ACCRETION OF THE OUTER PLANETS:

OLIGARCHY OR MONARCHY?

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In the terrestrial planet region, accretion of planets is "oligarchic," i.e., planetary embryos form with a characteristic mass and more or less uniform orbital spacing. Reasons for this are:

Runaway Growth: The largest planetesimal gains mass faster than the next-largest in some region near its orbit, due to the increase of gravitational cross-section with mass. The largest body becomes dominant and acquires most of the available mass.

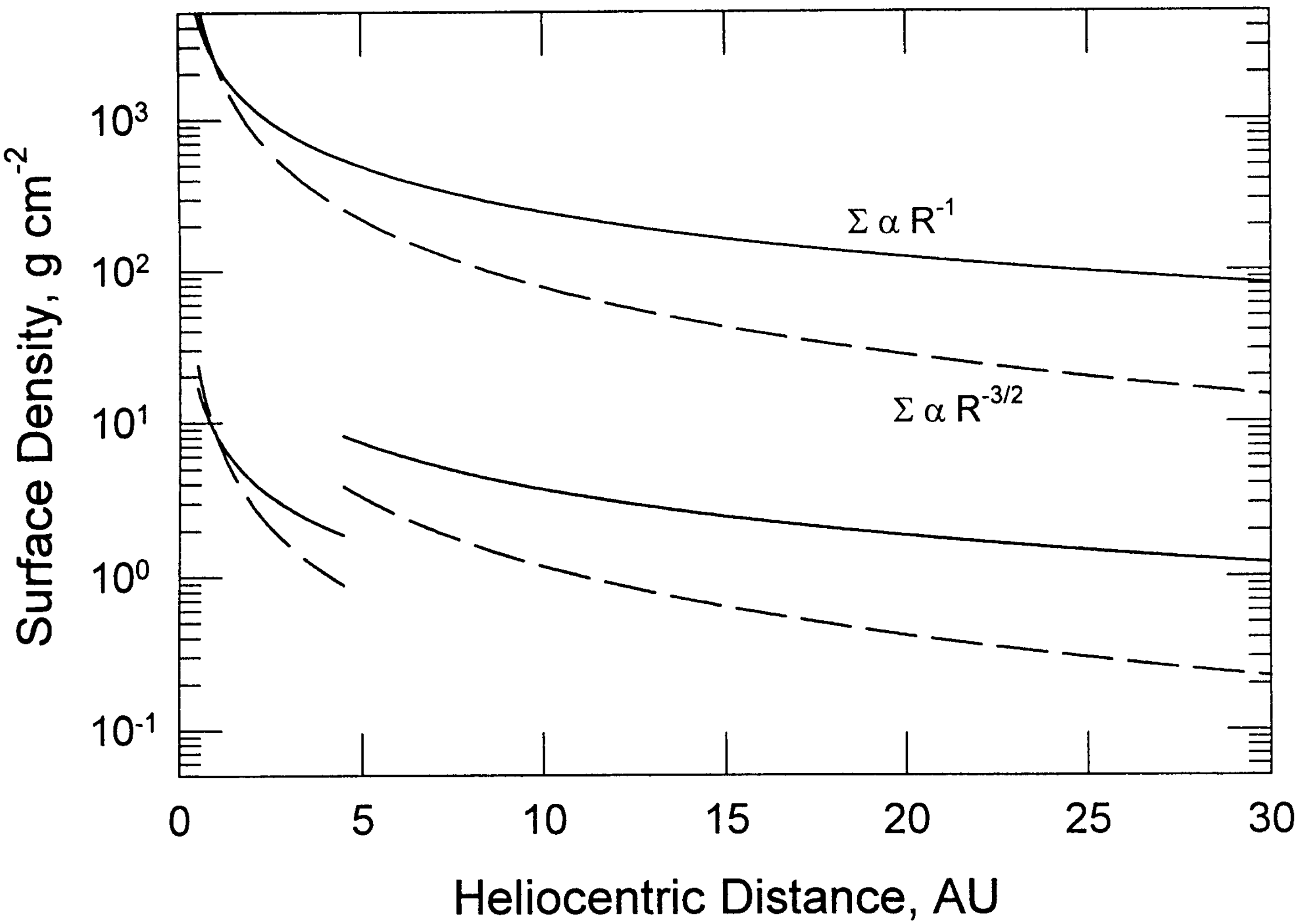
Dynamical Isolation: The amount of mass that a dominant embryo can accrete is limited by the Jacobi parameter in the restricted 3-body problem. Only small bodies within a certain distance of its orbit can collide with it. When these bodies are exhausted, the embryo's growth slows or halts. Typically, the limiting mass varies slowly with heliocentric distance.

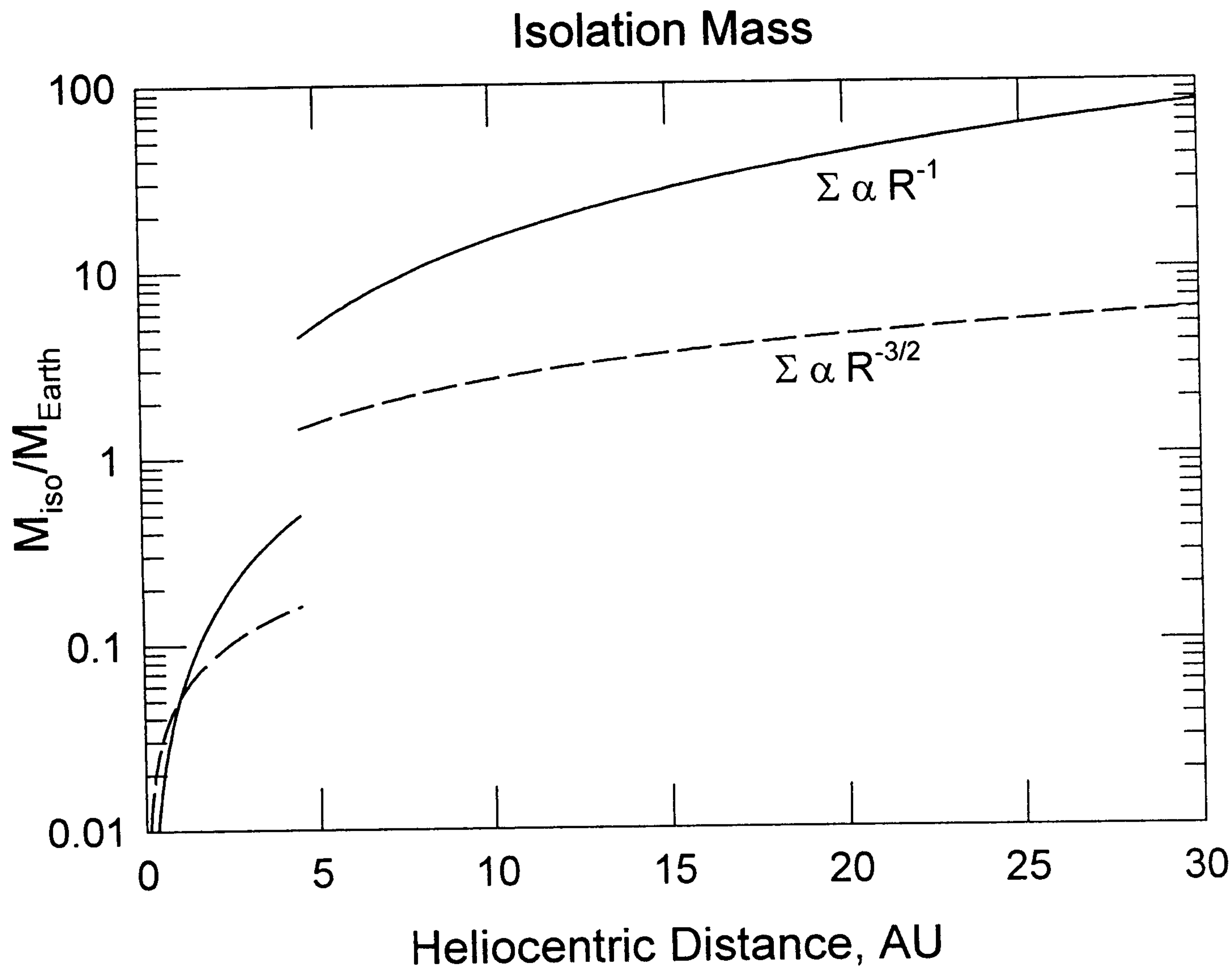
The "Isolation Mass" set by the Jacobi parameter is

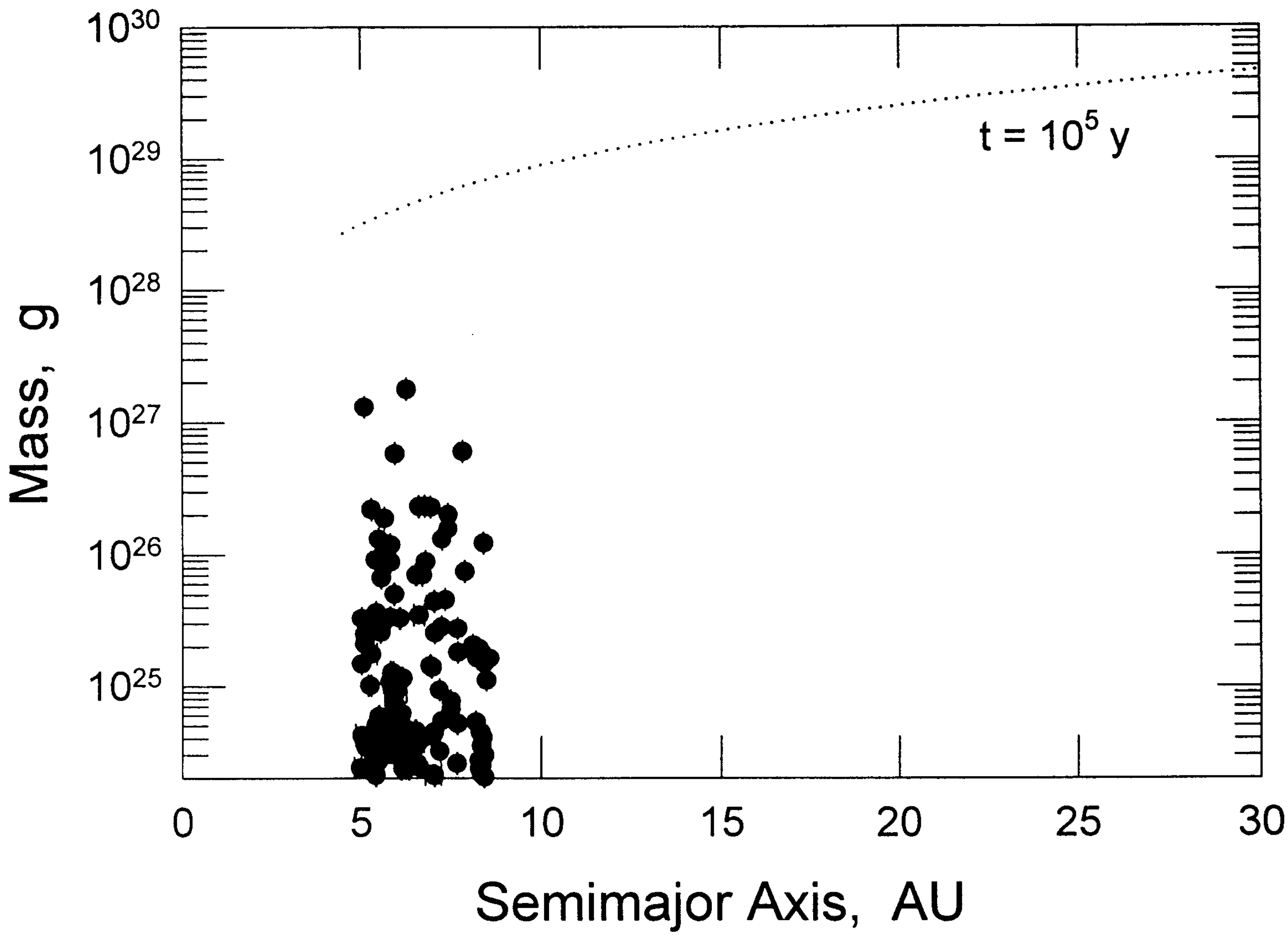
$$M_{iso} = 2.1\times10^{-3}\,\sigma^{3/2}\,R^{3}\,M_{\oplus}$$

where σ is in g cm^{-2} and R in AU. M_{iso} increases with R if σ decreases less steeply than R^{-2}.

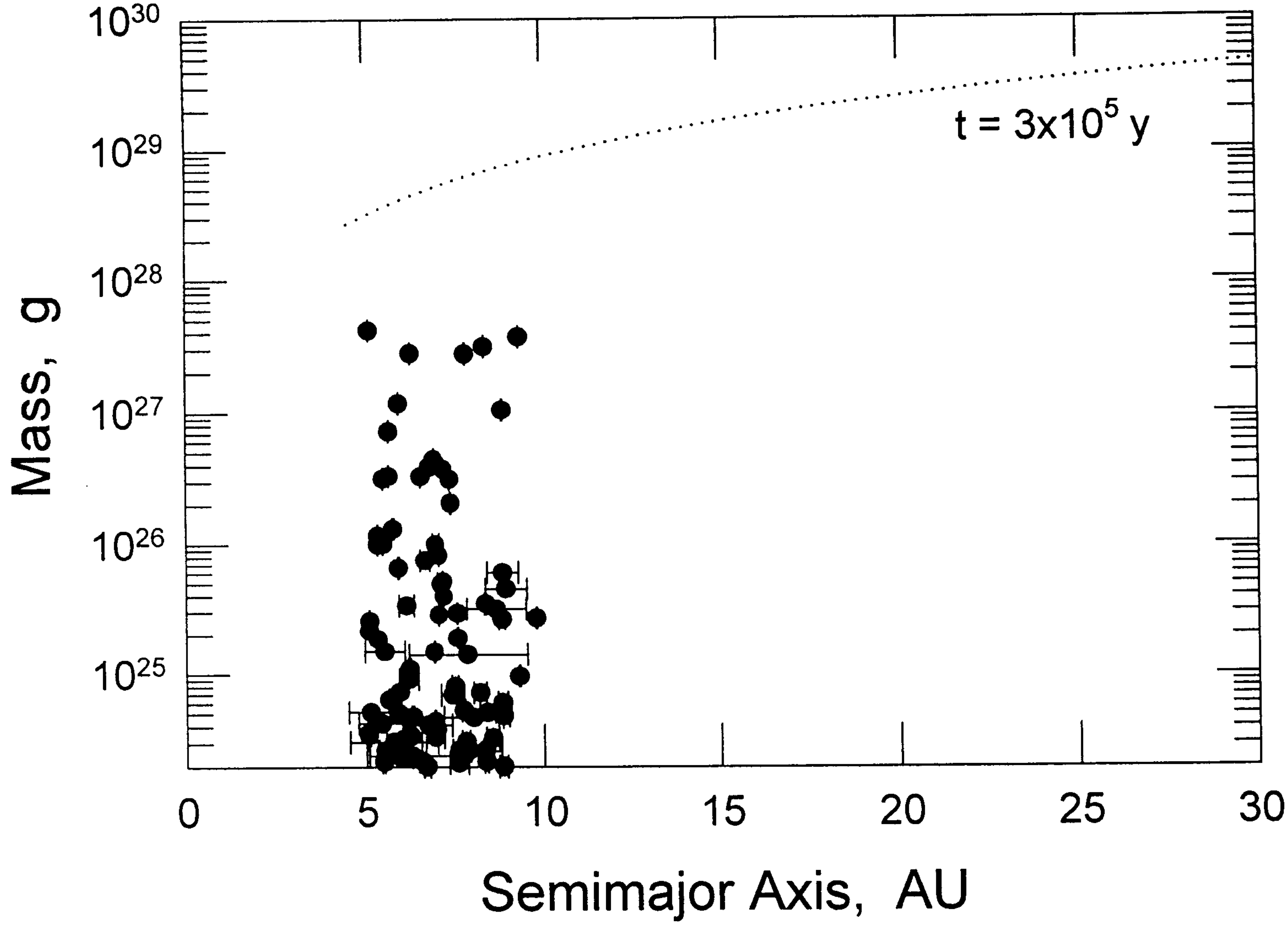
The surface density of the planetesimal swarm is fairly well constrained in the terrestrial region, ~ 10 g cm^{-2}. The "canonical" minimum-mass solar nebula has $\sigma \propto R^{-3/2}$, but ejection of comets and probable migration of the outer planets implies more mass in the outer nebula and a shallower gradient, consistent with R^{-1}.


Surface Density, g cm-2
10-1
10^0
10^1
10^2
10^3
Σ α R-1
Σ α R-3/2
0
5
10
15
20
25
30
Heliocentric Distance, AU

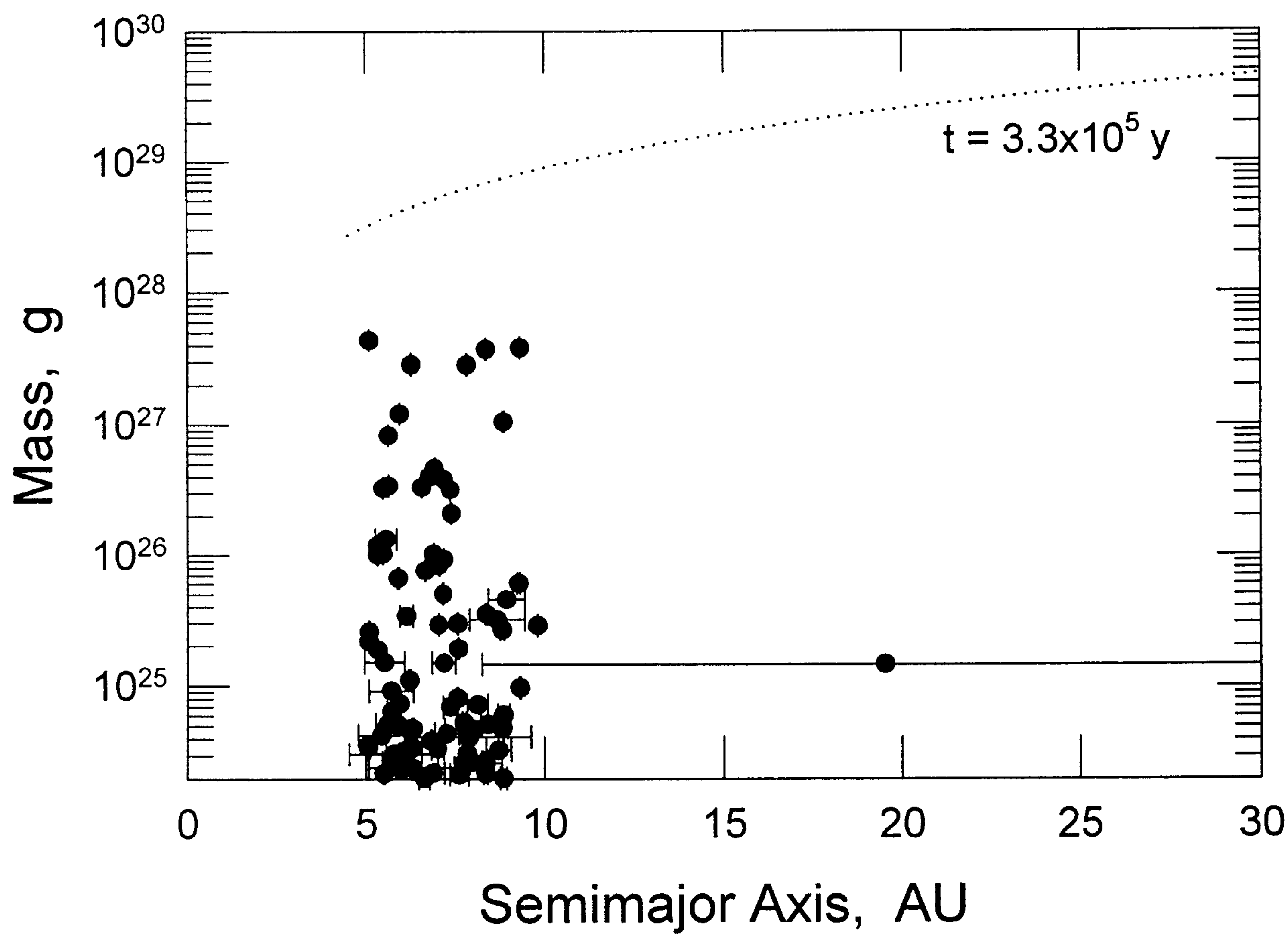

Isolation Mass
M_iso/M_Earth
100
10
1
0.1
0.01
0
5
10
15
20
25
30
Σ α R^-1
Σ α R^-3/2
Heliocentric Distance, AU



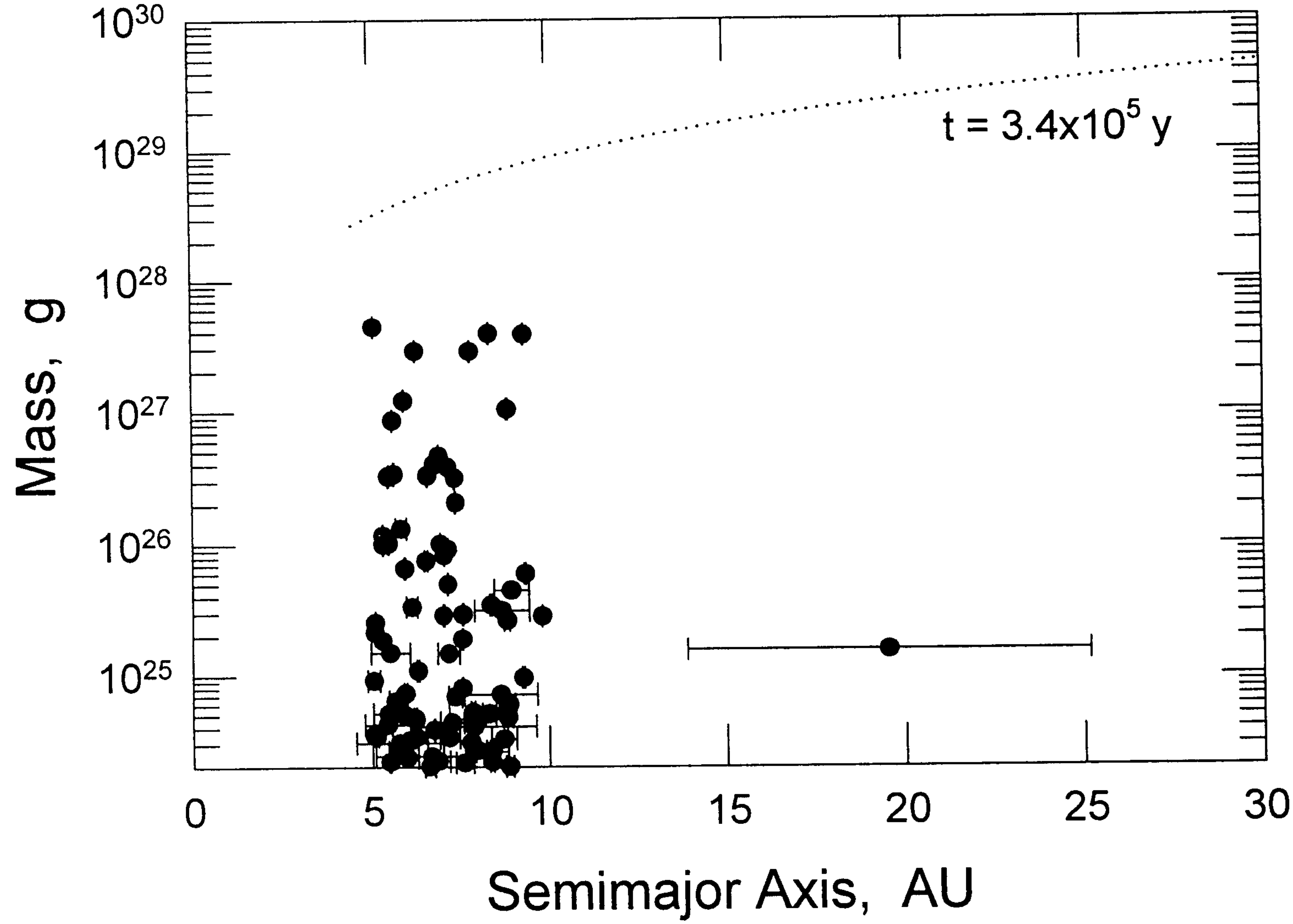
Mass, g
10^{30}
10^{29}
10^{28}
10^{27}
10^{26}
10^{25}
0
5
10
15
20
25
30
Semimajor Axis, AU
$t = 10^5$ y



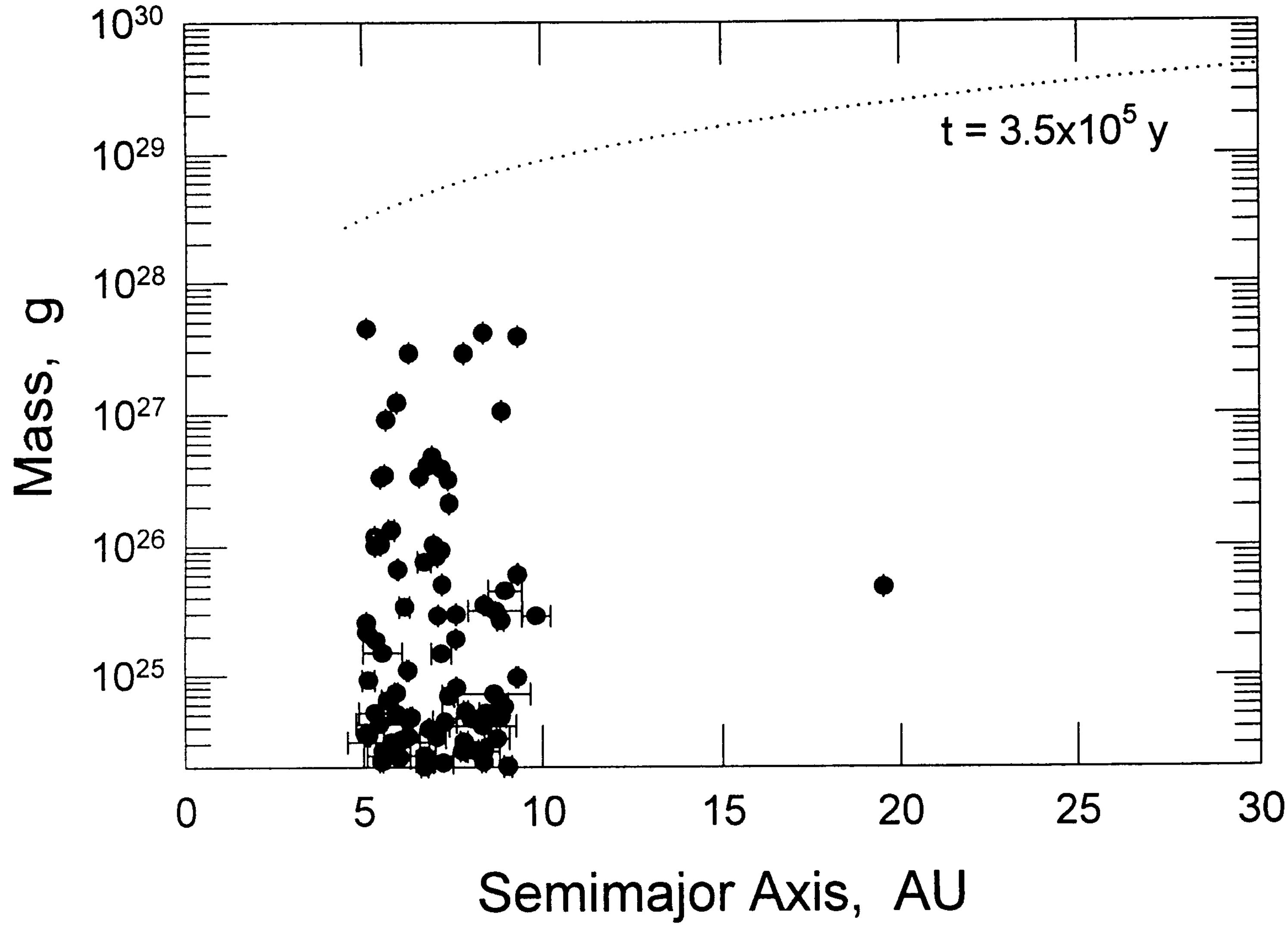
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
t = 3x10^5 y
0
5
10
15
20
25
30
Semimajor Axis, AU



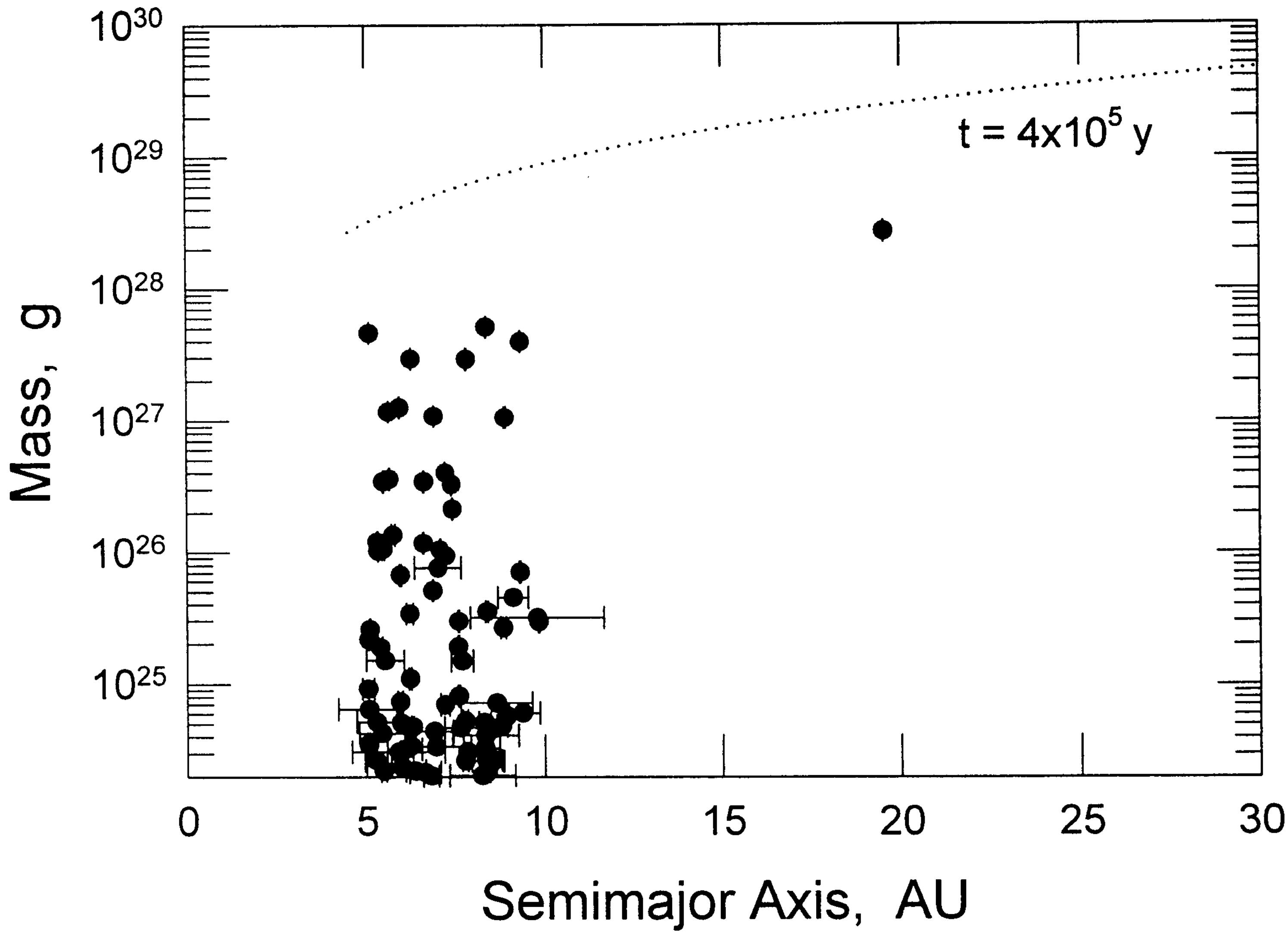
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
t = 3.3x10^5 y
0
5
10
15
20
25
30
Semimajor Axis, AU

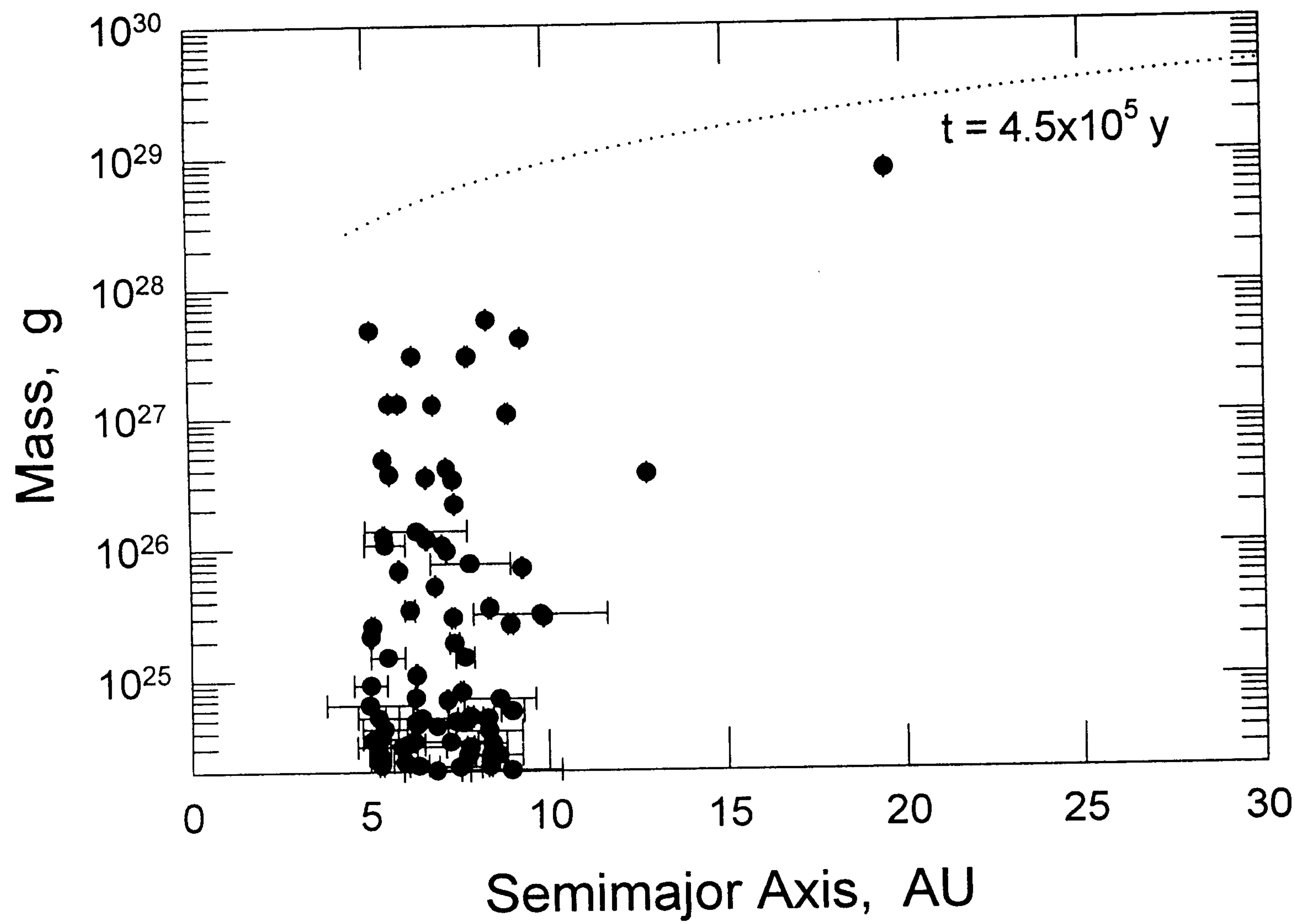


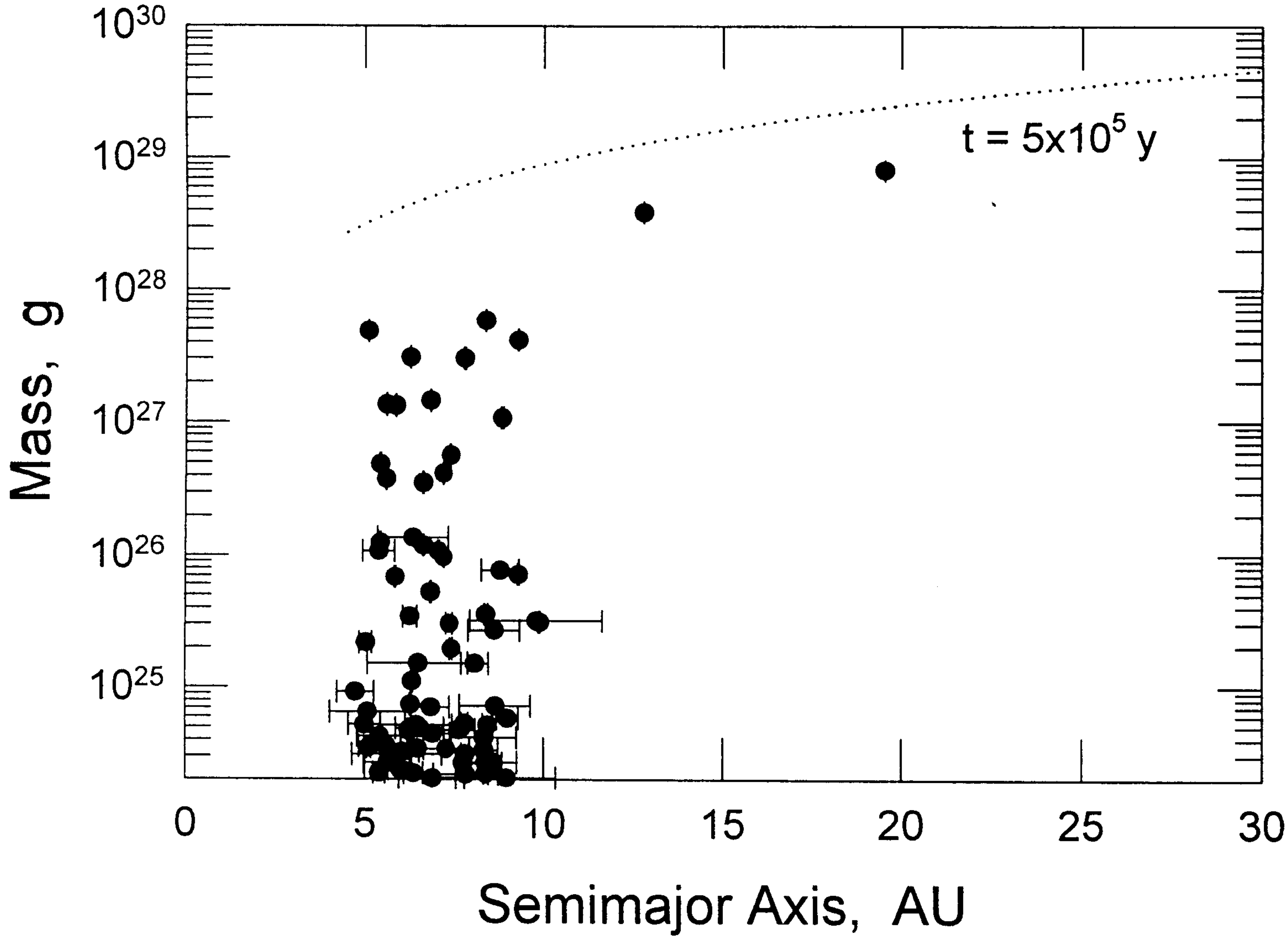
t = 3.4x10^5 y
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
0
5
10
15
20
25
30
Semimajor Axis, AU

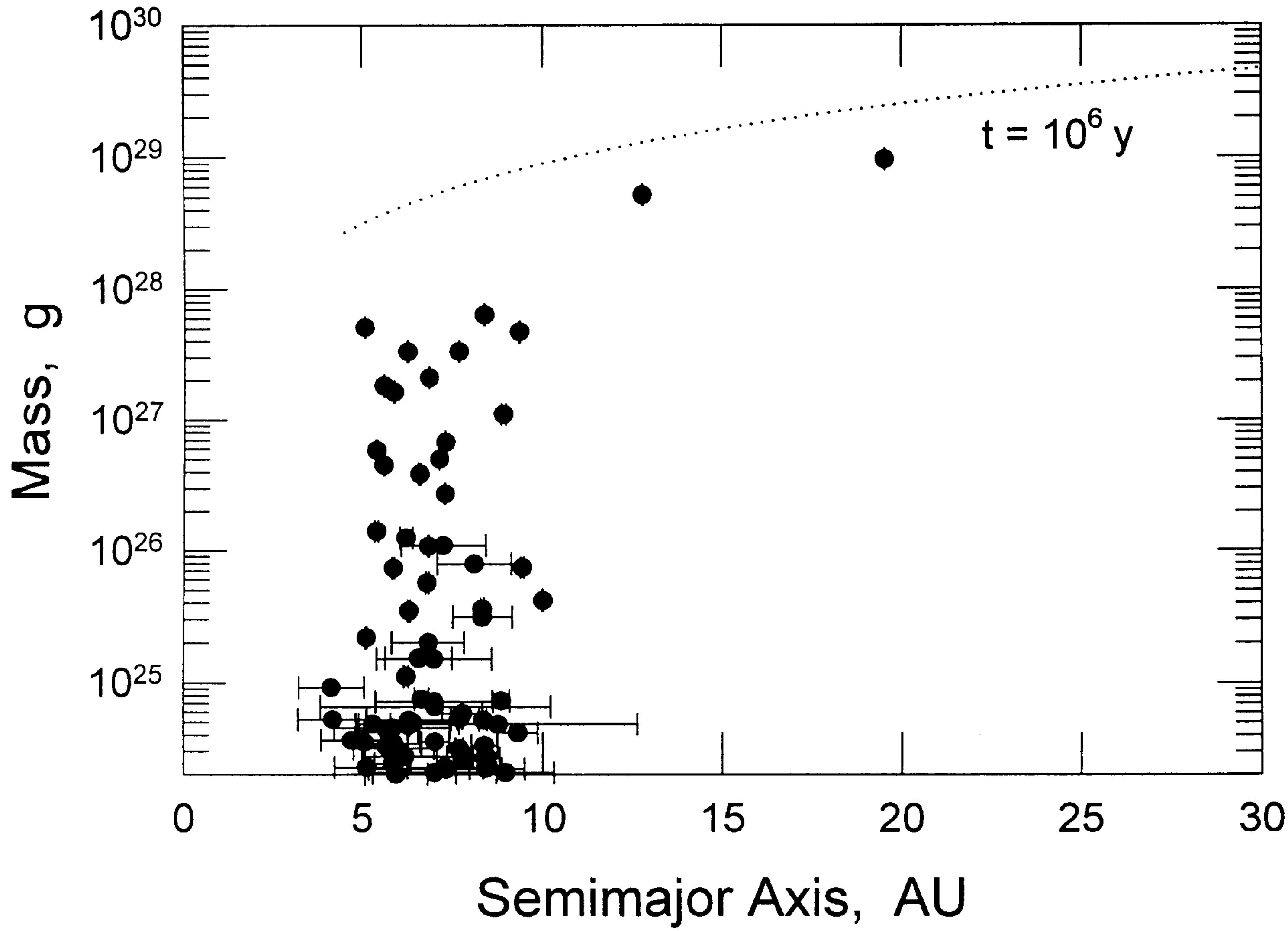


t = 3.5x10^5 y
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
0
5
10
15
20
25
30
Semimajor Axis, AU


t = 4x10^5 y
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
0
5
10
15
20
25
30
Semimajor Axis, AU


t = 4.5x10^5 y
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
0
5
10
15
20
25
30
Semimajor Axis, AU


t = 5x10^5 y
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
0
5
10
15
20
25
30
Semimajor Axis, AU



t = 10^6 y
Mass, g
10^30
10^29
10^28
10^27
10^26
10^25
0
5
10
15
20
25
30
Semimajor Axis, AU

Starting from small planetesimals of comparable mass, growth occurs in three stages:

1) Stochastic coagulation with collision rates set by random velocities, until a dominant body emerges with mass >> median $M/m \sim 10^6$.

2) Rapid growth of the dominant body with encounter velocities due to keplerian shear, until its mass approaches M_{iso}

3) Sweeping up of the remnants, coalescence of embryos

Stages 1 and 3 are slow; 2 is fast.

In terrestrial planet region, runaway growth is local. At larger heliocentric distances:

M_{iso} increases, so embryos perturb smaller planetesimals more strongly.

Surface density, and hence M_{iso}, increases at the "snow line."

Kepler velocity decreases, so a velocity perturbation of a given magnitude causes a larger change of orbit.

The result is that planetesimals are more mobile in the outer solar system.

An embryo can cause a velocity perturbation of the order of its escape velocity. Comparing V_e for a body of mass M_{iso} with V_K as a function of heliocentric distance,

$$\frac{V_e}{V_K} = 0.036\left(\frac{\rho}{1 g\, cm^{-3}}\right)^{1/6}\left(\frac{\sigma}{1\, g\, cm^{-2}}\right)^{1/2}\left(\frac{R}{1\ AU}\right)^{3/2}$$

If $\sigma \propto R^{-1}$, $V_e/V_K \propto R$; if $\sigma \propto R^{-3/2}$, $V_e/V_K \propto R^{3/4}$

At $R = 1$ AU, $\sigma = 10\, g\, cm^{-2}$: $M_{iso} = 0.07\ M_{\oplus}$, $V_e/V_K \sim 0.14$

At $R = 5$ AU, $\sigma = 8\, g$ cm^{-2}: $Miso = 6\, M_{\oplus}$, $V_e/V_K \sim 1$

Rapid runaway occurs when small bodies are brought to the growing embryo at low velocities by keplerian shear, rather than by random motions (e, i). The shear velocity is

$$\Omega \times R_H = (M/3M_\odot)^{1/3} V_K,$$

While the random velocity is comparable to the escape velocity of the small background population of mass m:

$$V_e = (2\, Gm/r)^{1/2}\,.$$

These are equal when

$$\frac{M}{m} = \left(\frac{9M_\odot}{4V_k^3}\right)\left(\frac{8G}{3}\right)^{3/2}(\pi\rho)^{1/2} \sim 2.3\times10^4\left(\frac{\rho}{1\ g\ cm^{-3}}\right)^{1/2}\left(\frac{R}{1\ AU}\right)^{3/2}$$

$\sim 10^4$ at 1 AU, $\sim 10^6$ at 10 AU, $\sim 2\times10^6$ at 20 AU.

e.g., a 100-km body among km-sized planetesimals.

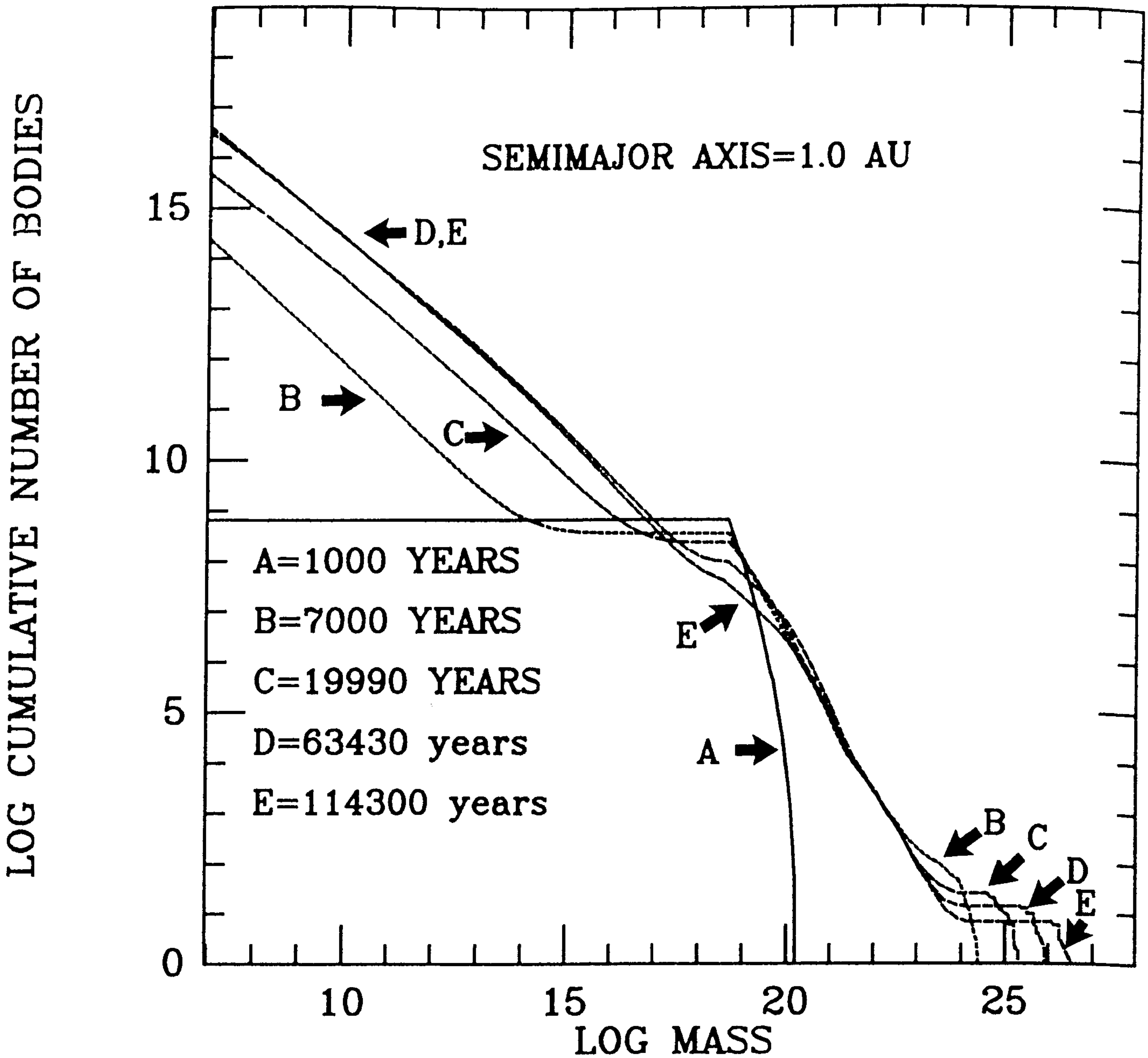
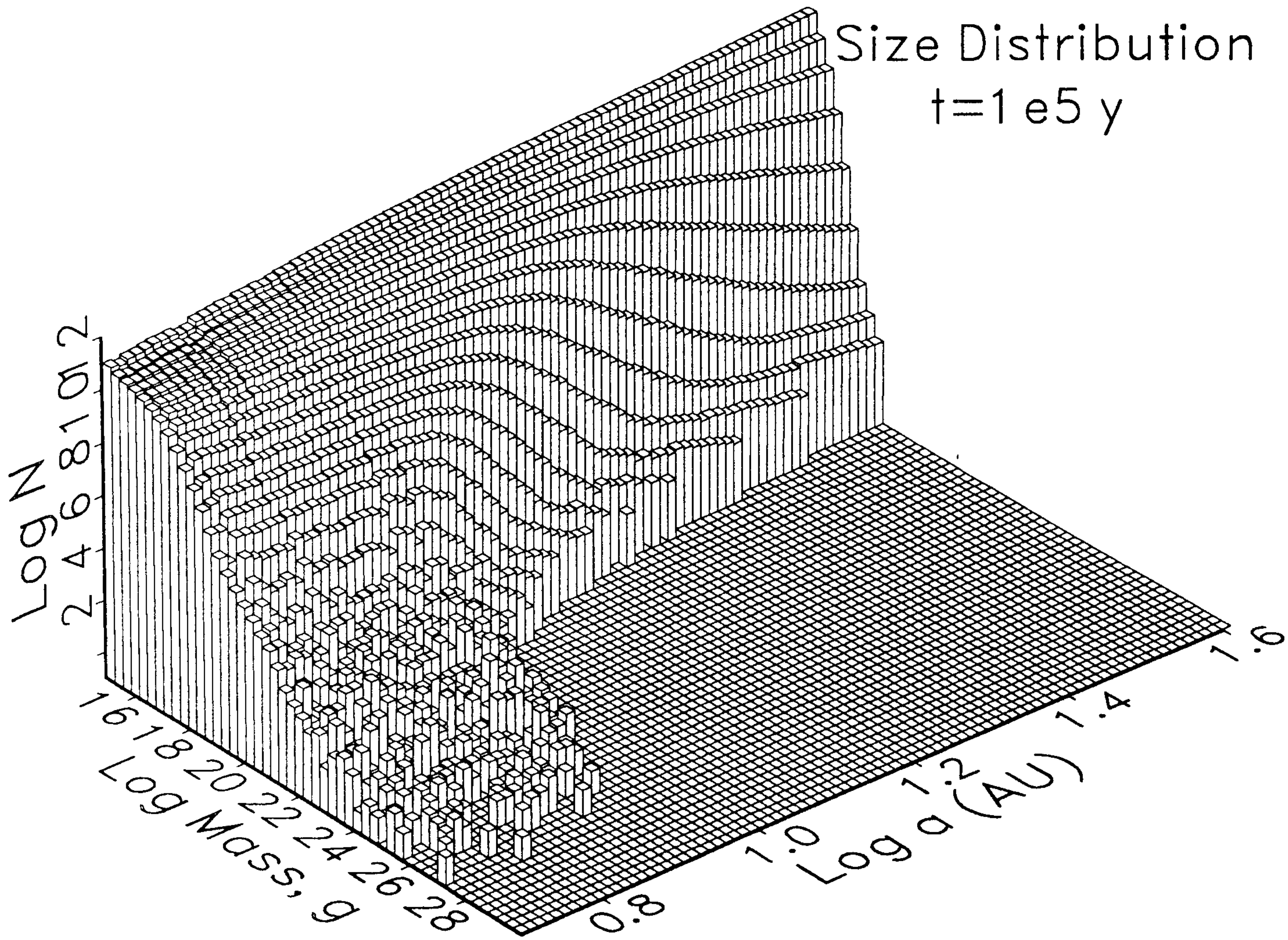
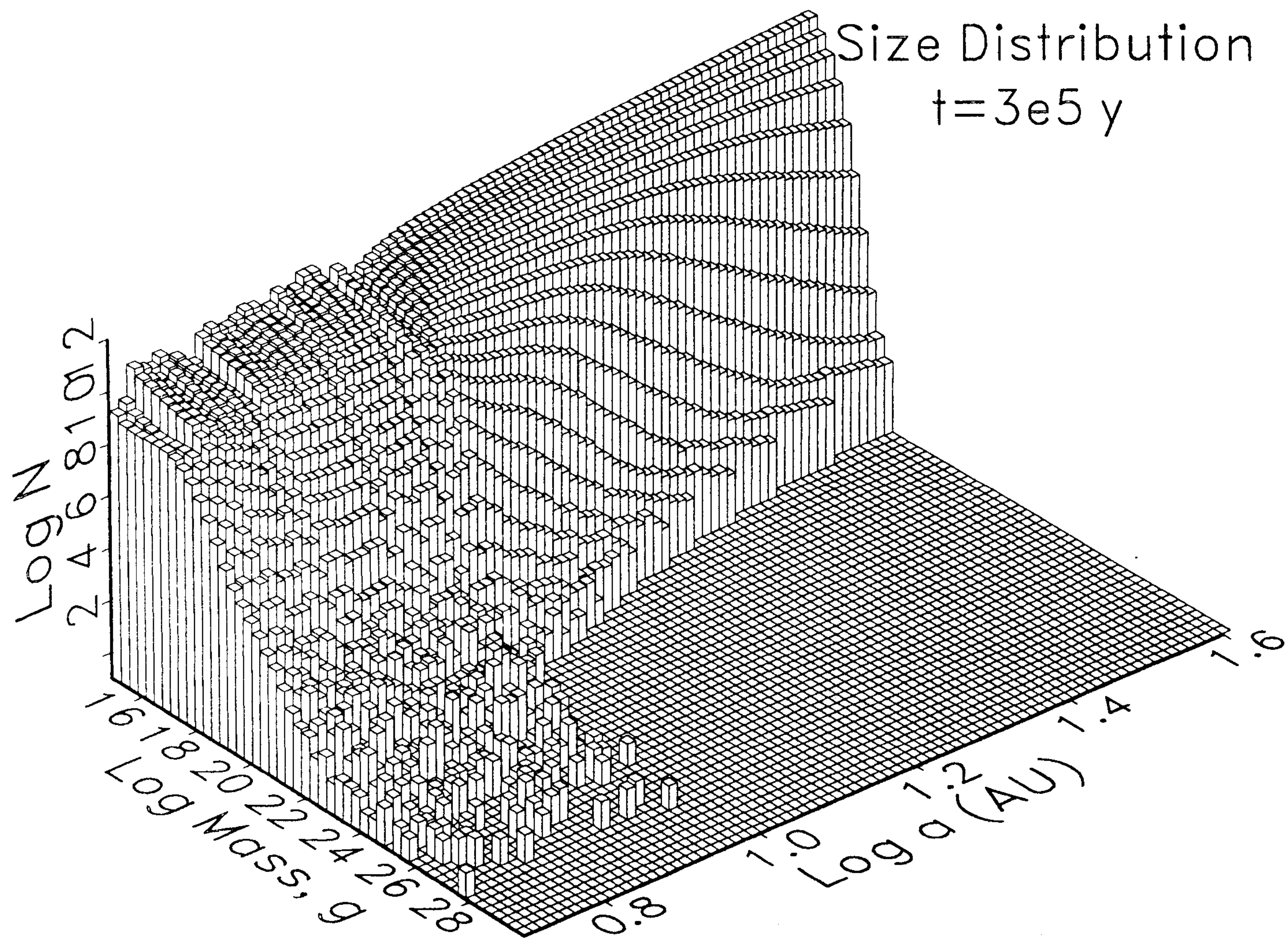
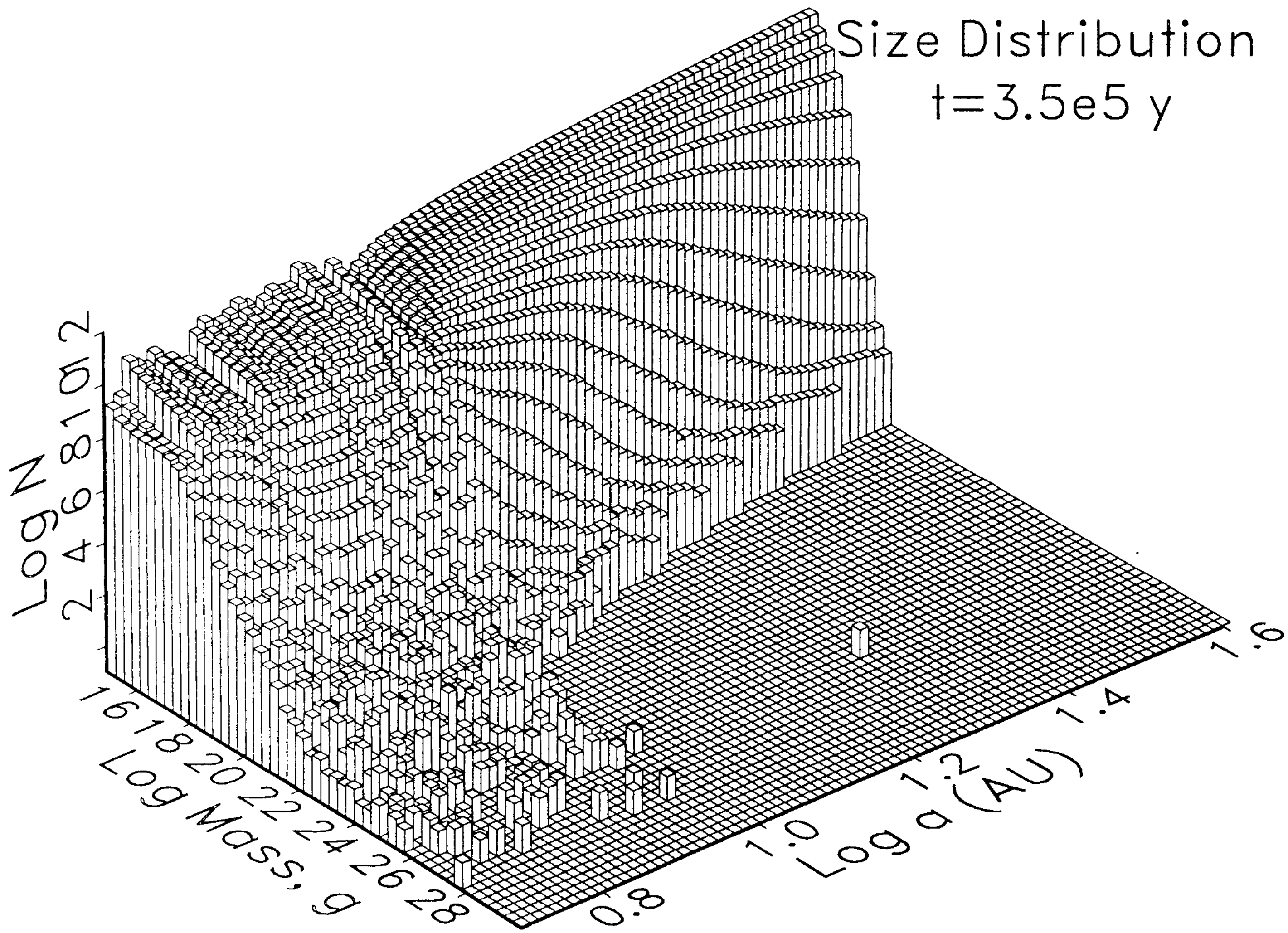
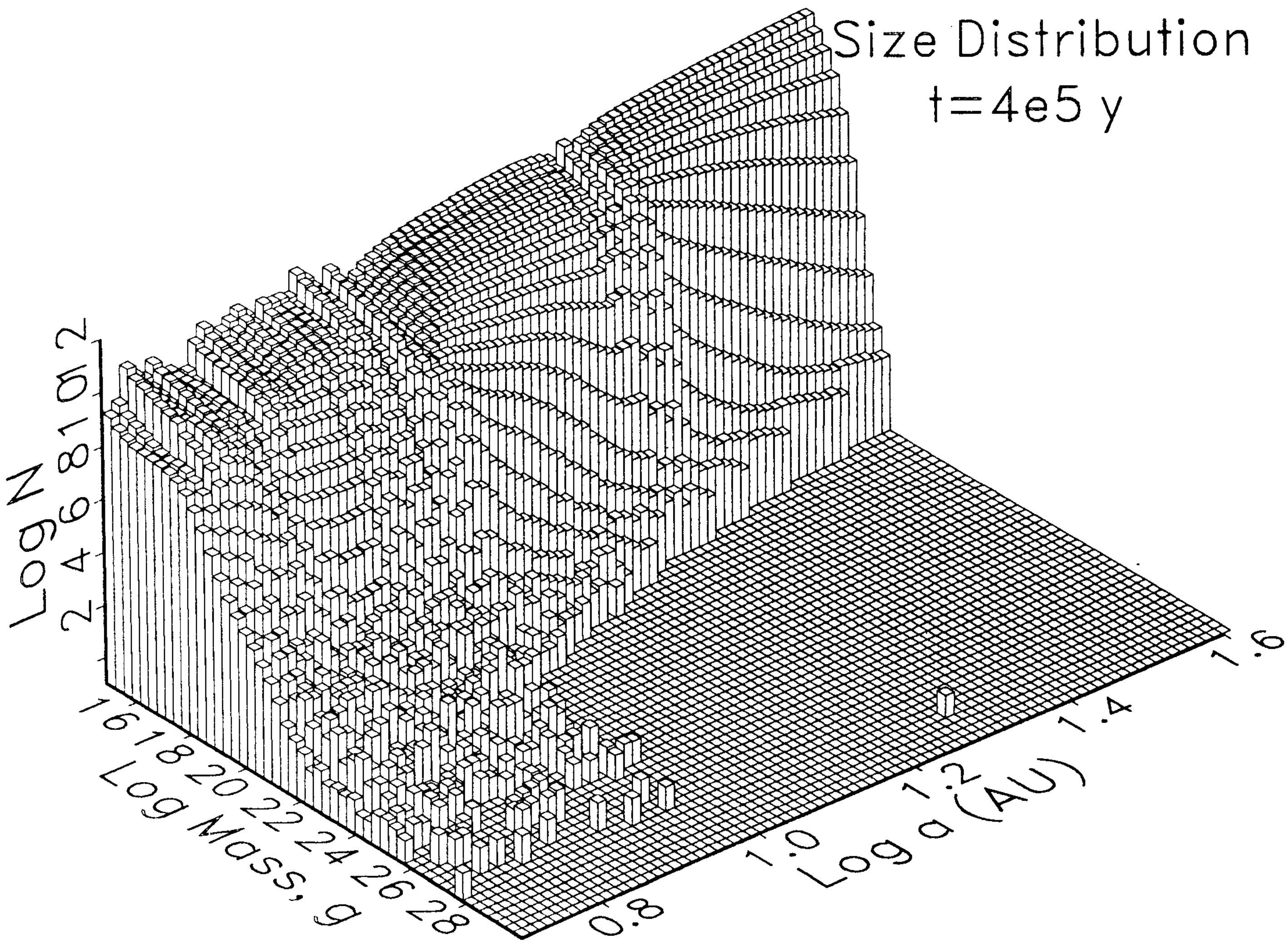


FIG. 1. Cumulative mass distributions for the nominal model. Runaway growth has begun by $\sim 2 \times 10^4$ years and continues at a fairly uniform rate of about 2×10^{21} g/year thereafter.

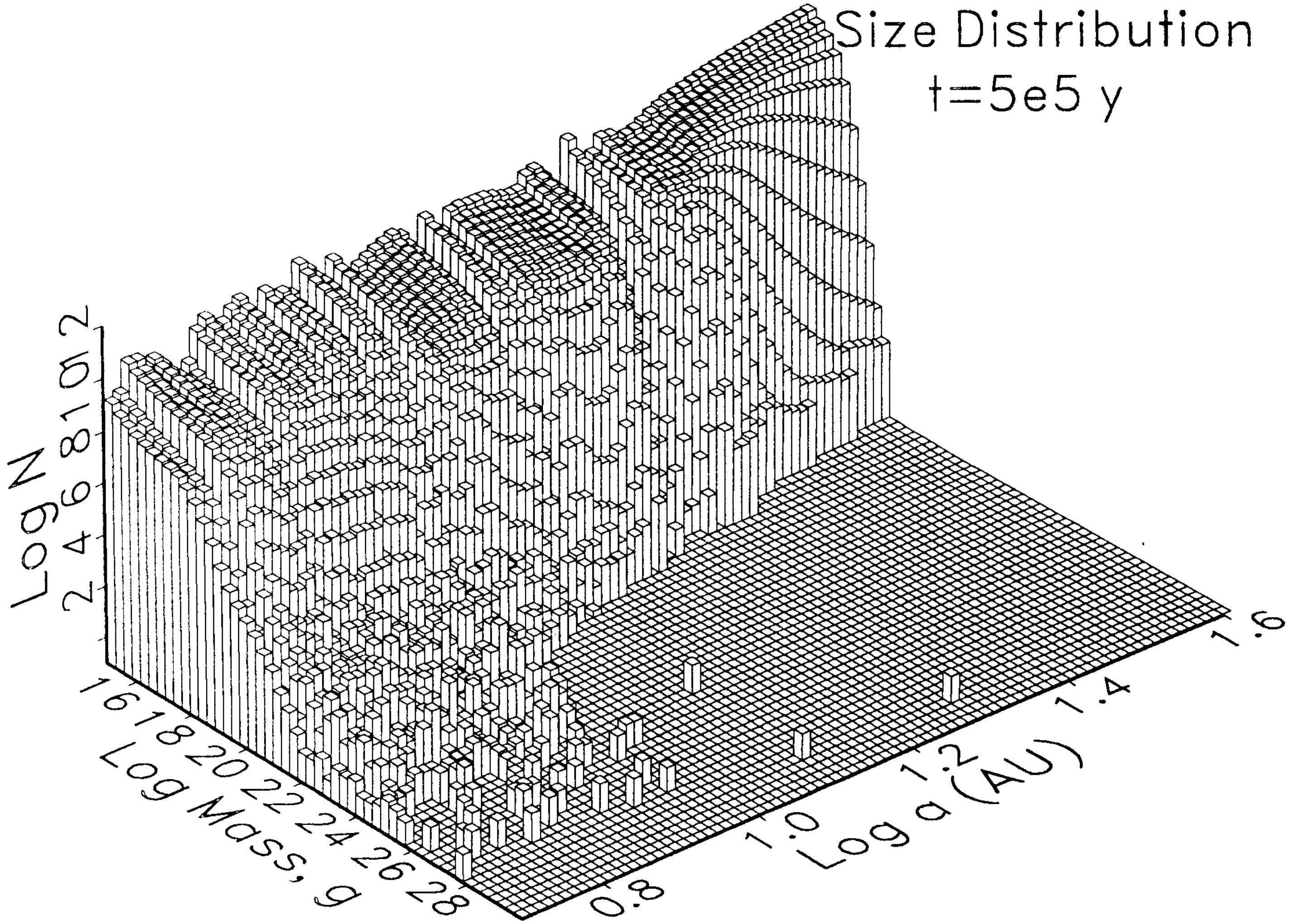

Size Distribution
t=1 e5 y
Log N
2
4
6
8
10
12
16
18
20
22
24
26
28
Log Mass, g
0.8
1.0
1.2
1.4
1.6
Log a (AU)

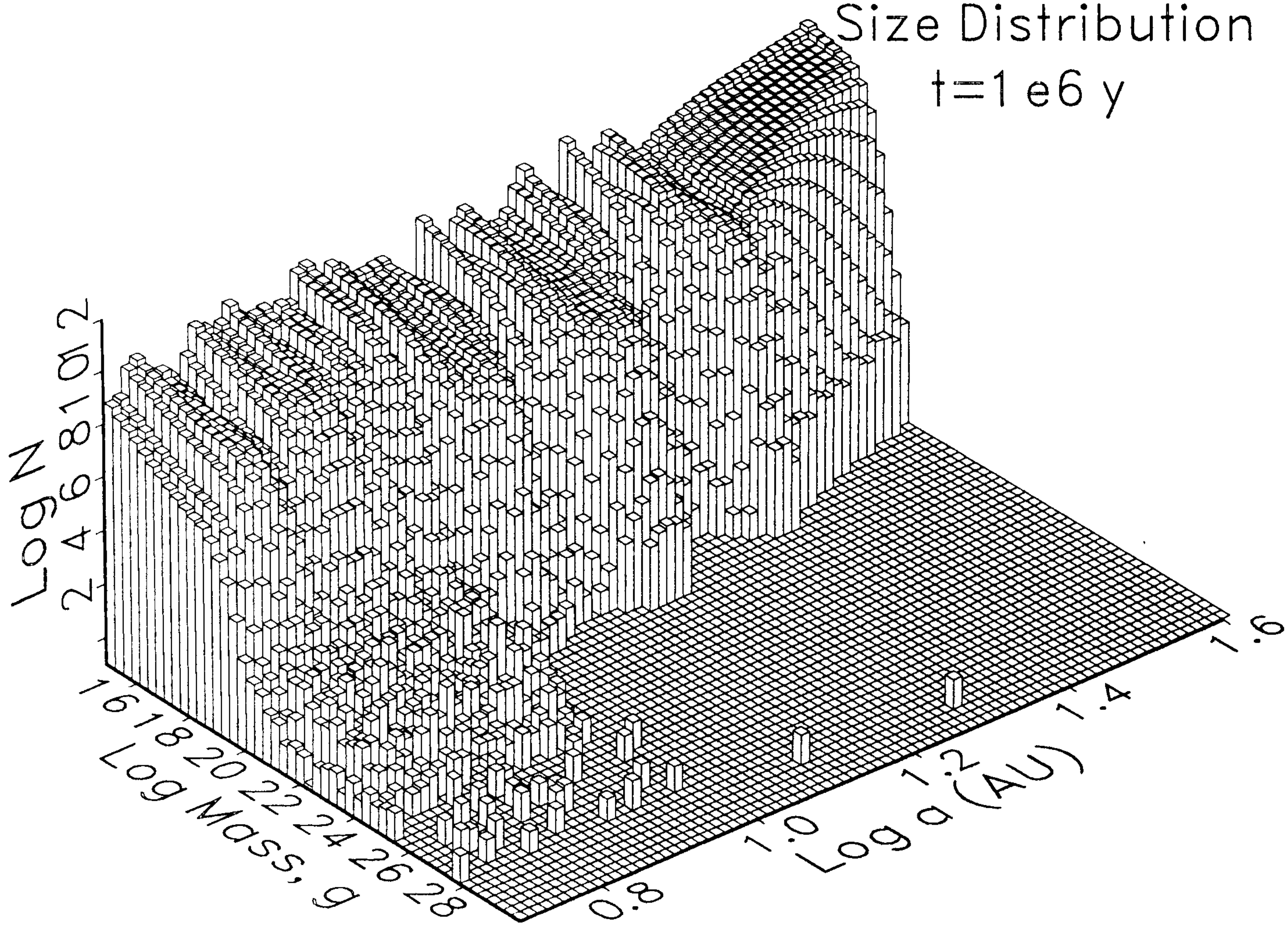

Size Distribution
t=3e5 y
Log N
2 4 6 8 10 12
16 18 20 22 24 26 28
Log Mass, g
0.8 1.0 1.2 1.4 1.6
Log a (AU)


Size Distribution
t=3.5e5 y
Log N
2
4
6
8
10
12
16 18 20 22 24 26 28
Log Mass, g
0.8
1.0
1.2
1.4
1.6
Log a (AU)

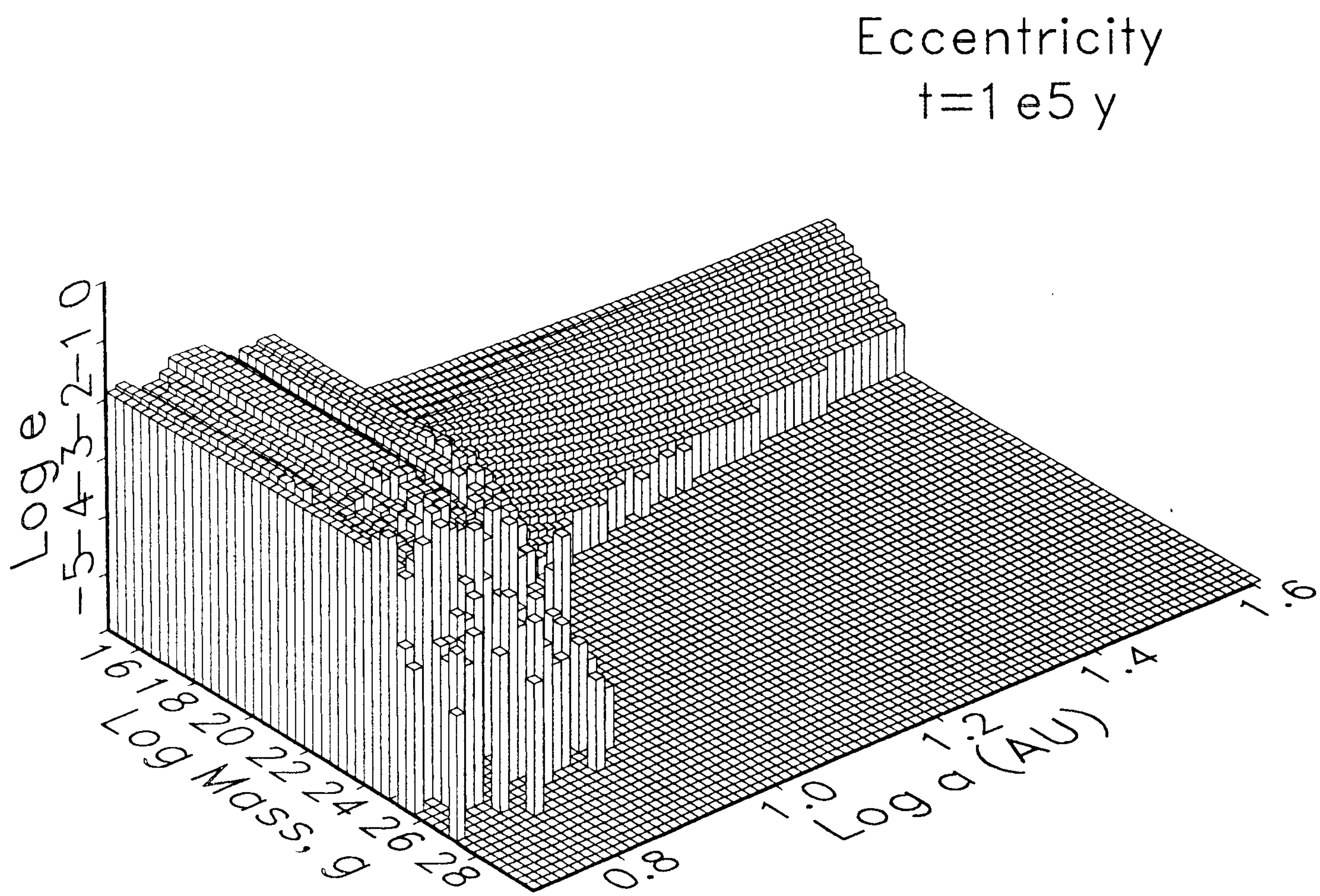


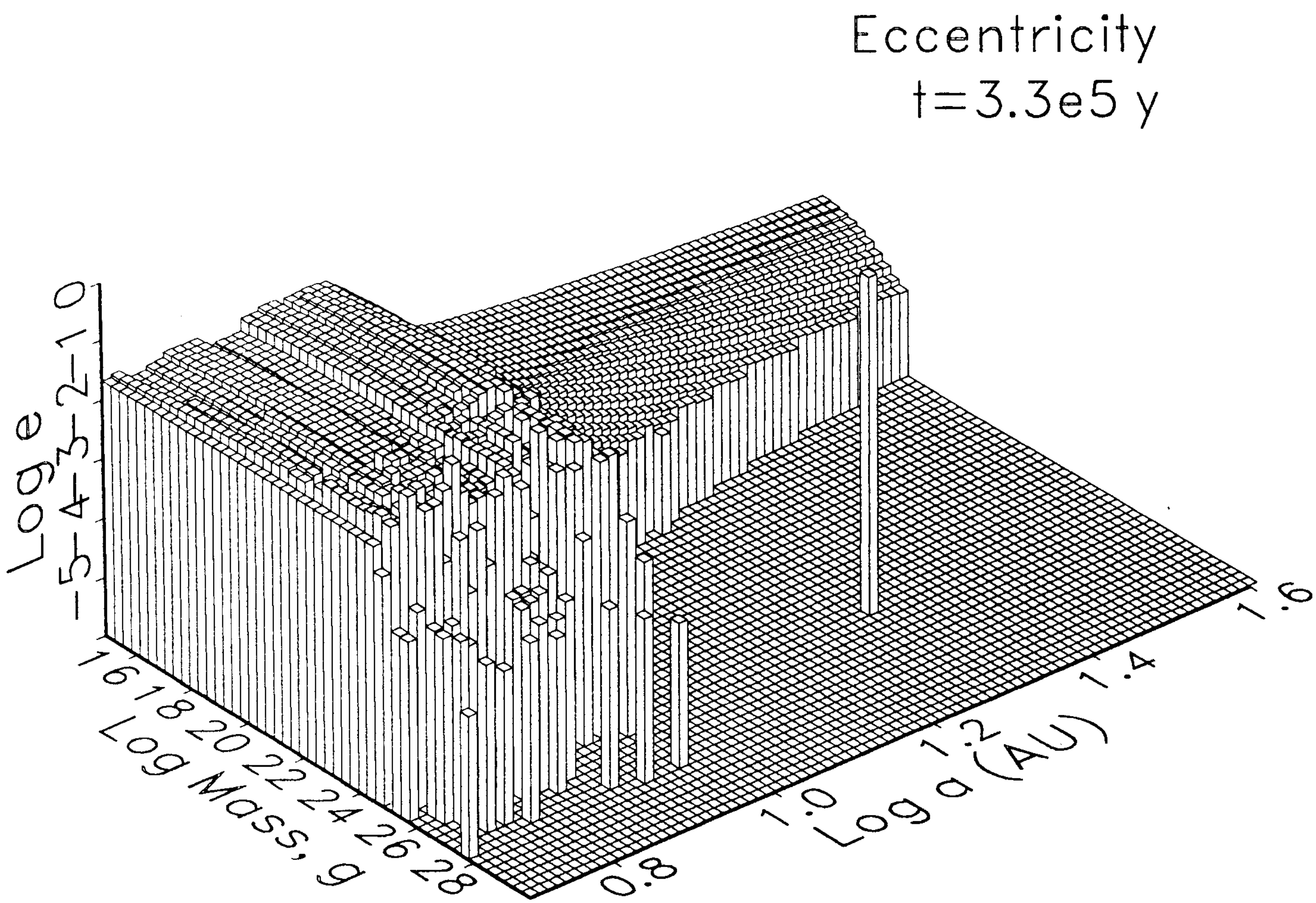
Size Distribution
t=4e5 y
Log N
2
4
6
8
10
12
16
18
20
22
24
26
28
Log Mass, g
0.8
1.0
1.2
1.4
1.6
Log a (AU)

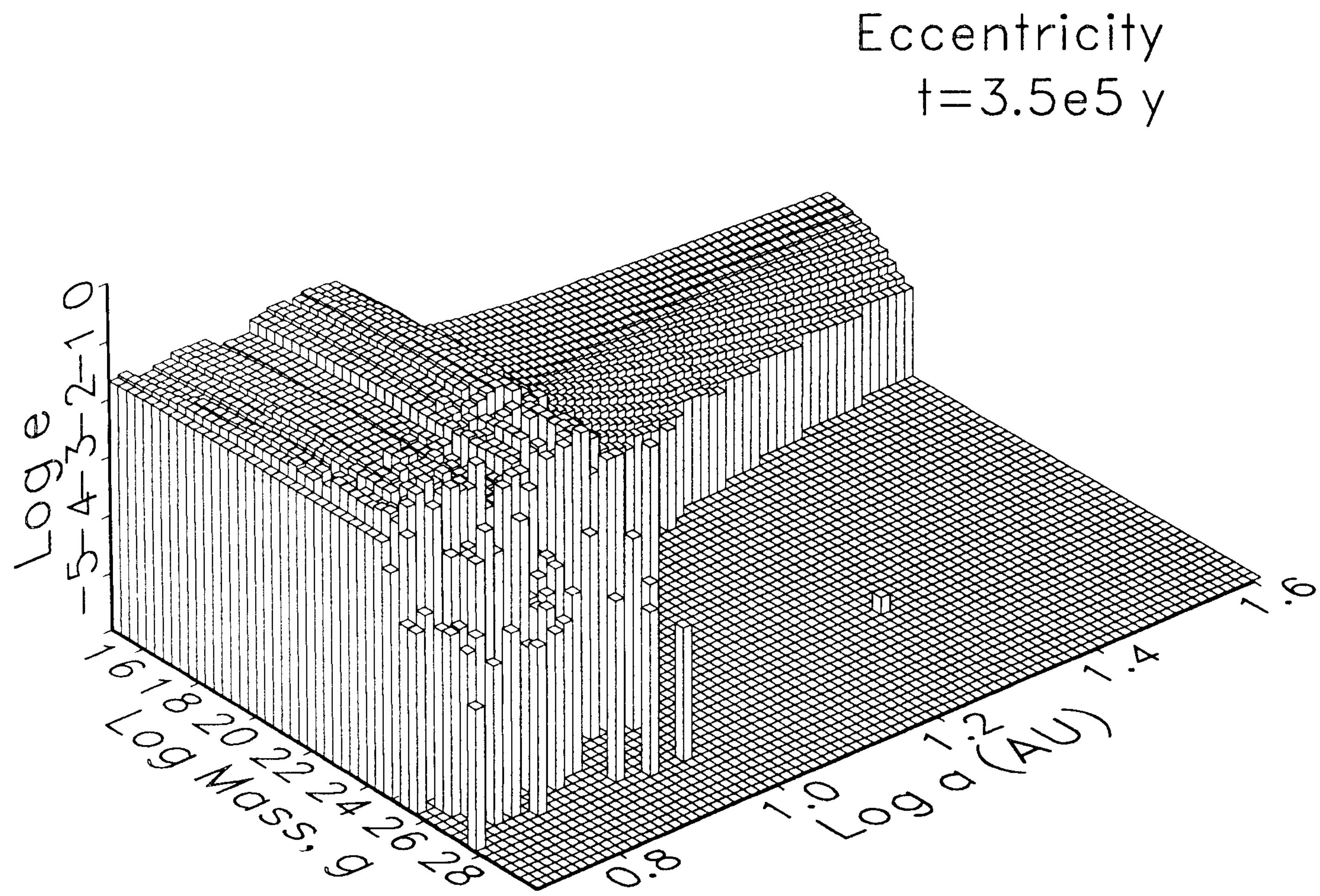

Size Distribution
t=5e5 y
Log N
2 4 6 8 10 12
16 18 20 22 24 26 28
Log Mass, g
0.8 1.0 1.2 1.4 1.6
Log a (AU)



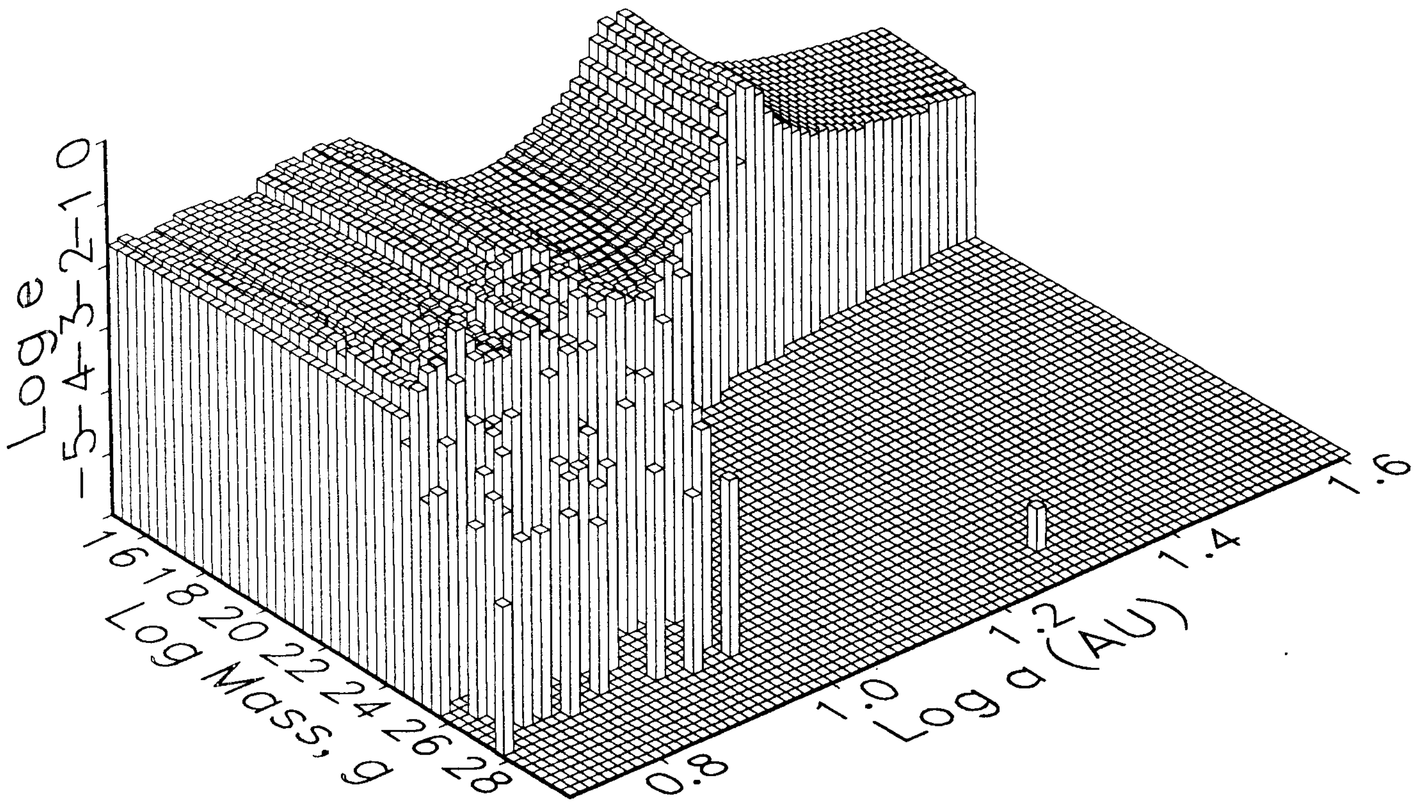
Size Distribution
t=1 e6 y
Log N
2
4
6
8
10
12
16
18
20
22
24
26
28
Log Mass, g
0.8
1.0
1.2
1.4
1.6
Log a (AU)

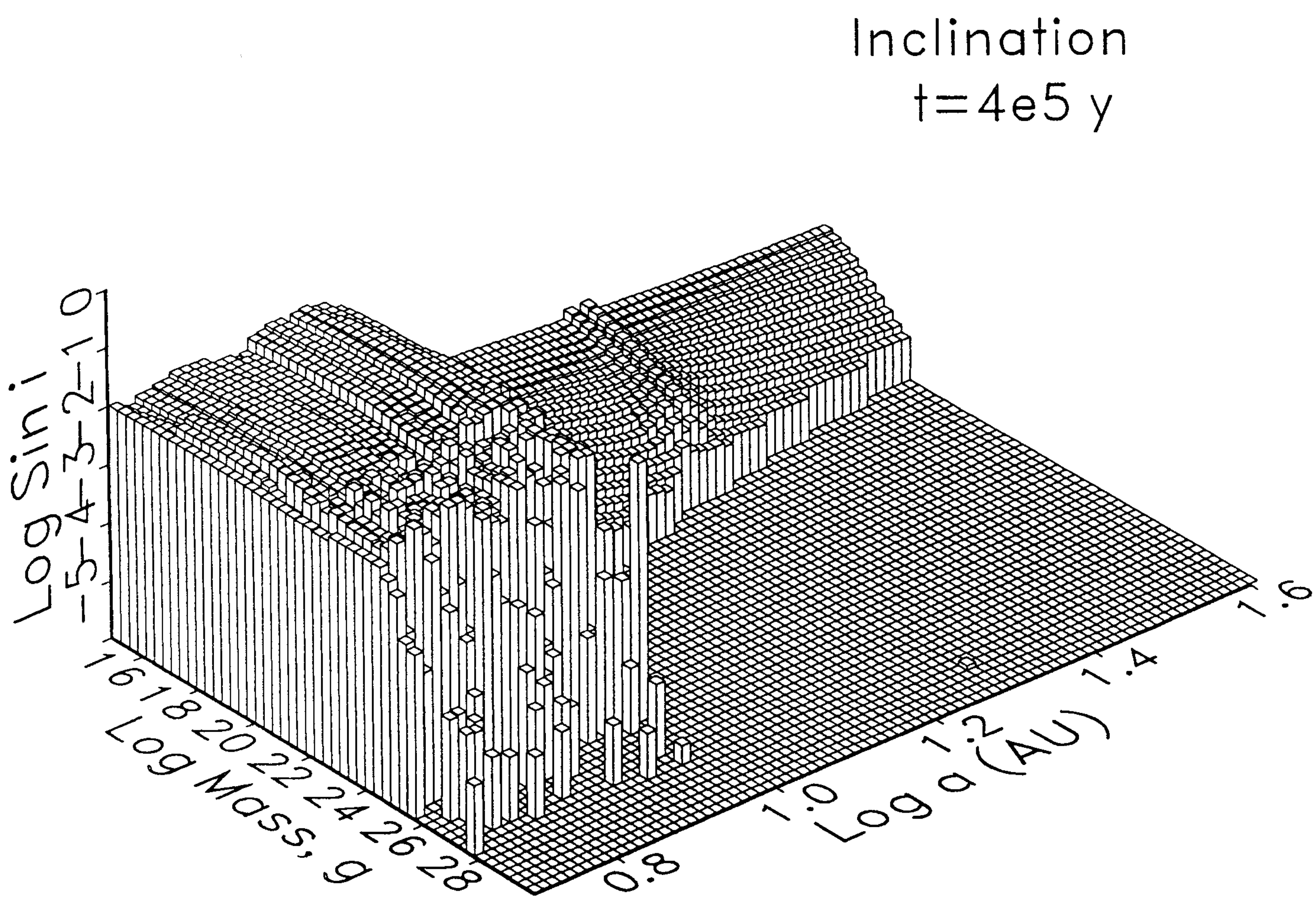

Eccentricity
t=1 e5 y
Log e
-5 -4 -3 -2 -1 0
16 18 20 22 24 26 28
Log Mass, g
0.8 1.0 1.2 1.4 1.6
Log a (AU)

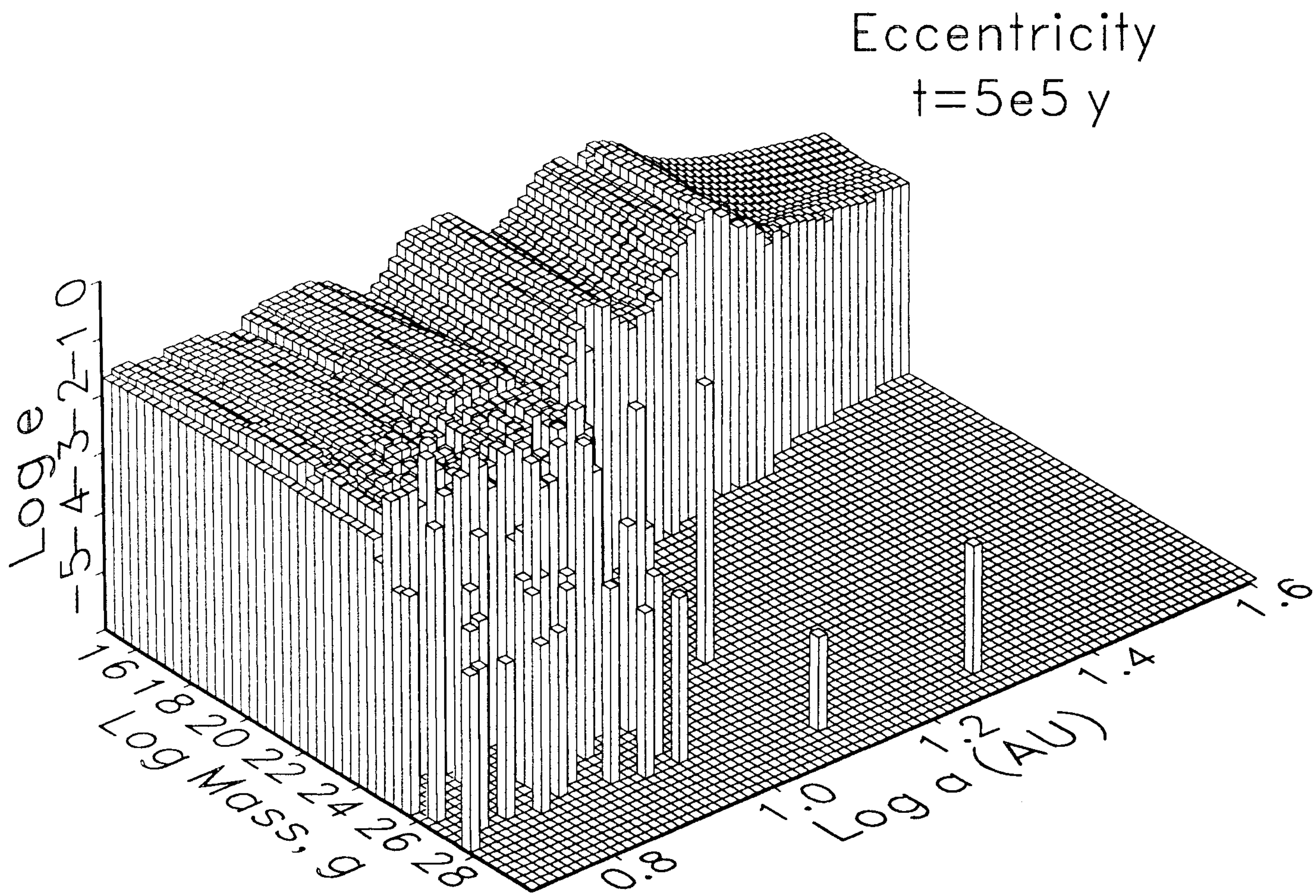

Eccentricity
t=3.3e5 y
Log e
0
-1
-2
-3
-4
-5
16 18 20 22 24 26 28
Log Mass, g
0.8
1.0
1.2
1.4
1.6
Log a (AU)


Eccentricity
t=3.5e5 y
Log e
-5
-4
-3
-2
-1
0
16
18
20
22
24
26
28
Log Mass, g
0.8
1.0
1.2
1.4
1.6
Log a (AU)

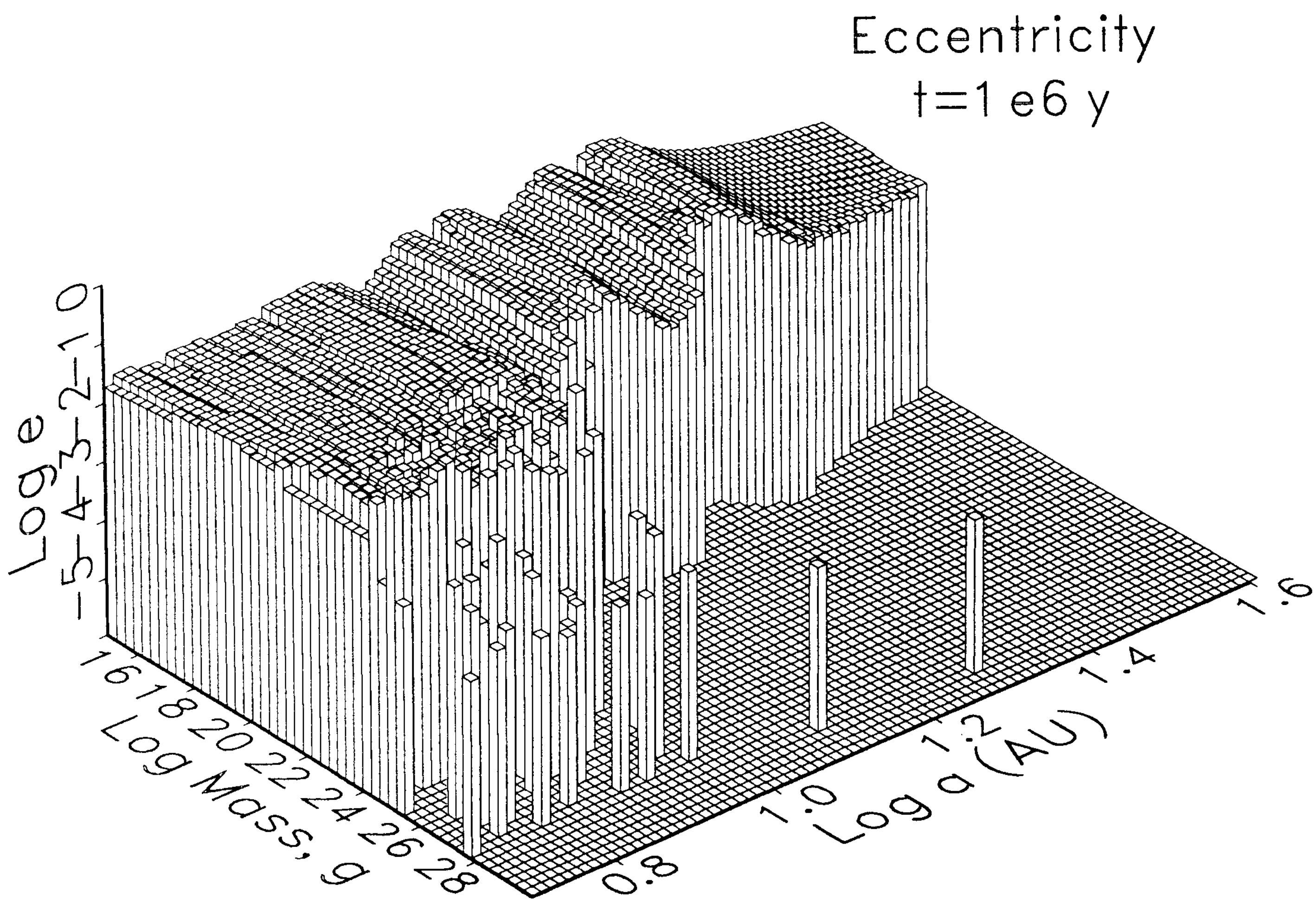
Eccentricity
t=4e5 y

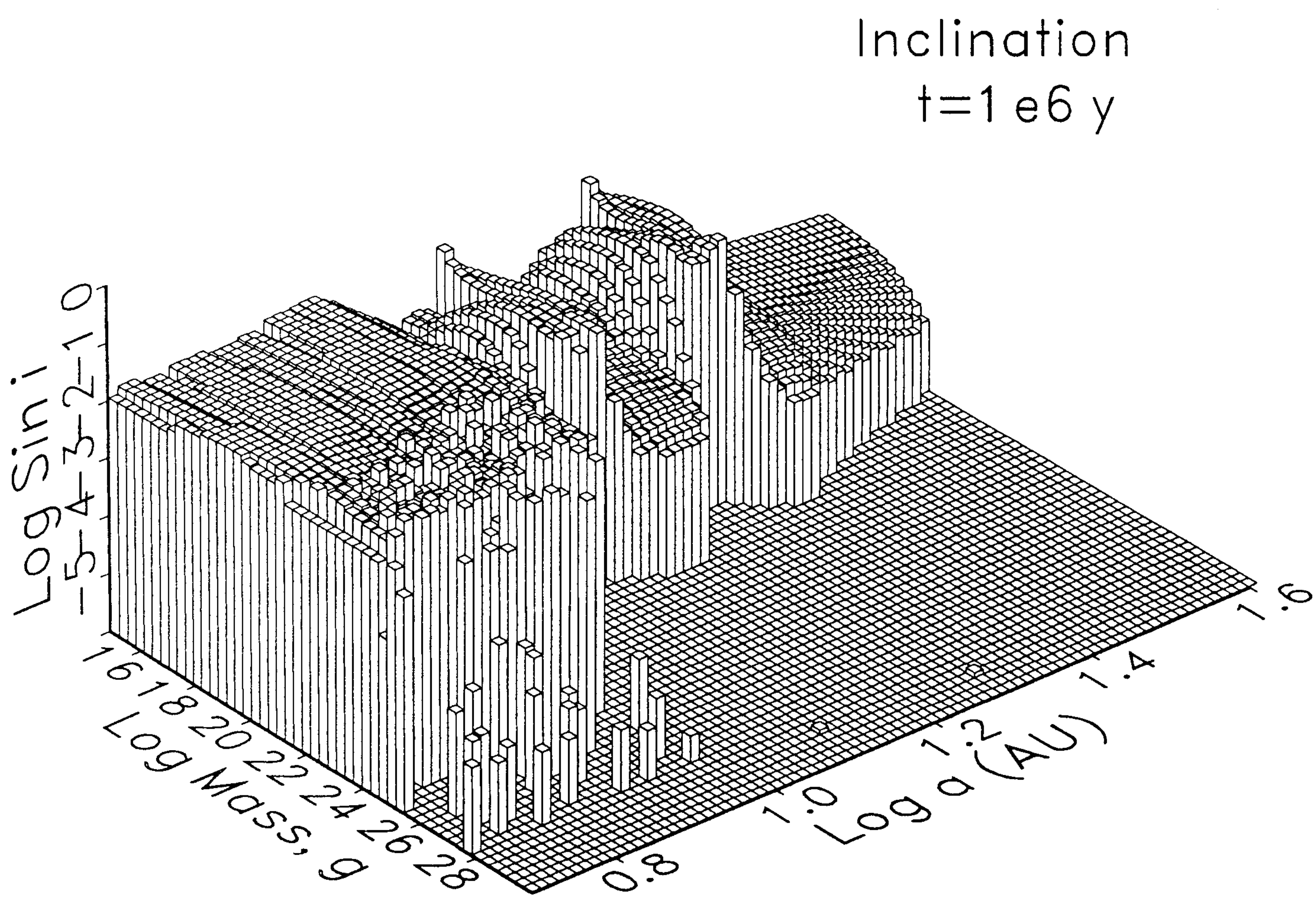



Inclination
t=4e5 y
Log Sin i
-5 -4 -3 -2 -1 0
Log Mass, g
16 18 20 22 24 26 28
Log a (AU)
0.8 1.0 1.2 1.4 1.6



Eccentricity
t=5e5 y
Log e
-5 -4 -3 -2 -1 0
16 18 20 22 24 26 28
Log Mass, g
0.8 1.0 1.2 1.4 1.6
Log a (AU)


Eccentricity
t=1 e6 y
Log e
-5
-4
-3
-2
-1
0
16
18
20
22
24
26
28
Log Mass, g
0.8
1.0
1.2
1.4
1.6
Log a (AU)


Inclination
t=1 e6 y
Log Sin i
-5 -4 -3 -2 -1 0
16 18 20 22 24 26 28
Log Mass, g
0.8 1.0 1.2 1.4 1.6
Log a (AU)

The timescale for growth by accretion increases with R. Oligarchic growth first produces embryos with masses $\gtrsim M_\oplus$ just outside the snow line, before significant growth occurs in the outer nebula. Smaller bodies ($\sim 10^{-4}$ - 10^{-3} $M_\oplus$) can be scattered into eccentric orbits, with large semimajor axes and perihelia near the snow line. They interact with the background population of planetesimals, which are much smaller due to the slower growth rate at larger R. Dynamical friction circularizes the orbits of the scattered bodies.

The scattered bodies, now in circular orbits, are much larger than the indigenous population. They are able to undergo rapid runaway growth.

Monarchical growth is caused by introduction of a single anomalously large body into a background population of small planetesimals. This bypasses the first stage of oligarchic growth, in which a dominant body emerges by stochastic coagulation. Growth in isolation keeps approach velocities low due to conservation of the Jacobi parameter; inclinations remain low.

Requirements:

Sufficient surface density to produce oligarchic bodies with masses ~ $M_\oplus$ just beyond the snow line.

Small mean size (~ 1-10 km) of planetesimals in the outer nebula, with correspondingly low eccentricities.

Scattering of one or more ~ lunar-sized bodies into the outer nebula, and decoupling from the oligarchs.

Implications

In situ (20-30 AU) accretion of Uranus and Neptune is possible on a reasonable timescale. The scattered seed bodies are a small fraction of the total mass, so most of the planetary mass is local in origin.

Only a few planets can be made this way; growth of a large body stirs up the swarm and inhibits another runaway in its vicinity.

Due to the nature of scattering, the numbers, orbital radii, and timing of formation of outer planets are stochastic rather than deterministic.