Line-defects of complex oscillatory spiral waves

Complex periodic spiral waves
(in a BZ system & in vitro cardiac system)

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Coupled network of nonlinear oscillators in biology

1) Bacterial colony
2) Firefly population
3) amoeba population
4) cardiac tissue
5) neural networks: eg., suprachiasmatic nucleus (the master biological clock)

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Simple system
but complex dynamics

\[
\begin{align*}
\dot{C}_x &= -C_y - C_z \\
\dot{C}_y &= C_x + AC_y \\
\dot{C}_z &= C_z C_x - CC_z + B
\end{align*}
\]

eg. Rössler system

- Period-1 cycle (C=C₁)
- Period-2 cycle (C=C₂)
- Period-4 cycle (C=C₃)

\[
\begin{align*}
C_x &\uparrow \\
C_y &\uparrow \\
C_z &\uparrow \\
\text{time} &\uparrow
\end{align*}
\]

\textit{cAMP waves} in signaling amoeba populations

\[
\begin{align*}
\text{spiral wave} &\quad \text{circular waves}
\end{align*}
\]

K. Lee et al., PRL (1996); PRL (2001); Nonlinearity (2002)
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**Period-1 spiral wave in a diffusively coupled Rössler system**

\[ \partial \tilde{c}(\tilde{r}, t)/\partial t = \tilde{R}[\tilde{c}(\tilde{r}, t)] + D \nabla^2 \tilde{c}(\tilde{r}, t) \]

\[ \begin{align*}
R_x &= -c_y - c_z \\
R_y &= c_x + Ac_y \\
R_z &= c_xc_z - Bc_z + B
\end{align*} \]

Parameters:
- A=0.2, B=0.2, C=3.84, D\Delta t/(\Delta r)^2=1.6\times10^{-2}

**References**

**Period-2 spiral wave with one line-defect in a Rössler system**

**References**
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**Period-2 spiral wave in the coupled Rössler system**

![Diagram of period-2 spiral wave]

**P1-P2 spiral wave transition in the coupled Rössler system**

![Diagram of P1-P2 spiral wave transition]

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**A pair of period-2 spiral waves**

*in BZ EXPERIMENT*  

Disk diameter: 20 mm

One-side fed reactor:
- 0.89 M HSO₄
- 0.06 M KBrO₃
- 0.27 M melonic acid
- 2.5 M ferroin
- 0.1 mM NaBr
- 0.1 mM SDS

**Schematic diagram of two-side fed gel reactor**

Output

Input

Anopore disk
Nitro-cellulose Membrane
Polyacrylamide Gel
Spacer

Chemistry
- A - HSO₄(0.97M), KBrO₃(0.12M)
- B - MA(0.54M), Ferroin(2.5ml), NaBr & SDS(0.1mM)

Volume : 10ml

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Period-1 to Period-2 spiral wave transition in a BZ system

$[H^+] = 3.02 \text{ M}$  $2.40 \text{ M}$  $2.12 \text{ M}$

Intermittent behaviors at a P-1 to P-2 spiral wave transition

Phase slips!!

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Breathing line-defect of a spiral pair at P-1 to P-2 transition

(a) (b)

[H+] = 2.80M
[BrO₃⁻] = 0.75M

Field of view: 8.2 x 4.8 mm²

Line-defect density fluctuation during a P-1 to P-2 spiral wave transition

(a) (b)

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Dynamic shape transition of a spiraling line-defect

\[ \text{[H+]}: 2.48 \text{ M} \rightarrow 2.22 \text{ M} \text{ at } t = 0 \]

J. Park et al., JKPS 41, 4 (2002)

Transverse instability of a line-defect

Diameter:

5.6 mm

Two-side fed reactor:

SIDE A

0.90 M H\textsubscript{2}SO\textsubscript{4}
0.50 – 0.64 M KBrO\textsubscript{3}

SIDE B

0.53 M MA,
2.5 M ferroin,
0.1 mM NaBr,
0.1 mM SDS

Flow rate = 120 ml/hr,
Volume = 10 ml,
Temp 22 °C

0.54 M
0.58 M
0.60 M

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**Analogy with bistable systems**

- Iodate-ferrocyanide-sulfite reaction-diffusion system, SCIENCE 1993, K. Lee et al.
- FitzHugh-Nagumo type model reaction-diffusion system, PRL 1994, A. Hagberg and E. Meron

**Dynamics of LINE-DEFECT in a Williamsokii- Rössler system**

Simulation by S. Woo (2002)  
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Heuristic picture & quantifications

Line-defect proliferation & chaotic domains

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Similar observations in a Williamskii- Rössler system

Complex periodic spiral waves: summary

- So far, we discussed several phenomena associated with diffusively (i.e., linearly) coupled nonlinear oscillators, mostly, in Period-1 & Period-1/Period-2 (mixed) regimes.

- Obviously, the system will become more complex, if it
  1) moves into more complex periodic regime,
  2) involves a more complex coupling mechanism, or
  2) becomes three-dimensional in space.

- Any complex periodic spiral wave in biology?
Line-defects of complex oscillatory spiral waves

- Jin-sung Park (BZ)
- Sung-jae Woo (Model)
- Seong-min Hwang
  (Cardiac waves)