

Hermite Regularization Scheme for the Common Man

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Motivation: Simple simulation code
for planetary systems

Basic integration: Time-symmetric Hermite
Kokubo et al. 1998

Close encounters: Two-body regularization
Burdet 1967, Heggie 1973

BH method: $t' = R, \underline{\Omega}, h$

Time-symmetric Hermite method

1. Predict all $\underline{r}_j, \dot{\underline{r}}_j$ to order $\dot{\underline{F}}_j$ at t
2. Predict $\underline{I}_i, \dot{\underline{I}}_i$ to order $\dot{\underline{F}}_i$
3. Obtain planetary perturbations
also indirect terms
4. Include solar contribution
5. Add fourth-order corrector
6. Perform one iteration of the Sun
7. Specify commensurate time-step
time quantization
8. Treat any other $t_j + \delta t_j = t$

Solar iteration may be helpful

Burdet-Heggie method

Relative motion $\ddot{\underline{R}} = -\frac{M}{R^3} \underline{R} + \underline{F}$

Time transformation $t' = R, \frac{dt}{dt'} = \frac{1}{R} \frac{d}{dt}$

New equation $\ddot{\underline{R}}' = \frac{\dot{R}' \dot{R}'}{R} - \frac{MR}{R^2} + \dot{R}^2 \underline{F}$

Elements $P = -\frac{2M}{R} + \dot{R}^2$

$$\ddot{\underline{B}} = \frac{MR}{R^2} - \dot{\underline{R}}^2 \underline{R} + (\underline{R} \cdot \dot{\underline{R}}) \dot{\underline{R}}$$

Equations of motion

$$\ddot{\underline{R}}' = P \dot{\underline{R}} + \ddot{\underline{B}} + \dot{R}^2 \underline{F}$$

$$\dot{P} = 2 \dot{\underline{R}} \cdot \underline{F}$$

$$\ddot{\underline{B}}' = -2(\dot{\underline{R}} \cdot \underline{F}) \dot{\underline{R}} + (\underline{R} \cdot \underline{F}) \ddot{\underline{R}} + (\underline{R} \cdot \dot{\underline{R}}') \dot{\underline{R}}$$

$$t' = R, t'' = R', t''' = R''$$

Runge-Lenz vector $\underline{\Omega} = -\dot{\underline{B}}/M$

Close encounters

Search for companion $\Delta t_i < \Delta t_{ce}$

Accept $m_j \quad R < R_{ce}, \dot{R} < 0$

Form elements $\underline{R}, \underline{B}, P$

Define c.m. as $i = N+1$

Initialize c.m. $E_{cm}, \dot{E}_{cm}, \Delta t_{cm}$

Advance regularized solution

up to $\min(t_j + \Delta t_j) = T^*$

Treat other particles up to T^*

Note correct form of \underline{E}_{cm}

Termination $R > R_0$ flyby

Collisions $R < R_{coll}$

Escape $r_i > r^* \text{ or } e_i > 0.99\dots$

Conclusions

A practical and simple scheme

Reasonably high accuracy

Brute force method $N \lesssim 10^2$

Close encounter strategy
Solar perturbation

Uniform integration method