Chaos, Black Holes and Quantum Mechanics

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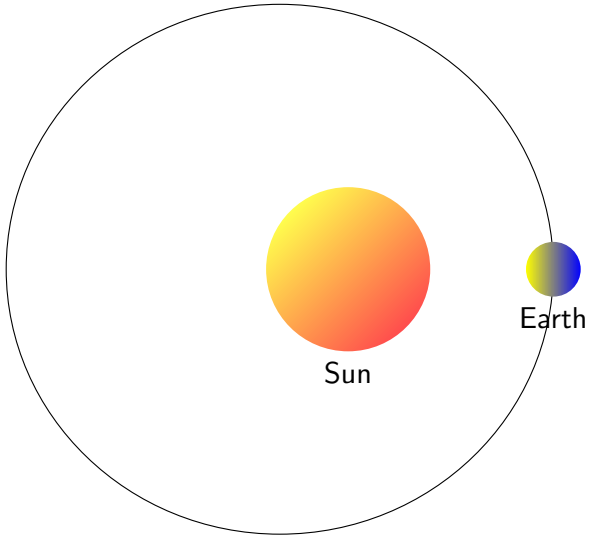
Regular motion

- Newtonian view of the world:
- Specify initial configuration, then evolve by dynamical laws.



- Future is easily predictable.

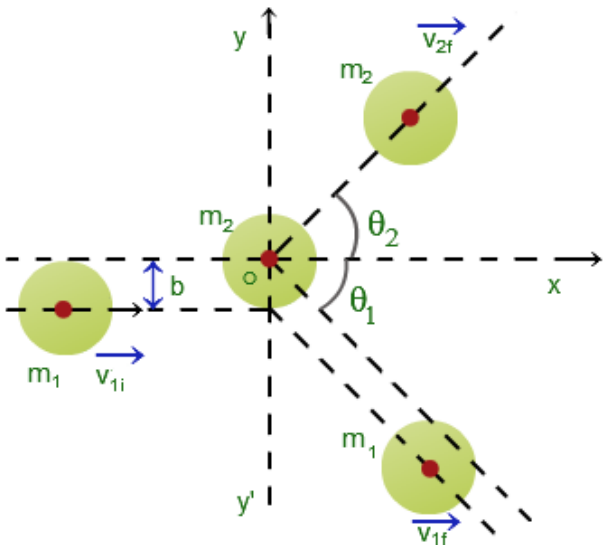
Chaos

- Not typical.
- It is usually hard to predict the future.

- It is hard to be a good pool player!

Sensitive dependence on initial conditions

- "Sensitive dependence on initial conditions "



- A small change in initial impact parameter makes a large change in final trajectory.

Sensitive dependence on initial conditions, contd.

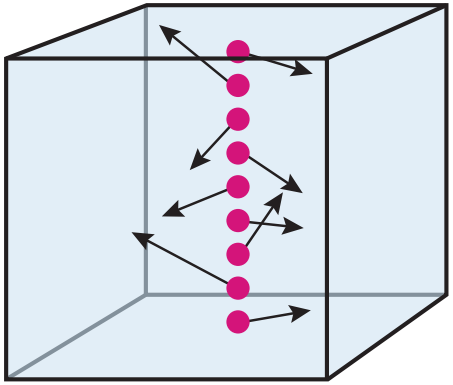
- Sensitive dependence on initial conditions $\leftrightarrow$ strong chaos.
- The weather.
- The "butterfly effect."
- A fundamental source of uncertainty, even in classical physics.

Sensitive dependence on initial conditions, contd.

- Sensitive dependence on initial conditions does **not** mean that nothing is predictable.
- The summer is hotter than the winter.
- Dumping large amounts of carbon dioxide into the atmosphere warms the planet.
- The detailed size and time course of these effects is hard to predict.

Chaos and thermal behavior

- Chaos leads to simplicity.



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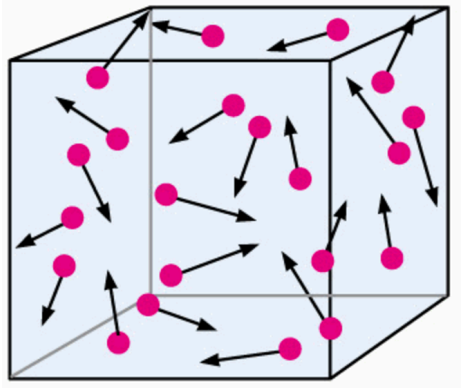
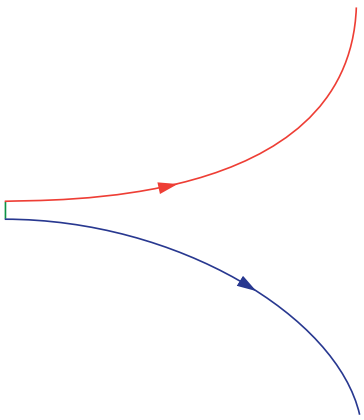


- Atoms spread out uniformly – constant density.
- Energy becomes uniformly shared – constant energy density.
- Hallmarks of thermal behavior.
- Uniformity $\leftrightarrow$ thermal equilibrium.
- Kinetic energy per particle $= \frac{3}{2} k_B T$
- Thermal behavior in such systems is a consequence of chaos.

Quantifying chaos

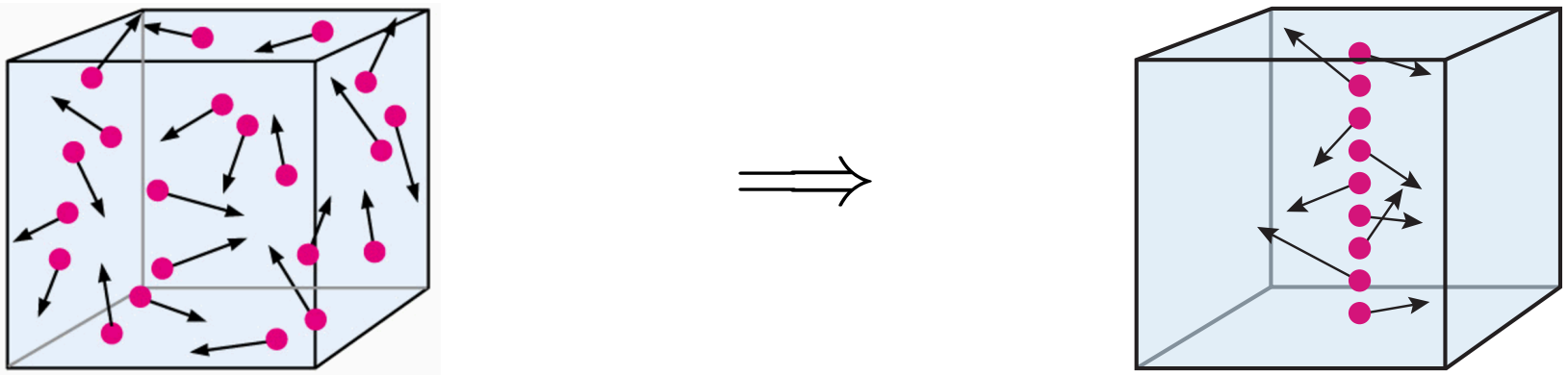
- How sensitive to initial conditions?



- Trajectories diverge **exponentially** in time.
- 1 inch apart, 2 inches, 4, 8, $\rightarrow$ 85 feet in 10 seconds!
- distance grows like $e^{\lambda_L t}$
- λ_L is a Lyapunov exponent
- Large λ_L means trajectories diverge more quickly – stronger chaos.
- Weather: one of the λ_L gives a divergence time of $\sim$ 5 days.

Quantifying chaos, contd.

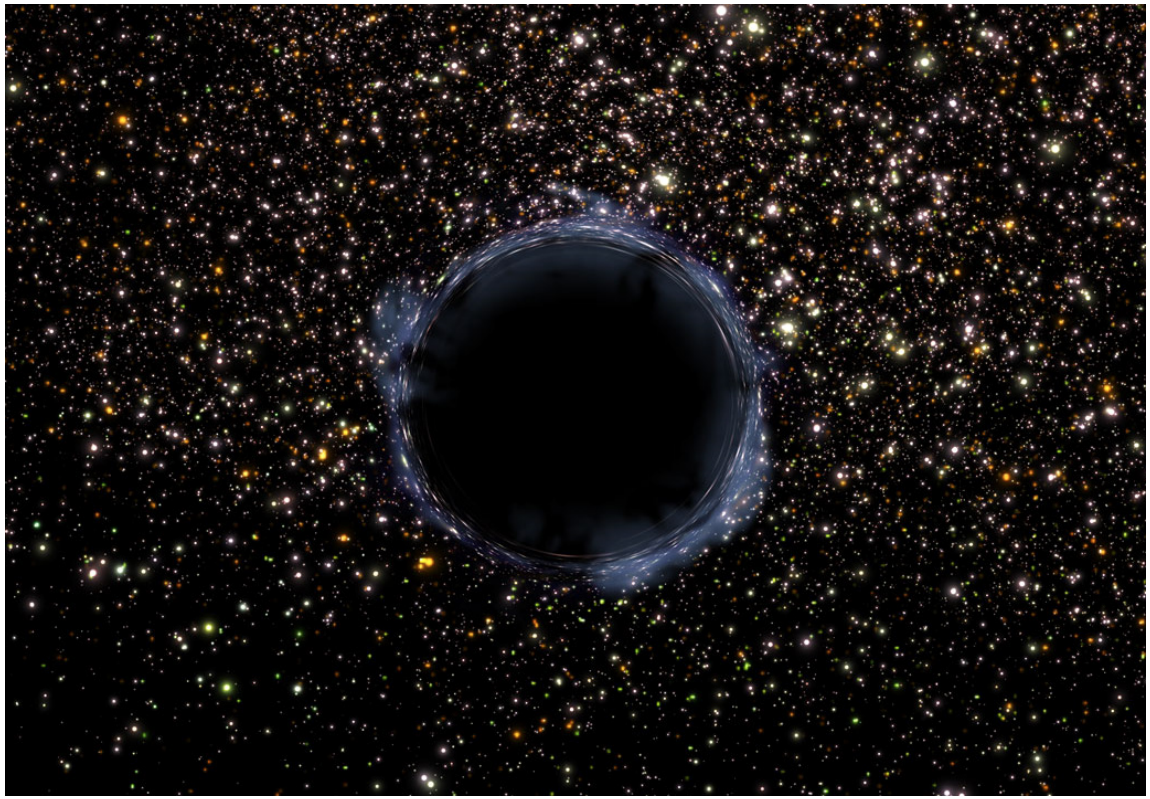
- Another way – "run the movie backwards."
- Fine tune initial conditions to end up in a special configuration after a time t.



- How much can you disturb the initial conditions and still end up in the special configuration? An exponentially small amount at late time, $e^{-\lambda_L t}$.
- Can arrange such a fine tuned situation in a black hole.

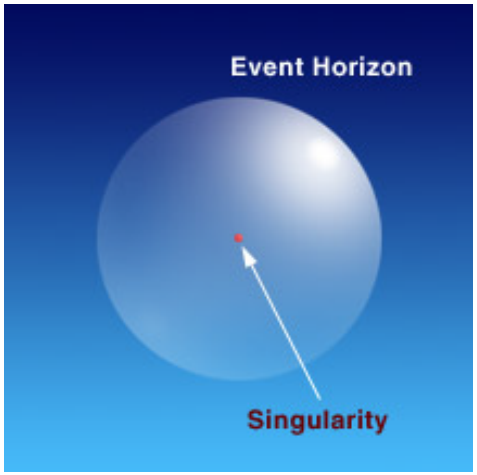
Black holes

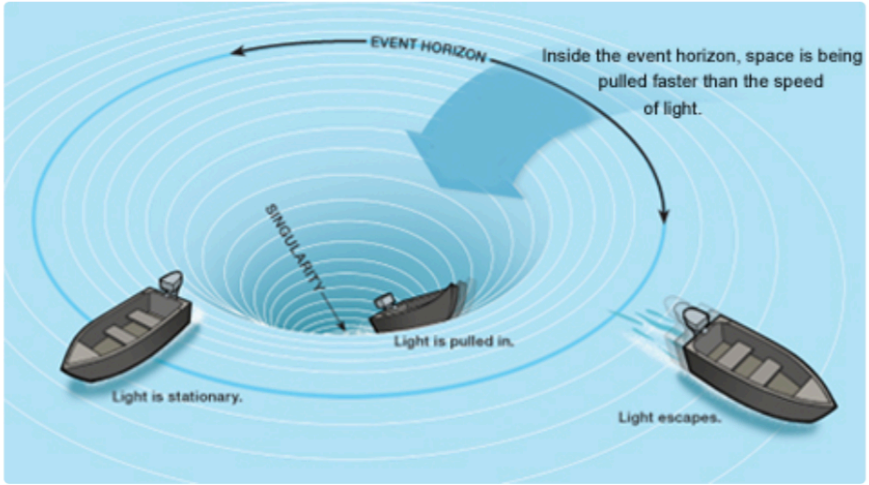
- A striking consequence of General Relativity.
- A profound distortion of space-time geometry.
- Classically nothing, including light, can leave a black hole.



Black holes, contd.

- Black holes are hard to visualize:





- Horizon: once inside you cannot get out, no matter how much energy you use. You inevitably hit the singularity.
- The horizon size grows with mass.
- From just outside the horizon you can escape if you use enormous energy.

Chaos in black holes

- Set up a fine tuned initial condition to end up in a special configuration:

- A photon (particle of light) starting just outside the horizon, tuned to stay close for a long time t and finally depart.
- Requires starting exponentially close to the horizon, because the distance increases exponentially in time.
- Requires an exponentially large amount of energy, which "redshifts" away.

Chaos in black holes, contd.

- Now perturb this initial condition a bit by dropping a particle into the black hole [‘t Hooft]

- The black hole becomes more massive, the horizon grows and swallows the photon.

Chaos in black holes, contd.

- The photon is trapped behind the horizon so it inevitably hits the singularity, a dramatically different outcome than before.
- If t is large the photon starts exponentially close to the horizon, so an exponentially small mass increase is enough to cause this.
- This exponential sensitivity is a signal of Lyapunov behavior.

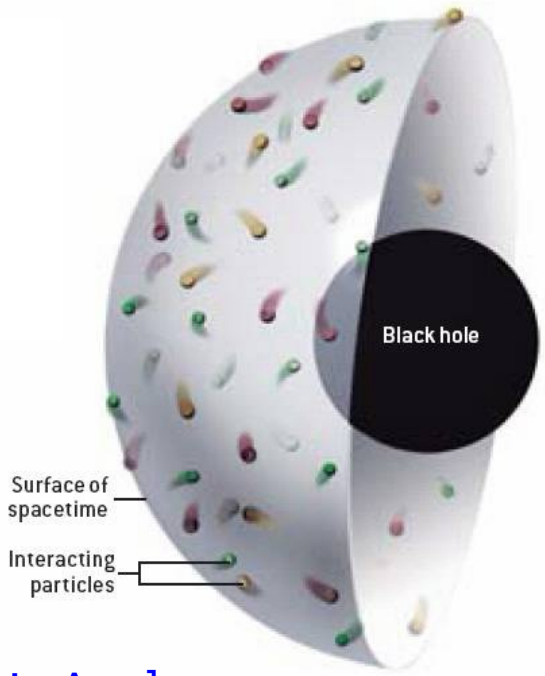
Toward quantum chaos

- What we have just described is a kind of **classical** chaos present in classical general relativity. What about quantum mechanics?
- "Gauge-gravity duality" maps this to quantum chaos in a "dual" system.

Gauge/gravity duality

- Gauge/gravity duality: [Maldacena]
- A precise mapping (a duality) between an ordinary quantum mechanical system without gravity (like QCD) and a quantum gravitational one.
- The duality provides a dictionary to translate between the two systems.

Gauge/gravity duality, contd.



[Maldacena, Sci. Am.]

- The grey shell is the boundary of space.
- The ordinary quantum system lives on the boundary (the "boundary theory").
- Quantum gravity lives inside the shell (the "bulk" theory).
- The quantum black hole is thermal, at the Hawking temperature, so the boundary theory is thermal too.

Gauge/gravity duality and quantum chaos

- When the boundary system has many degrees of freedom, the gravitational system – the black hole – behaves classically, while the boundary system remains strongly quantum mechanical.
- So classical gravity can give results, via the duality dictionary, about certain quantum systems with many degrees of freedom.
- The classical Lyapunov behavior of black holes we discussed above can be used to determine the quantum Lyapunov behavior of a boundary system with many degrees of freedom – a calculation of the quantum butterfly effect in a strongly interacting many-body system. [SS - Stanford; Kitaev]
- The quantum Lyapunov exponent is given by

$$\lambda_L = \frac{2\pi k_B T}{\hbar}$$

A bound on chaos

- This calculation has taught us some lessons about quantum chaos in many-body physics.
- For example, it motivated the derivation of
- "The chaos bound." [Maldacena–SS–Stanford]
- The quantum Lyapunov exponent of any quantum system with many degrees of freedom is bounded by the black hole value.

$$\lambda_L \leq \frac{2\pi k_B T}{\hbar}$$

with reasonable physical assumptions

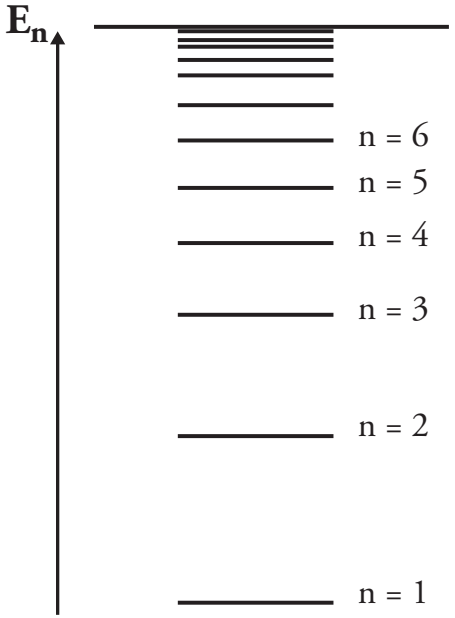
- "Black holes are the fastest scramblers in nature." [Sekino-Susskind]

Quantum gravity

- These results are satisfying, but the gravitational physics involved, the classical behavior of black holes in general relativity, is not new.
- Can we extend beyond the limit where gravity is classical and use quantum chaos to learn new things about quantum gravity?

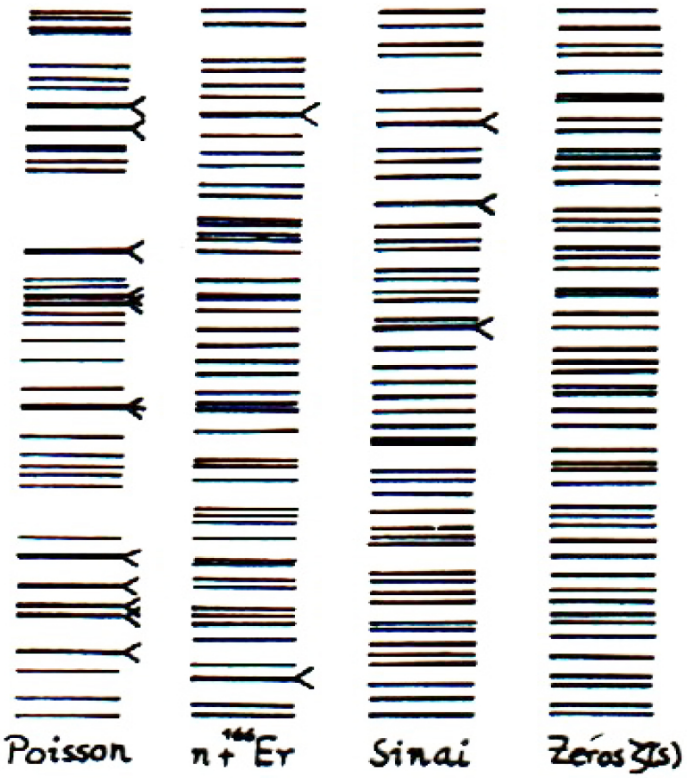
Quantized energy levels

- A defining feature of quantum systems (whose motion is bounded) is that the energies can only assume discrete values.
- The energy is "quantized."
- Discrete energy levels correspond to independent quantum states.
- The Hydrogen atom is an example:



Energy levels in quantum chaotic systems

- Quantized energy levels in quantum chaotic systems follow a rather different pattern:



- In chaotic systems neighboring energy levels "repel:" "short range level repulsion."
- The "gas" of energy levels is hard to compress: "long range spectral rigidity."

Energy levels in quantum chaotic systems

- Chaos gives rise to simplicity in the energy level pattern.
- A remarkable conjecture: quantum chaotic energy levels obey **random matrix statistics**. [Wigner, Dyson, Mehta, Bohigas-Giannoni-Schmit, Berry, ...]
- A **universal** pattern

Energy levels and eigenvalues

- The dynamical rules of a quantum system determine a matrix, the Hamiltonian matrix, which can be quite complicated:

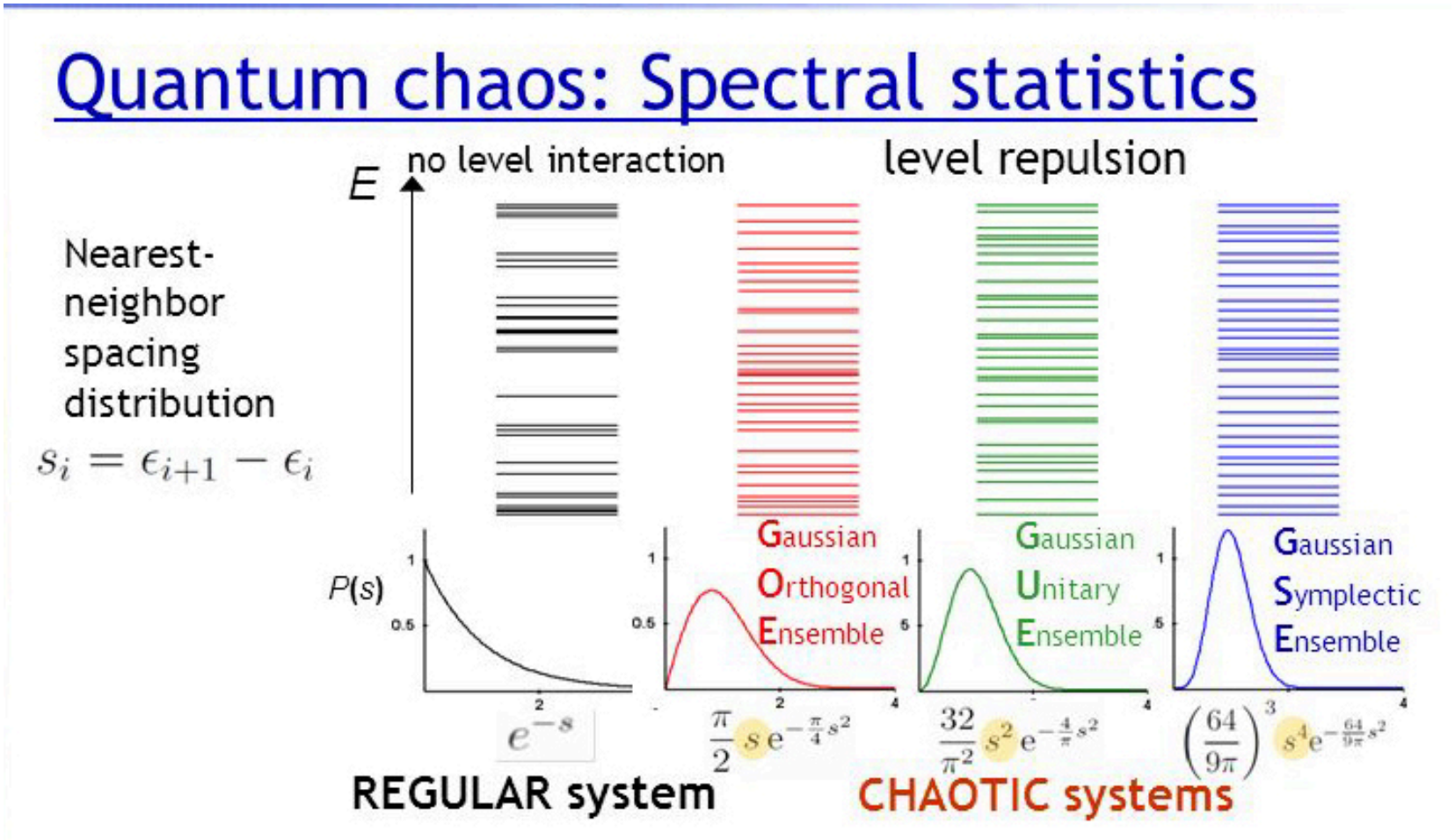
$$\begin{pmatrix} 1.7 & 2.14 & -3.56 & .71 \\ 2.14 & 5.9 & .04 & 95 \\ -3.56 & .04 & -3.8 & 66 \\ .71 & 95 & 66 & 7.2 \end{pmatrix}$$

- "Diagonalize" this matrix to find its "eigenvalues." These are the energy levels (Matrix mechanics).
- A quantum chaotic system has energy levels corresponding to the eigenvalues of a **random** matrix

$$\begin{pmatrix} +1 & -1 & -1 & +1 \\ -1 & +1 & +1 & +1 \\ -1 & +1 & -1 & -1 \\ +1 & +1 & -1 & -1 \end{pmatrix}$$

Random matrix eigenvalues

- Easy to compute properties of eigenvalues of a random matrix (chaos leads to simplicity, again...)



- Long range spectral rigidity and short range level repulsion are clearly visible.

Black holes have energy levels

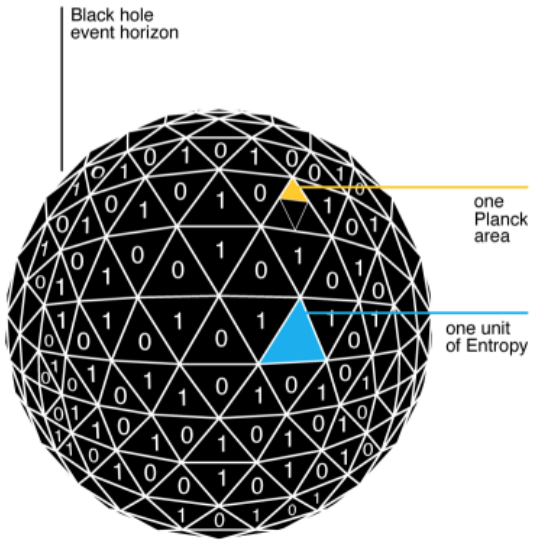
- Black holes in quantum gravity are bound quantum systems.
- They should have quantized energy levels.
- How many?
- In thermal systems (like a black hole) the logarithm of the number of energy levels (in an energy band) is called the "entropy."

The entropy of quantum black holes

- The entropy of a quantum black hole: [Bekenstein, Hawking]

$$S_{\rm BH} = \frac{1}{4}\frac{\text{Horizon area}}{\text{Planck area}}$$

The Planck area is the Planck length squared, where the Planck length is $\sim 10^{-33}$ cm.



- One "qubit" per Planck area

Size of black hole entropy

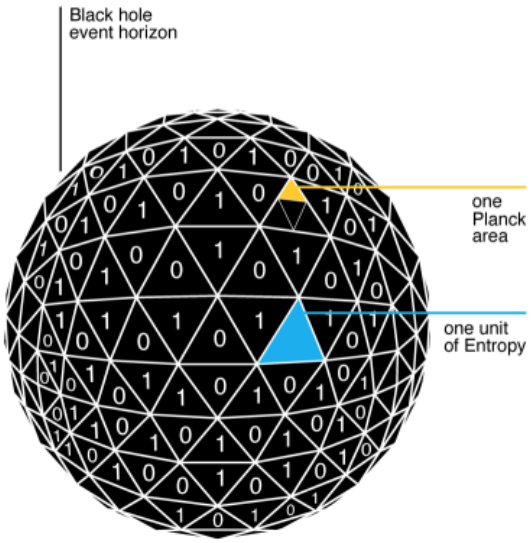
- For a solar mass black hole the horizon radius is $\sim 3\mathrm{km}$. Entropy $\sim 10^{78}$ qubits, a large number.
- But the number of energy levels is $10^{10^{78}}$, a very large number.
- Black holes are chaotic systems so the levels should look like the eigenvalues of a very large random matrix.
- In the limit of classical gravity this quantization will go away and the energies will become continuous.
- Discreteness and random matrix behavior are distinctly quantum gravitational phenomena.

A simple model

- These ideas have been extremely difficult to study until recently.
- The Hamiltonian of the boundary theory is complicated, and the matrices very large.
- But recently a simple model of a quantum black hole has been introduced, enabling us to make progress in understanding these questions . [Sachdev-Ye, Kitaev]

The Sachdev-Ye-Kitaev model

- Make a model of "pure quantum information."



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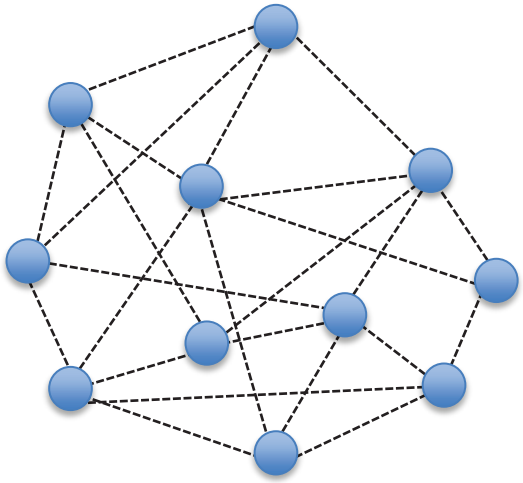
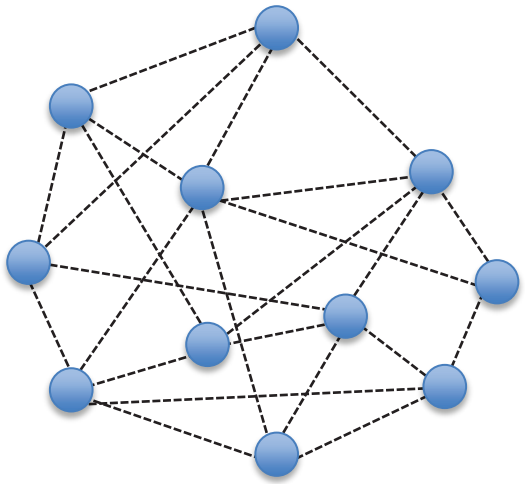


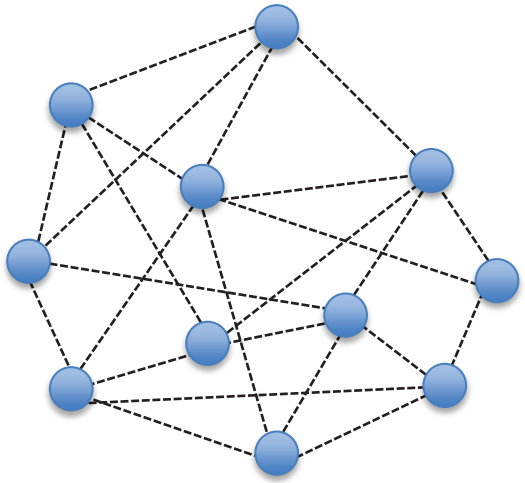
- N qubits coupled in groups of 4, with random couplings. The Sachdev-Ye-Kitaev (SYK) model.

The Sachdev-Ye-Kitaev model, contd.



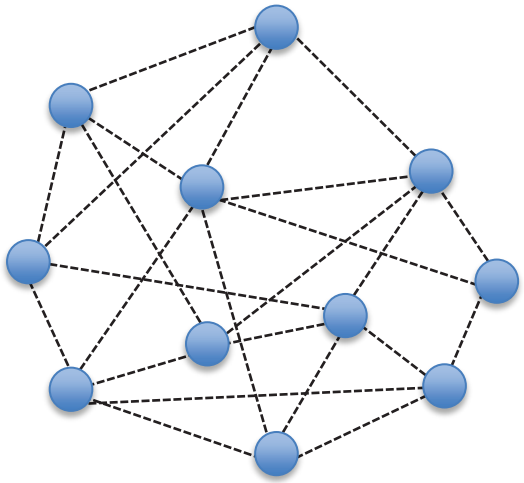
- There is a spirited competition going on to determine the most practical kind of qubit for quantum computation.
- Santa Barbara is a hotbed of this:
  Google/UCSB (Martinis group) – superconducting qubits;
  Microsoft Station Q/UCSB – Majorana fermions.
- The SYK model uses Majorana fermions. [Kitaev]

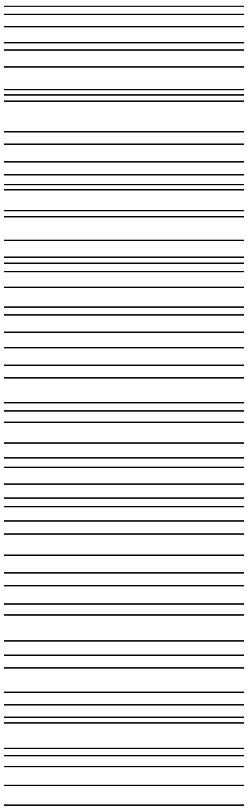
Good things about the Sachdev-Ye-Kitaev model



- The SYK model is maximally chaotic, like a black hole. [Kitaev]
- It has a sector in its bulk dual that behaves like gravity, in 1 space and 1 time dimension, so lessons can be learned about quantum gravity.
- For N Majorana fermions the number of energy levels is $2^{N/2}$. $N = 34$ has $\sim 128,000$ levels, large enough to be interesting, small enough to study on modern (classical) computers.

Energy levels of the Sachdev-Ye-Kitaev model





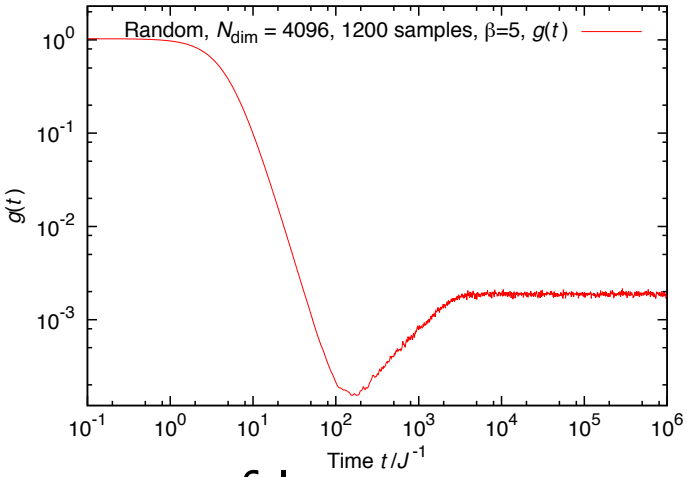
- The energy levels of SYK display long range spectral rigidity and short range eigenvalue repulsion, like random matrix levels.
- We need a more quantitative test.

Spectral form factor

- The spectral form factor: a quantitative diagnostic of long range spectral rigidity and short range energy level repulsion.

$$\sum_{n,m} e^{i(E_n - E_m)t}$$

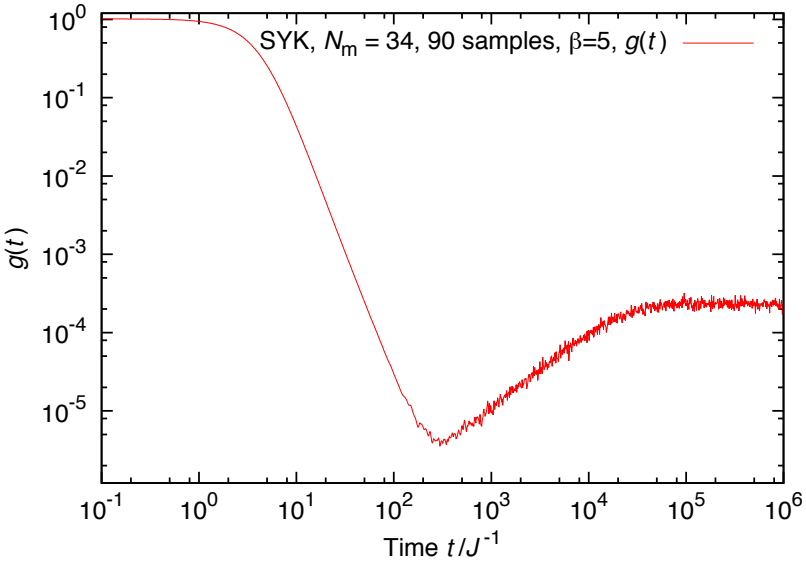
- The Fourier transform of the energy difference distribution.
- Try it out on random matrices:

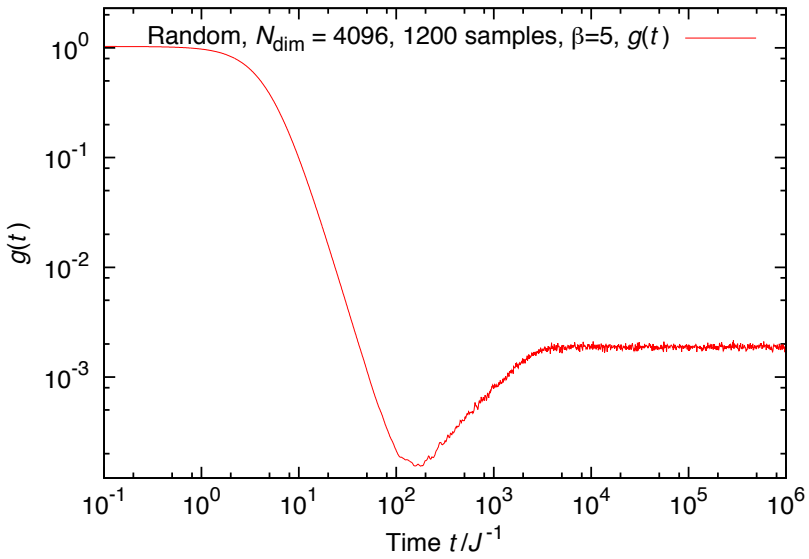


- The "ramp" is a signature of long range spectral rigidity.
- The "plateau" is a signature of short range energy level repulsion.

Spectral form factor

- Compute the spectral form factor for the SYK model [Jordan Cotler, Guy Gur-Ari, Masanori Hanada, Joe Polchinski, Phil Saad, Stephen Shenker, Douglas Stanford, Alex Streicher, Masaki Tezuka].

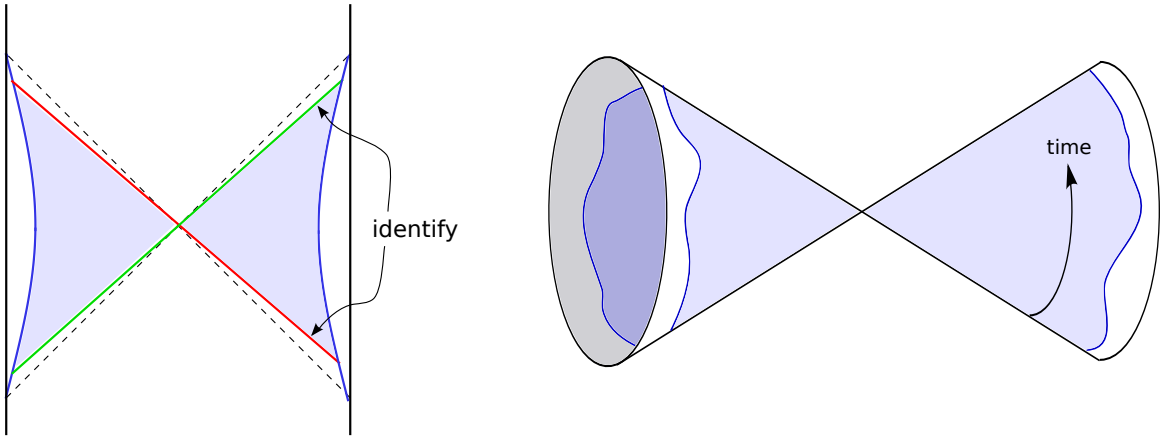




- Quantitative agreement with random matrix statistics.
- A reasonable conjecture that this is the case for general black holes.
- Should have an explanation in quantum gravity.

The ramp in quantum gravity

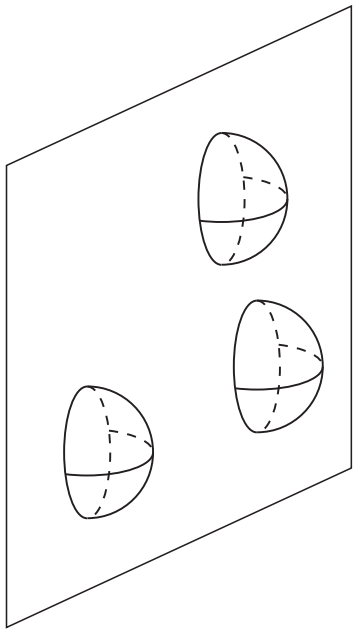
- In **quantum** gravity the space-time geometry is not definite, because of the uncertainty principle.
- The contribution of multiple geometries must be included.
- The ramp can be explained by the contribution of a different geometry than that of the original black hole – the "double cone." [Phil Saad-SS-Douglas Stanford].



- Represents a one dimensional universe (a line) evolving in time.
- Has a natural generalization to higher dimensional black holes.

The plateau as a D-brane

- The plateau is the "smoking gun" for quantized energy levels. It is a much more subtle phenomenon than the ramp.
- We believe its explanation in quantum gravity requires effects beyond geometry [Phil Saad-SS-Douglas Stanford, **in progress**].
- Arbitrary numbers of "baby universes" ending on a D-brane [Polchinski].



- We will keep you posted...

Thank you

Thank You!