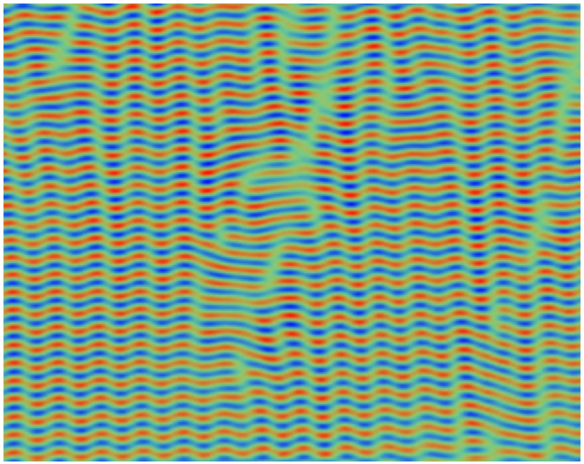
Order, Chaos, and Defects in Inclined Layer Convection



Karen E. Daniels

Experiments:

Eberhard Bodenschatz
LASSP, Cornell University

Simulations:

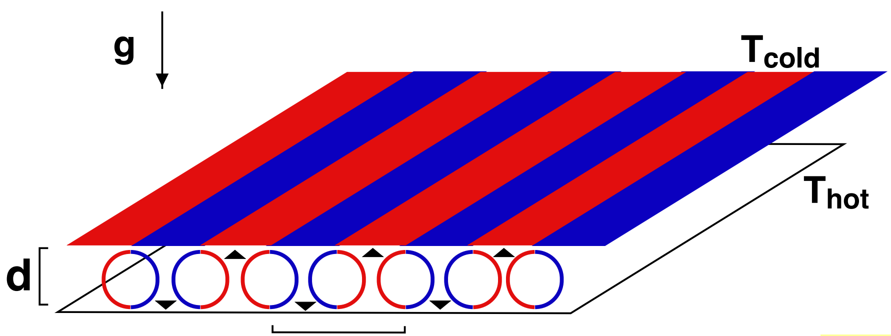
Oliver Brausch, Werner Pesch
University of Bayreuth

KITP
19 August 2003

Supported under DMR−0072077

Rayleigh-Bénard Convection

□T < □T$_C$: conduction



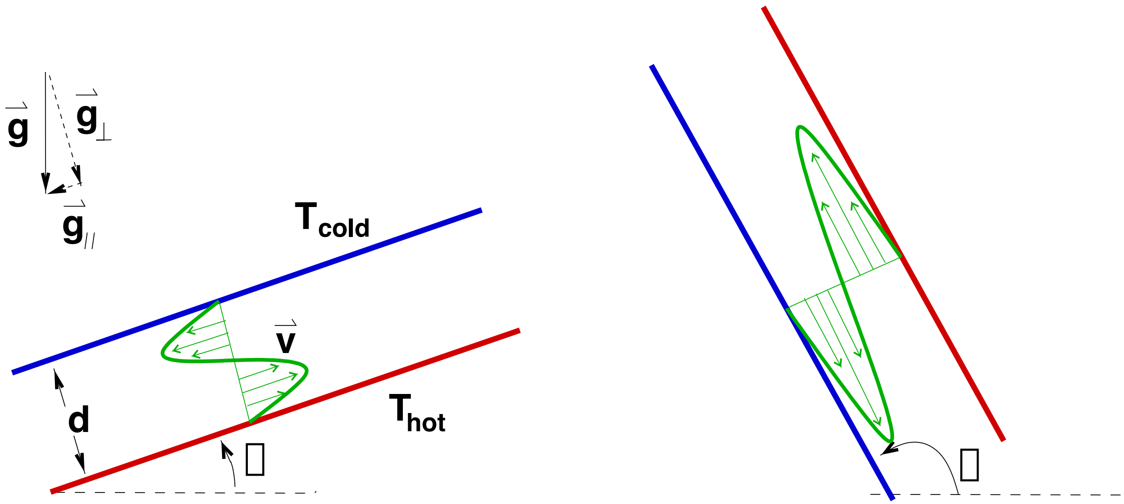
□T > □T$_C$: convection

□ □ 2d

k □ □/d

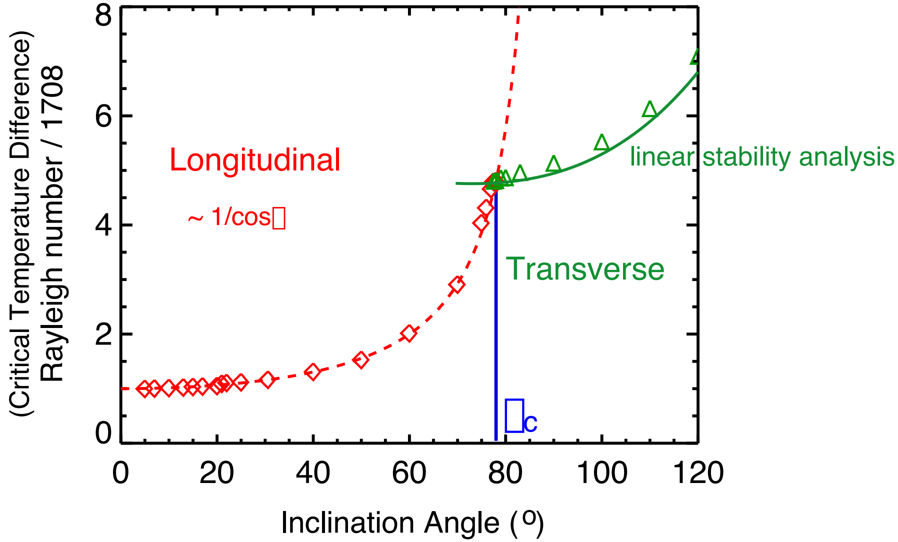
□= $\frac{□T − □T_C}{□T_C}$

Inclined Layer Convection



- shear flow ⇛ anistropic
- two control parameters: □and □

Onset of Inclined Layer Convection

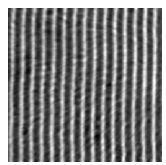


buoyancy instability
(Rayleigh-Benard)

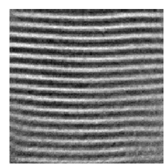
shear instability

uphill

downhill



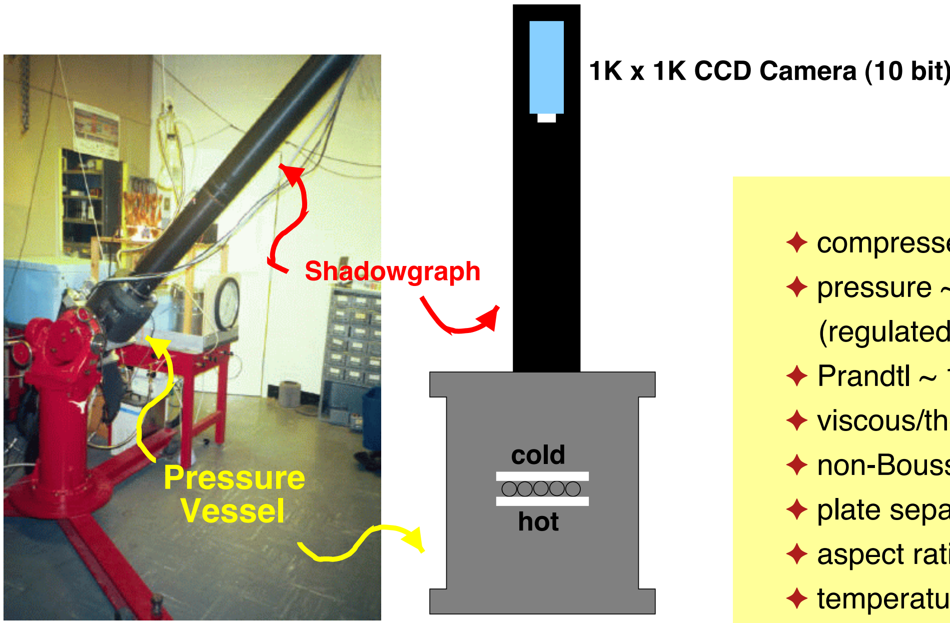
longitudinal



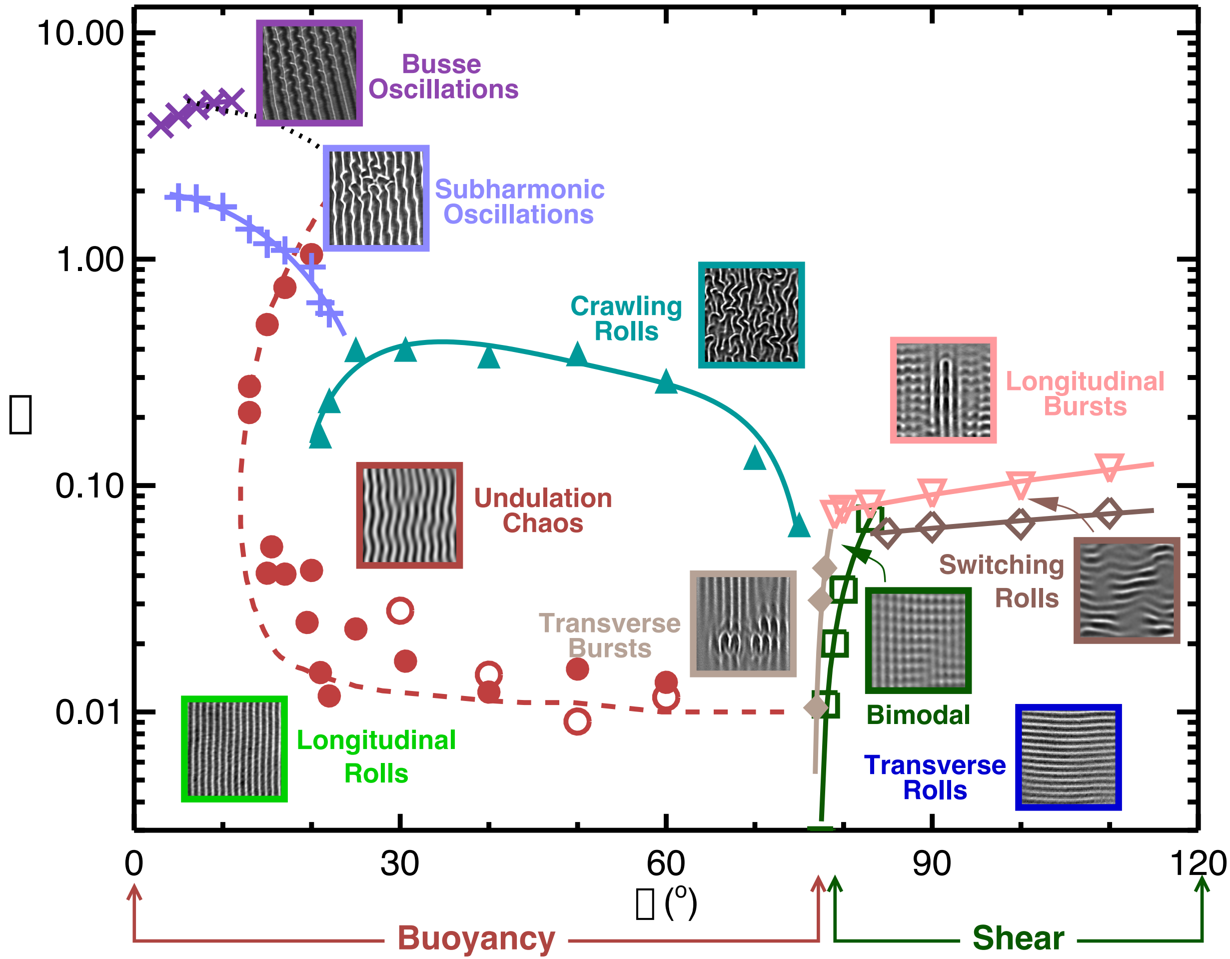
transverse
(travelling)

Cloud Bands

High Pressure Gas Experiment

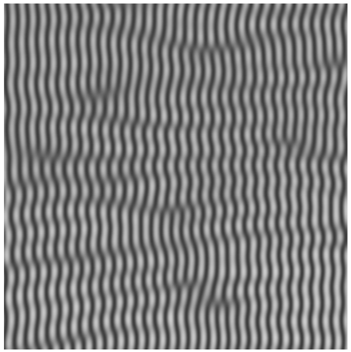


- compressed carbon dioxide
- pressure ~ 40 to 55 bar (regulated to +/– 0.005 bar)
- Prandtl ~ 1
- viscous/thermal times ~ 1 to 3 sec
- non-Boussinesq number < 1
- plate separation ~ 0.3 to 1 mm
- aspect ratio ~ 10 to 100 rolls
- temperature difference ~ 1 to 10 ^{o}C (regulated to +/– 0.0003 ^{o}C)
- mean temperature ~ 28 ^{o}C

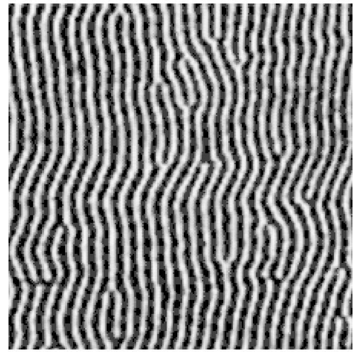

10.00
1.00
0.10
0.01
0
30
60
90
120
(o)
Busse Oscillations
Subharmonic Oscillations
Crawling Rolls
Longitudinal Bursts
Undulation Chaos
Switching Rolls
Transverse Bursts
Bimodal
Longitudinal Rolls
Transverse Rolls
Buoyancy
Shear

Defect-Turbulence in Striped Patterns

Inclined Layer Convection

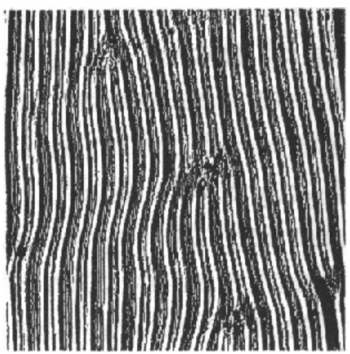


Optical Chaos



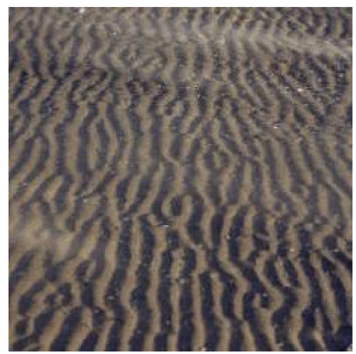
Ducci, Ramazza, Gonzalez-Vinas, Arecchi (1999)

Liquid Crystal Convection

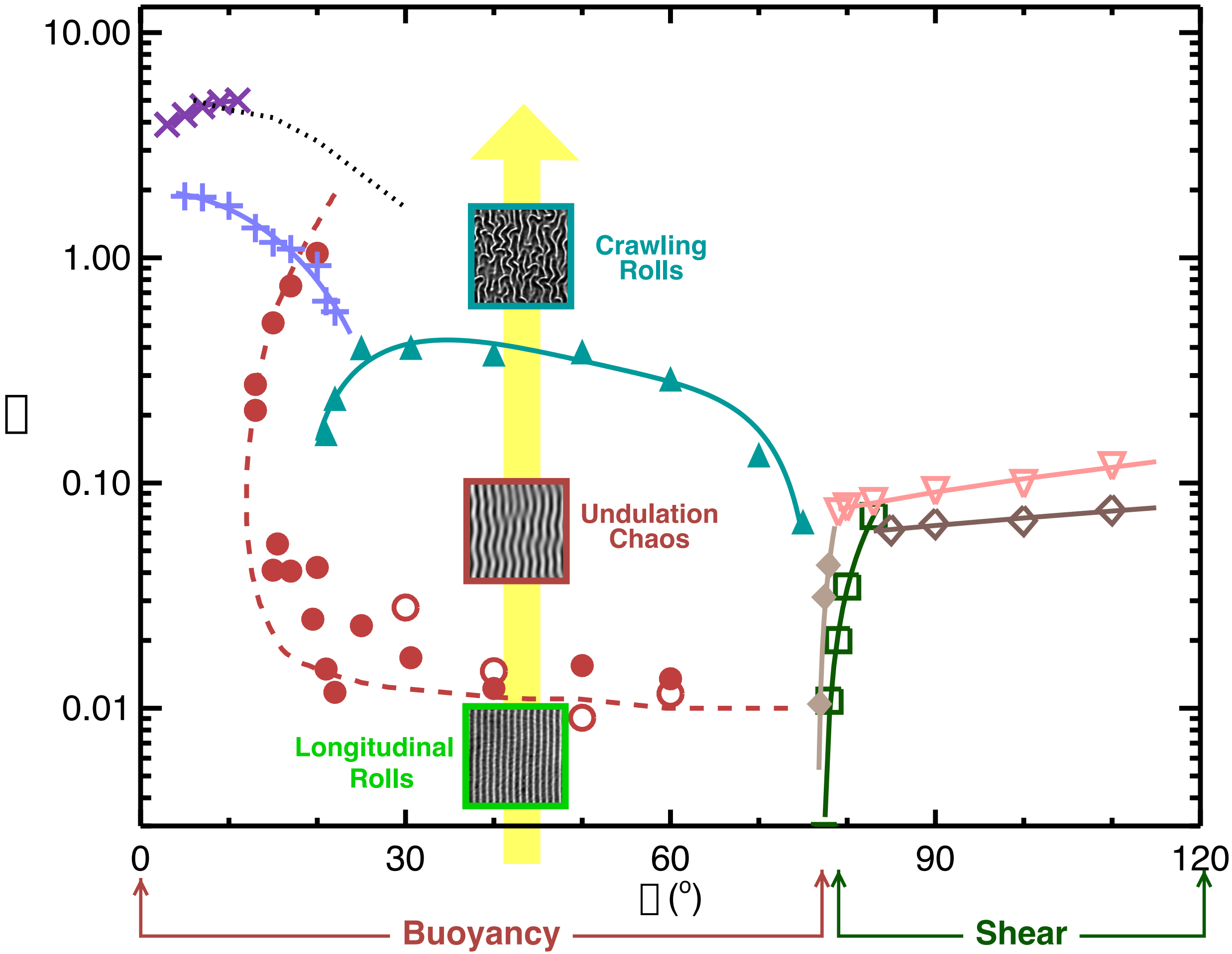


Rehberg, Rasenat, Steinberg (1989)

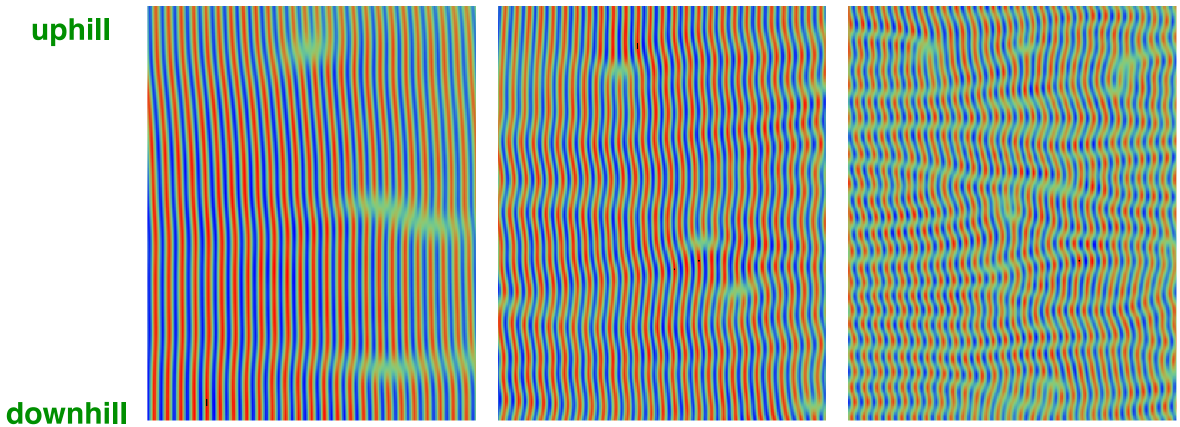
Sand Ripples



J. Land


10.00
1.00
0.10
0.01
0
30
60
90
120
(°)
Crawling Rolls
Undulation Chaos
Longitudinal Rolls
Buoyancy
Shear

Undulation Chaos



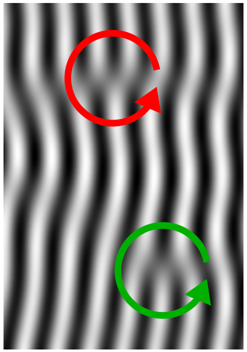
□= 0.03 □= 0.06 □= 0.12

- Two complementary descriptions: defects and undulations
- Spatiotemporally chaotic motion of defects, undulations
- Defect density and undulation wavenumber increase with □

$$\epsilon = 0.08$$

Topological Defects

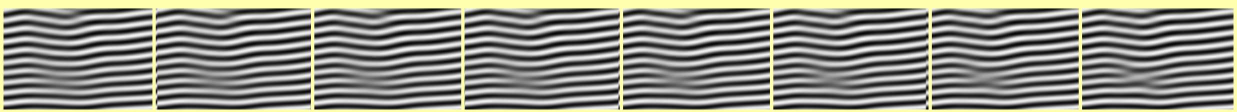
–2□



+2□

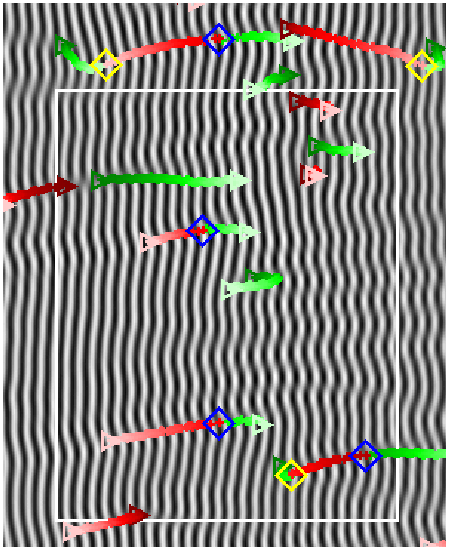
$$\oint \vec{\nabla}\Phi \cdot d\vec{s} = n2\pi$$

- defects occur where a roll-pair ends and convection amplitude is zero
- sign of phase jump determines topological charge
- defects climb and glide through pattern, changing orientation and wavenumber
- defects create and annihilate as pairs of +/– defects
- +/– defects enter and leave through the boundaries of the system



t=0 sec t=1 sec t=2 sec

Tracking Defects

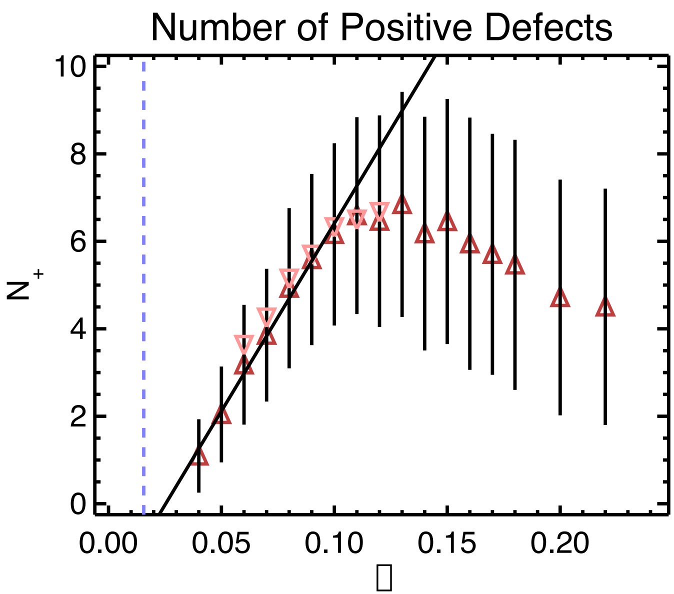


□ = 30 □ = 0.07

- negative defect track
- positive defect track
- creation
- annihilation

- trim and Fourier-filter shadowgraph images
- recover phase information from Re/Im parts
- locate defects using zero-crossings and +/–2□ phase jumps
- connect defects in adjacent frames to form tracks
- two types of tracks:
  100 frames x 400-600 runs at each □
  80000 frames x 2-4 runs at each □
- link tracks to find creations/annihilations
- determine velocities along track by local linear fit to x(t), y(t)
- total raw data: 1.5 terabytes

Defect Density



How can we explain defect density in terms of defect gain and loss rates?

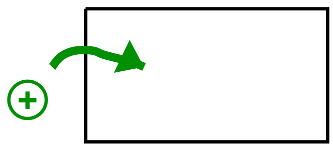
Detailed balance/equilibrium requires:

$$\mathcal{P}(N)\ \text{loss}(N) = \mathcal{P}(N-1)\ \text{gain}(N-1)$$

Defect Event Rates

N = # of defect pairs ~ # of positive defects ~ # of negative defects

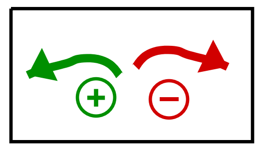
Entering



random, indepedent of # defects

$E(N) = E_0$

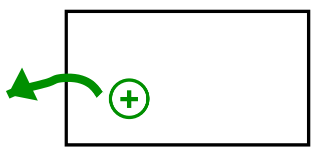
Creation



random, indepedent of # of defects

$C(N) = C_0$

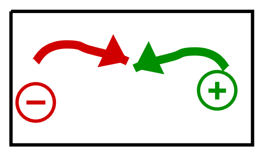
Leaving



depends on # of defects in region

$L(N) = L_0 N$

Annihilation



takes two defects of opposite sign

$A(N) = A_0 N^2$

Poisson-like Distributions

$$P(N) = \frac{gain(N-1)}{loss(N)} P(N-1)$$

Poisson

defects are gained/lost individually
N = number of defects in system

defects enter: $E(N) = E_0$
defects leave: $L(N) = L_0 N$

$$\mu = \frac{E_0}{L_0} = \langle N \rangle \qquad P(N) = \frac{\mu}{N} P(N-1)$$

$$P(N) = \frac{\mu^N}{e^\mu N!}$$

Squared Poisson

defects are gained and lost as pairs
requires infinite system or periodic B.C.
N = number of defect pairs

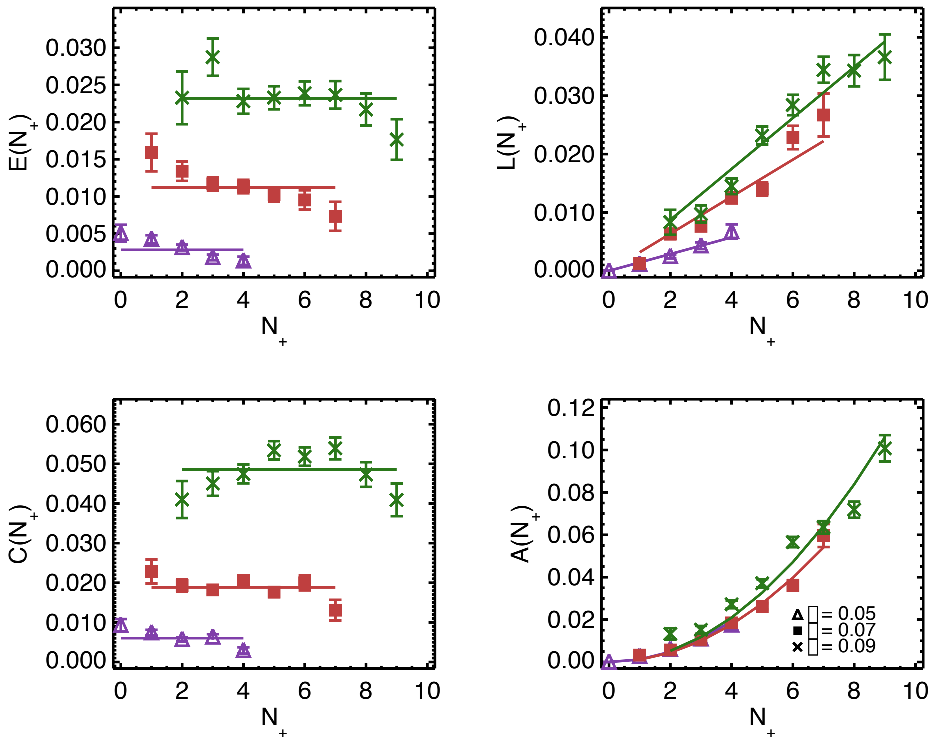
creation: $C(N) = C_0$
annihilation: $A(N) = A_0 N^2$

Gil, Lega, Meunier (1990) PRA

$$\gamma = \frac{C_0}{A_0} = \langle N^2 \rangle \qquad P(N) = \frac{\gamma}{N^2} P(N-1)$$

$$P(N) = \frac{\gamma^N}{I_0(2\sqrt{\gamma}) N!^2}$$

Measured Defect Event Rates



△ □ ×
empirical rates

expected behavior

Single-defect effects are significant: can't leave them out!

Modified Poisson Distribution

- accounts for both single defects and pairs
- doesn't require periodic boundary conditions

N = number of positive defects
= number of negative defects
= number of defects pairs

creation: $C(N) = C_0$
entering: $E(N) = E_0$
annihilation: $A(N) = A_0N^2$
leaving: $L(N) = L_0N$

- two parameters, □ and □, determined from rate equations

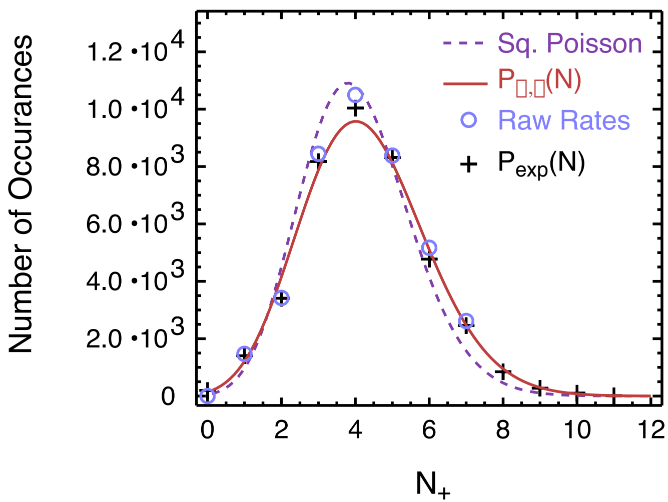
$$P(N) = \frac{gain(N-1)}{loss(N)} P(N-1)$$

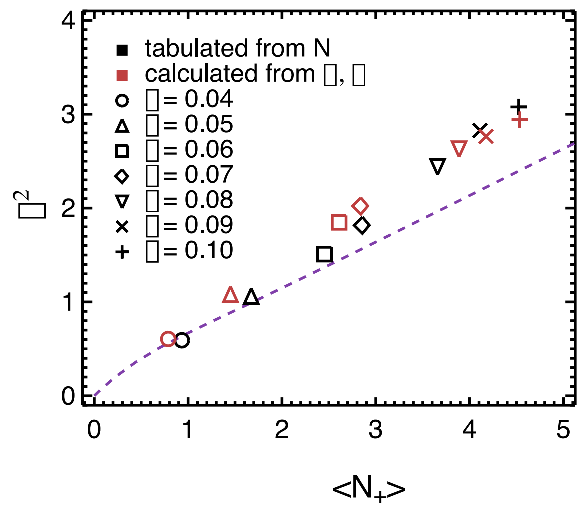
$$P(N) = \frac{E_0 + C_0}{L_0 N + A_0 N^2} P(N-1) = \frac{\alpha}{\beta N + N^2} P(N-1)$$

$$\alpha \equiv \frac{E_0 + C_0}{A_0} \qquad \beta \equiv \frac{L_0}{A_0}$$

$$P(N) = \frac{\alpha^{\frac{\beta}{2}+N}}{I_\beta(2\sqrt{\alpha})\Gamma(1+\beta+N)N!}$$

Comparison of Distributions

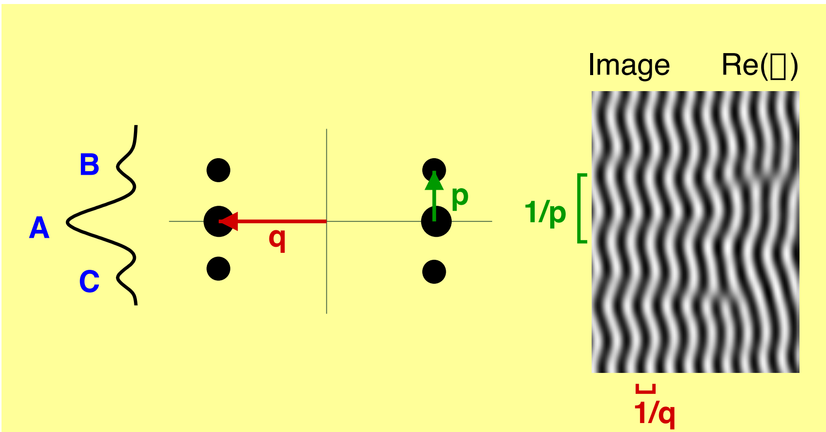


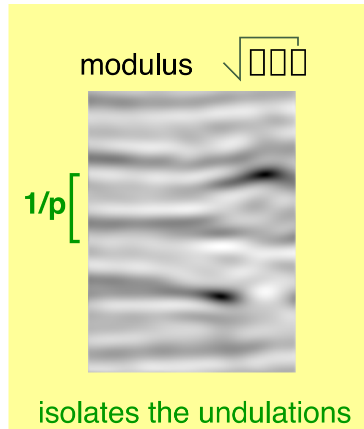


Characterizing Undulations

First-order undulations:

$$\psi(x,y) = e^{iqx}\left(A + iBe^{ipy} + iCe^{-ipy} + \cdots\right)$$

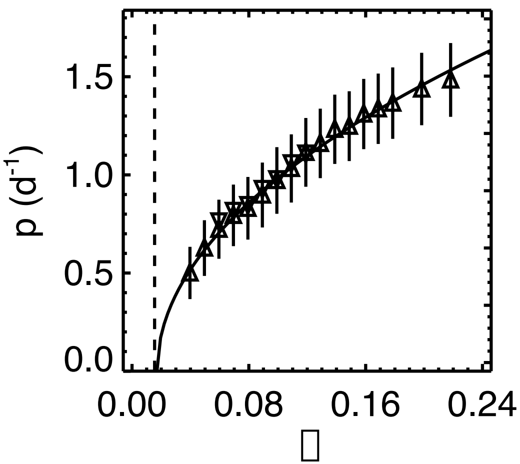




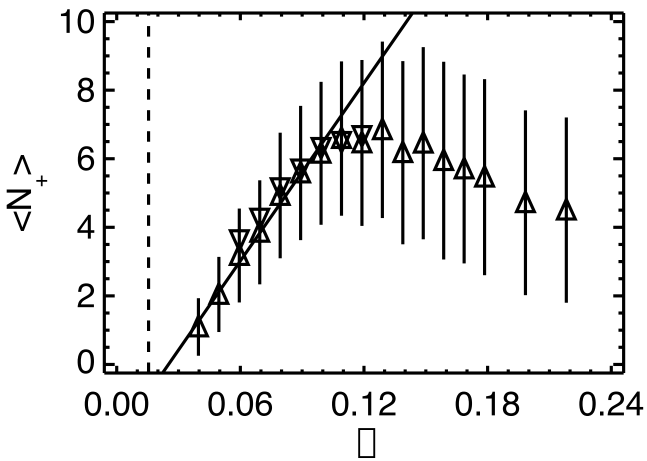
measure q by local wavenumber technique* on Fourier-filtered (complex) data
measure p by local wavenumber technique on modulus of data
measure A, B, C from local minima and maxima of □□□

* *Egolf, Melnikov, Bodenschatz. PRL. 80: 3228 (1998)*

Onset of Undulation Chaos

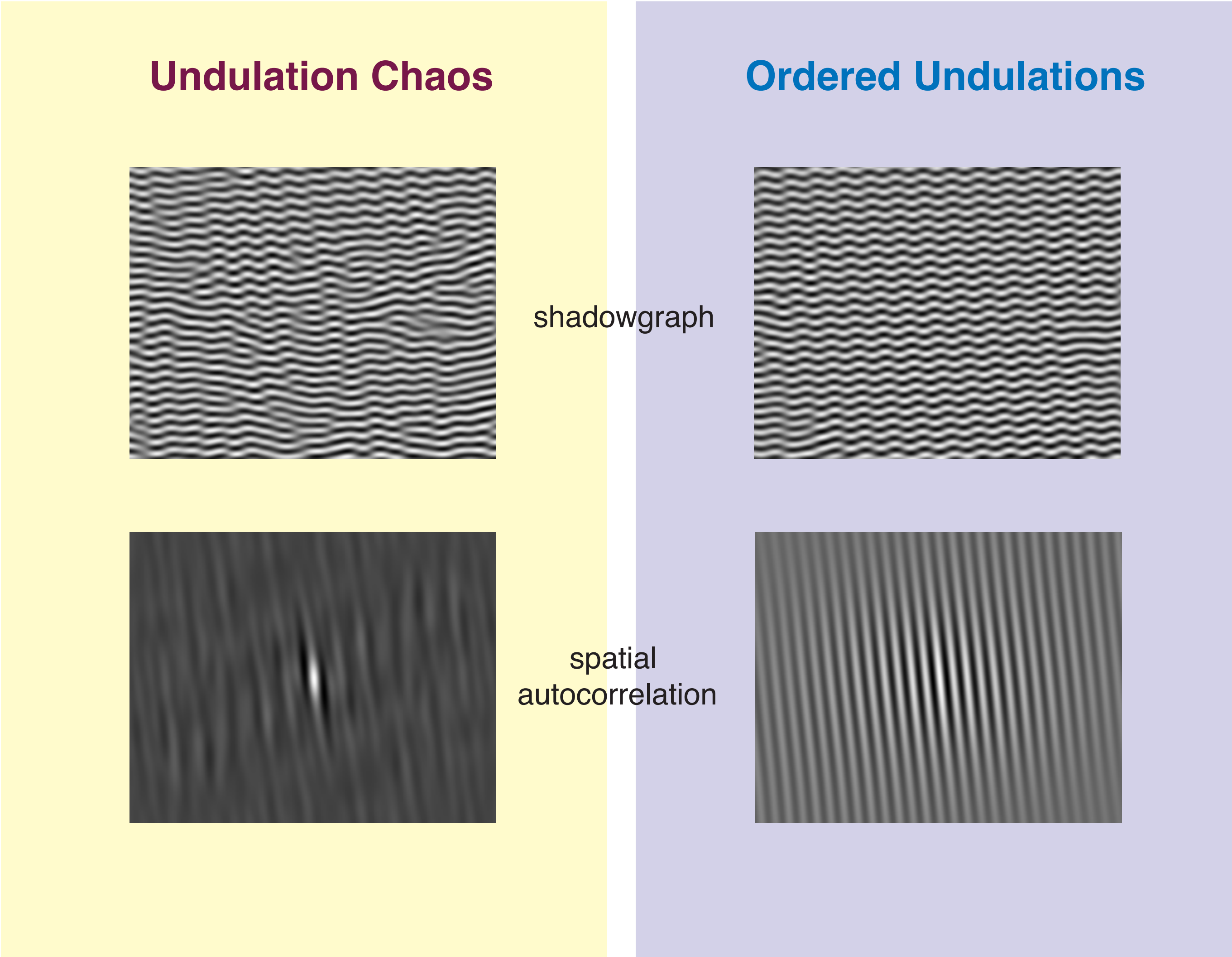


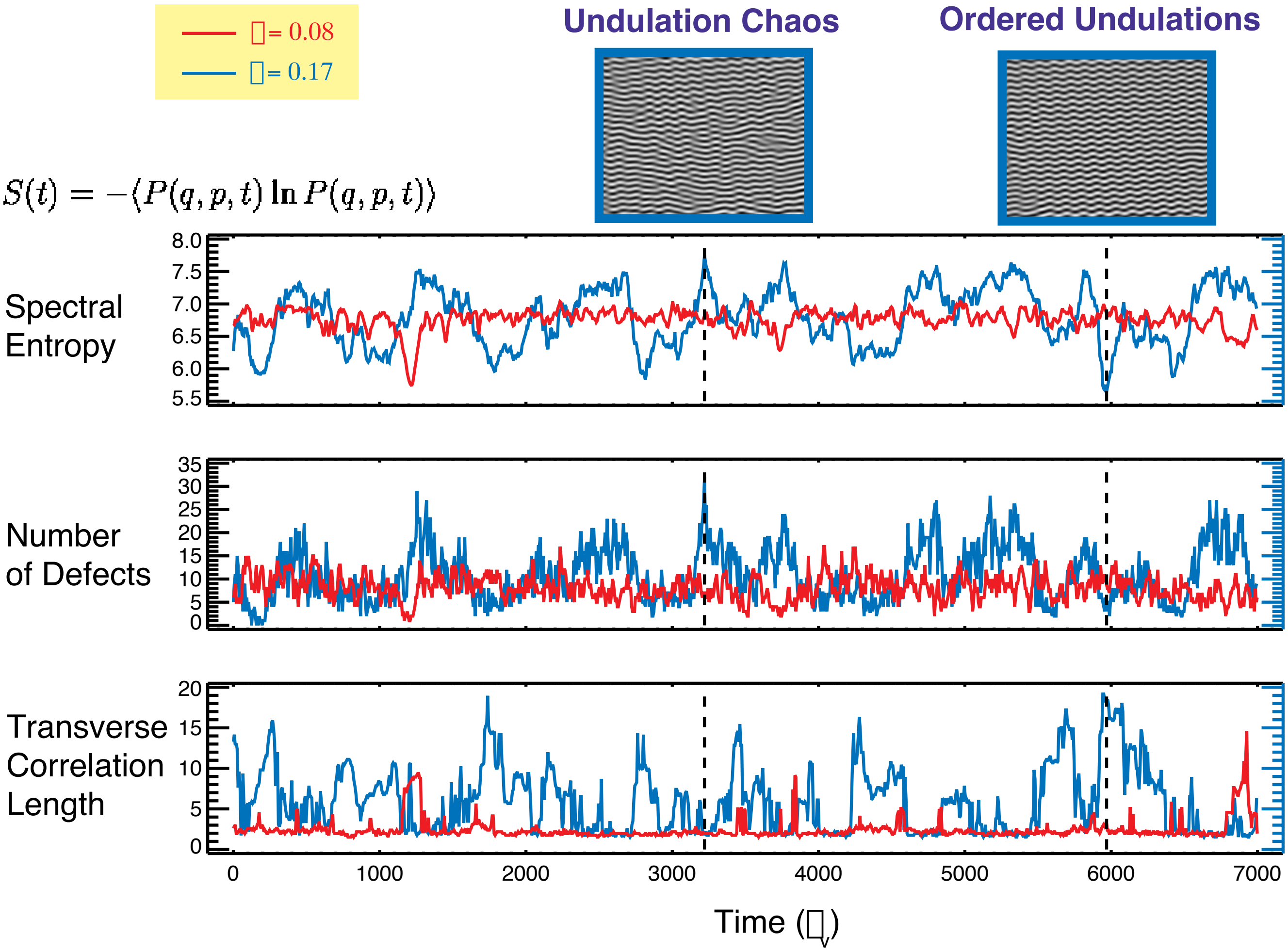
Undulation Wavenumber



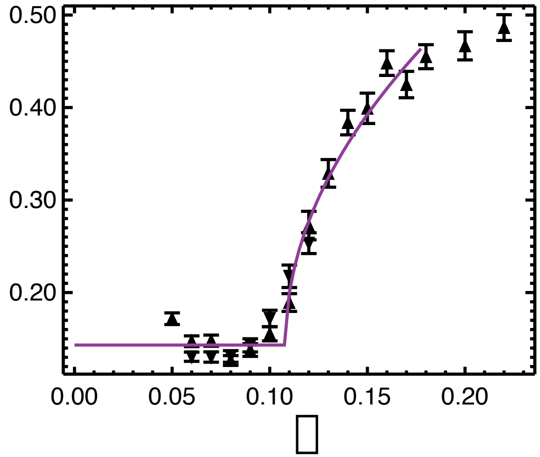
Number of Defects

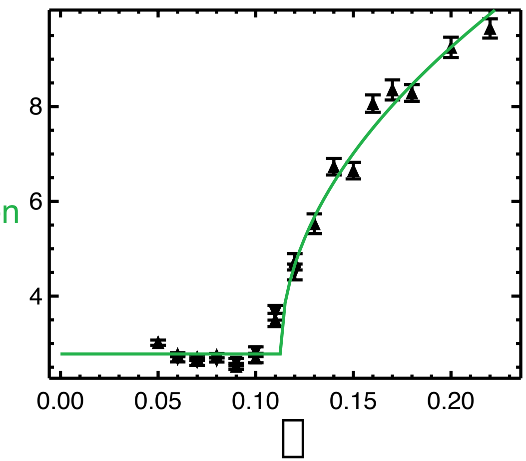
$$\epsilon = 0.17$$


Undulation Chaos
Ordered Undulations
shadowgraph
spatial
autocorrelation

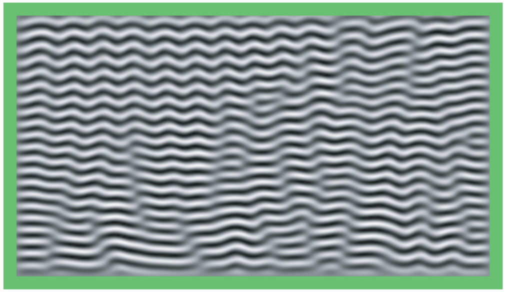

= 0.08
= 0.17
Undulation Chaos
Ordered Undulations
$S(t) = -\langle P(q,p,t) \ln P(q,p,t) \rangle$
Spectral Entropy
Number of Defects
Transverse Correlation Length
Time

Transition to Order/Chaos Competition

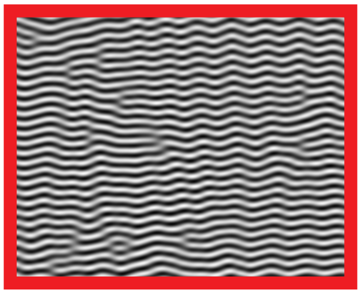




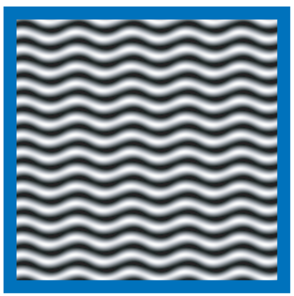
Stability of Wavenumbers



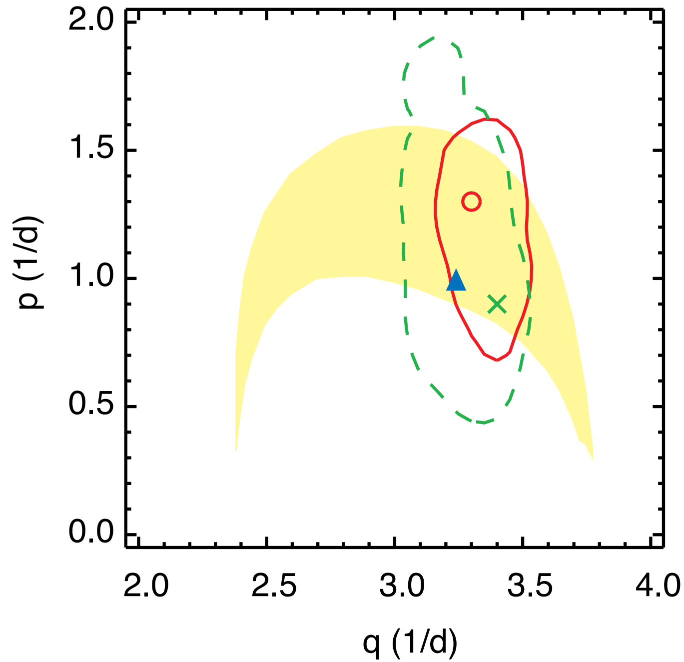
Simulation ×



Experiment ○



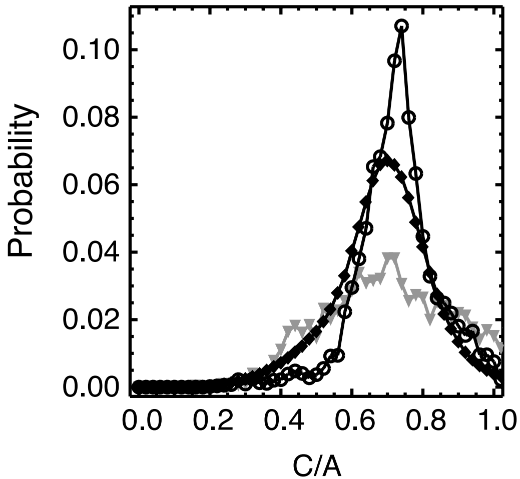
Galerkin ▲



□= 0.10

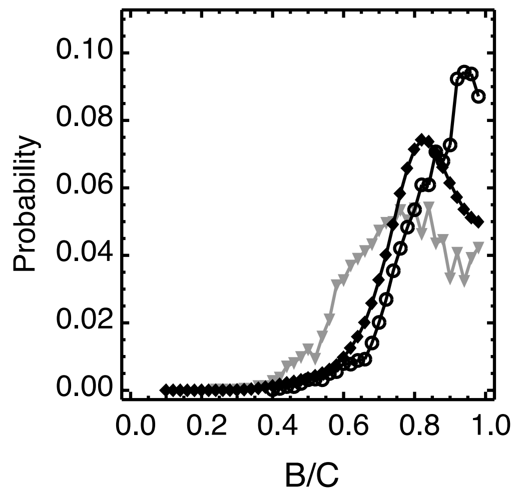
Order and Chaos in 3-mode Ansatz

$$\psi(x,y) = e^{iqx}\left(A + iBe^{ipy} + iCe^{-ipy} + \cdots\right)$$



▼ single Undulation Chaos image
○ single Ordered Undulation image
◆ average for 500 images (OU & UC)

□= 0.17

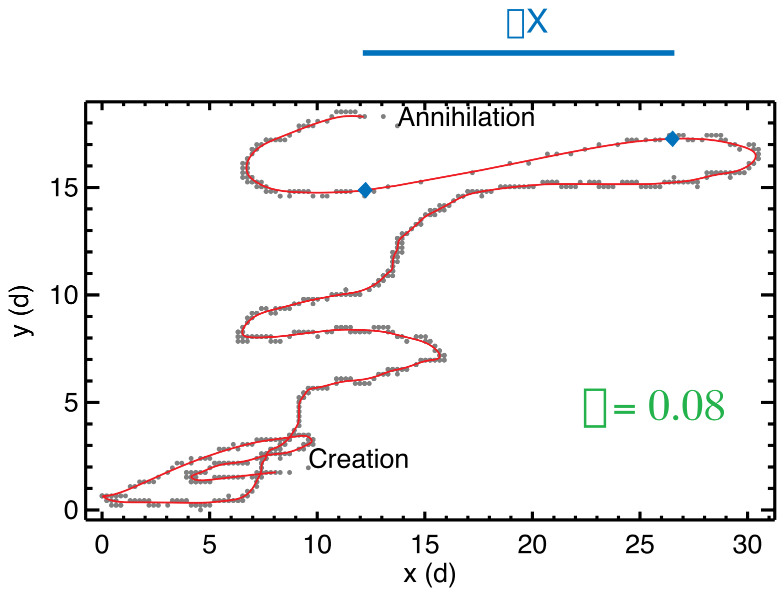


✦ undulation chaos: broad C/A
ordered undulations: sharp peak in C/A

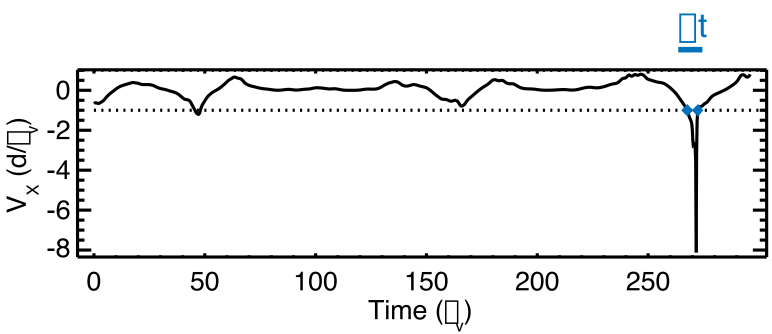
✦ undulation chaos: B/C ➔ 1

$$\epsilon = 0.08$$

Tracking Defects



weight neighbors by (x, y, t) to obtain smoothed trajectory



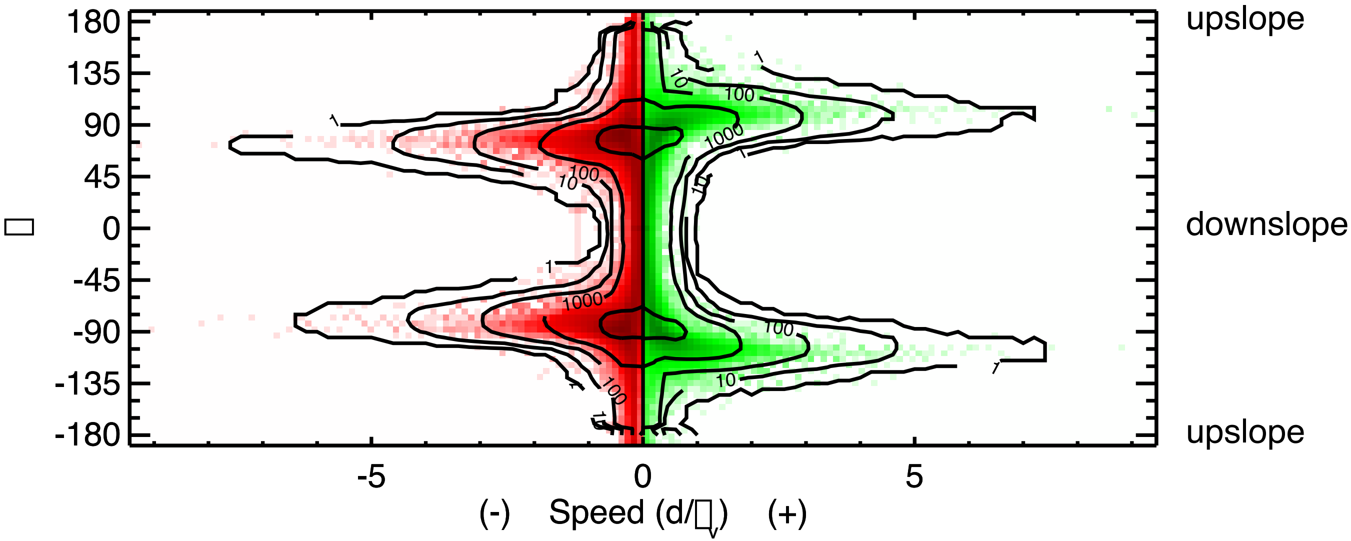
local linear fit with weights in (x, y, t) to obtain $v_x(t)$ and $v_y(t)$

flights: located where $|v_x| > 1d/\tau_v$

ΔX = displacement

Δt = duration

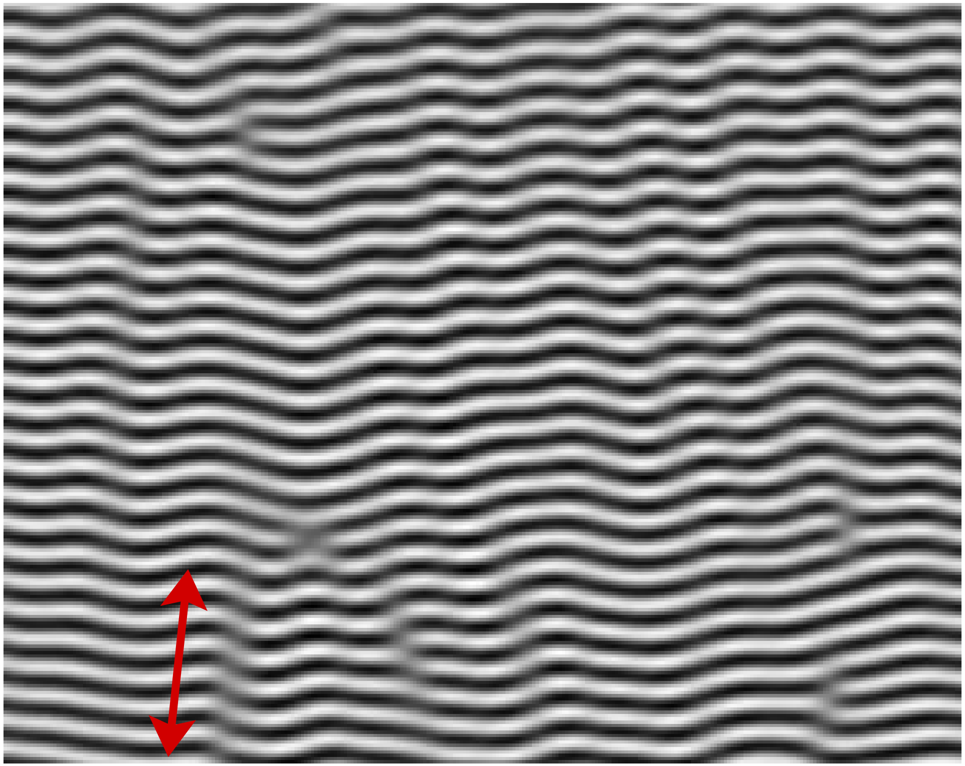
Defect Velocity and Direction



Negative Defects

Positive Defects

Defect Flights

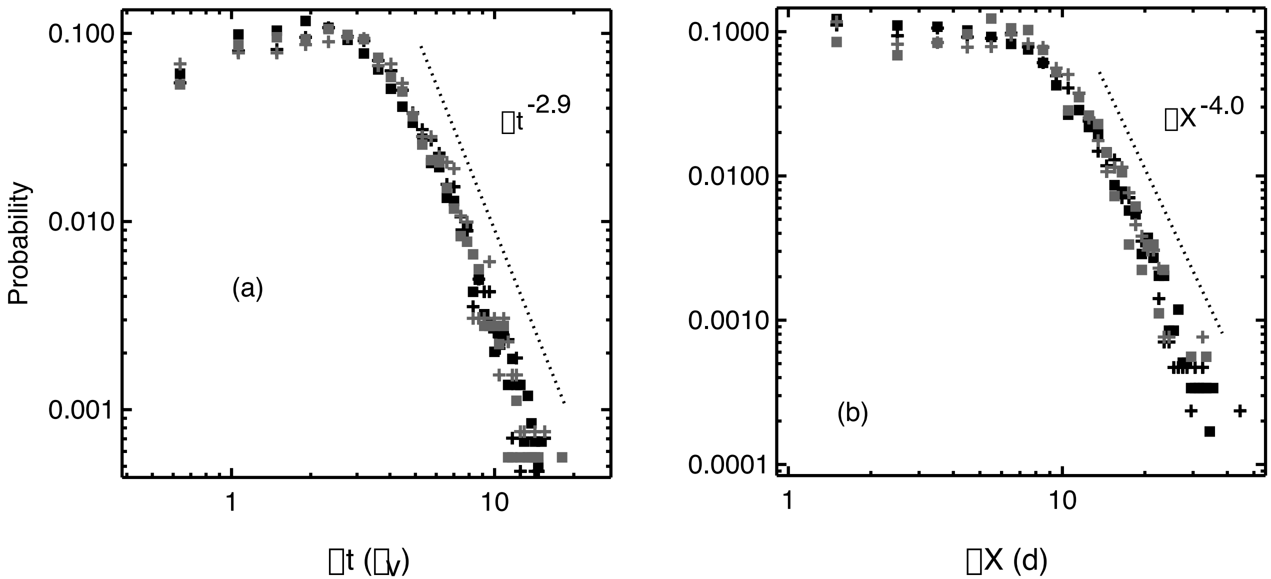


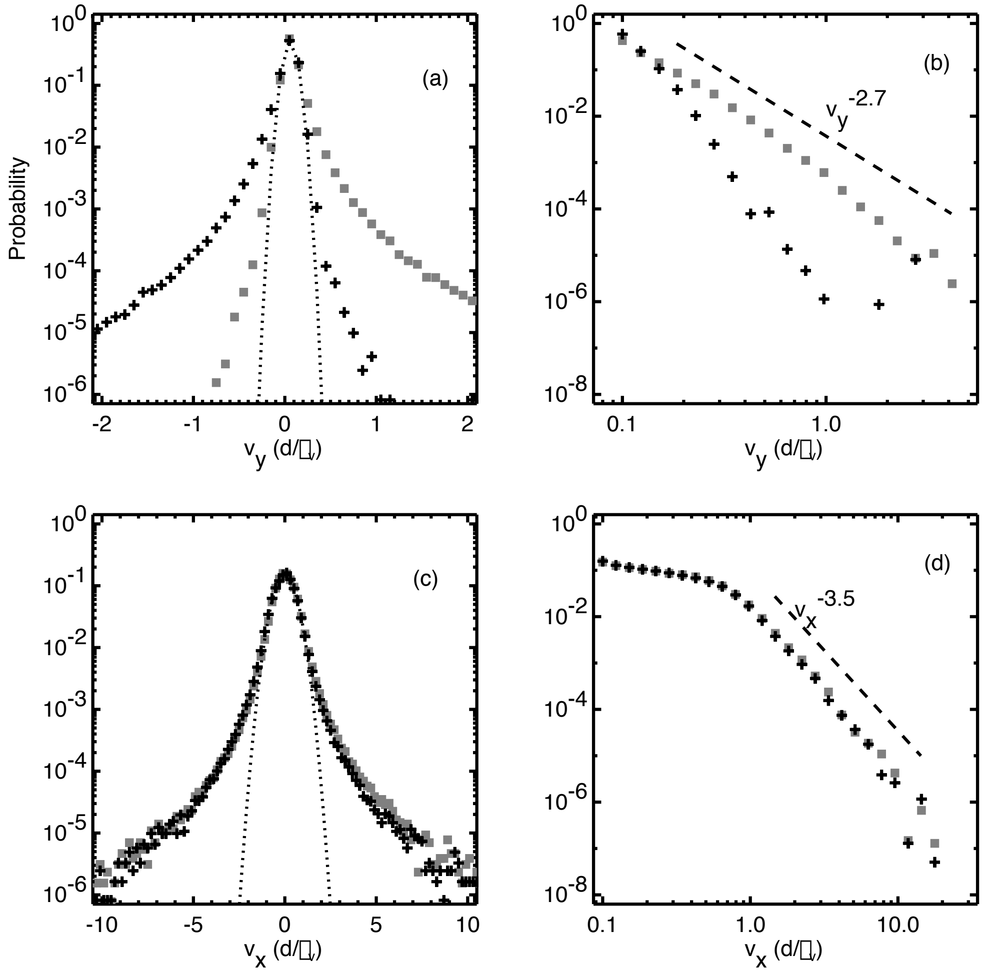
defects "zip" along lines of weak convection

Defect Flight Distributions

□= 0.08

Duration | **Displacement**

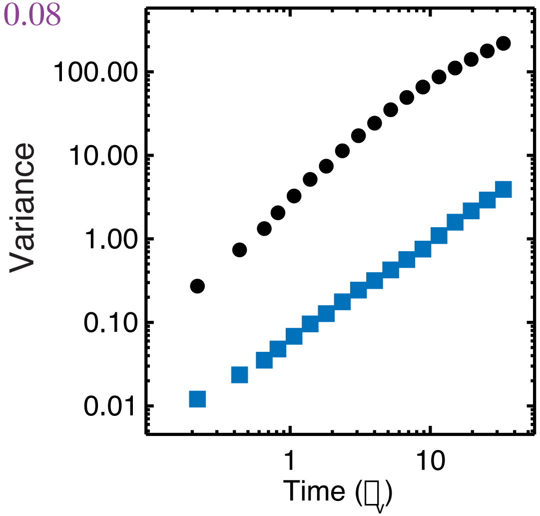


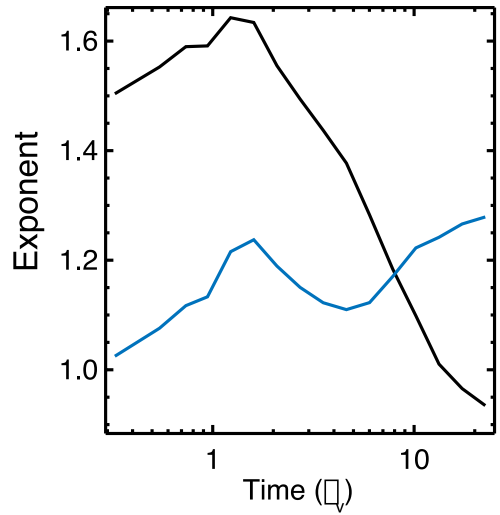


Velocity PDFs

Defect Diffusion

□= 0.08





$$\langle x^2(t)\rangle - \langle x(t)\rangle^2 = 2D_x t^\alpha$$

$$\langle y^2(t)\rangle - \langle y(t)\rangle^2 = 2D_y t^\alpha$$

□ = 1 diffusion

□ < 1 subdiffusion

□ > 1 superdiffusion

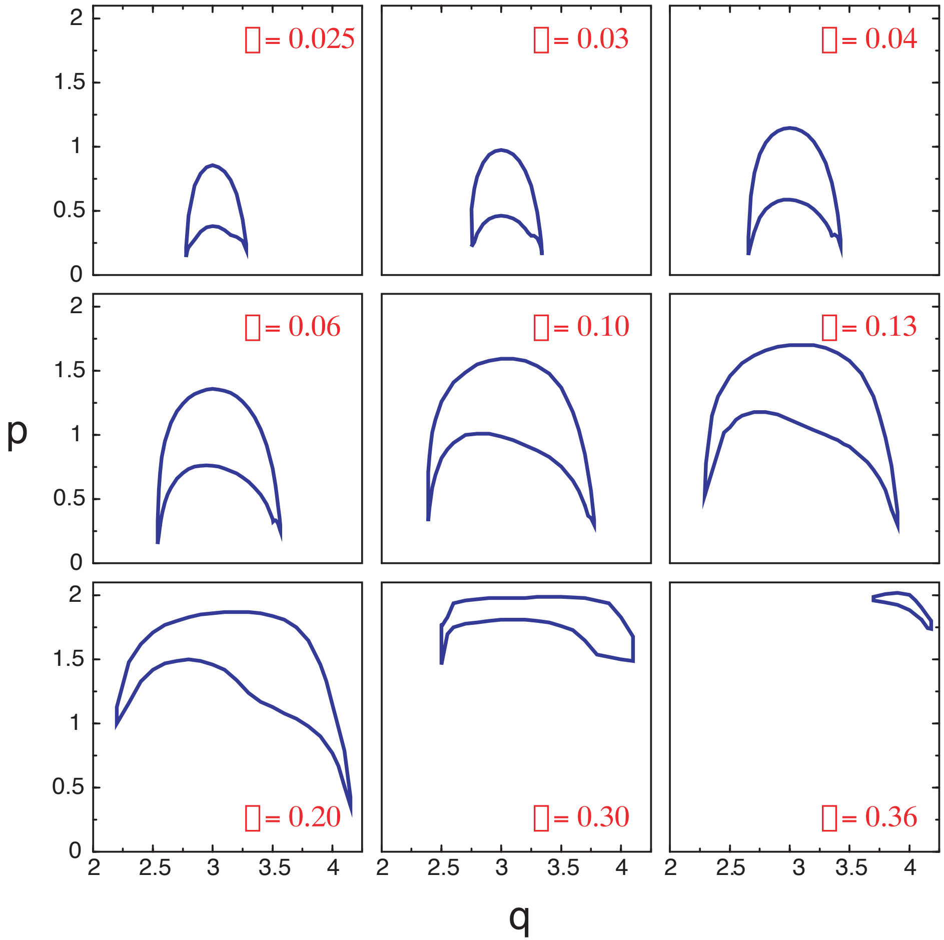
Conclusions

- exhibits a rich phase space with many spatiotemporally chaotic patterns
- provides for the study of spatiotemporal chaos in general and defect-turbulence in particular
- defects and undulations provide complementary descriptions
- measured defect creation and annihilation rates behave as expected, providing parameters for a modified Poisson distribution for the number of defects
- competing attractors observed for ordered undulations and undulation chaos (ordered undulations at *stronger* driving)
- defect motion exhibits Lévy-flight-like behavior associated with tearing of convection rolls

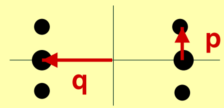
Outlook: Defect Turbulence

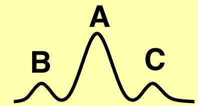
- What relationship exists between the relative position and velocity of neighboring defects?
- In intermittent phase, how do defect motions relate to the excursions between order and chaos?
- How does defect motion depend upon the local wavenumber of undulations?
- Can we obtain long-lived stable undulations in the experiment? At what (p, q, A, B, C)?
- To what extent are these results applicable to other defect-turbulent systems?

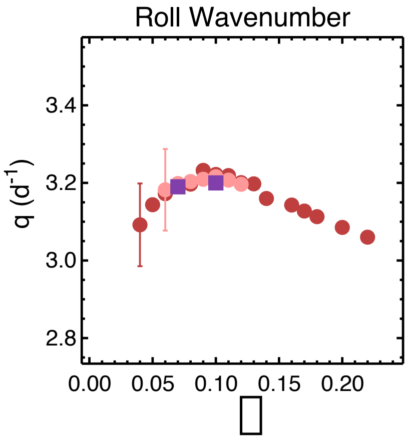
Stability Balloons for Ordered Undulations

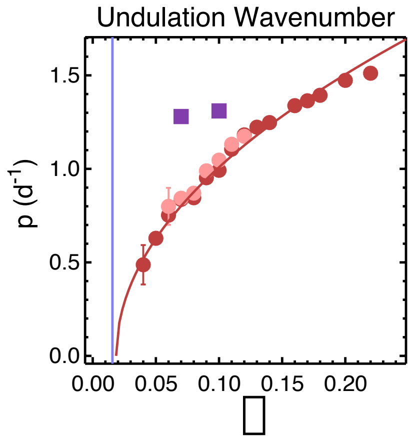


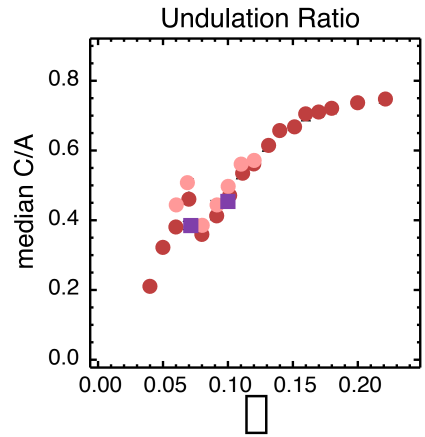
Comparing Undulations

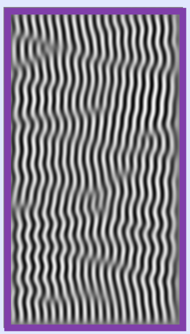




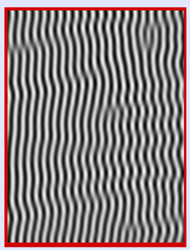






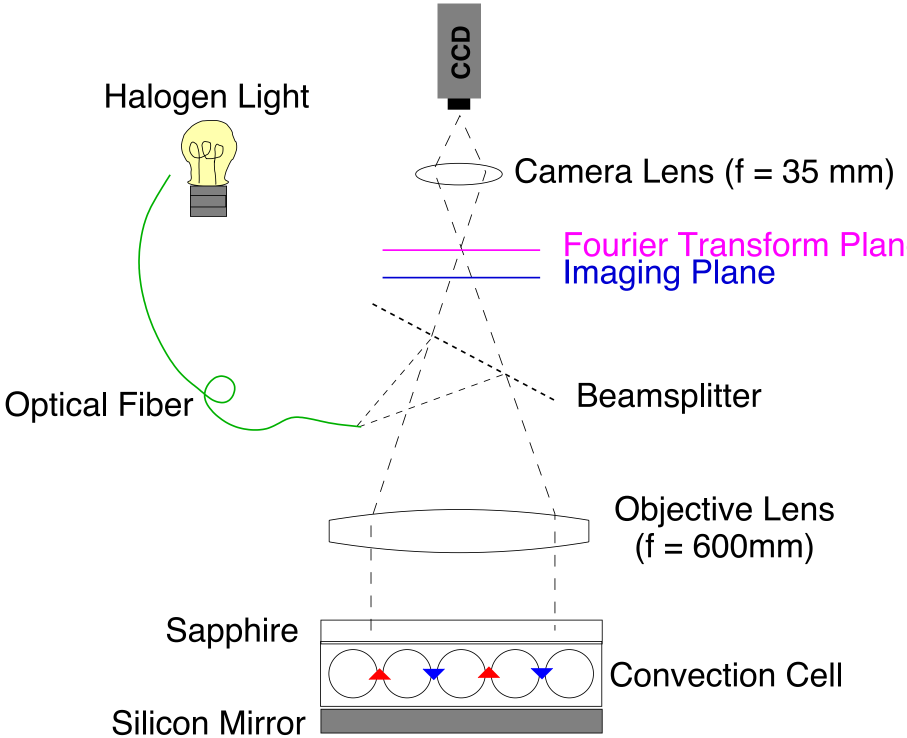


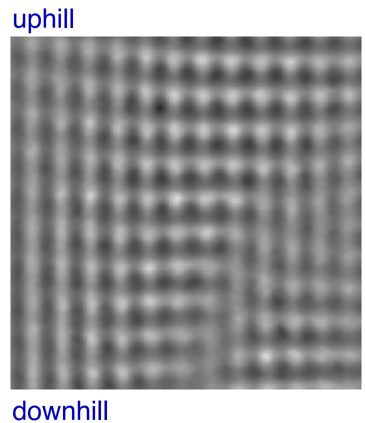
□= 30^{o} □= 0.07



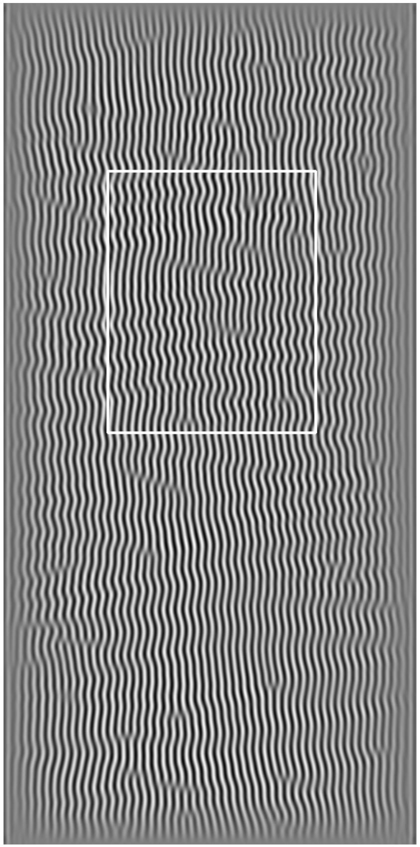
- experiment (quasistatic inscrease)
- (quasistatic decrease)
- pseudospectral simulation
- square root fit
- Galerkin prediction for undulation onset

Shadowgraph Visualization

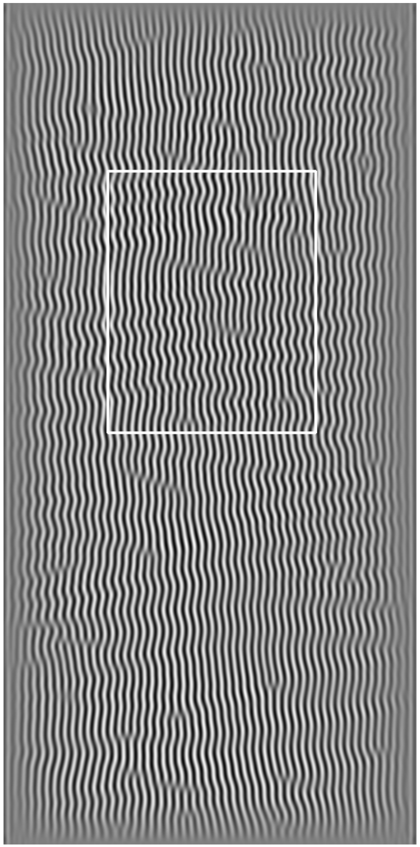


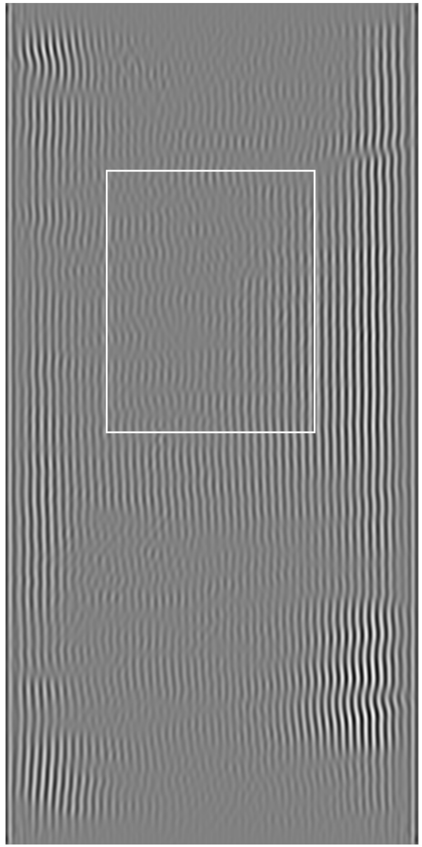


Cell Homogeneity



Cell Homogeneity





Cell Homogeneity