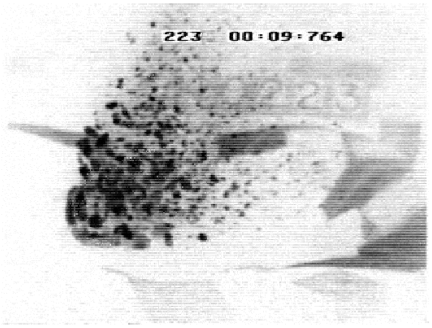
Fragmentation of Planetesimals

and

Ongoing Work on Planetesimal Coagulation

(in collaboration with S.Ida, Tokyo, Japan)

Patrick Glaschke
Astronomisches Rechen–Institut
Heidelberg, Germany



Characteristics of a Collision

- Mass of the largest fragment M_L, or dimensionless: $f_l = \frac{M_L}{M}$.
- Technical terms: $f_l < \frac{1}{2}$: *Fragmentation*, otherwise: *Cratering*. The case $f_l = \frac{1}{2}$ refers to *critical fragmentation*.
- Energy/Volume S which yields $f_l = \frac{1}{2}$ (without reaccumulation). This is called *Impact strength*.
- $f_{\mathrm{KE}} := 2E_{\mathrm{kin}}^{\mathrm{frag}}/E_{\mathrm{kin}}$: Fraction of the impact energy that is converted into the kinetic energy of the fragments.

Fragment size distribution:

- $N_m(m)$: Number of all fragments with a mass $m_i \geq m$.
- $N_D(D)$: Number of all fragments with a diameter $D_i \geq D$.

  If all fragment masses m_i are sorted in decreasing order, starting with $i = 1$, then $N_m(m_i) = i$ holds.
- Velocity distribution $N(v)$: Number of all Fragments with $v_i \geq v$.

Experimental Results

- Impact strength S :

$$S = S_0 \left(\frac{R}{1\text{m}}\right)^{-0.24} (1 + 1.6612 \times 10^{-7} \left(\frac{R}{1\text{m}}\right)^{1.89})$$
$$S_0 = 1.726 \times 10^6 \text{ Jm}^{-3}$$

(House and Holsapple 1990)

- Largest fragment :

$$M_L = M - \alpha_m E_{\text{kin}} \quad \text{for } (M - M_L) \ll M$$
$$M_L = \frac{1}{2} M \left(\frac{\rho E_{\text{kin}}}{2SM}\right)^{-K} \quad \text{for } M_L \leq \frac{1}{2} M$$

Reasonable choice: $K = 1.24$ (Fujiwara et al. 1977)

- Common fragment size distribution :

$$N(\geq m) = \begin{cases} c\, m^{-b} & \text{for} \quad m \leq M_L \\ 0 & \text{otherwise.} \end{cases}$$

Conditions to determine c and b:

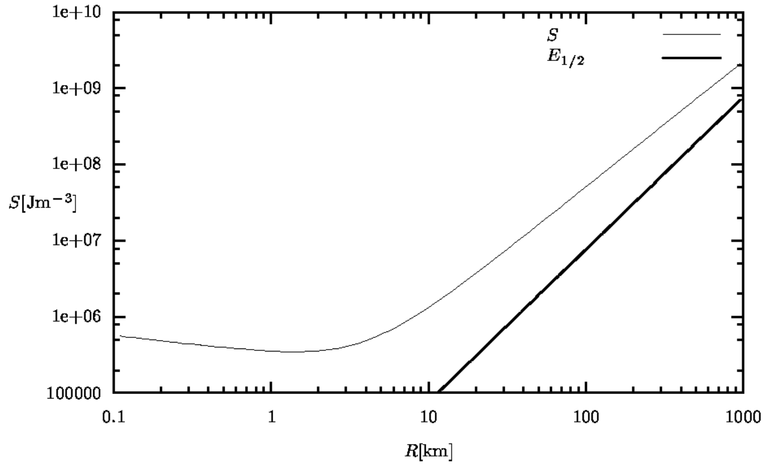
$$M = \int_0^{M_L} \left| \frac{\mathrm{d}N(x)}{\mathrm{d}x} \right| x\, dx$$
$$N(M_L) = 1$$

- Energy partition coefficient:

$$f_{\text{KE}} = 2 \frac{E_{\text{kin}}^{\text{frag}}}{E_{\text{kin}}} \approx 0.01 \quad \text{for lab experiments.}$$

- Velocity distribution:

$$< v >_m = V_0 \left(\frac{m}{M}\right)^{-k}$$



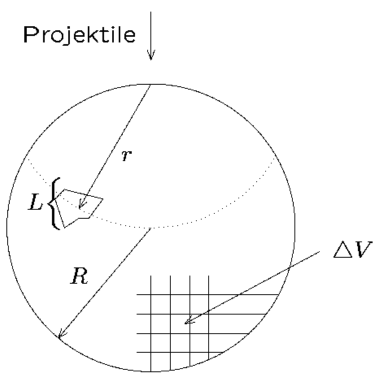
The impact strength used in this work. As a comparison the minimal energy $E_{1/2}$ is given, which is necessary to remove one half of the target mass (solely due to gravitation).

Semiempirical Fragmentation Model

- It is not a calculation based on first principles (like SPH) $\rightarrow$ the model needs to be gauged with experiments and/or more sophisticated simulations.
- Can be easily adapted to new parameters within the limits of this model.
- Fast calculation allows for the scanning of a large parameter space.

The model is derived from the observation, that fragment size increases with increasing distance to the impact site.

$$\boxed{L(r) = CR\left(\frac{r}{R}\right)^n \ , \ C > 0 \ , \ n > 0}$$

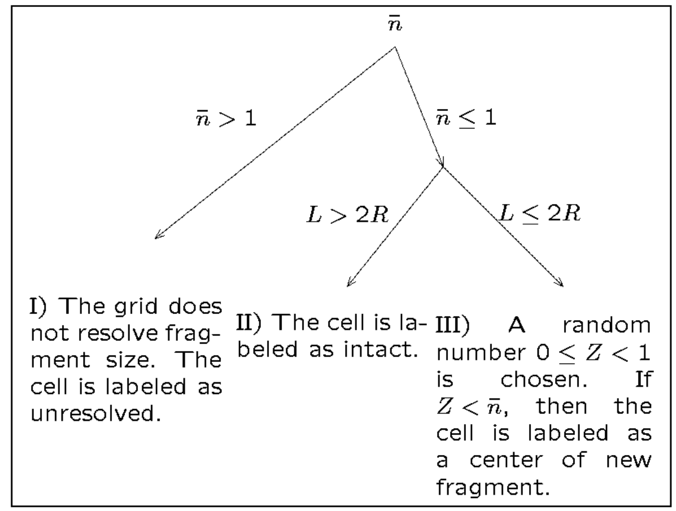


Numerical Decomposition

The mean number of fragments in a subvolume ΔV is:

$$\bar{n}(r) = \frac{\Delta V}{L(r)^3}$$

Each grid cell is subject to a case distinction:



Non–labeled cells are assigned to the nearest center (Voronoi–Decomposition).

Integration of Physics:

- **Input** : Radius R, Density ρ, Impact strength $S(R)$, kinetic energy of the projectil E_{kin} and f_{KE}.

- The exponent n is regarded as a material property. (arises from comparison with experimental results)

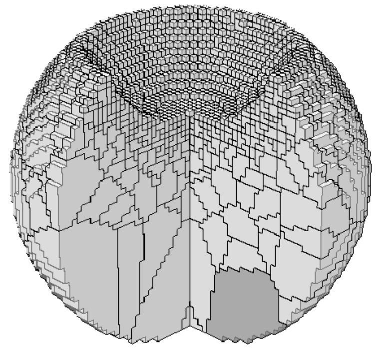
- The parameter C depends solely on f_l for a fixed material: $C = f(f_l, n)$.

- f_l is calculated as follows:

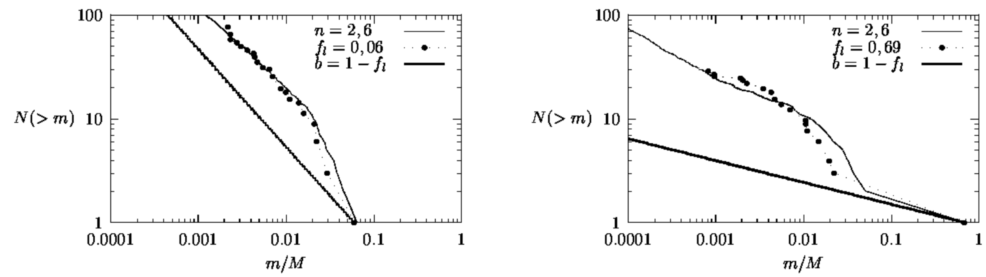
$$\epsilon := \frac{\rho E_{\mathsf{kin}}}{2SM}$$

$$f_l(\epsilon) = \begin{cases} 1 - \frac{1}{2}\epsilon & \text{for} \quad \epsilon < 1 \\ \frac{1}{2}\epsilon^{-K} & \text{otherwise.} \end{cases}$$

- Use f_{KE} to calculate $E_{\mathsf{kin}}^{\mathsf{frag}}$.



Section for $f_l = 0.04$ and $n = 3$. In this calculation $60 \times 60 \times 60$ grid cells were used.



Lab experiments with $f_l = 0.06$ (left) and $f_l = 0.69$ (right). The data is represented by a dotted line with black dots, the new model by a thin line. The thick line represents a simple power law.

(Data from Davis and Ryan 1990)

Velocity Distribution Models

- Direction: The velocity vectors point radially outwards in all models.
- Modulus:
  1. All Fragments have the same velocity $v_i = v_0$.
  2. Each Fragment has $v_i = C_v m_i^{-1/6}$. Fragments of the same mass have the same velocity.
  3. The fragment velocities are derived from a maxwell distribution. The mean square velocity is $v_0^2 \propto \left(m_i^{-k}\right)^2$.

     $$p(v, v_0) = \frac{6\sqrt{3}}{\sqrt{2\pi}} \frac{v^2}{v_0^3} \exp(-\frac{3}{2}\frac{v^2}{v_0^2})$$

  4. The fragment velocities are superimposed with a scatter of the form: $v_i = v_i^0(1 + R_v)$. R_v is gaussian distributed with a fixed dispersion 0.25 for all fragments.

     $$p(x, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{x^2}{2\sigma^2})$$

If necessary, the velocities are rescaled to obtain the desired total kinetic energy $E_{\mathrm{kin}}^{\mathrm{frag}}$.

Numerical Integration

Equations of motion:

$$\frac{d^2\vec{x}_i}{dt^2} = \sum_{j \neq i} \frac{G m_j (\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|^3} \quad \text{Sumation over } j$$

Integrator: Second order Leapfrog-Method.

Collisions

- Fragments are regarded as spheres.
- Rotation of the fragments is neglected.
- *No* further fragmentation due to secondary collisions.
- The restitution parameter is $\alpha = 0.8$

Analysis

After the calculation all gravitationally bound fragments are merged into a "rubble pile". In the case of two bound fragments orbital parameters corresponding to an unperturbed keplerian orbit are calculated.

Ongoing Work on the Formation of protoplanets

Integrator: Nbody6++ with Hermite Scheme

Hermite Iteration

First step: Particle prediction:

$$x_p = x_0 + v_0 \Delta t + \frac{1}{2} a_0 (\Delta t)^2 + \frac{1}{6} \dot{a_0} (\Delta t)^3$$
$$v_p = v_0 + a_0 \Delta t + \frac{1}{2} \dot{a_0} (\Delta t)^2$$

Hermite Step:

$$v_c = v_0 + \frac{1}{2}(a_p + a_0)\Delta t - \frac{1}{12}(\dot{a_p} - \dot{a_0})(\Delta t)^2$$
$$x_c = x_0 + \frac{1}{2}(v_c + v_0)\Delta t - \frac{1}{12}(a_p - a_0)(\Delta t)^2$$

The iteration is achieved by recalculating $a_p, \dot{a_p}$ with x_c, v_c.
Two iterations are sufficient in practice.

Additional Features of NBody6:

- Neighbor Scheme: Split force in irregular component (near neighbors) and regular component (distant particles and central force)
- KS (Kustaanheimo-Stiefel) Integration of close encounters.

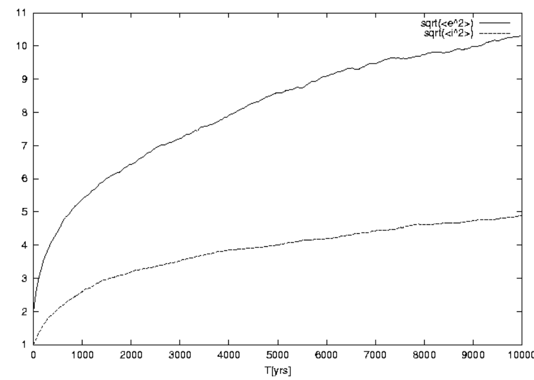
Feasible is:

(assuming an elapsed time of one week and $10,000$ Orbits)

Hardware	Code	N
One GRAPE-6	NBODY4	70.000
GRAPE-6 (B) in Hydra-Cluster (ARI)	NBODY6++	300.000
Cray T3E - 10 Processors	NBODY6++	22.000
Cray T3E - 50 Processors	NBODY6++	50.000
Cray T3E - 100 Processors	NBODY6++	71.000
Cray T3E/512	NBODY6++	160.000

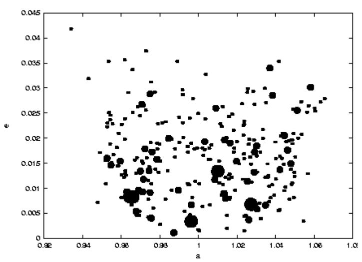
Direct simulation of planetesimals in a narrow ring (1 $\pm$ 0.04 AU).

Testing the Code



1000 collisionless particles, $, < a >= 1$ AU, $\Delta a = 0.07$ AU, $\Sigma = 10 \frac{\mathrm{g}}{\mathrm{cm}^2}$. e,i in units of $\frac{r_{\mathrm{Hill}}}{<a>}$.

Including Collisions with $F = 10$.



Mass Bins:
m≤1E-9 1E-9<m≤1E-8
1E-8<m.

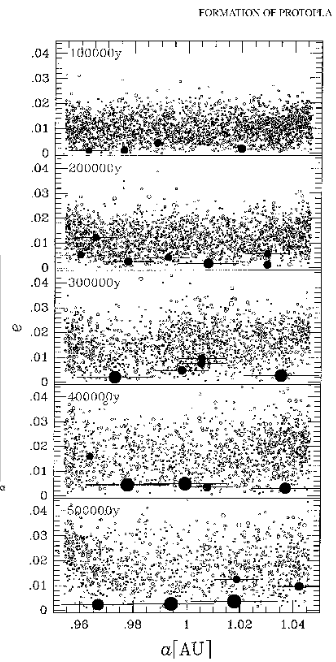


FIG. 7. Snapshots of a planetesimal system on the a–e plane. The circles represent planetesimals and their radii are proportional to the radii of planetesimals. The system initially consists of 4000 planetesimals whose total mass is 1.3×10^{27} g. The initial mass distribution is given by the power law mass distribution with the power index $\alpha = -2.5$ with the mass range $2 \times 10^{23} < m < 4 \times 10^{24}$ g. The numbers of planetesimals are 2712 ($t = 100{,}000$ years), 2200 ($t = 200{,}000$ years), 1784 ($t = 300{,}000$ years), 1488 ($t = 400{,}000$ years), and 1257 ($t = 500{,}000$ years). The filled circles represent protoplanets with mass larger than 2×10^{25} g and lines from the center of the protoplanet to both sides have the length of $5r_H$.

(Kokubo & Ida 2000)

Including Collisions with $F = 10$.