Critical Phenomena near Bifurcations in Non-Equilibrium Systems

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Rayleigh-Benard convection: Jaechul Oh (now at NRL), Nathan Becker
Electroconvection: Michael Scherer, Xinliang Qiu, Sheng-Qi Zhou

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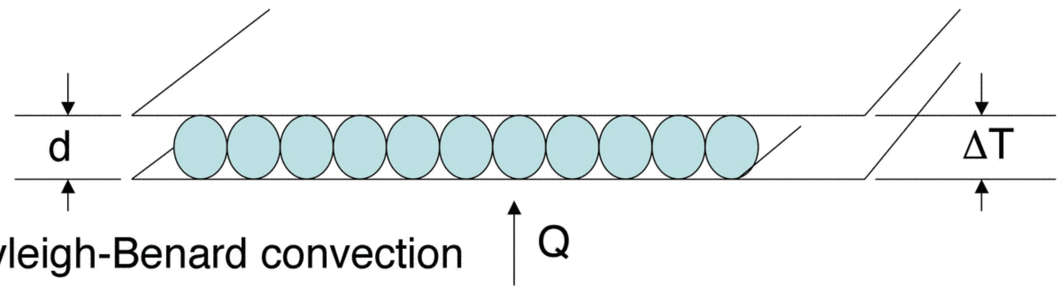


Rayleigh-Benard convection

Prandtl number

$Pr = \nu / \kappa$

ν = kinematic viscosity

κ = thermal diffusivity

$\varepsilon = \Delta T/\Delta T_c - 1$

Supported by the US National Science Foundation Grant DMR0243336

$$F_{th} = \frac{k_B T}{\rho \nu^2 d} \, 0.19(\nu / \kappa)$$

J. Swift, P. C. Hohenberg, Phys. Rev. A **15**, 319 (1977).
P. C. Hohenberg, J. Swift, Phys. Rev. A **46**, 4 773 (1992)

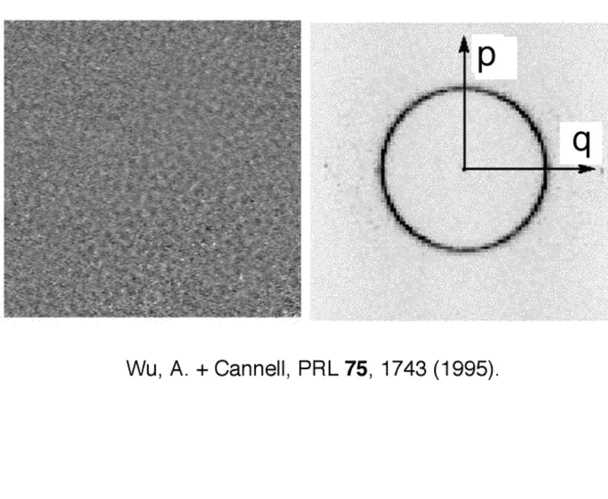
H. van Beijeren, E. G. D. Cohen, J. Stat. Phys. **53**, 77 (1988)

Water $F_{th} = 10^{-9}$

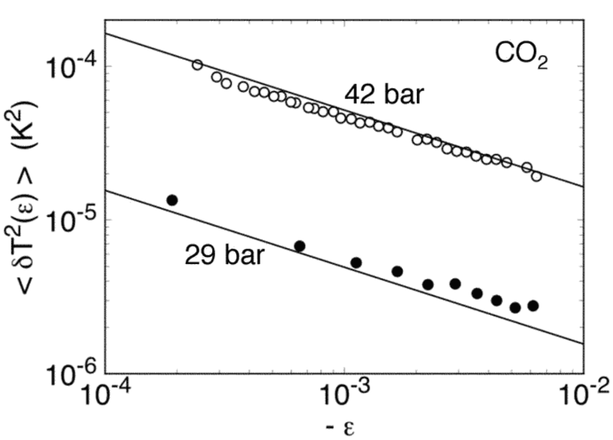
CO_2 20 bar $F_{th} = 10^{-7}$

Linear theory :

$$< \Delta T^2 > \sim F_{th} / |\varepsilon|^{\gamma}, \; \gamma = 1/2$$



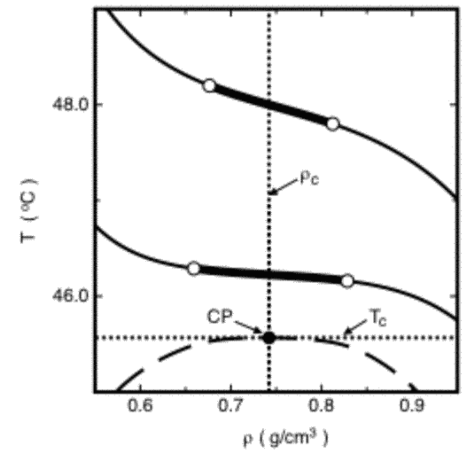
Wu, A. + Cannell, PRL **75**, 1743 (1995).



CP of SF_6

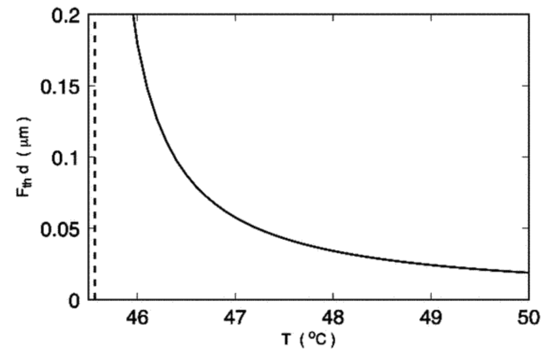
On critical isochore:

46.5 ^oC, d = 34.3μm, $F_{th} = 5.1x10^{-4}$
48.0 ^oC, d = 59.0μm, $F_{th} = 0.8x10^{-4}$

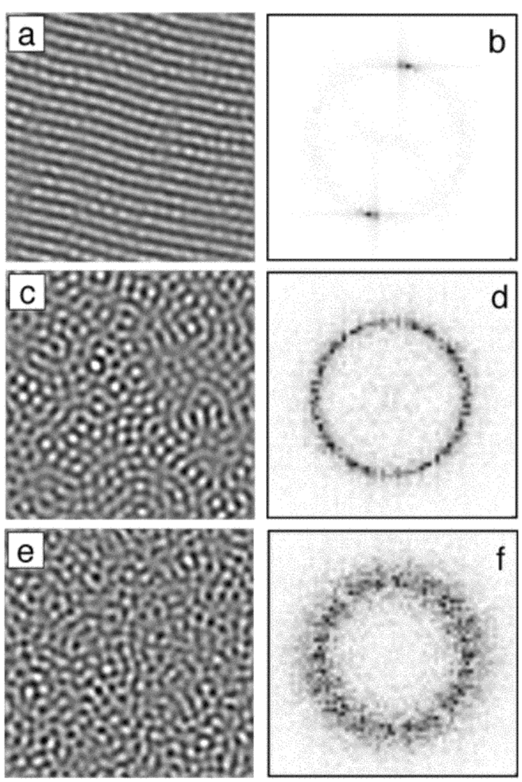


$$F_{th} = \frac{k_B T}{\rho \nu^2 d} 0.19(\nu / \kappa)$$

$dn / dT \sim 0.1 K^{-1}$



J. Oh and G.A., Phys. Rev. Lett., In print.

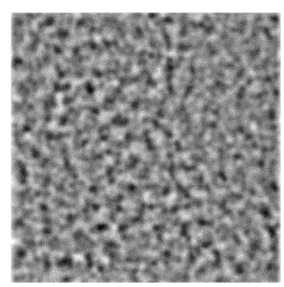


$\Delta T = 0.1320$ K
$\varepsilon = 0.007$

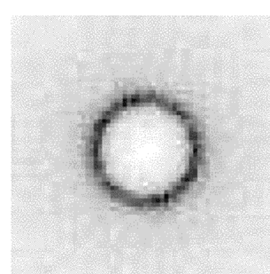
0.1311 K
-0.007

0.1248 K
-0.054

1.28x1.28x0.0343 mm^3



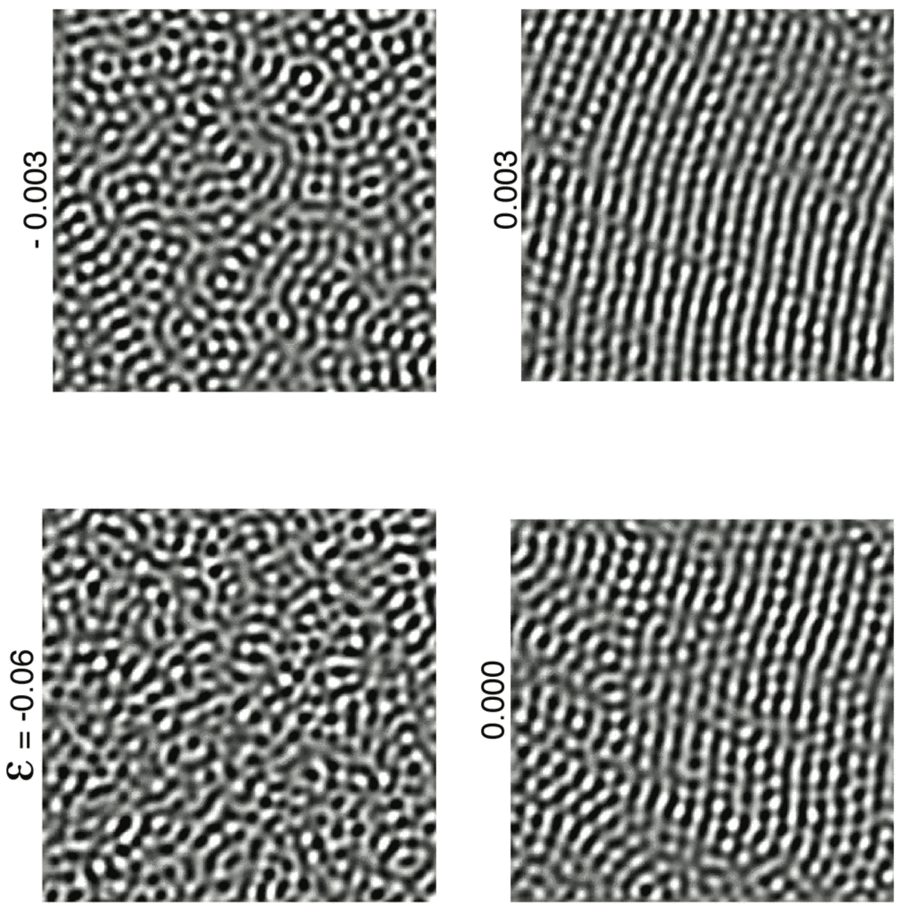
snapshot

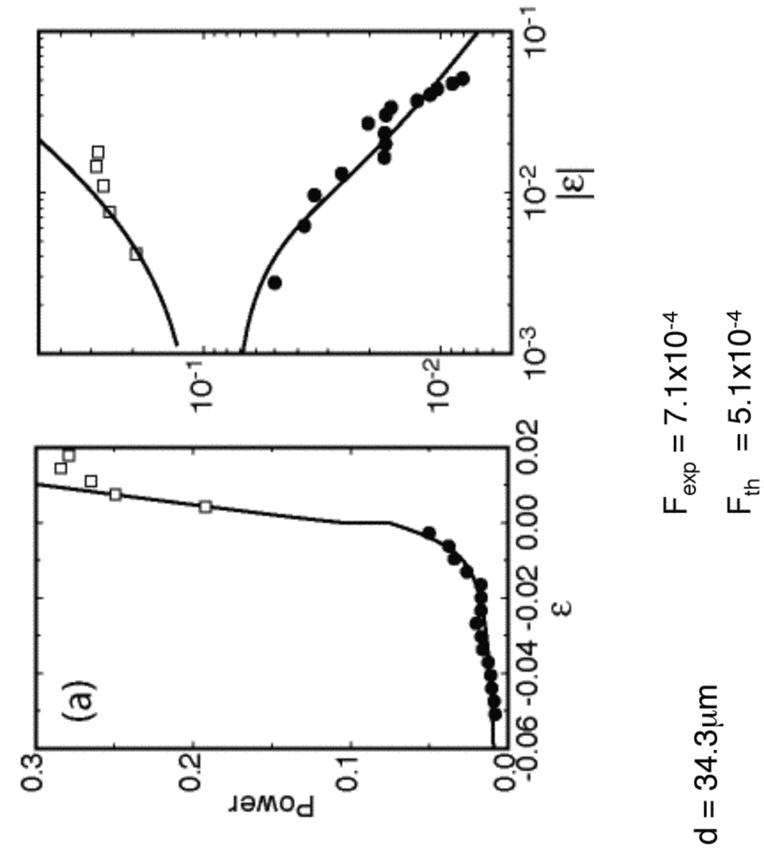


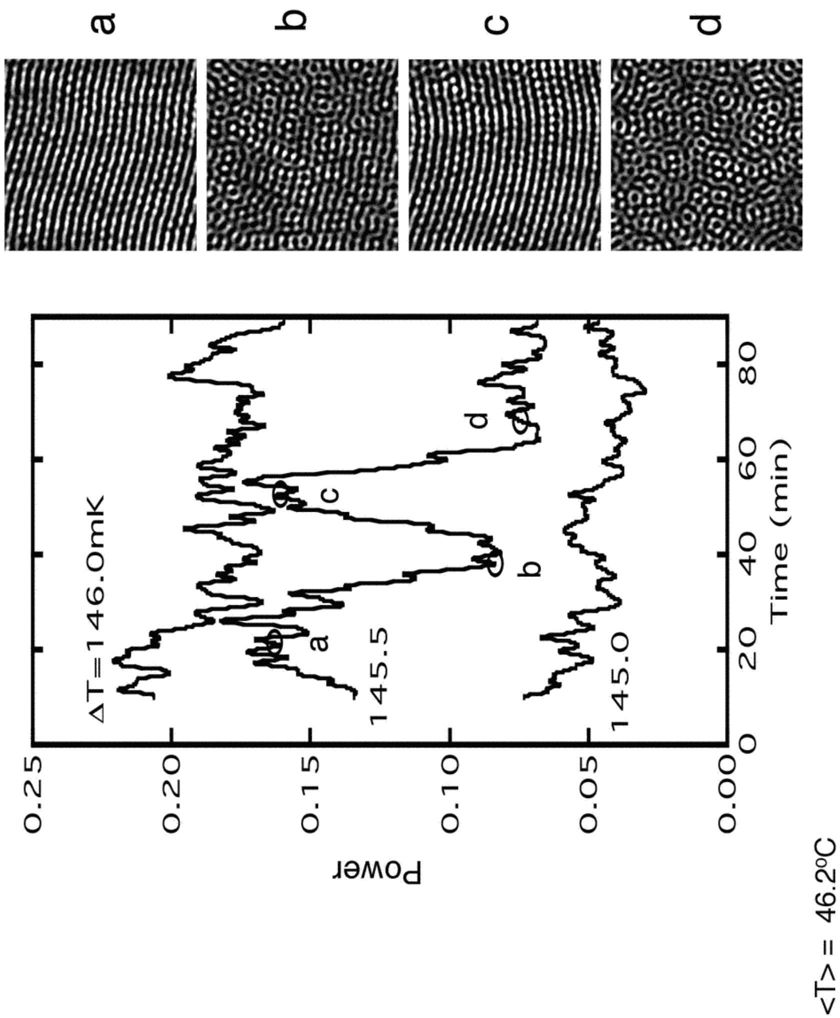
Avg. of 100 structure factors

Fluct. In oscillated granular Fluid, Goldman, Swift, and Swinney, preprint July 31 03.

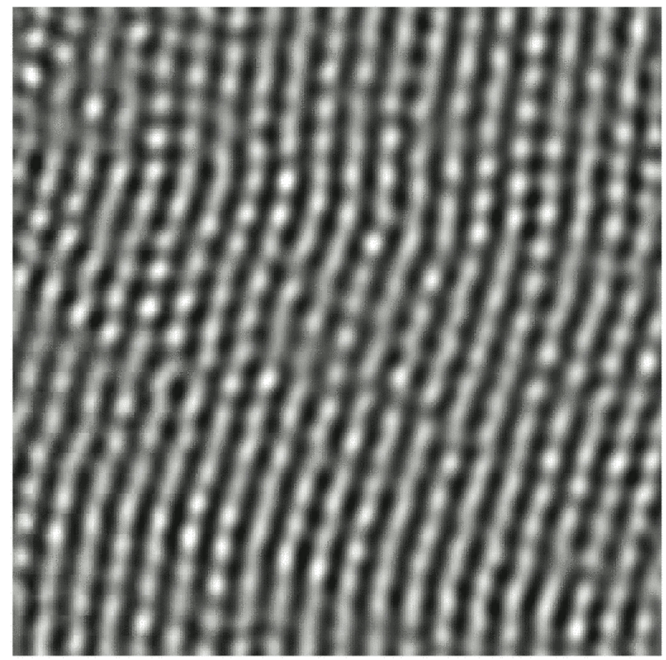
$\varepsilon = -0.06$
6.25x6.25 cm^2


ε = -0.06
- 0.003
0.000
0.003

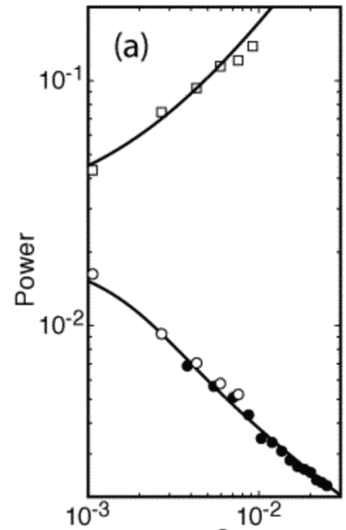

(a)
Power
ε
|ε|
F_exp = 7.1x10^-4
F_th = 5.1x10^-4
d = 34.3μm


a
b
c
d
ΔT=146.0mK
145.5
145.0
Power
Time (min)
0.00
0.05
0.10
0.15
0.20
0.25
0
20
40
60
80
<T> = 46.2°C

ε = 0



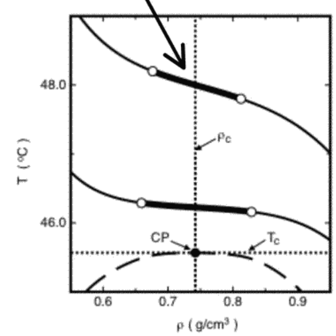
291



$F_{exp} = 1.4 \times 10^{-4}$

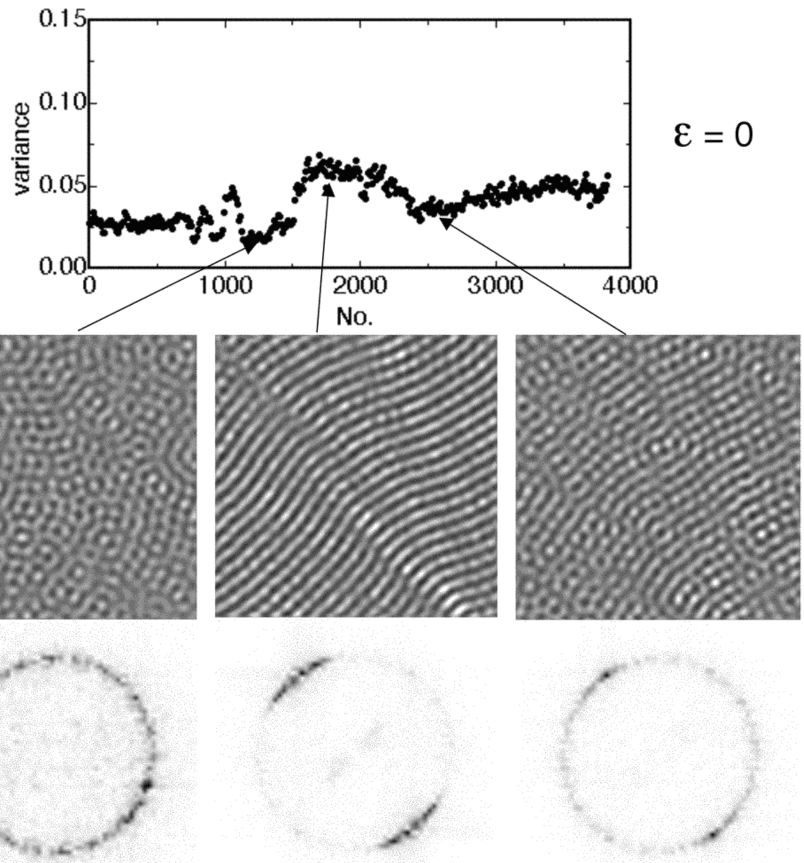
$F_{th} = 0.8 \times 10^{-4}$

d = 0.059 mm



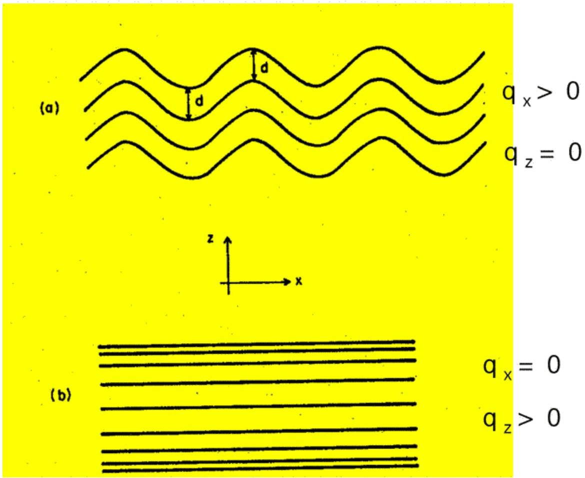
<T> = 48.0°C

ε = 0

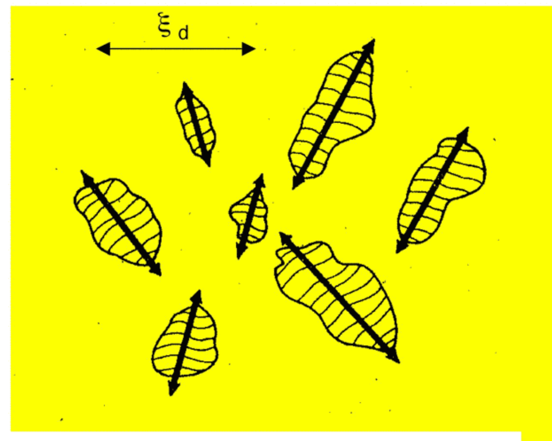


1245 1777 2580

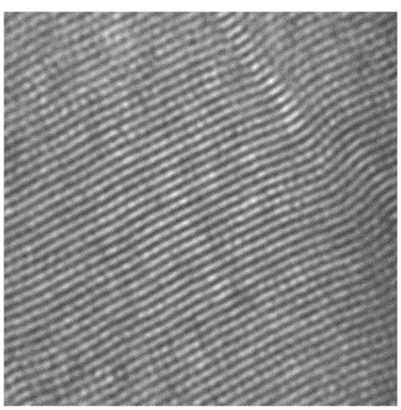
two types of
"phonons" :



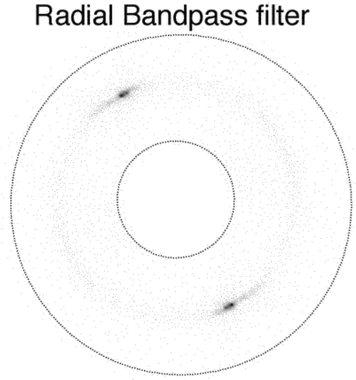
dislocations disclinations



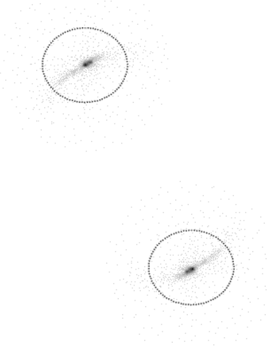
J. Toner and D.R. Nelson, Phys. Rev. B **23**, 316 (1981).

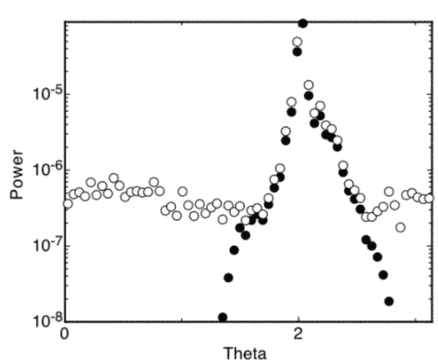


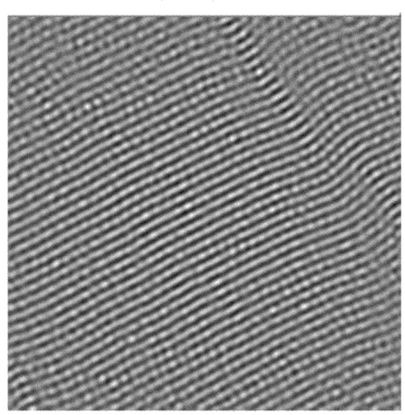
Radial Bandpass filter

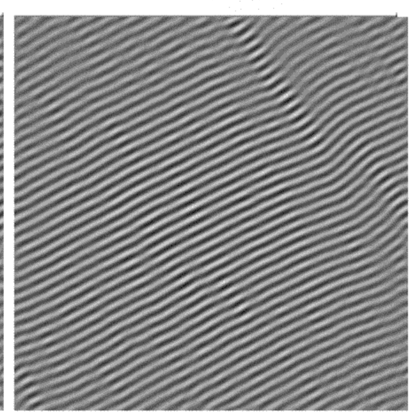


Peak filter



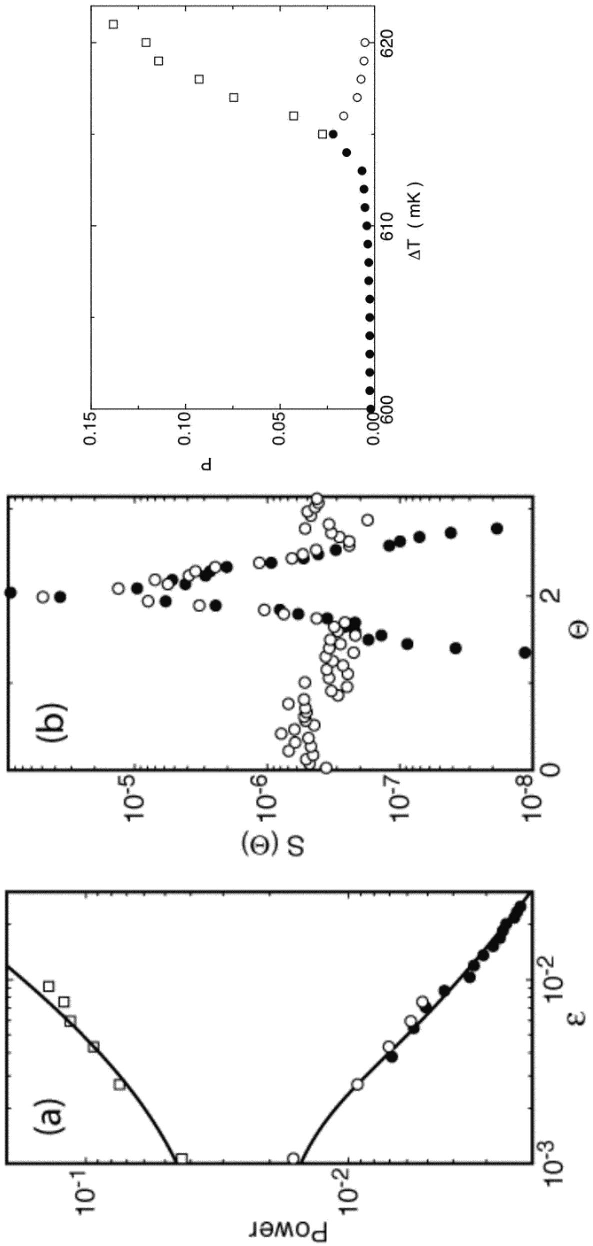


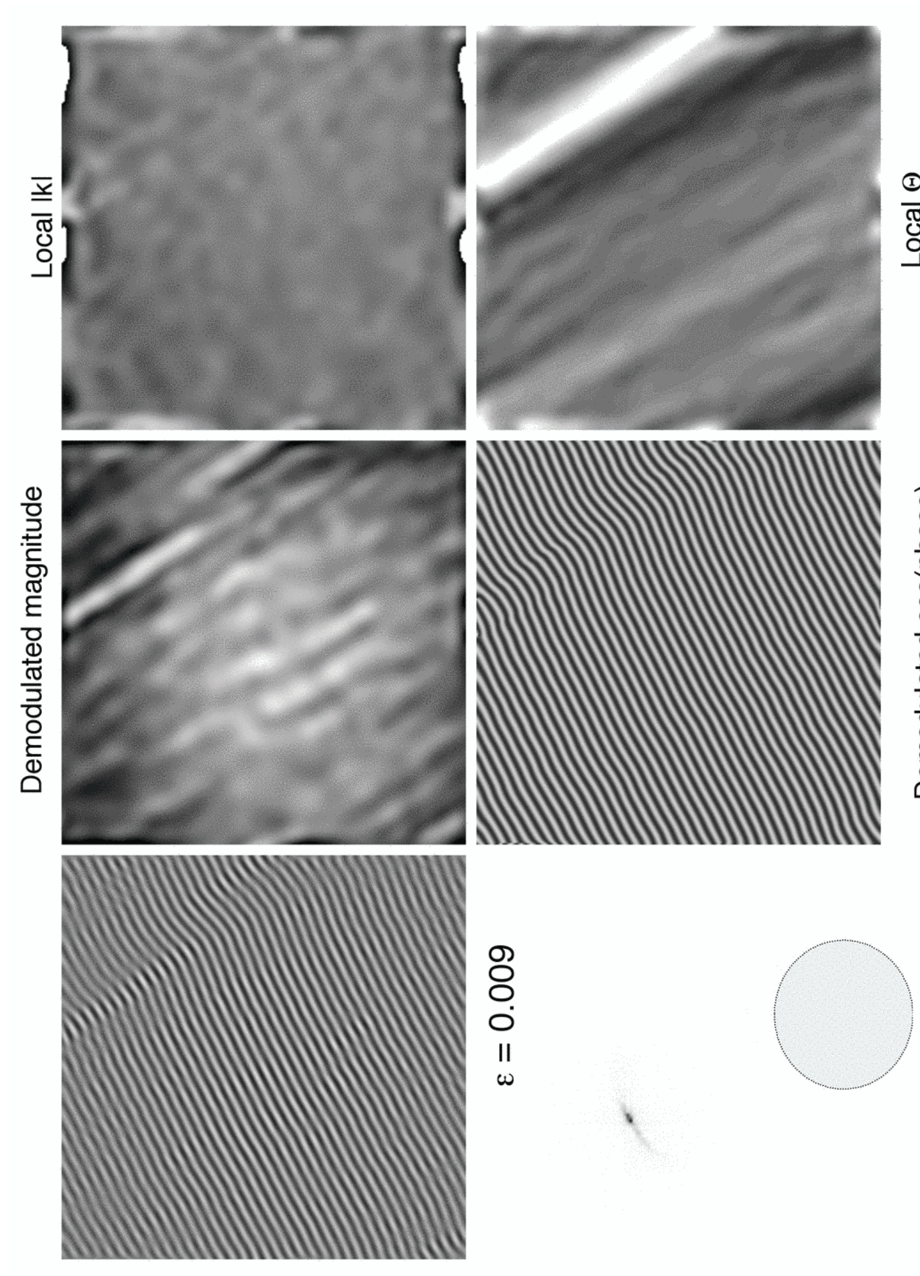


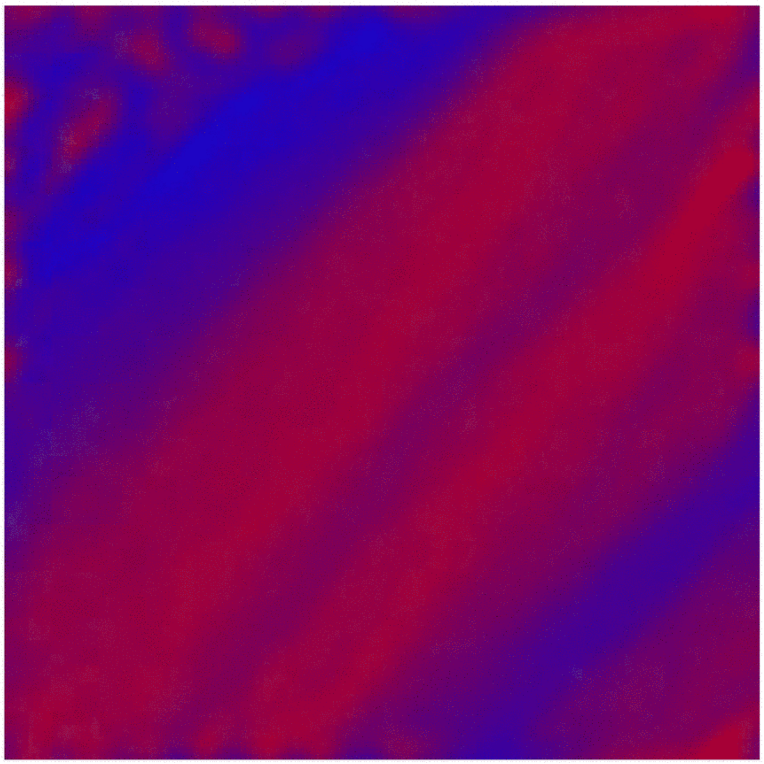


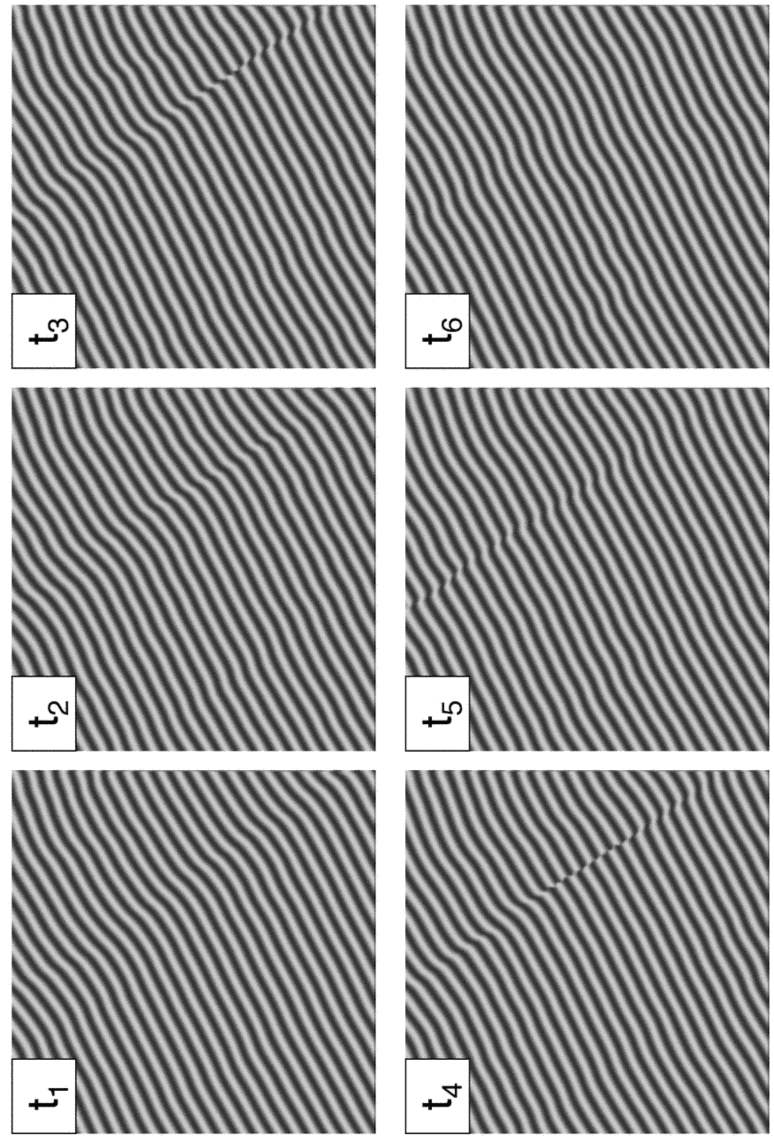
021212a/621/*.999*

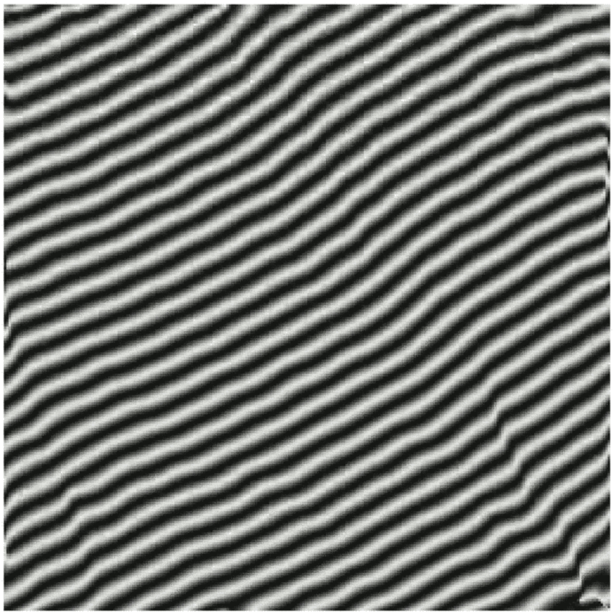
$\varepsilon = 0.009$


(a)
Power
ε
(b)
S (Θ)
Θ
P
ΔT (mK)

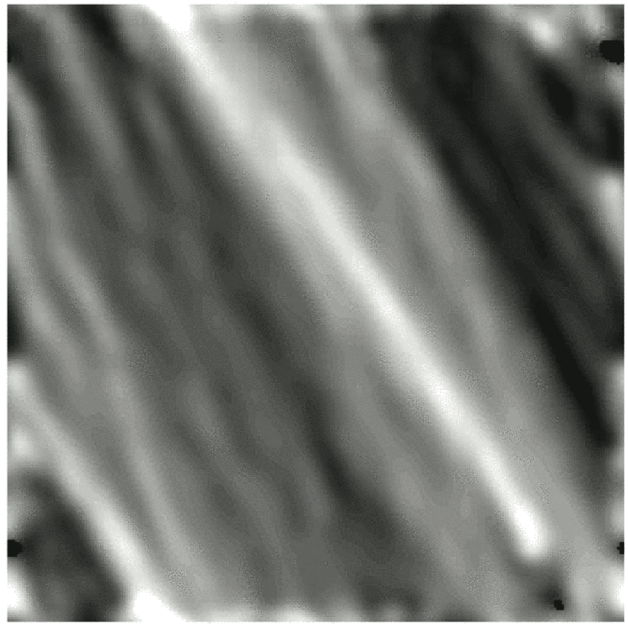

Local |k|
Local Θ
Demodulated magnitude
Demodulated cos(phase)
ε = 0.009


ε = 0.004
620/617.5


t1
t2
t3
t4
t5
t6
021212a/619 138, 140, 142, 144, 146, 148
0.0024

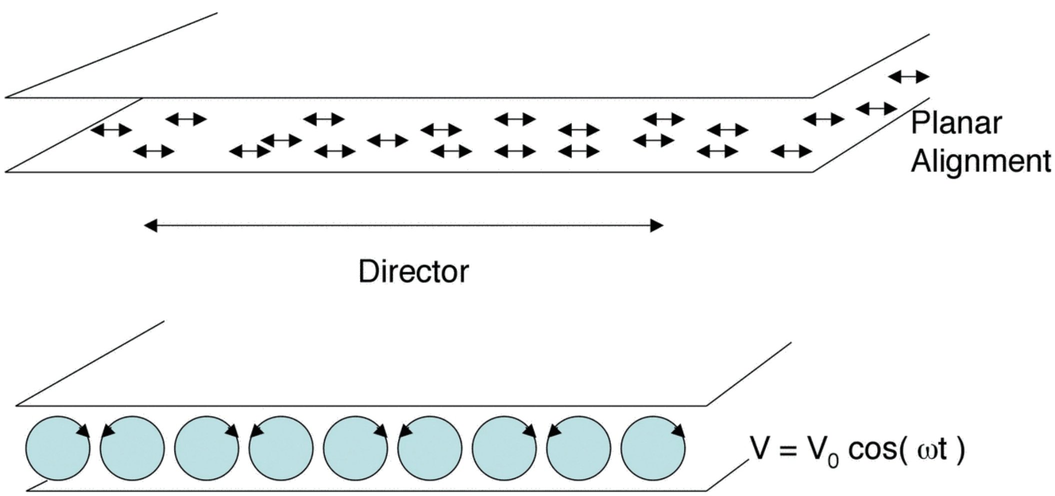


cos(phase)



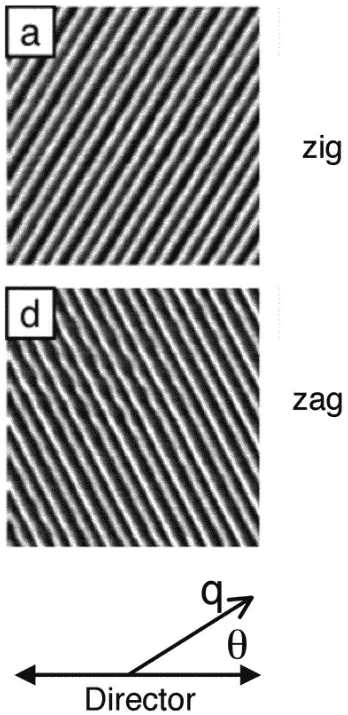
roll angle

Electroconvection in a nematic liquid crystal

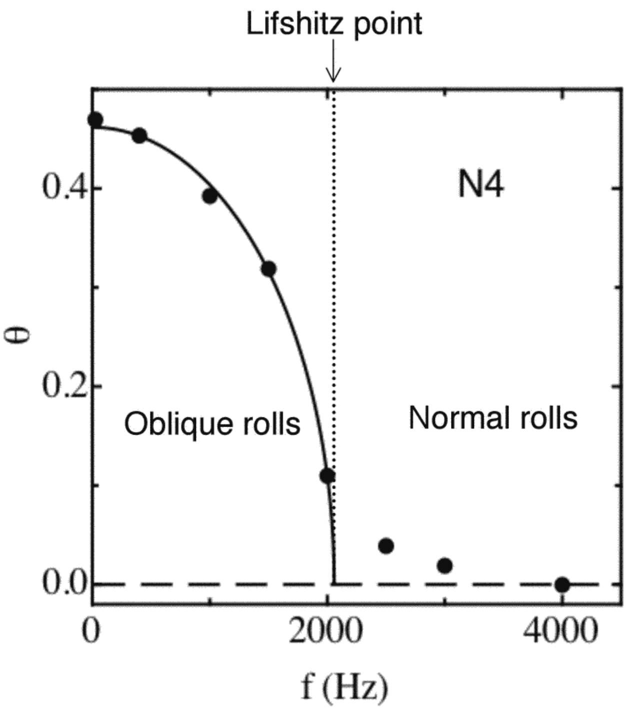


Convection for $V_0 > V_c$ $\varepsilon = (V_0 / V_c)^2 - 1$

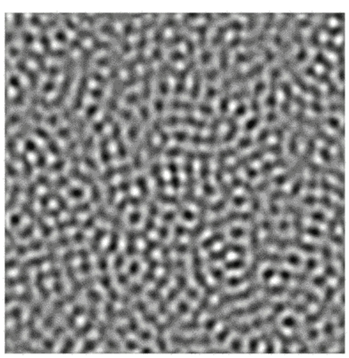
Anisotropic !



Stationary Bifurcation



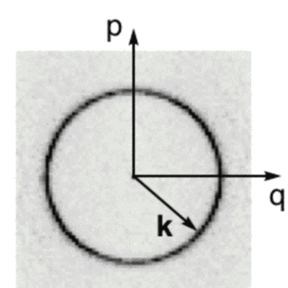
$$f = \omega / 2\pi$$

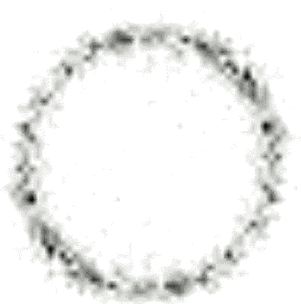


Rayleigh-Benard :

$$S(k) = \frac{S_0}{\tilde{\xi}^2(k^2 - k_0^2)^2 + \epsilon}$$

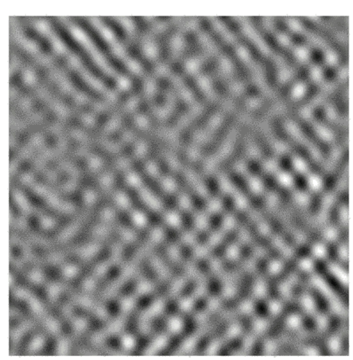
$$k^2 = q^2 + p^2$$

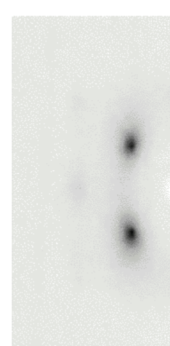


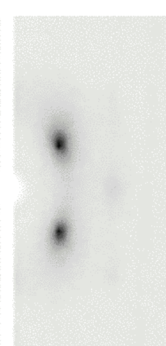


Electroconvection :

$$S(\mathbf{k}) = \frac{S_0}{\xi_q{}^2(q - q_0)^2 + \xi_p{}^2(p - p_0)^2 + \xi_{qp}{}^2(q - q_0)(p - p_0) + \varepsilon}$$







Possible Universality Classes for EC:

Planar alignment:

Stationary Bifurcations

- normal rolls
- *oblique rolls*
- Lifshitz point

Hopf Bifurcations

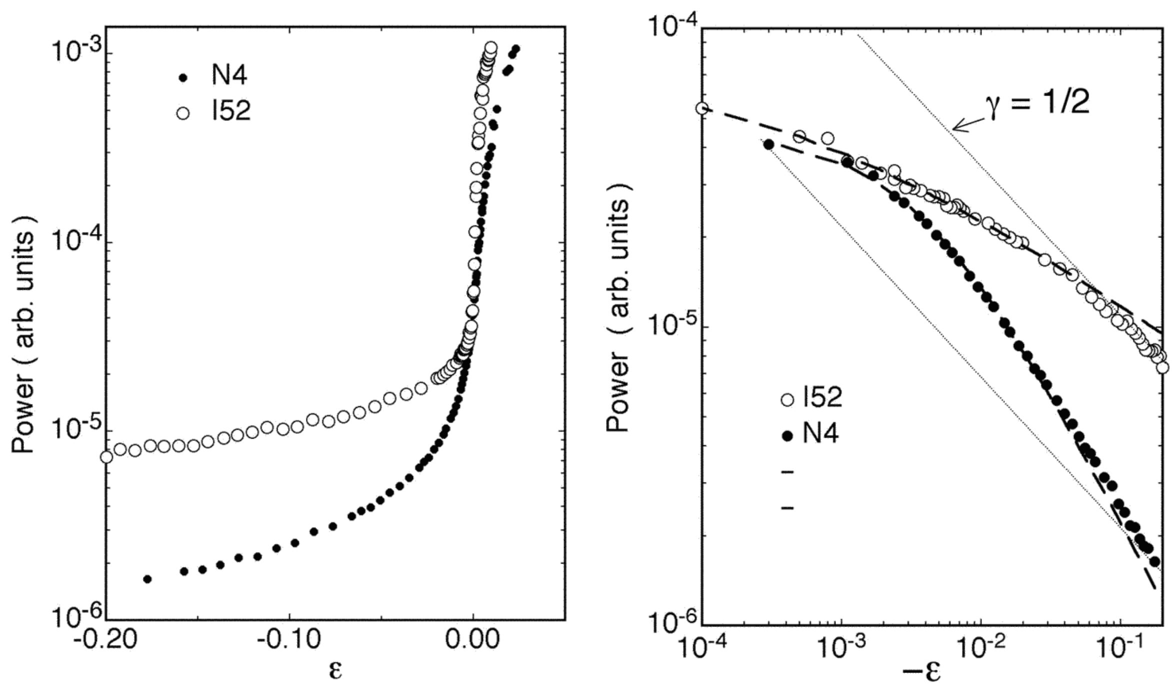
- normal traveling rolls
- *oblique traveling rolls*
- Lifshitz point

Codimension-two points

Homeotropic alignment:

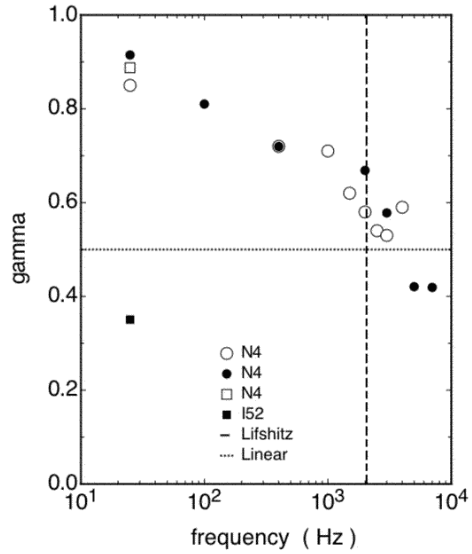
Preceded by Fredericz transition

Directly to EC (Brazovskii)



○ I52: Oblique traveling waves
● N4: Stationary oblique rolls

N4: stationary bifurcation to oblique rolls at small f, normal rolls at large f
I52: Hopf bifurcation to traveling oblique rolls at small f.



$$P = (1 / P_1 + 1 / P_0)^{-1}, \quad P_1 = P_{10} |\varepsilon|^{-\gamma}, \quad P_0 = \text{const}$$

X. Qiu + G.A., unpublished.

Summary :
- Fluctuations near RB instability (Brazovskii)
  - fluctuation induced 1st order transition to rolls (or stripes)
  - fluctuations in striped phase
  - “phonons” in striped phase
    - amplitude modulation
    - roll angle modulation
  - dislocations in striped phase
- Fluctuations near oblique traveling wave bif. In EC (I52)
- Fluctuations near oblique and normal stationary bif. In EC (N4)
- Fluctuations near Lifshitz point of stationary rolls (N4)

Possible universality classes:
- **RBC: Brazovskii**
- **EC Normal stationary rolls**
- **EC Oblique stationary rolls**
- EC Normal traveling rolls
- **EC Oblique travelling rolls**
- EC codimension-two points
- **EC Lifshitz stationary rolls**
- EC Lifshitz traveling rolls
- EC Homeotropic after Fredericsz transition
- EC Homeotropic w/out Fred.: Brazovskii

Other systems:

micro-crystallization of diblock co-polymers (Fredrickson et al.), equilibrium phase transition of the Brazovskii universality class

stripe phases in 2-d Coulomb gases
- 2-d high Landau levels.
  - Brazovskii? Perhaps not because of coupling to lattice anisotropy.

stripes or squares in vertically vibrated granular layers