Tides in Gas Giant Planets

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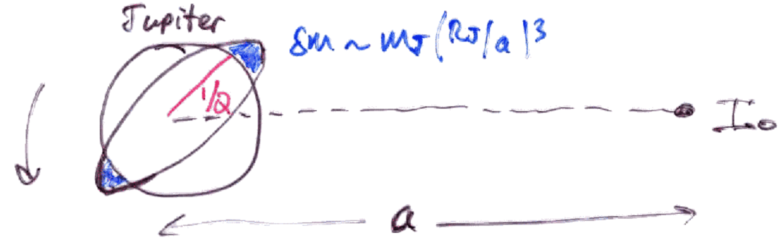
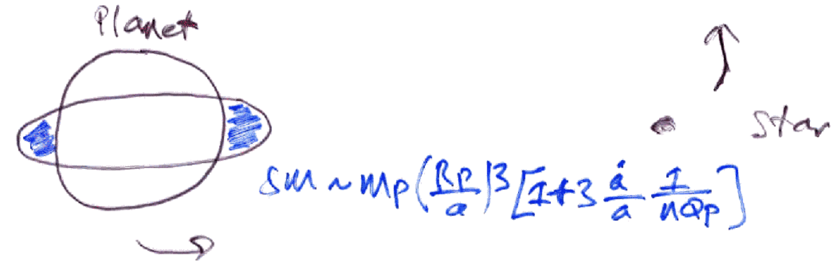
Orbit Evolution by Tides

- Expansion of Io's orbit



$$\text{torque} \sim \frac{G M_J}{R_J^2} \cdot R_J \cdot M_J \left[\frac{M_{Io}}{M_J} \left(\frac{R_J}{a} \right)^3 \right]^2 \frac{1}{Q_J}$$

- Circularization of extrasolar planet orbits



$$\frac{d h_e}{dt} \sim - \frac{M_\star}{M_p} \left(\frac{R_p}{a} \right)^5 \sqrt{\frac{G M_\star}{a^3}} \frac{1}{Q_p}$$

Q for giant planets

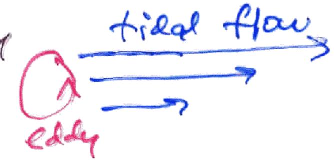
$$Q \sim \frac{\omega_{tide} E_{Tide}}{\dot{E}_{tide}}$$

measures dissipation rate

- tides push Io into resonance with Europa & Ganymede $\Rightarrow Q_J \lesssim 10^6$ (Goldreich & Soter)
- Circularization of extrasolar planet orbits at 3 day orbits $\Rightarrow Q_p \lesssim 10^6$ (e.g. Marcy et al 1997)

Theory of Q

Hubbard: turbulent viscosity



$$\dot{E}_{viscous} \sim E \frac{\nu}{R^2}$$

$$\nu \sim V_{conv} H \sim \left(\frac{F}{\rho}\right)^{1/3} \frac{P}{\rho g} \sim \left(\frac{L R^4}{M}\right)^{1/3}$$

$$\Rightarrow Q \sim \left[\frac{\frac{GM^2}{R}\sqrt{\frac{GM}{R^3}}}{L}\right]^{1/3} \sim 10^5$$

Goldreich & Nicholson: reduction factor

$\tau_{eddy} \sim H/V_{conv} \sim yr$ for large scale eddy

$P_{tide} \sim 5$ hr

$\Rightarrow$ energy bearing eddies too slow! Can't transport momentum. Need small scale eddies ($V_\ell \sim \ell^{1/3}$).

$$\nu \sim V_{conv} H \left(\omega \tau_{eddy}\right)^{-2} \sim \frac{L}{GM^2/R^3}$$

$$\Rightarrow Q \sim \frac{\frac{GM^2}{R}\sqrt{\frac{GM}{R}}}{L} \sim 10^{15} \quad !!$$

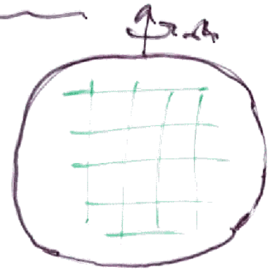
Disagrees with observations by $\sim 10^7$

Waves in giant planets

Convective core: inertial modes

$$\omega \simeq 2\Omega_{spin} \frac{k_z}{k}$$

$\delta v \propto \rho^{-1/2}$ amplitude



radiative envelope: rotation modified g-modes

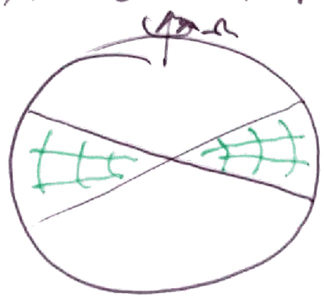
$$\omega^2 \simeq 4\Omega_{spin}^2 \frac{k_z^2}{k^2} + N^2 \frac{k_{hor}^2}{k^2}$$

$k_r \gg k_\theta, k_\phi$, $k_z \simeq k_r \cos\theta \Rightarrow$

$$k_{hor}^2 \simeq k_r^2 \left[\omega^2 - 4\Omega_{spin}^2 \cos^2\theta\right]$$

$\Rightarrow$ horizontal propagation for $\cos\theta \le \frac{\omega}{2\Omega_{spin}}$.

Amplitude $\delta v \propto \left(\frac{M}{\rho}\right)^{\frac{1}{2}}$.



How are tides different in a Rotating Star/Planet?

1) $\delta P_m(r,\theta,\phi) = \sum_{\ell \ge m} \delta P_{\ell m}(r)\, Y_{\ell m}(\theta,\phi)$

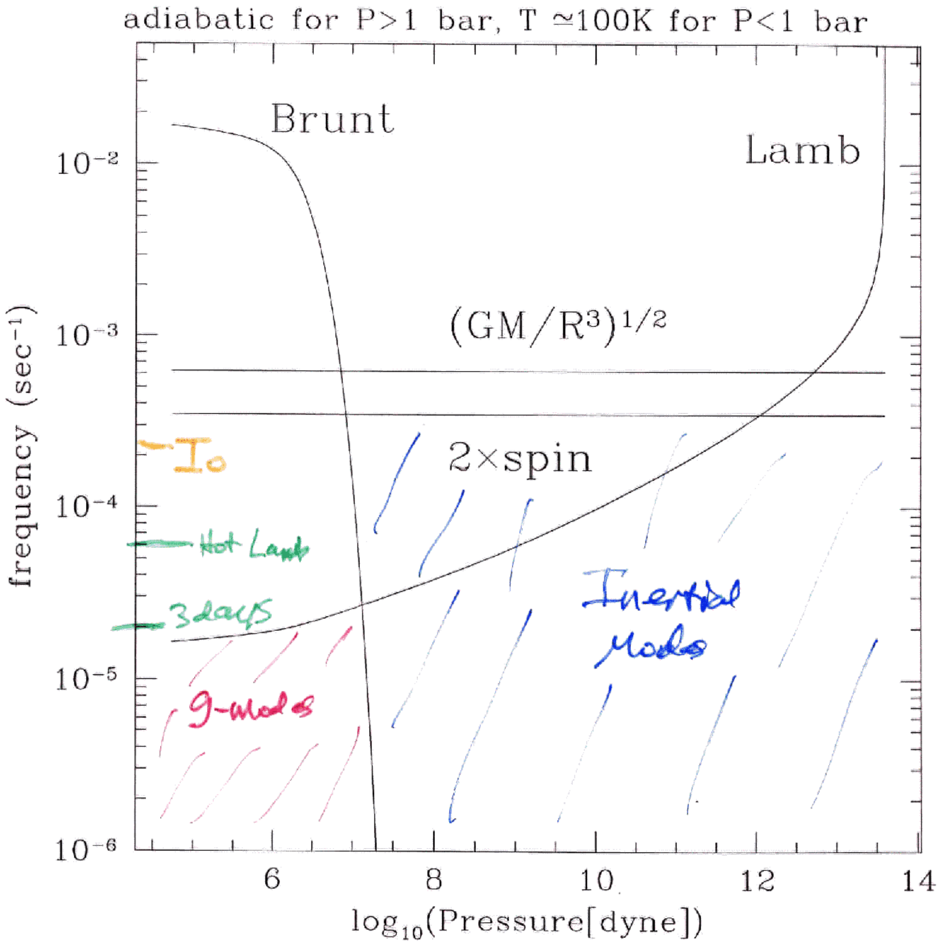
Coriolis force couples all spherical Harmonics.

Tidal potential $\sim Y_{2m}$ overlaps with modes better (only $\ell=2$ in nonrotating star).

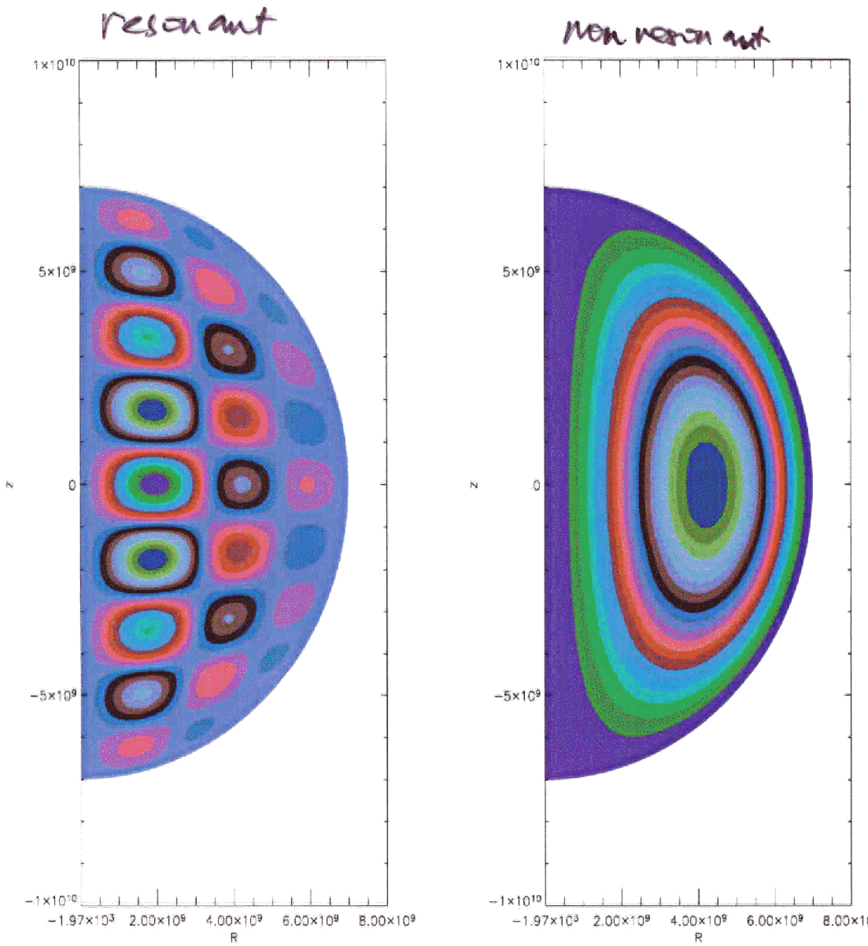
2) Can adjust two quantum #s to find good resonance (in nonrotating case, ℓ & m fixed; only n can vary).

Ex: $\omega \simeq 2\Omega_{spin} \frac{n_z}{n_R}$

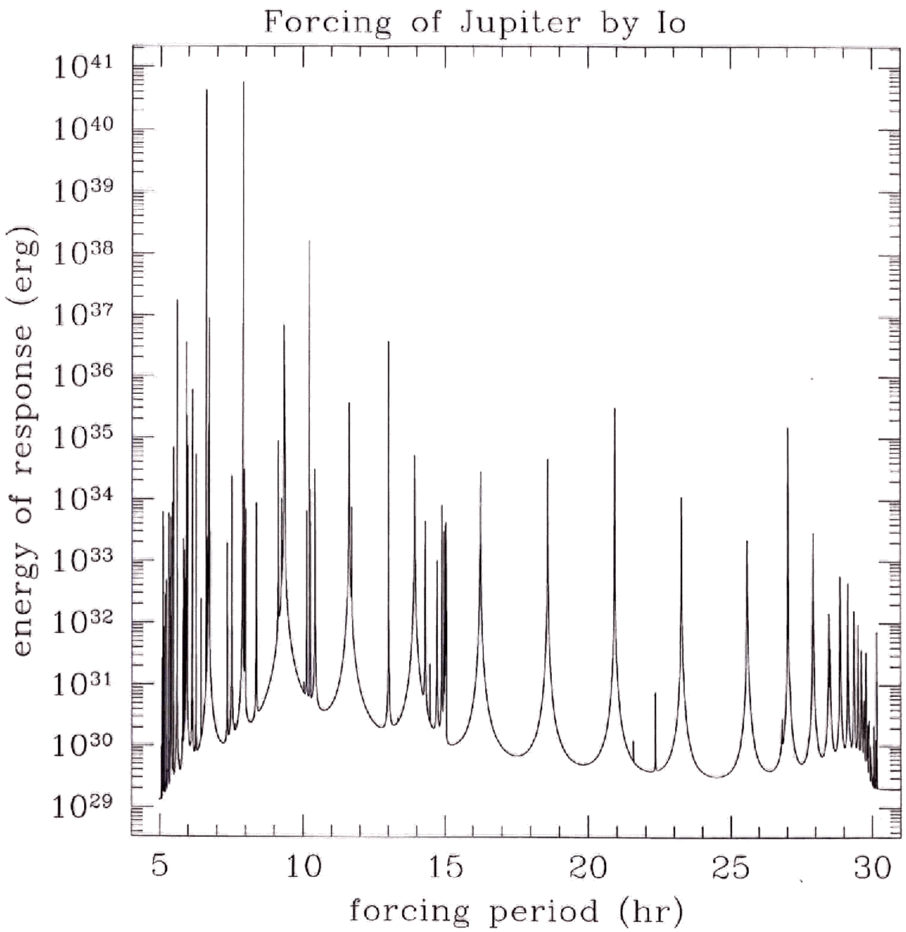
Where do waves propagate?



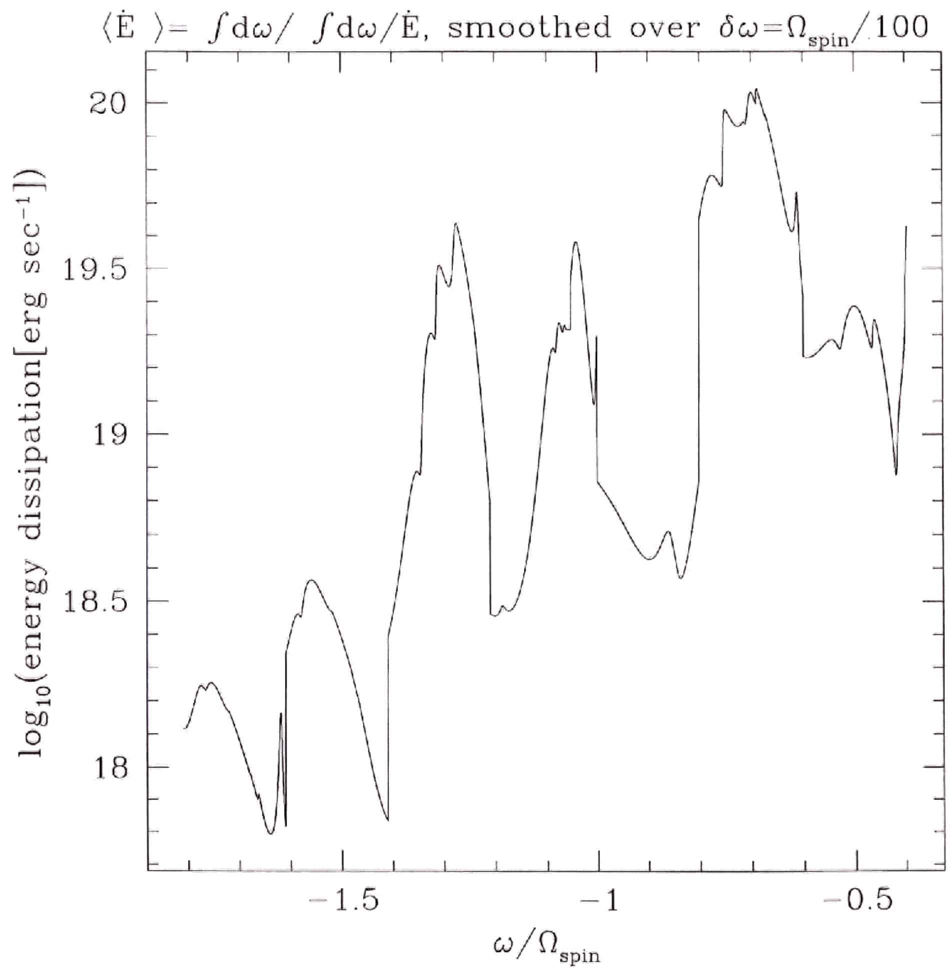
Tidal Response of Jupiter to Io



Plotted: Pressure perturbation


Forcing of Jupiter by Io
energy of response (erg)
forcing period (hr)
5
10
15
20
25
30
10^{41}
10^{40}
10^{39}
10^{38}
10^{37}
10^{36}
10^{35}
10^{34}
10^{33}
10^{32}
10^{31}
10^{30}
10^{29}

Nr=164 , Ne = 8


$\langle \dot{E} \rangle = \int d\omega / \int d\omega/\dot{E}$, smoothed over $\delta\omega = \Omega_{spin}/100$
$\log_{10}(\text{energy dissipation[erg sec}^{-1}])$
ω/Ω_{spin}
−1.5
−1
−0.5
18
18.5
19
19.5
20

Estimate Q for turbulent viscosity

torque $\sim \frac{\dot{E}}{\omega} \sim \left(\frac{m_{io}}{m_J}\right)^2 \left(\frac{R_J}{a}\right)^6 \frac{G M_J^2}{R_J} Q_J^{-1}$

time spent in $(\omega, \omega + d\omega) \propto \dot{E}^{-1}$

$$\Rightarrow \langle \dot{E} \rangle_{time} = \frac{\int d\omega}{\int d\omega / \dot{E}}$$

$\Rightarrow$ off-resonant torque dominates.

Plug in #s from plot

$$\boxed{\langle Q \rangle_{turb} \simeq 10^{7-9} \frac{\langle \dot{E} \rangle}{10^{18-20}\ \mathrm{erg/sec}}}$$

Needs more work!

Resolution, better Jup. models, etc

Apply to hot Jup. next.