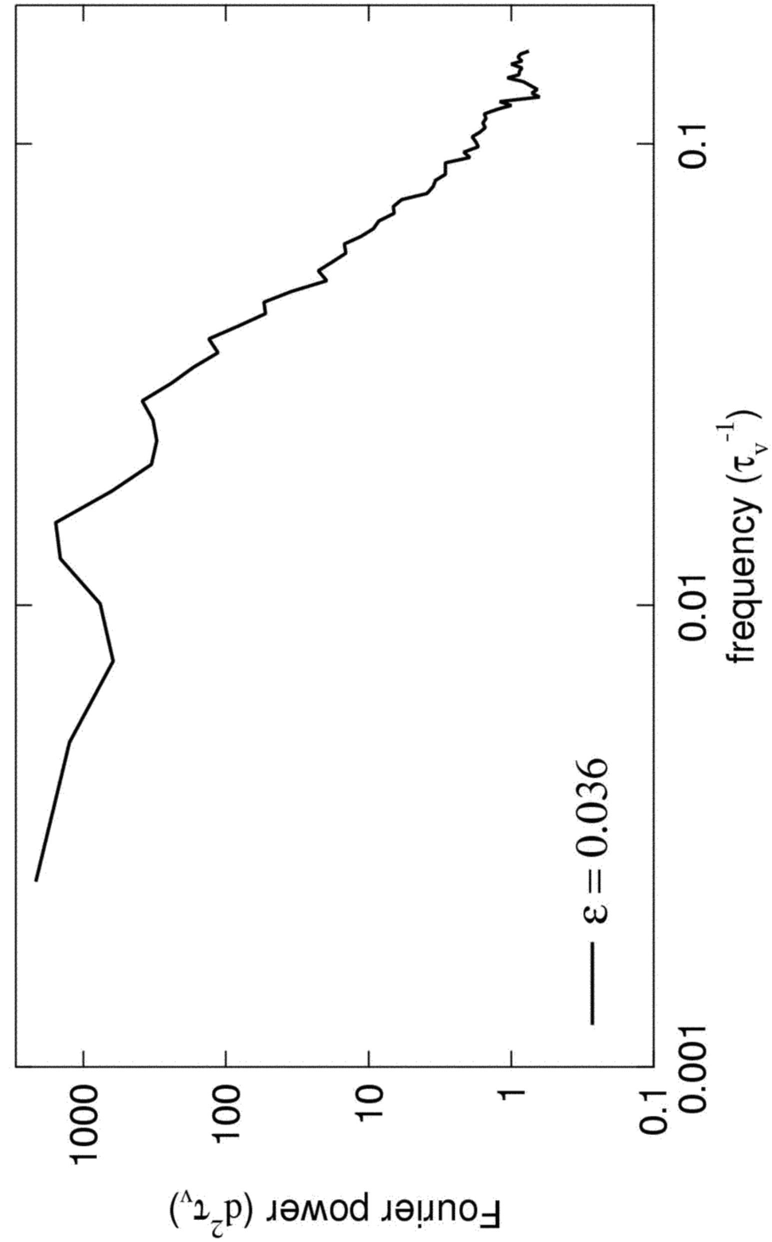


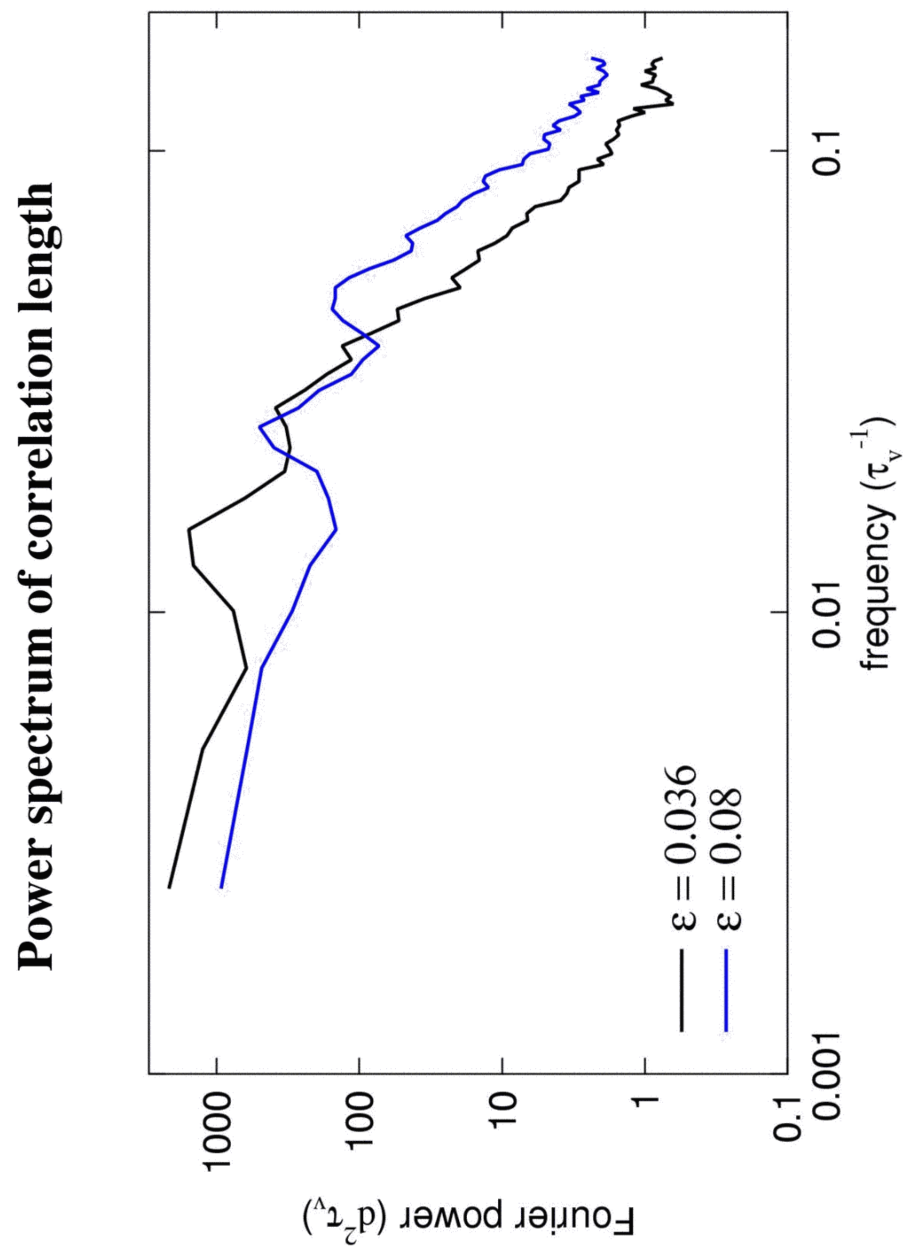


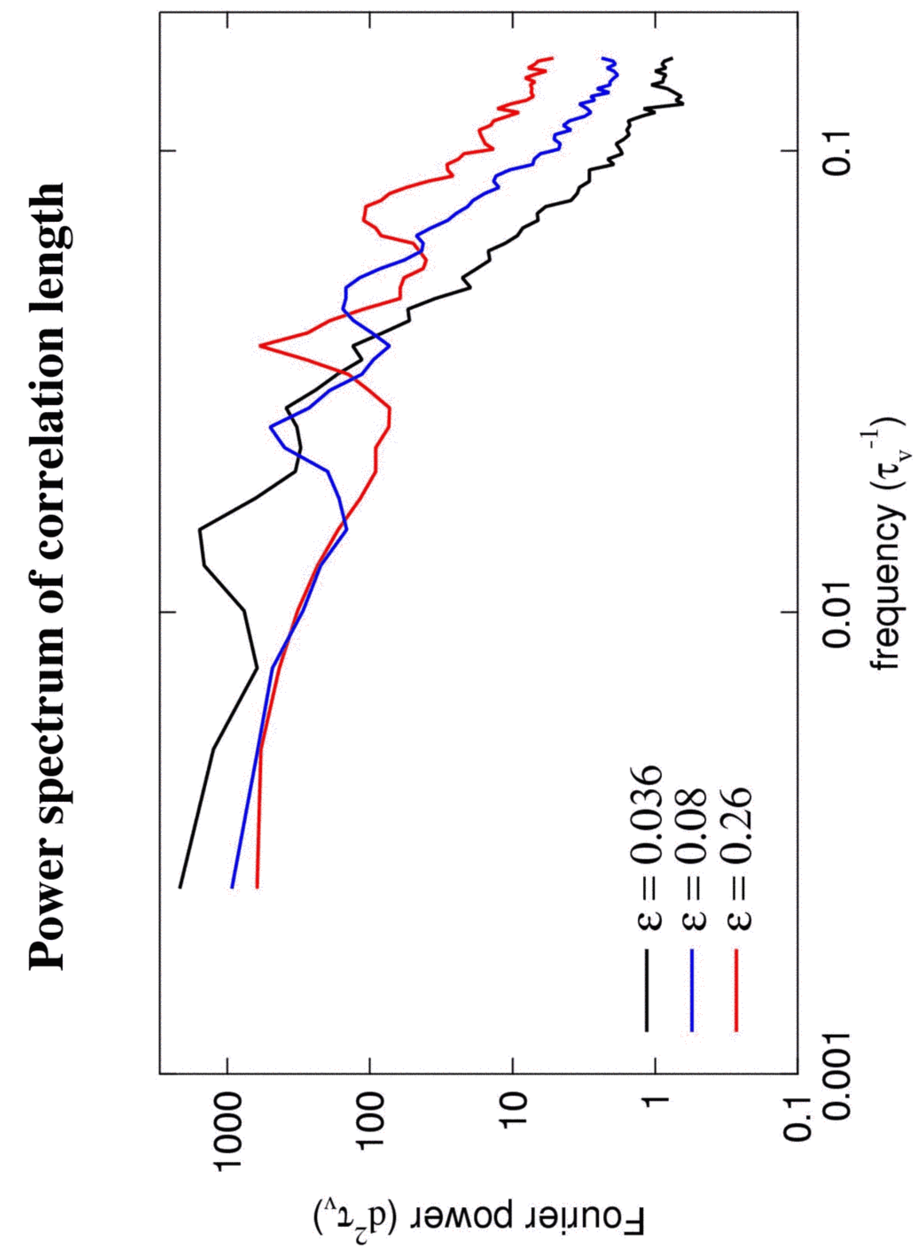
$\varepsilon = 0.025$

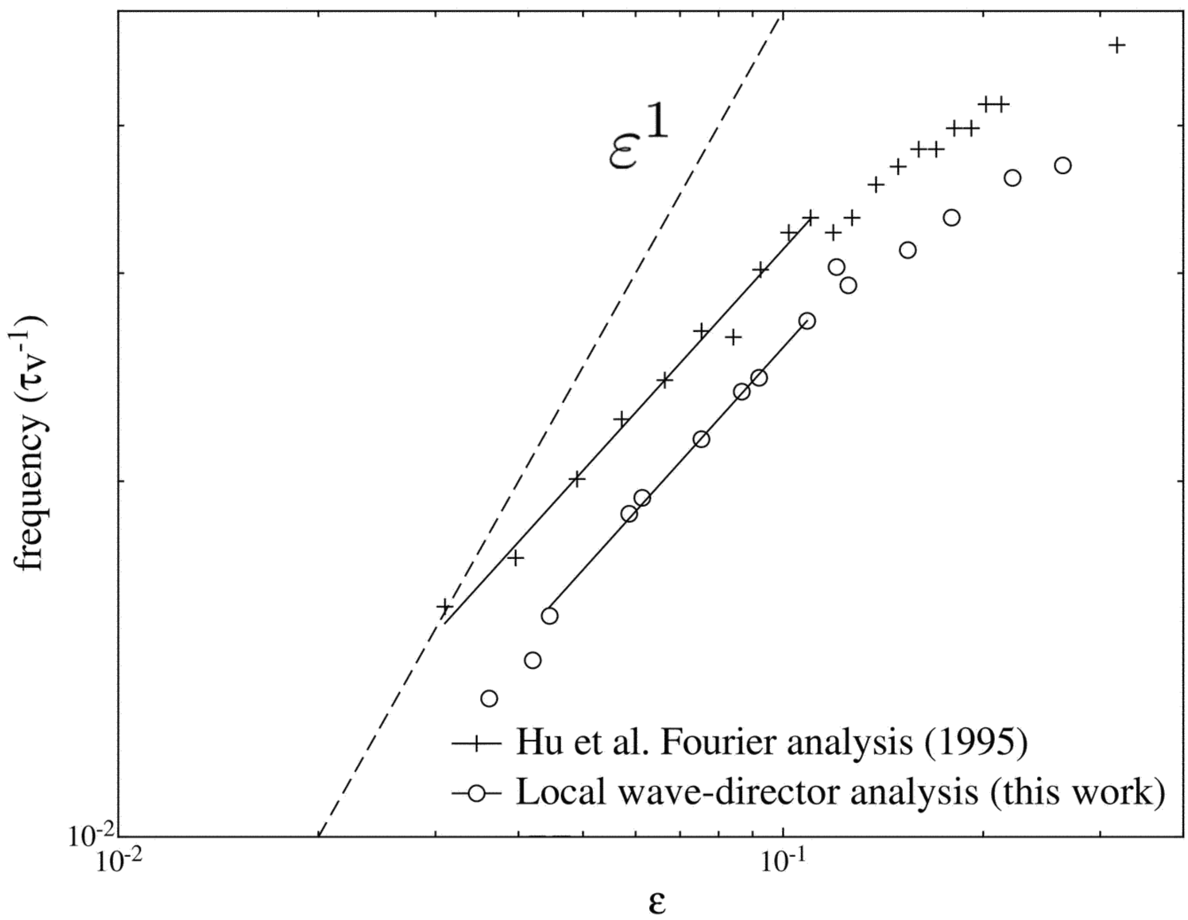
$\varepsilon = 0.125$

Power spectrum of correlation length




Power spectrum of correlation length
ε = 0.036
ε = 0.08
Fourier power ($d^2\tau_v$)
1000
100
10
1
0.1
0.001
0.01
0.1
frequency (τ_v^{-1})


Power spectrum of correlation length
ε = 0.036
ε = 0.08
ε = 0.26
Fourier power ($d^2\tau_v$)
1000
100
10
1
0.1
0.001
0.01
0.1
frequency (τ_v^{-1})



Summary

- Experimentally measured time and length scales do not obey the predicted power law dependence. Thus, weakly nonlinear theory in its usual form seems inapplicable.
- The correlation length, measured using local wave-director analysis, exhibits unexplained periodic time dependence. This means that the domain size oscillates. The frequency is roughly the same as the domain switching frequency.