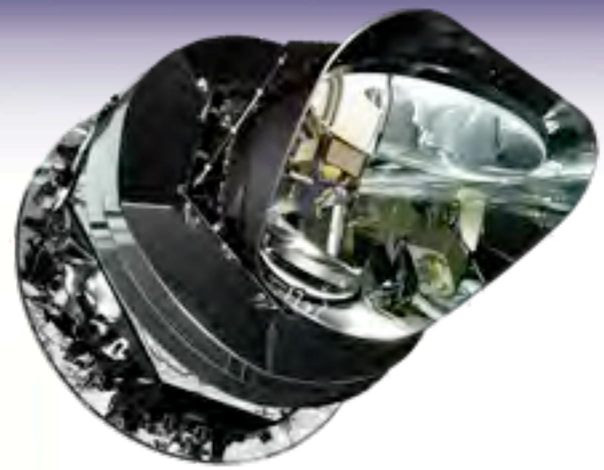


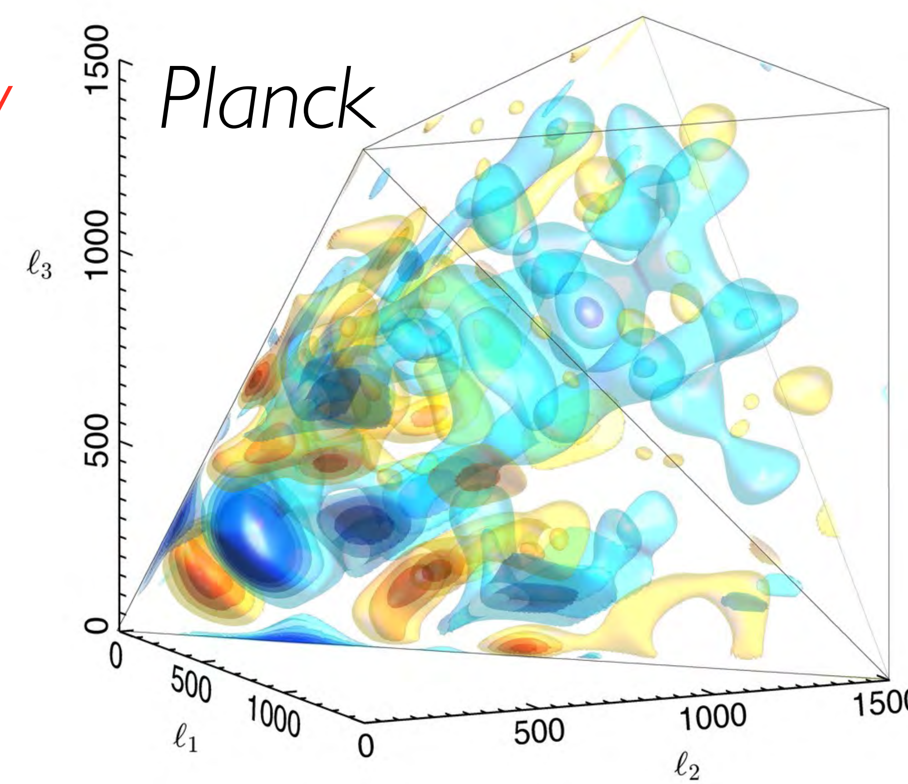
Planck Cosmology Results 2013

Non-Gaussian Inflation & Planck

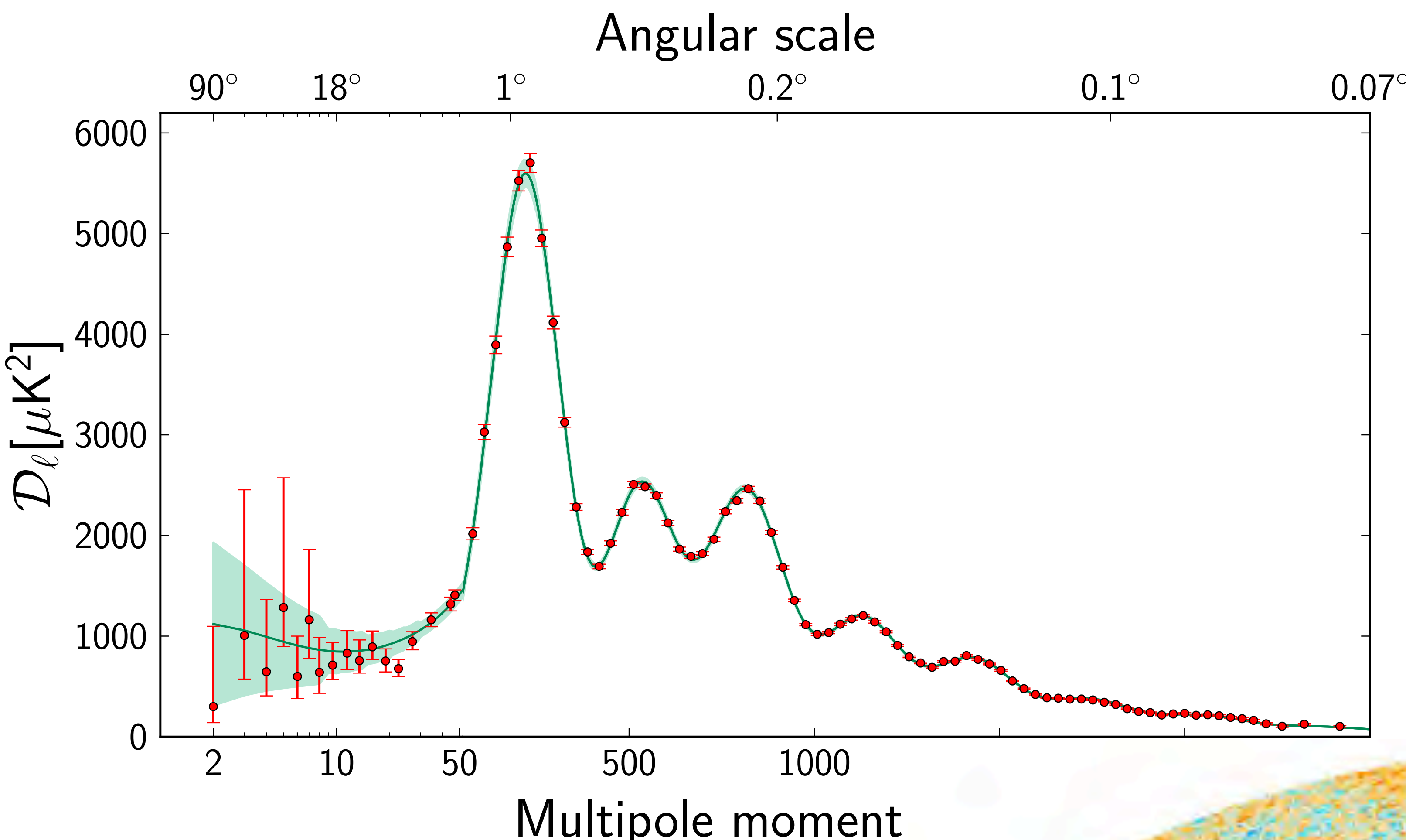
James Fergusson and Paul Shellard
on behalf of Planck collaboration
(Centre for Theoretical Cosmology, DAMTP, Cambridge University)

XXIV. Constraints on primordial non-Gaussianity
(XXV. Searches for cosmic strings & other defects)

Primordial Cosmology Conference
Kavli Institute for Theoretical Physics
Santa Barbara, 22 April 2013



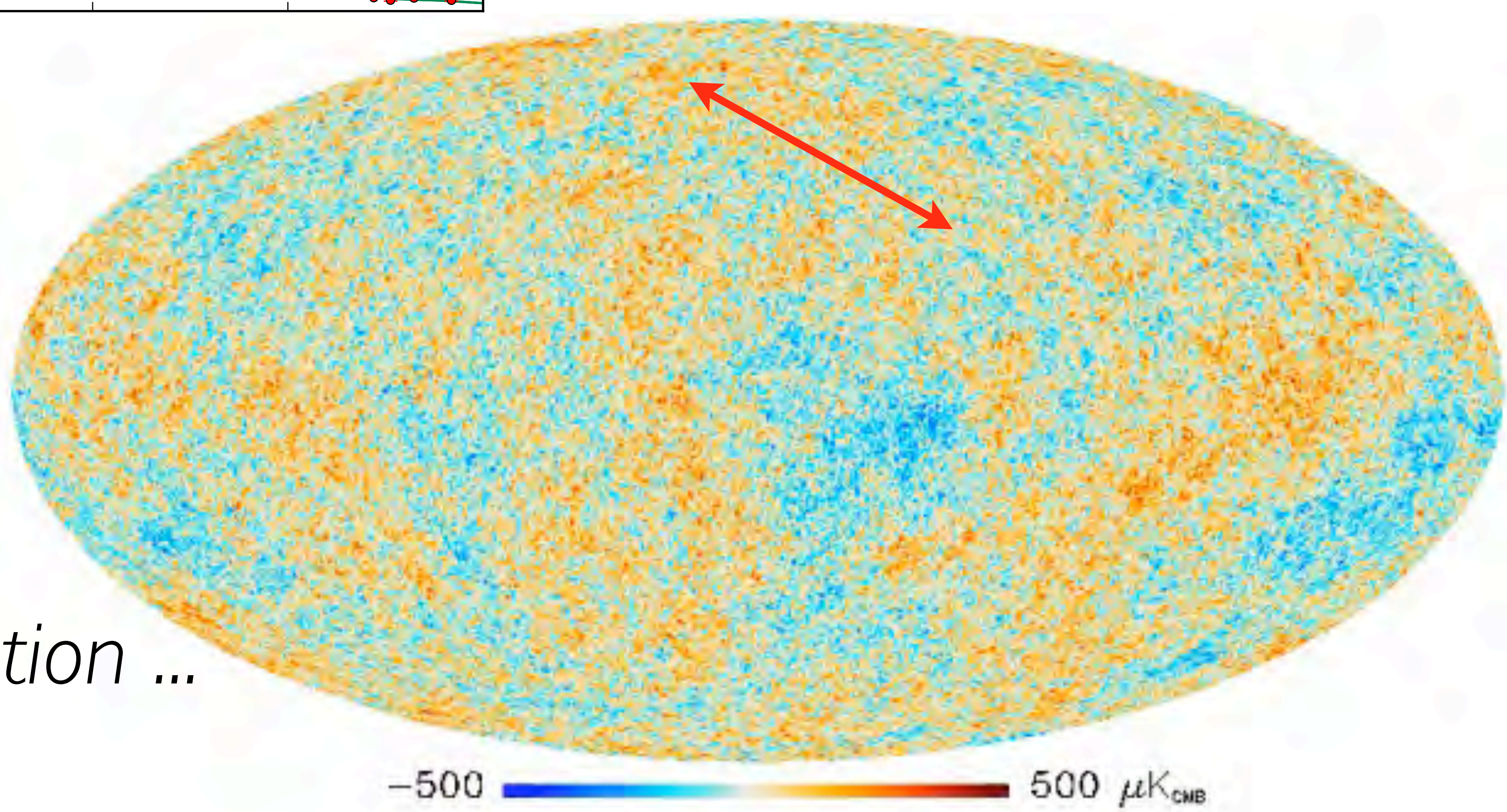
The triumph of inflation



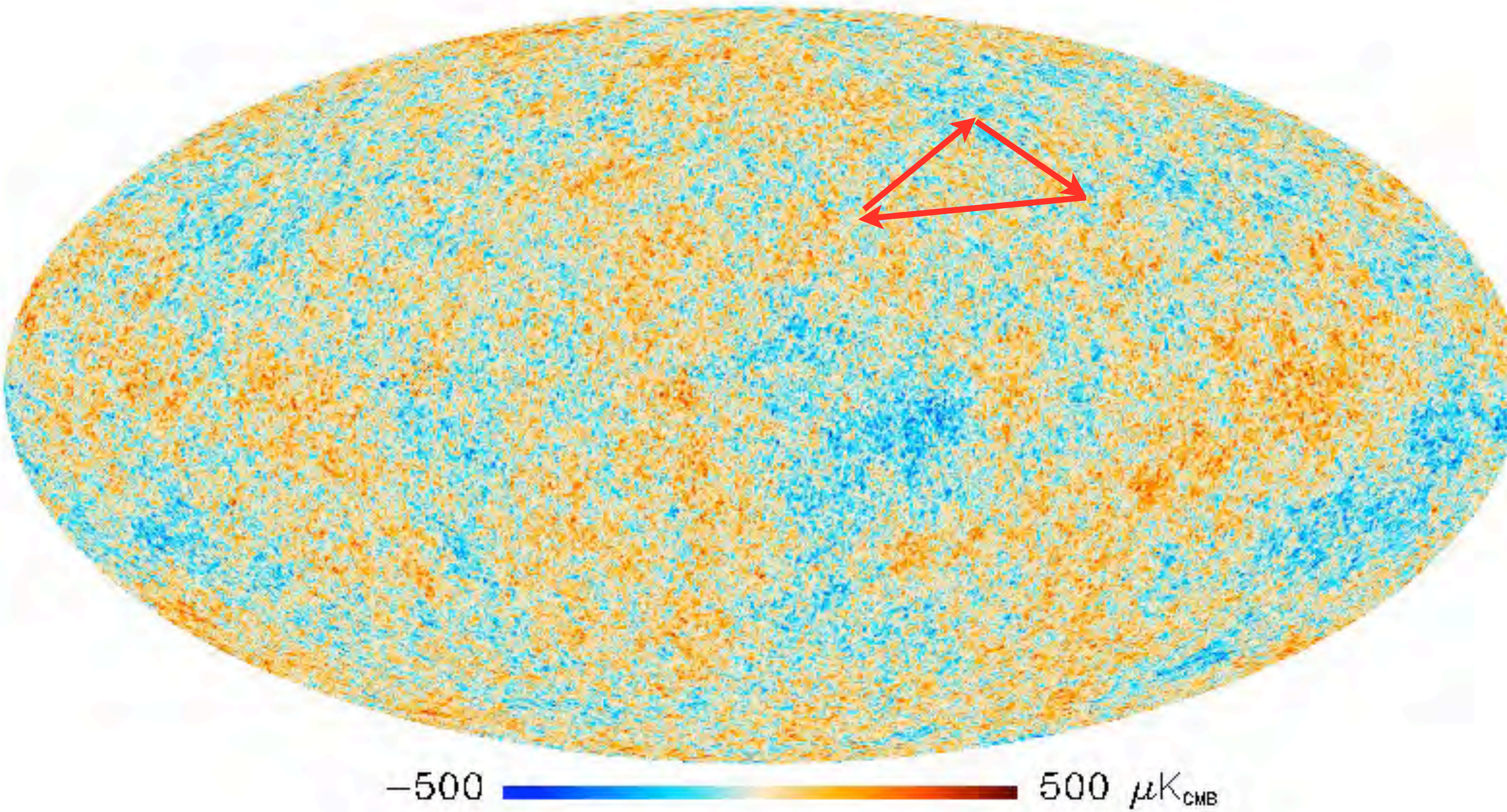
A self-consistent concordance model with 68.3% dark energy, 26.8% dark matter, and 4.9% ordinary matter.

Based on the two-point correlator or angular power spectrum C_l

But there is more information ...



Triangles in the Sky



The CMB Bispectrum

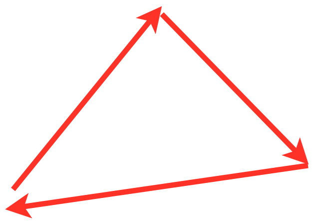
Tetrapyd - bispectrum domain

Allowed multipoles l_1,l_2,l_3 for the CMB bispectrum live in the domain

Resolution: $l_1, l_2, l_3 \leq l_{\max}\,, \quad l_1, l_2, l_3 \in \mathbb{N}\,,$

Triangle condition: $l_1 \leq l_2 + l_3$ for $l_1 \geq l_2,\, l_3, \quad +$ cyclic perms.

Parity condition: $l_1 + l_2 + l_3 = 2n\,, \quad n \in \mathbb{N}\,.$



Reduced bispectrum $b_{l_1l_2l_3}$ from primordial bispectrum $B(k_1,k_2,k_3)$

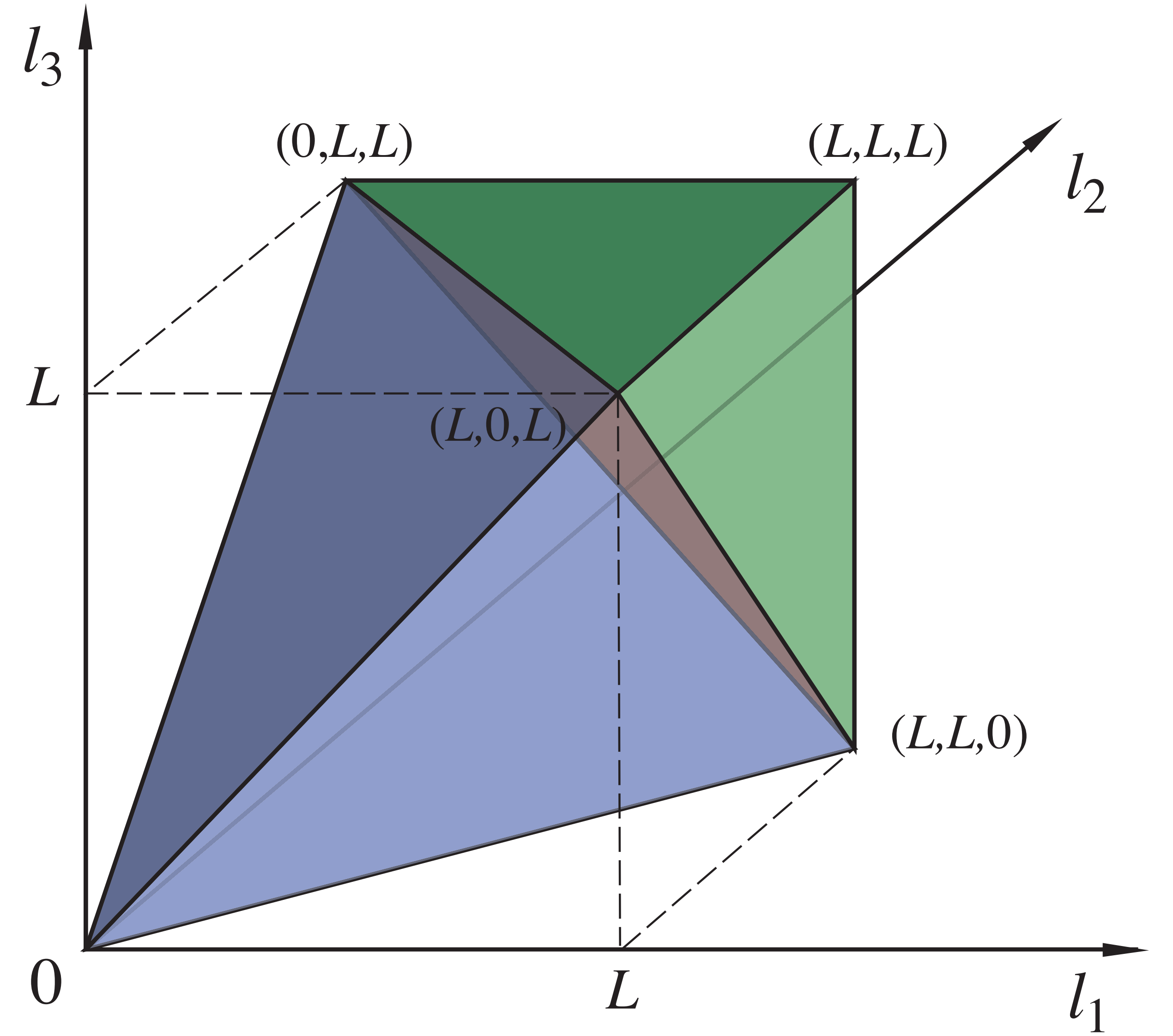
$$b_{l_1l_2l_3} = \left(\frac{2}{\pi}\right)^3 \int x^2 dx \int dk_1 dk_2 dk_3 (k_1k_2k_3)^2 B(k_1,k_2,k_3)\Delta_{l_1}(k_1)\Delta_{l_2}(k_2)\Delta_{l_3}(k_3) j_{l_1}(xk_1) j_{l_2}(xk_2) j_{l_3}(xk_3)$$

Inner product:

Defined by estimator sum

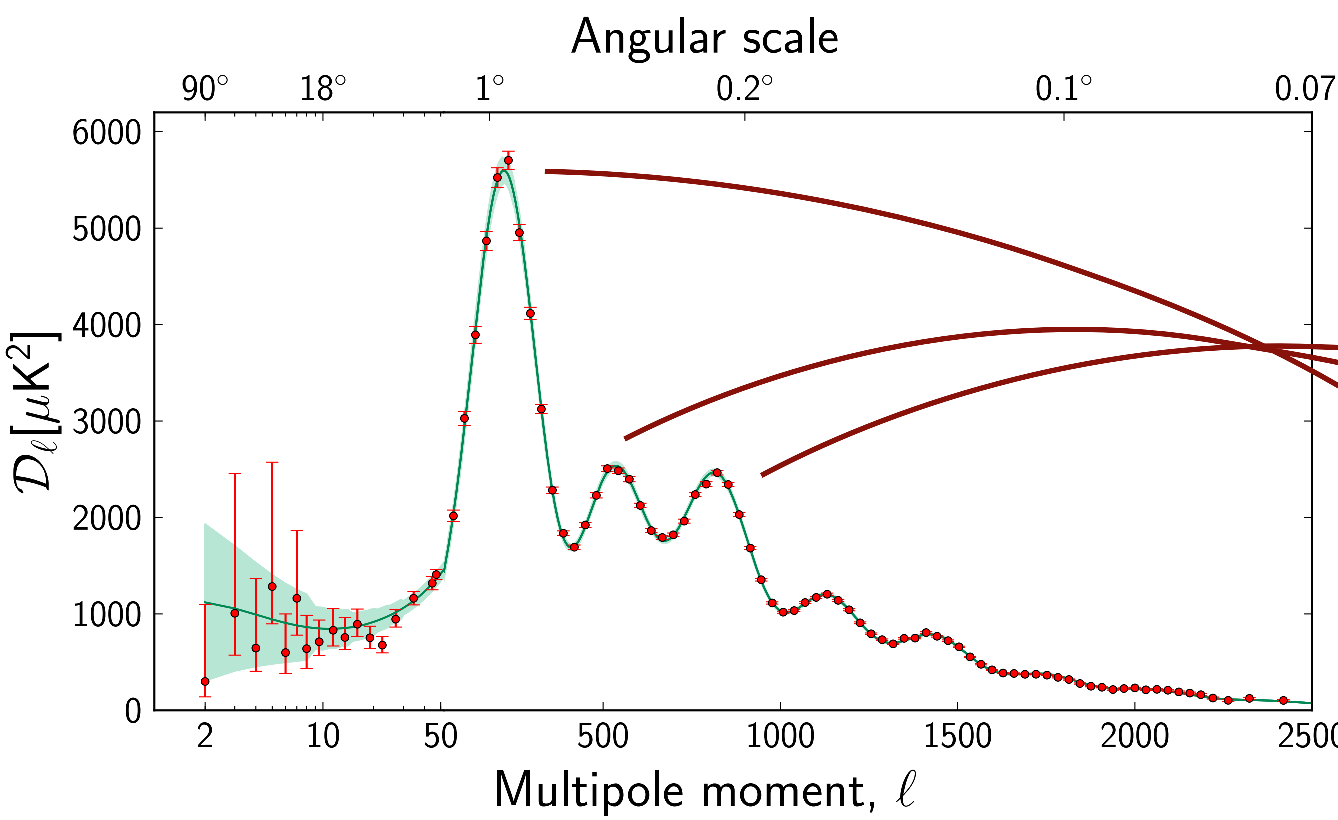
$$\langle b,\, b' \rangle \equiv \sum_{l_1,l_2,l_3 \in \mathcal{V}_{\mathcal{T}}} w_{l_1l_2l_3}\, b_{l_1l_2l_3}\, b'_{l_1l_2l_3}$$

with weight $w_{l_1l_2l_3} = h^2_{l_1l_2l_3}$

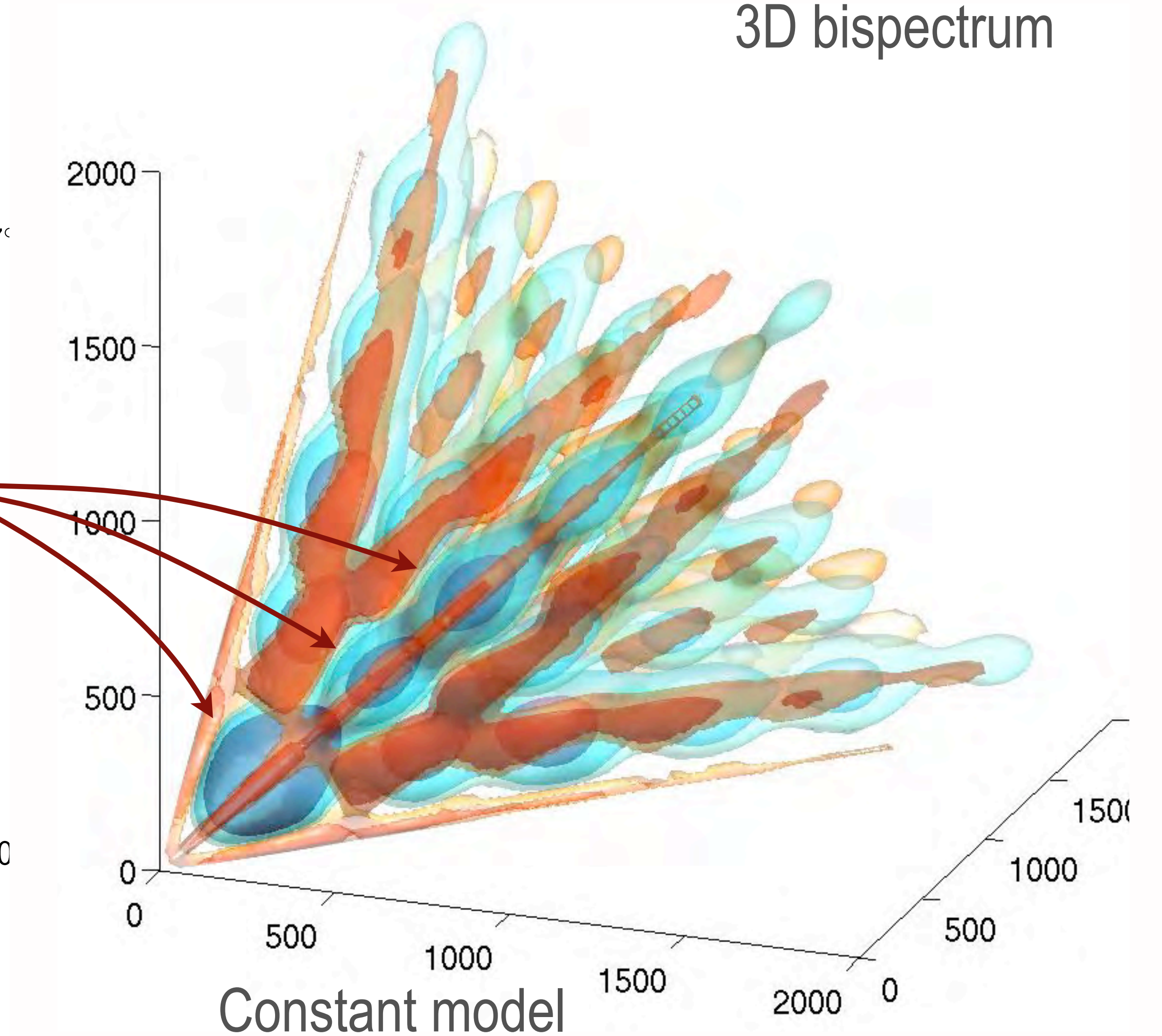


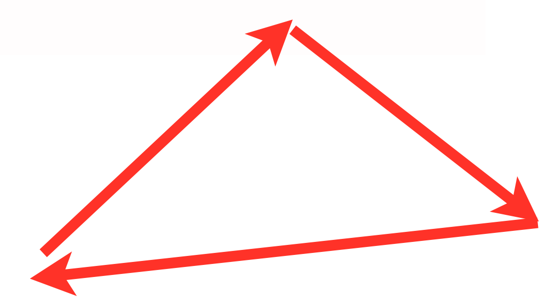
Inflation and the bispectrum

Hot plasma oscillations:



Power spectrum (2pt correlator)



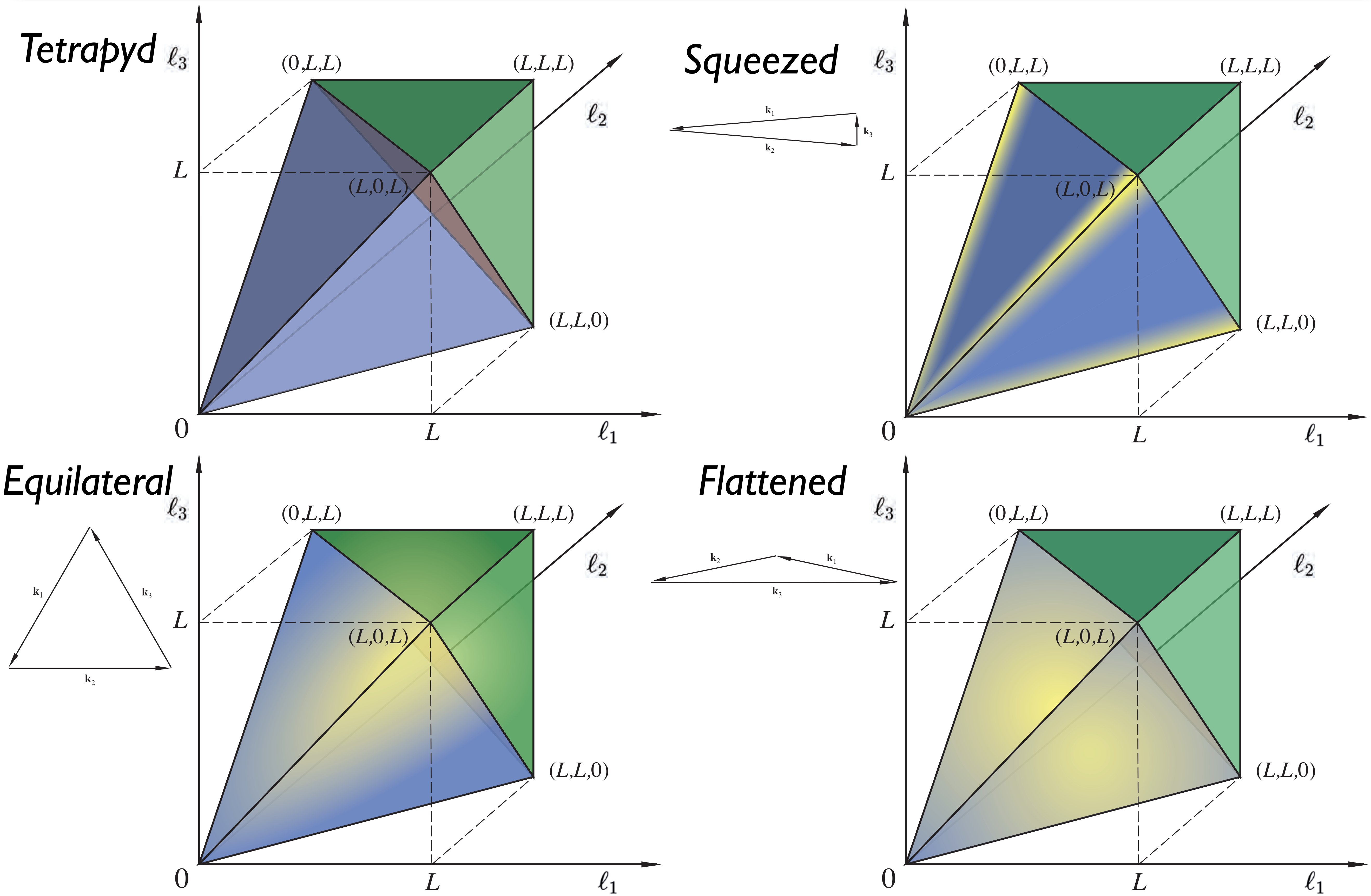


- Simple inflation predicts no (observable) randomness

  $B \sim P^{3/2} / 1{,}000{,}000$

- Deviations less than 1 part in a million! Most stringent inflation test ....

Aside: tetrapyd triangles



No-Go for Inflation

Simple inflation models <u>*cannot*</u> generate observable non-Gaussianity:

- single scalar field
- canonical kinetic terms
- always slow roll
- ground state initial vacuum
- standard Einstein gravity

Non-Gaussianity is arguably the most stringent test of the standard picture

But simple inflation model-building faces rigorous challenges in fundamental theory (e.g. *eta problem and super-Planckian field values*). Many new ideas/ solutions violate these conditions!

Multifield inflation

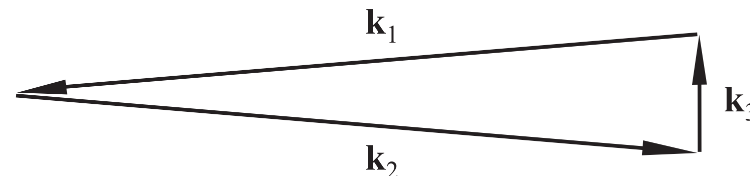
NG from interacting potentials

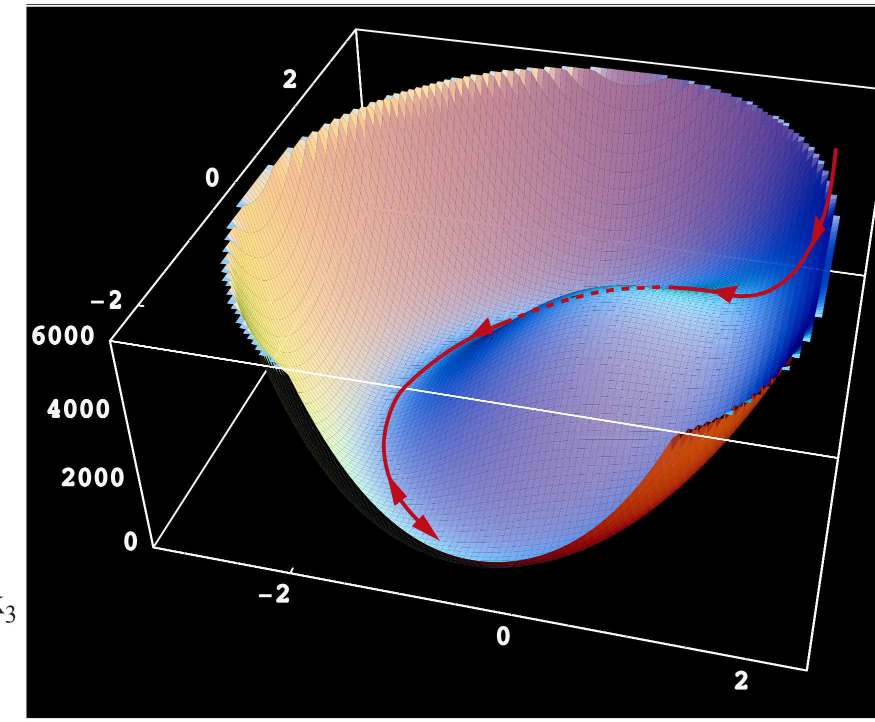
$$V(\phi_1, \phi_2) = \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2 - m^2)^2 + \nu(\phi_1 + m)^3$$

Significant final f_{NL} ingredients:

- *corner turning*
- *nontrivial potential*
- *or breakout (hybrid models)*

Rigopoulos, EPS, van Tent 05, 06; see also Vernizzi & Wands 06, and Bernadeau & Uzan 02 etc etc





- **Curvatons** - *post-inflation eqn of state domination*

  e.g. Linde & Mukhanov 96; Enqvist & Sloth 01; Lyth & Wands 01; Moroi &Tak

- **End of inflation, reheating and preheating**

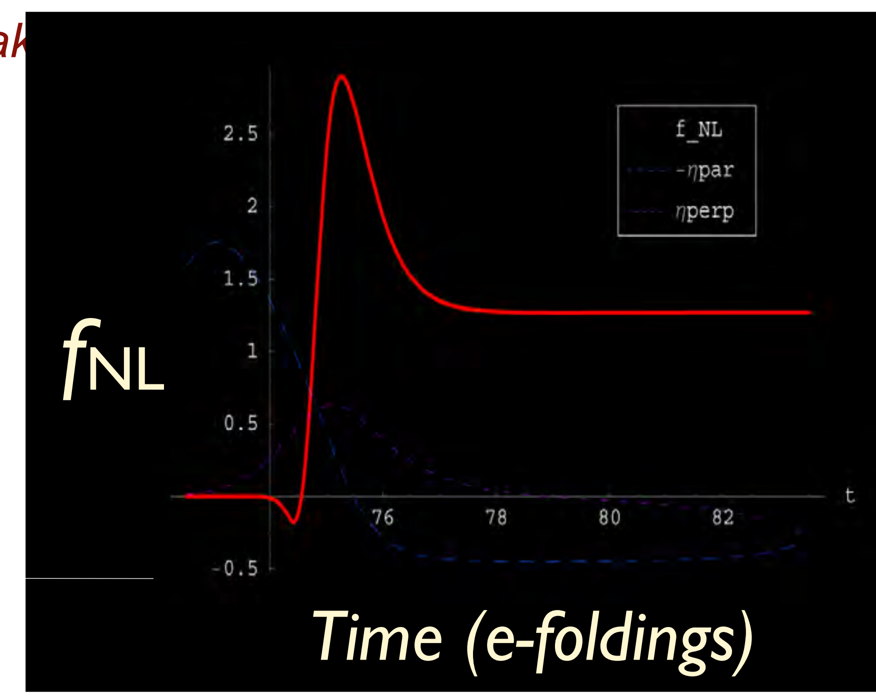
  Modulated reheating *e.g. Kofman et al 05; Dvali et al 06; etc*

  Nonlinear perturbations from preheating
  e.g. Chambers & Rajantie 07,08; Bond, Frolov, Huang & Kofman, 09.

- *Particle production during inflation*

  (incl. warm inflation) *Moss & Xiong, 07; Moss & Graham, 07.*

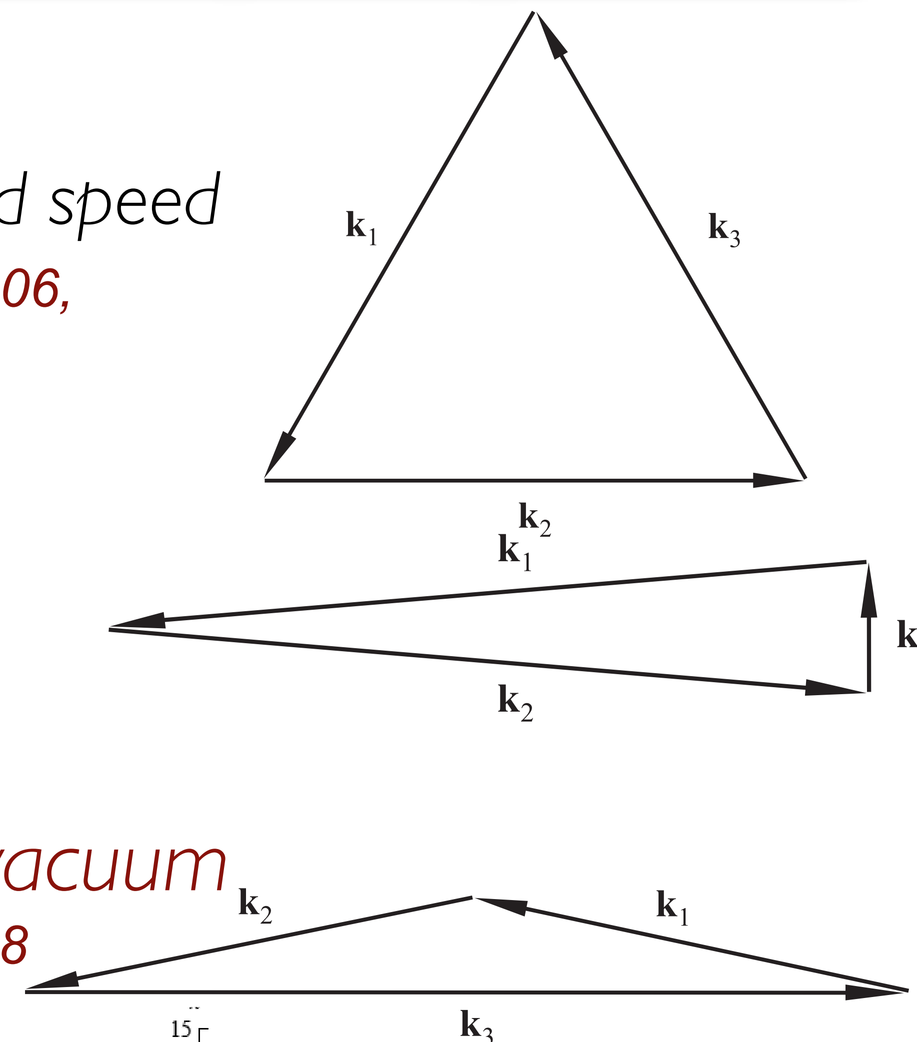
- *Scale-dependent bispectra* *e.g. Byrnes et al, 08; Liguori & Sefusatti et al, 09.*



Non-Gaussian Sources (cont.)

Non-Canonical Inflation

- *Single field: K-inflation, DBI inflation* - modified sound speed
  e.g. Silverstein & Tong 2003; Alishaha et al 2004; Chen et al 2006, Burrage et al, 2011 etc.
- *Multifield DBI inflation* *e.g. Chen,10; Renaux-Petel, 10.*
- *NG effects from Galileons* *e.g. Renaux-Petel, 10.*
- *Vector inflation (anisotropy), Modified gravity etc.*
  e.g. Shiraishi et al, 10, Bartolo et al 11 etc..

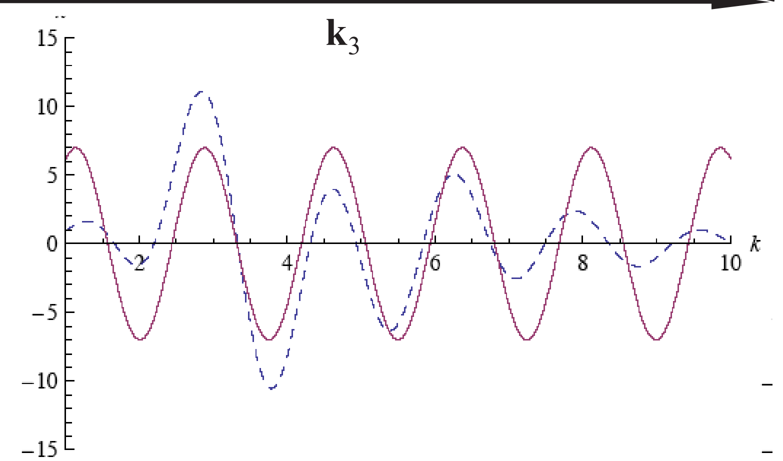


Excited initial states - non-Bunch-Davies vacuum

e.g. Chen, et al, 2006; Holman & Tolley, 2008; Meerburg et al 2008

Feature and periodic models

e.g. Chen, Easther & Lim, 2005; Meerburg, 2010; Westerval et al 2009
Interesting work on polyspectra correlations - Chen, 2011.



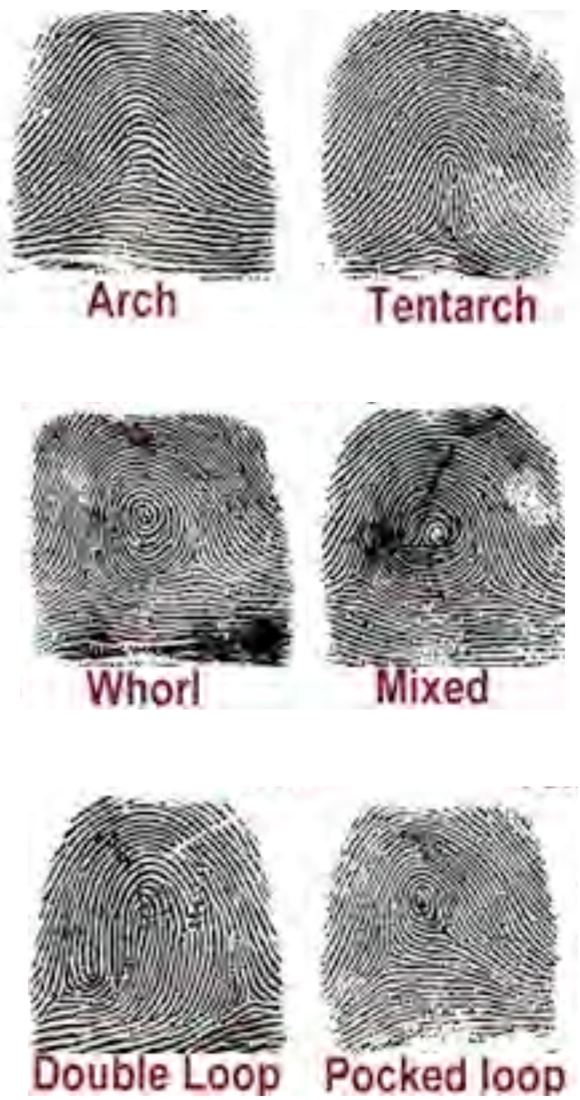
Alternative primordial scenarios -

e.g. cosmic superstrings, textures, ekpyrotic models etc

Secondary NGs - second-order Einstein-Boltzmann eqns, ISW etc.

Bartolo, Mataresse et al, 08; Pitrou & Bernardeau, 10; Pitrou, 11; Zichi et al 12; Su et al 12, etc;
Important contaminant at Planck sensitivity; dominant for LSS (see later).

Alternative models: Fingerprints of the very early Universe?

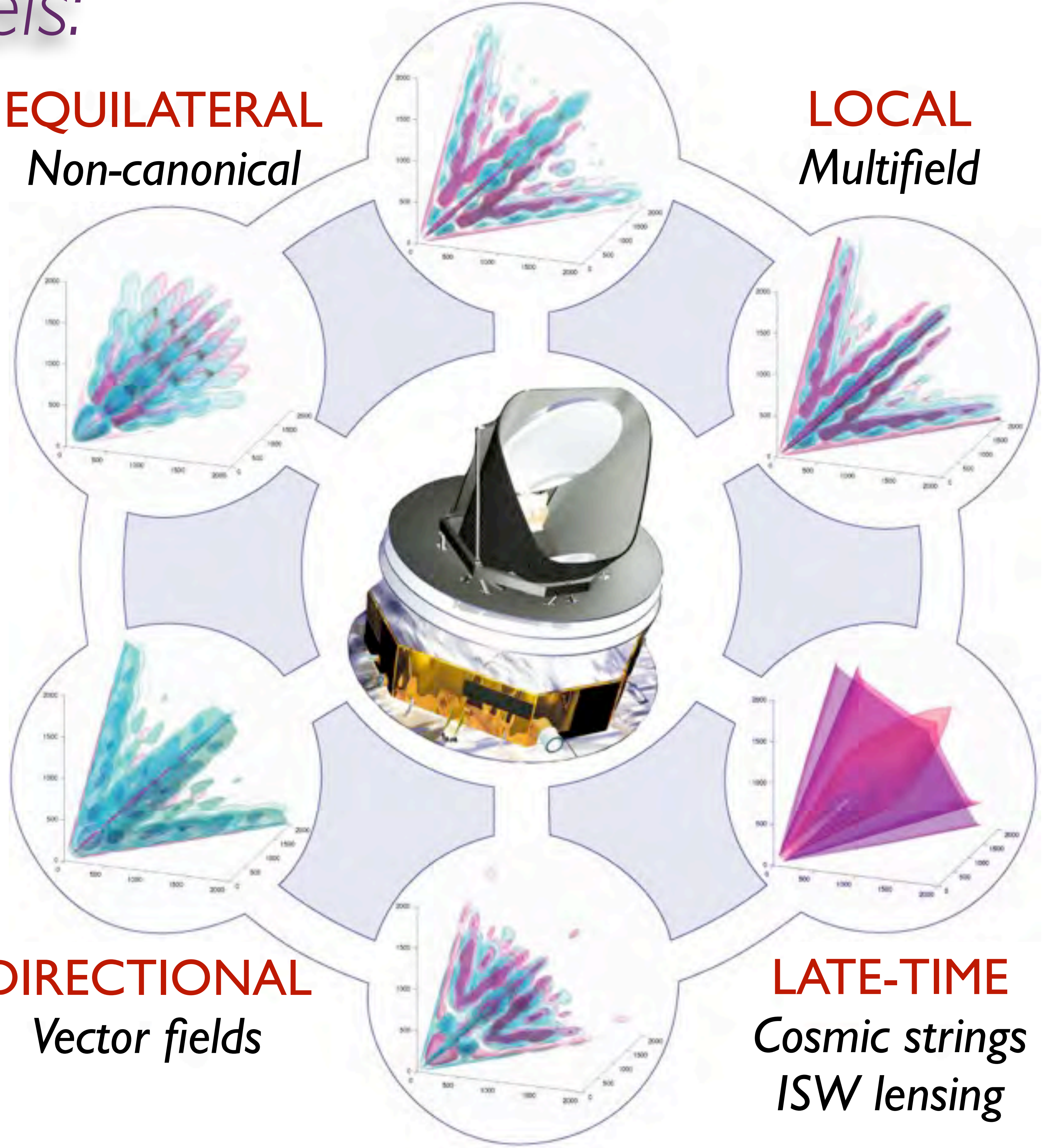


Fergusson and EPS, 2008

FLAT *Excited states*

EQUILATERAL *Non-canonical*

LOCAL *Multifield*



DIRECTIONAL *Vector fields*

LATE-TIME *Cosmic strings ISW lensing*

NON-SCALING *Oscillatory features*

Bispectrum estimator

Purpose: Test a model with predicted theoretical bispectrum

$$b^{\rm th}_{l_1 l_2 l_3} = \sum_{m_i} \mathcal{G}^{\,l_1\; l_2\; l_3}_{m_1 m_2 m_3} \langle a^{\rm th}_{l_1 m_1} a^{\rm th}_{l_2 m_2} a^{\rm th}_{l_3 m_3} \rangle$$

Estimator gives a least squares fit to the data

$$\begin{aligned}\mathcal{E} &= \frac{1}{N^2} \sum_{l_i, m_i} \langle a^{\rm th}_{l_1 m_1} a^{\rm th}_{l_2 m_2} a^{\rm th}_{l_3 m_3} \rangle (C^{-1}a)_{l_1 m_1} (C^{-1}a)_{l_2 m_2} (C^{-1}a)_{l_3 m_3} \\ &= \frac{1}{N^2} \sum_{l_i m_i} \frac{\mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3}\, b^{\rm th}_{l_1 l_2 l_3}\; a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}\end{aligned}$$

Model

Signal

Noise

Babich, 2005; see also KSW etc

with covariance matrix $\quad C_{lm,l'm'} = \langle a_{lm} a_{l'm'} \rangle$

with inverse weighting $\quad (C^{-1}a)_{lm} = C^{-1}_{lm,l'm'} a_{l'm'} \approx \dfrac{a_{lm}}{C_l}$ (ideal case)

(Neglected discussion of 'linear term', incorporating systematic effects.)

Separable Q modes

Need complete set of separable eigenmodes spanning tetrapyd

Define **separable basis functions Q_n** *Fergusson, Liguori and EPS, 2009, 2010*

$$\begin{aligned}\overline{\mathcal{Q}}_n(l_1,l_2,l_3) &= \tfrac{1}{6}[\bar{q}_p(l_1)\,\bar{q}_r(l_2)\,\bar{q}_s(l_3) + \bar{q}_r(l_1)\,\bar{q}_p(l_2)\,\bar{q}_s(l_3) + \text{cyclic perms in } prs] \\ &\equiv \bar{q}_{\{p}q_r q_{s\}} \quad \text{with} \quad n \leftrightarrow \{prs\}\,,\end{aligned}$$

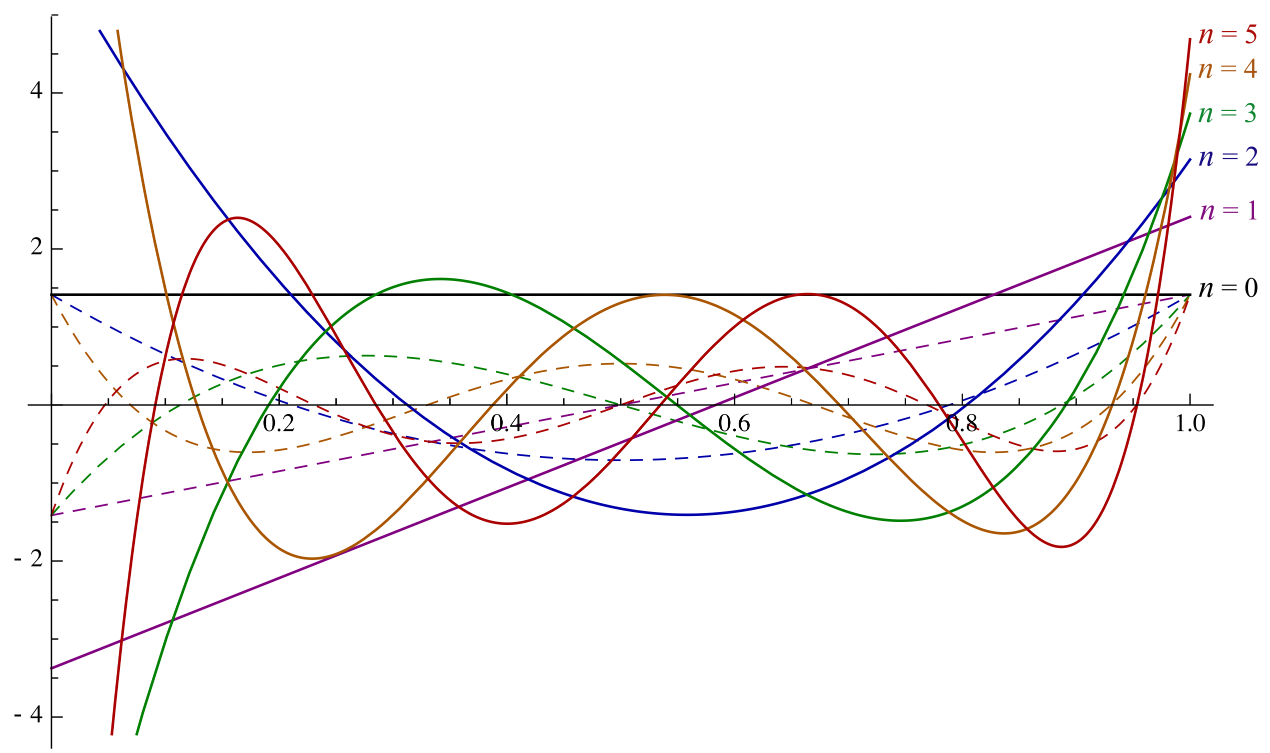
where $q_p(l)$ can be like Legendre polynomials (or trigonometric etc.)

Inner product:

Defined by estimator sum

$$\langle b,\, b'\rangle \;\equiv\; \sum_{l_1,l_2,l_3\in\mathcal{V}_\mathcal{T}} w_{l_1l_2l_3}\, b_{l_1l_2l_3}\, b'_{l_1l_2l_3}$$

with weight $w_{l_1l_2l_3} = h^2_{l_1l_2l_3}$



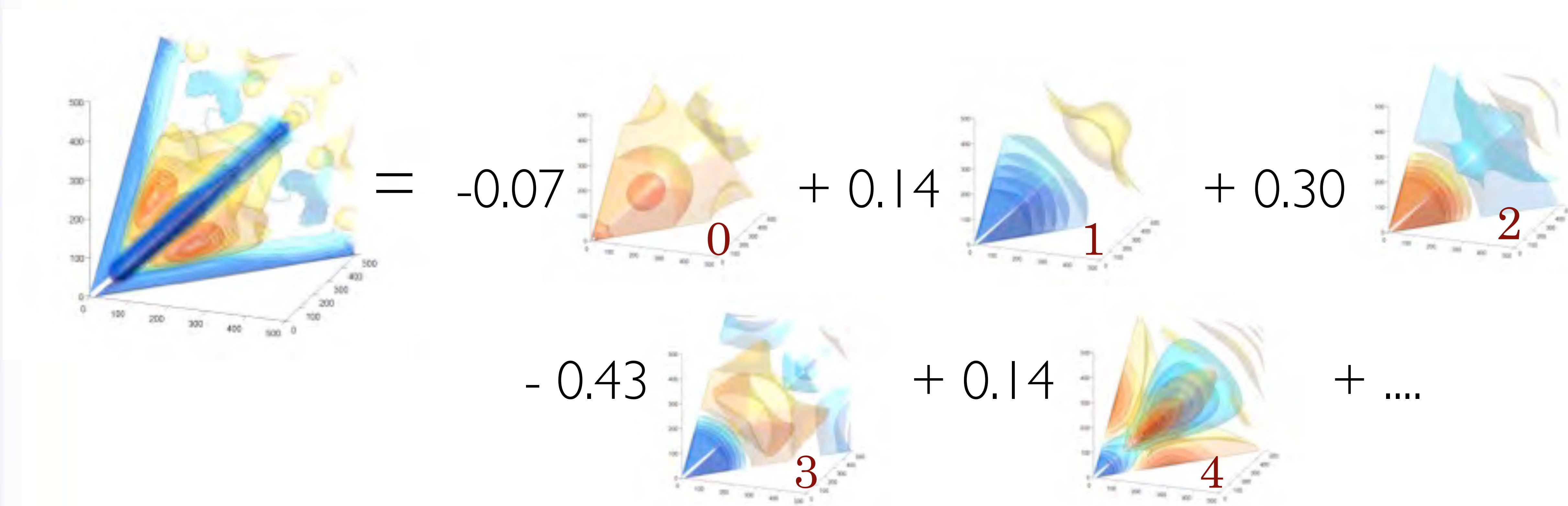
Note: $\langle \overline{\mathcal{Q}}_n,\, \overline{\mathcal{Q}}_p\rangle \equiv \gamma_{np} \neq \delta_{np}$ not orthogonal

$B_{l_1 l_2 l_3}$ reconstruction

Expand any (nonseparable) bispectrum signal strength in modes as

$$\frac{v_{l_1} v_{l_2} v_{l_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}}\, b_{l_1 l_2 l_3} = \sum_n \bar{\alpha}_n^{\mathcal{R}} \overline{\mathcal{R}}_n$$

E.g. Local f_{NL} Model expansion for the a_n coefficients:



Fergusson, Liguori and EPS, 2009

CMB modal decomposition

$$\mathcal{E} = \sum_{l_i,m_i}\sum_{n\leftrightarrow prs}\bar{\alpha}^{\mathcal{Q}}_n \bar{q}_{\{p}\bar{q}_r\bar{q}_{s\}}\int d^2\hat{\mathbf{n}}\, Y_{l_2m_2}(\hat{\mathbf{n}})Y_{l_1m_1}(\hat{\mathbf{n}})\,Y_{l_3m_3}(\hat{\mathbf{n}})\,\frac{a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}}{v_{l_1}v_{l_2}v_{l_3}\sqrt{C_{l_1}C_{l_2}C_{l_3}}}$$

$$= \sum_{n\leftrightarrow prs}\bar{\alpha}^{\mathcal{Q}}_n\int d^2\hat{\mathbf{n}}\left(\sum_{l_1,m_1}\bar{q}_{\{p}\frac{a_{l_1m_1}Y_{l_1m_1}}{v_{l_1}\sqrt{C_{l_1}}}\right)\left(\sum_{l_2,m_2}\bar{q}_r\frac{a_{l_2m_2}Y_{l_2m_2}}{v_{l_2}\sqrt{C_{l_2}}}\right)\left(\sum_{l_3,m_3}\bar{q}_{s\}}\frac{a_{l_3m_3}Y_{l_3m_3}}{v_{l_3}\sqrt{C_{l_3}}}\right)$$

$$\bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} q_p(l)\frac{a_{lm}}{v_l\sqrt{C_l}}Y_{lm}(\hat{\mathbf{n}})$$

$$\bar{\mathcal{M}}_n(\hat{\mathbf{n}}) = \bar{M}_p(\hat{\mathbf{n}})\bar{M}_r(\hat{\mathbf{n}})\bar{M}_s(\hat{\mathbf{n}})$$

$$\beta_n = \int d^2\hat{\mathbf{n}}\mathcal{M}_n(\hat{\mathbf{n}})$$

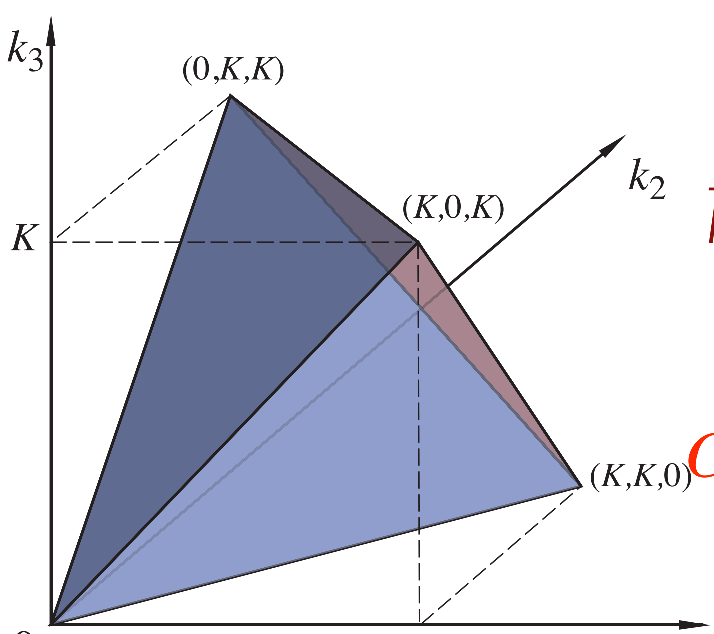
$$\boxed{\mathcal{E} = \frac{1}{N}\sum_{n=0}^{n_{\max}}\bar{\alpha}^{\mathcal{Q}}_n\bar{\beta}^{\mathcal{Q}}_n}$$

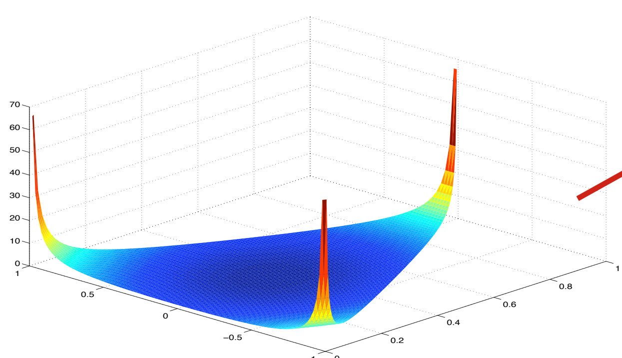
Now the projection is in alpha rather than beta

Fergusson, Liguori and EPS, 2009, 2010; see also KSW

Modal Polyspectra Estimation

<u>THEORY</u>
Primordial bispectra
(k-space)



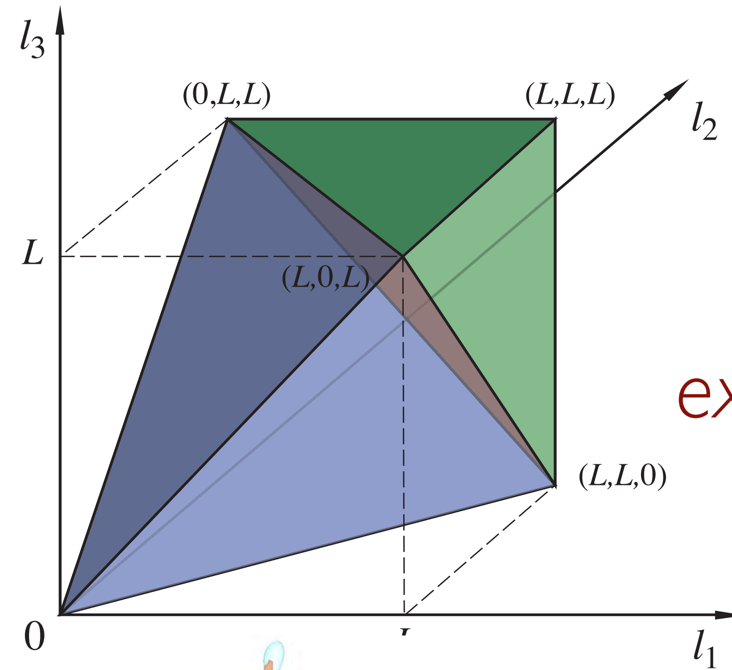


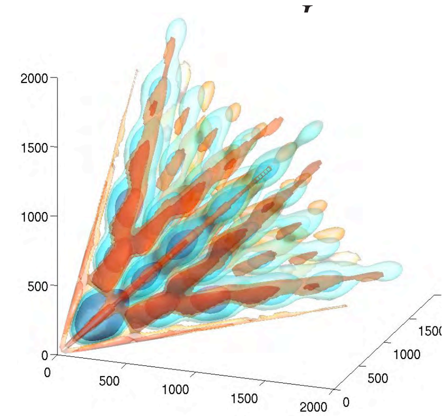
Expand <u>any</u> model with primordial modes α_n

Mode transfer functions

$\alpha_n \rightarrow \bar{\alpha}_n$

<u>CMB bispectra</u>
(l-space)

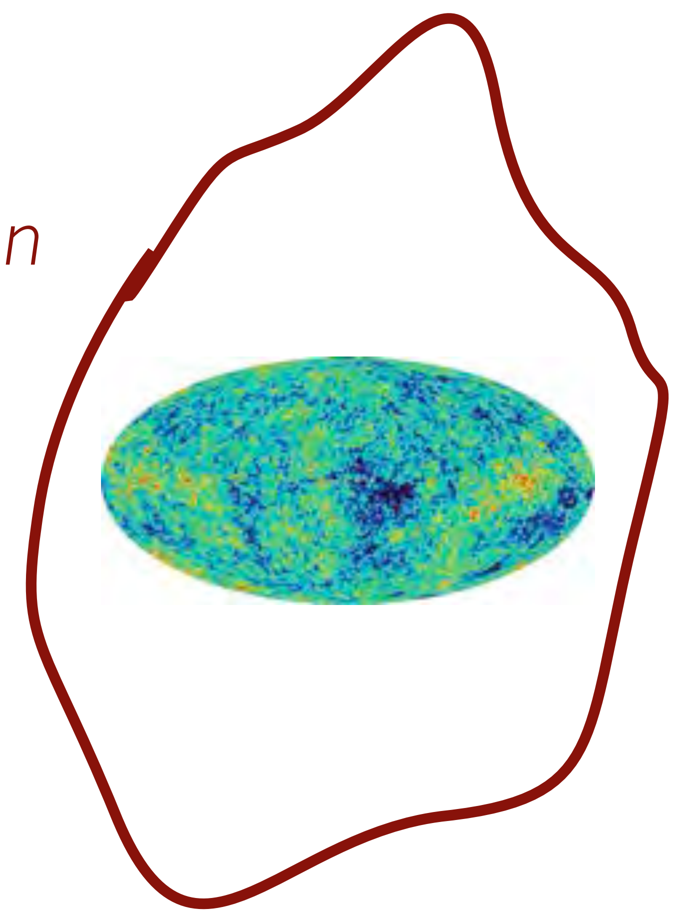




Map extraction

$\bar{\beta}$

<u>OBSERVATION</u>
CMB map



Modal estimator

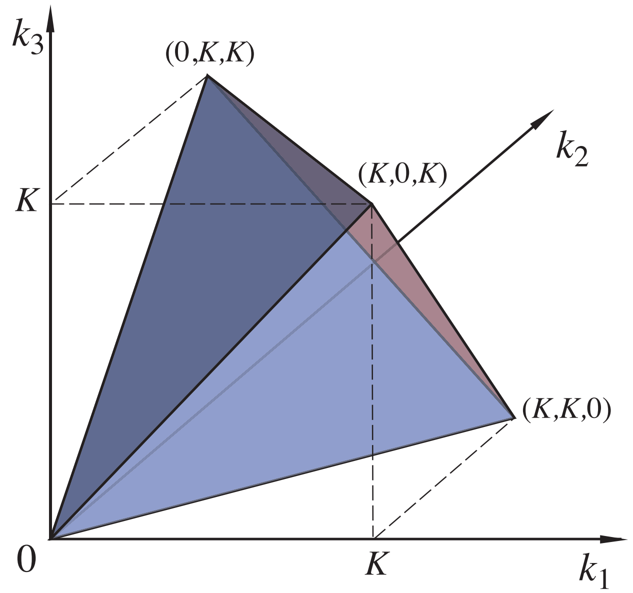
$$\mathcal{E} = \frac{\sum_n \bar{\alpha}_n^R \bar{\beta}_n^R}{\sum_n (\bar{\alpha}_n^R)^2}$$

Fergusson, Liguori and EPS, 2009

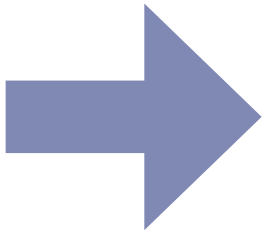
Filter with sufficient separable eigenmodes

Primordial to CMB basis

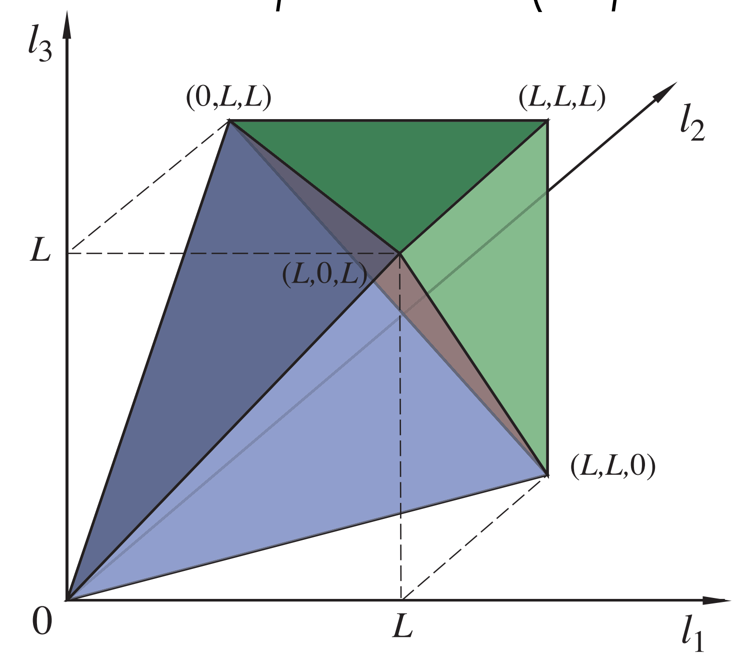
Primoridial bispectrum (k-space)



Transfer functions
$\Delta_l(k)$



CMB bispectrum (l-space)



Use transfer functions <u>*once*</u> to project forward primordial modes so we calculate

$$\Gamma_{nm} = \left\langle \bar{Q}^n \frac{vvv\tilde{Q}^m}{\sqrt{CCC}} \right\rangle$$

Then we can transform between the primordial and CMB expansions

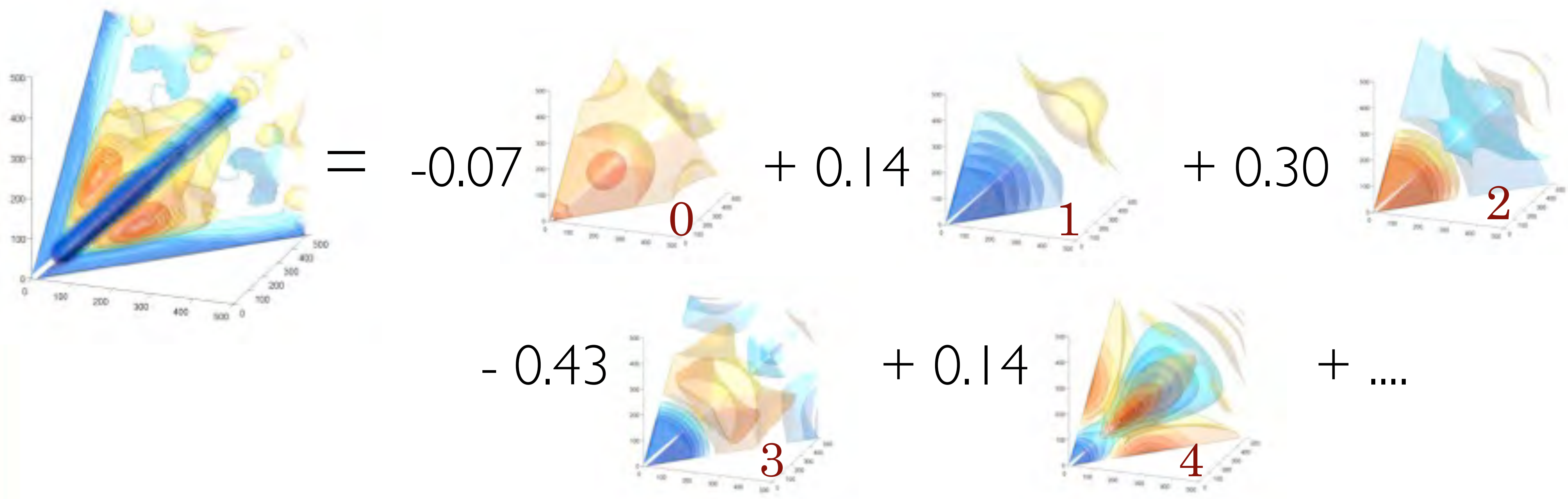
$$\bar{\alpha}^Q = \bar{\gamma}^{-1} \Gamma \alpha^Q$$

$B_{l_1 l_2 l_3}$ reconstruction

Expand any (nonseparable) bispectrum signal strength in modes as

$$\frac{v_{l_1} v_{l_2} v_{l_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}} b_{l_1 l_2 l_3} = \sum_n \bar{\alpha}_n^{\mathcal{R}} \overline{\mathcal{R}}_n$$

E.g. Local f_{NL} Model expansion:

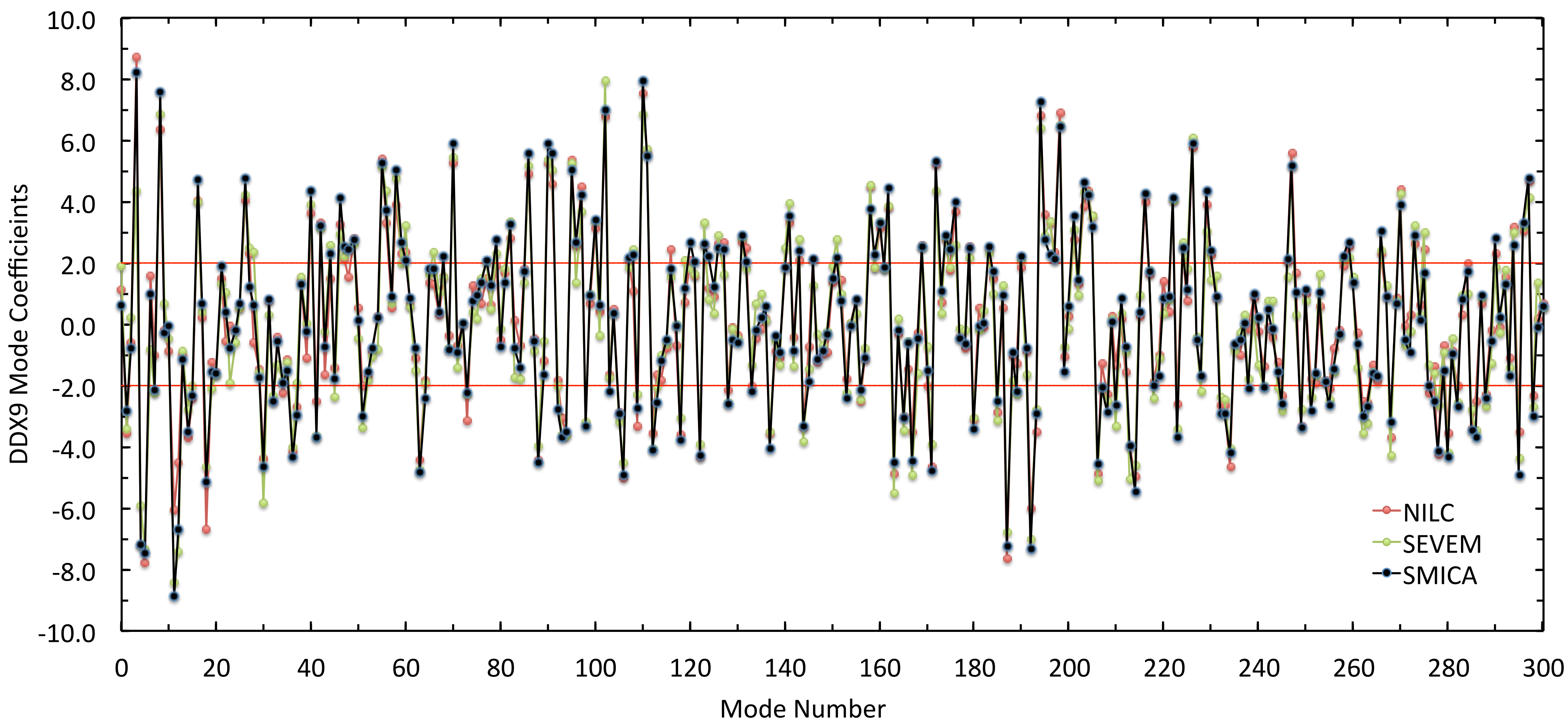


OR filter the Planck data with these modes and *reconstruct* bispectrum

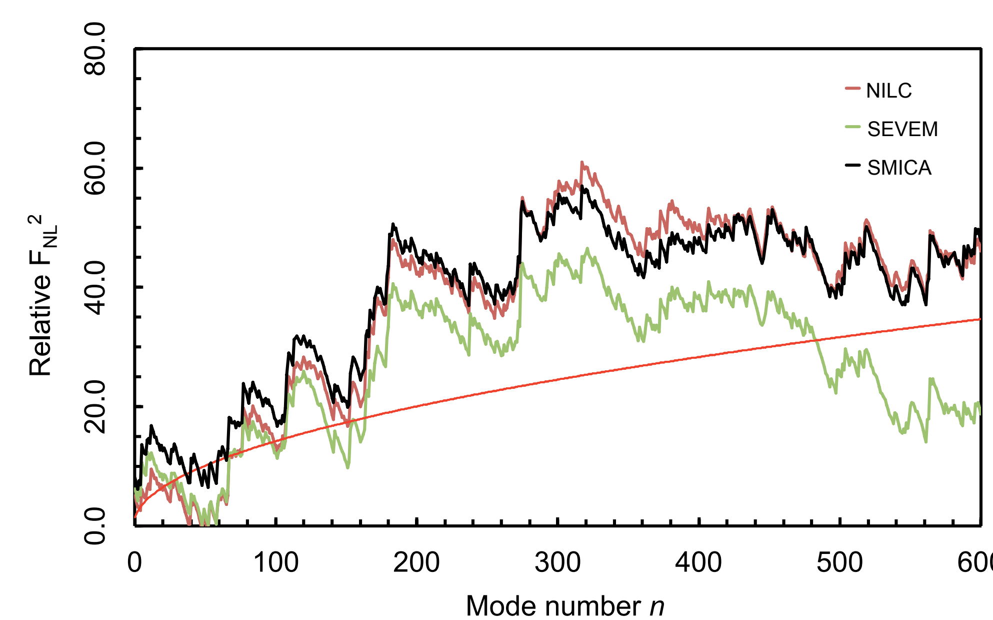
Fergusson, Liguori and EPS, 2009

Bispectrum reconstruction modes

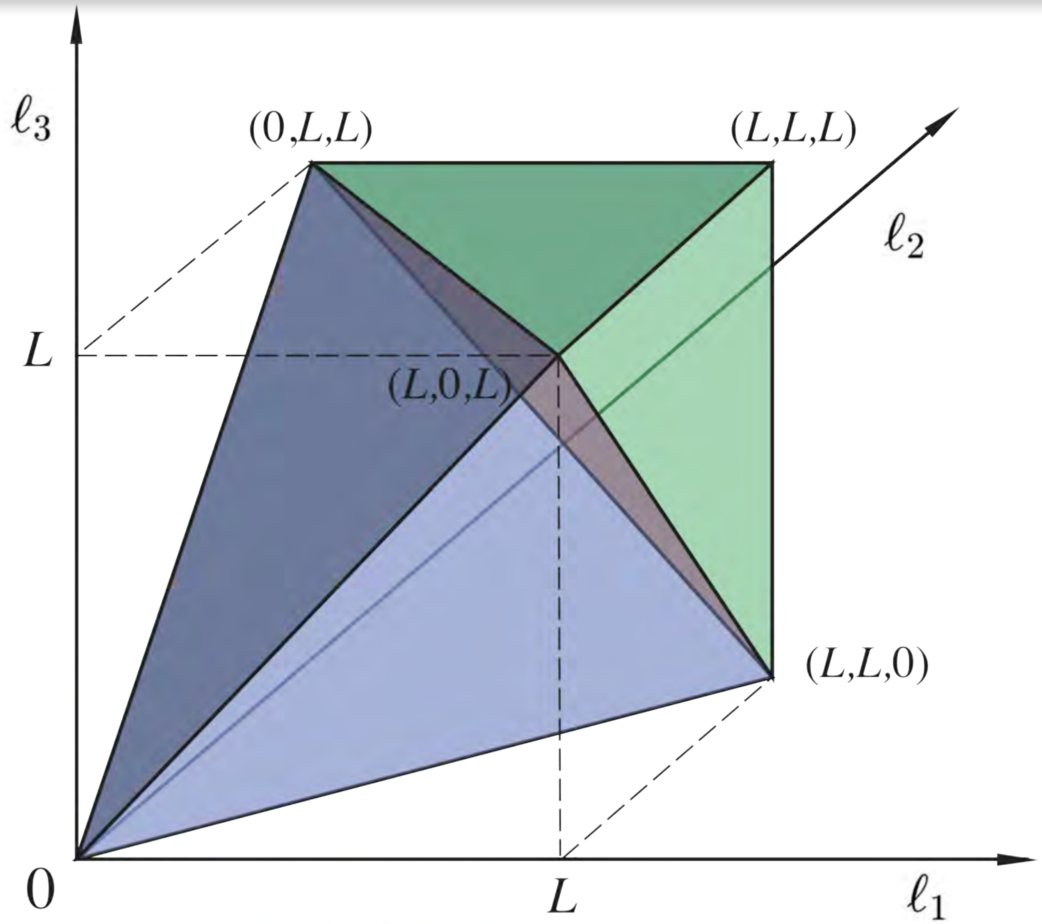
Reconstructed α_n modes from filtering Planck data

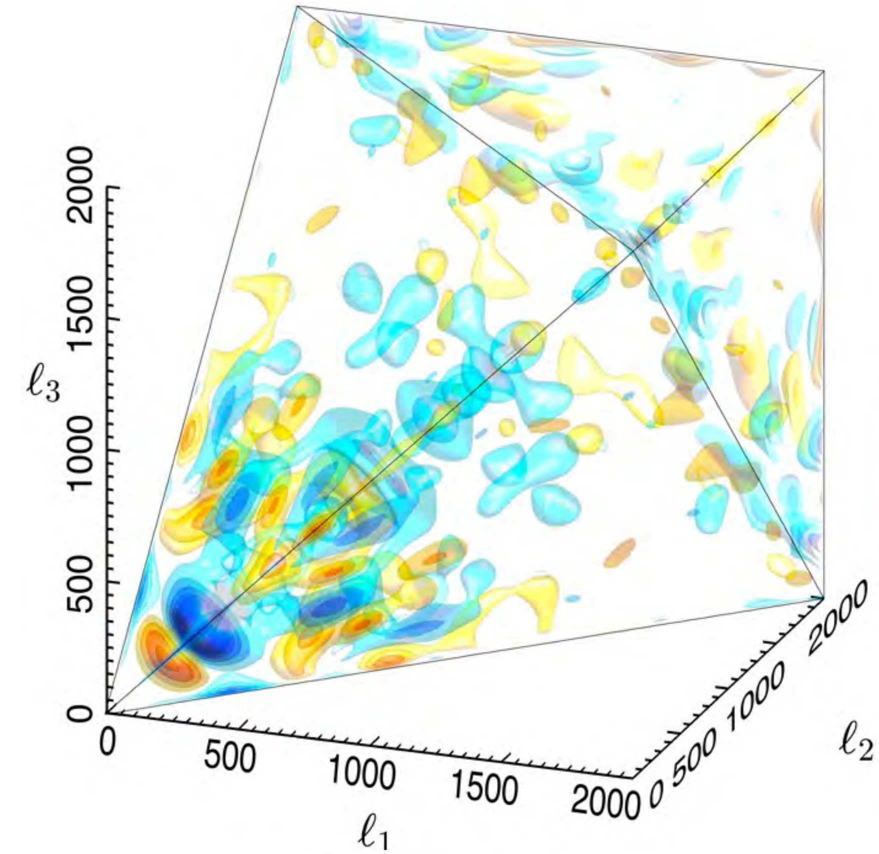


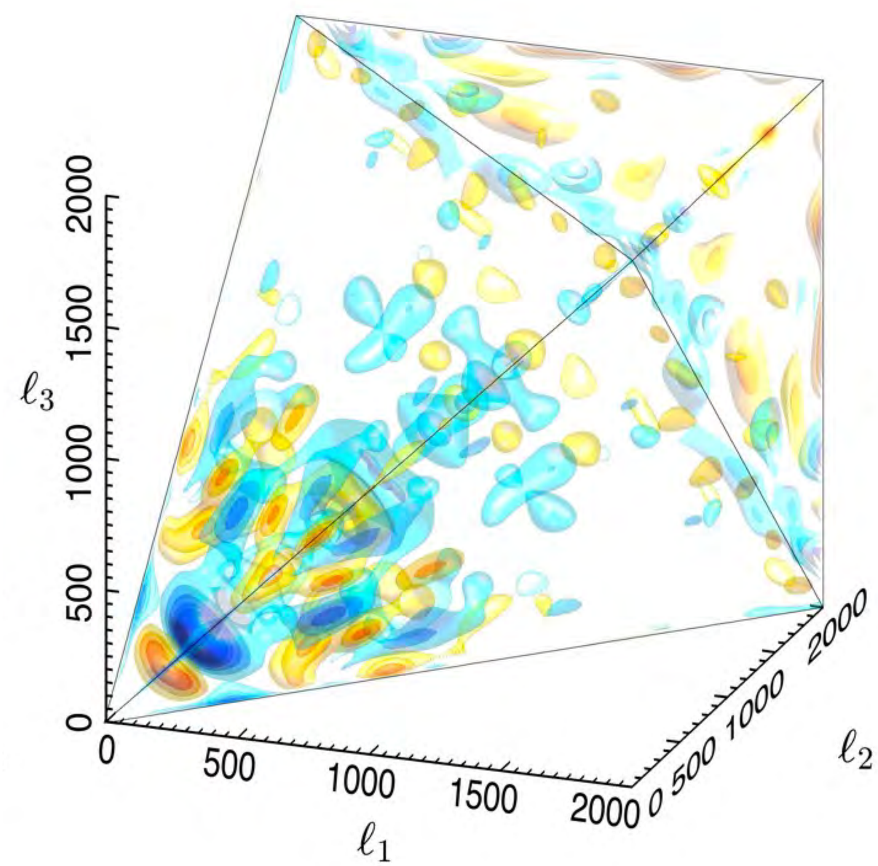
High signal from comparison with 200 Gaussian maps (χ^2-tests see later)

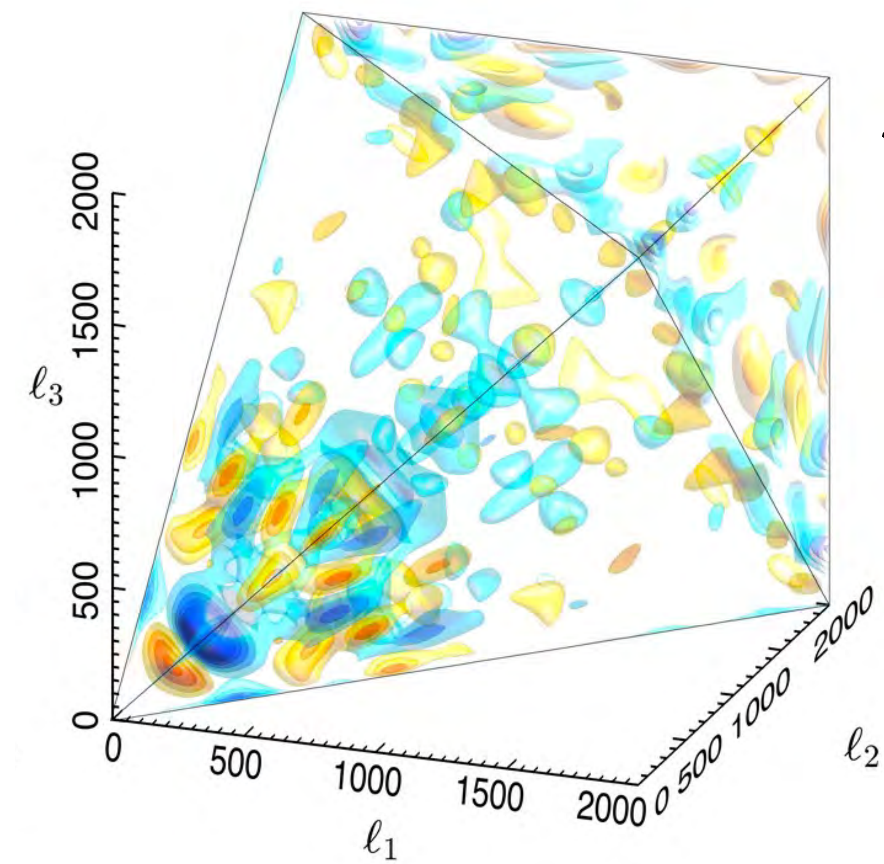


Planck Bispectrum Reconstruction



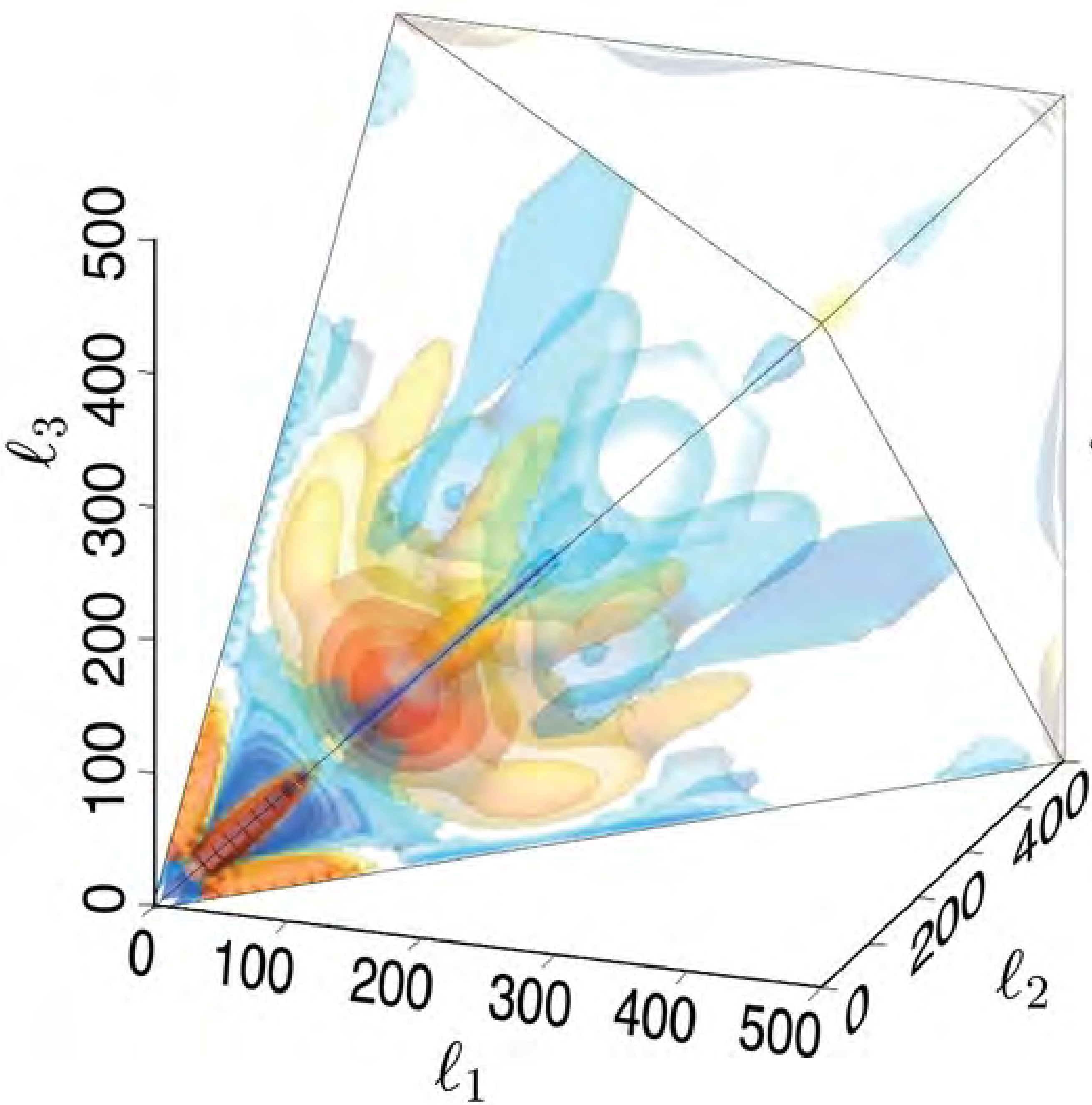




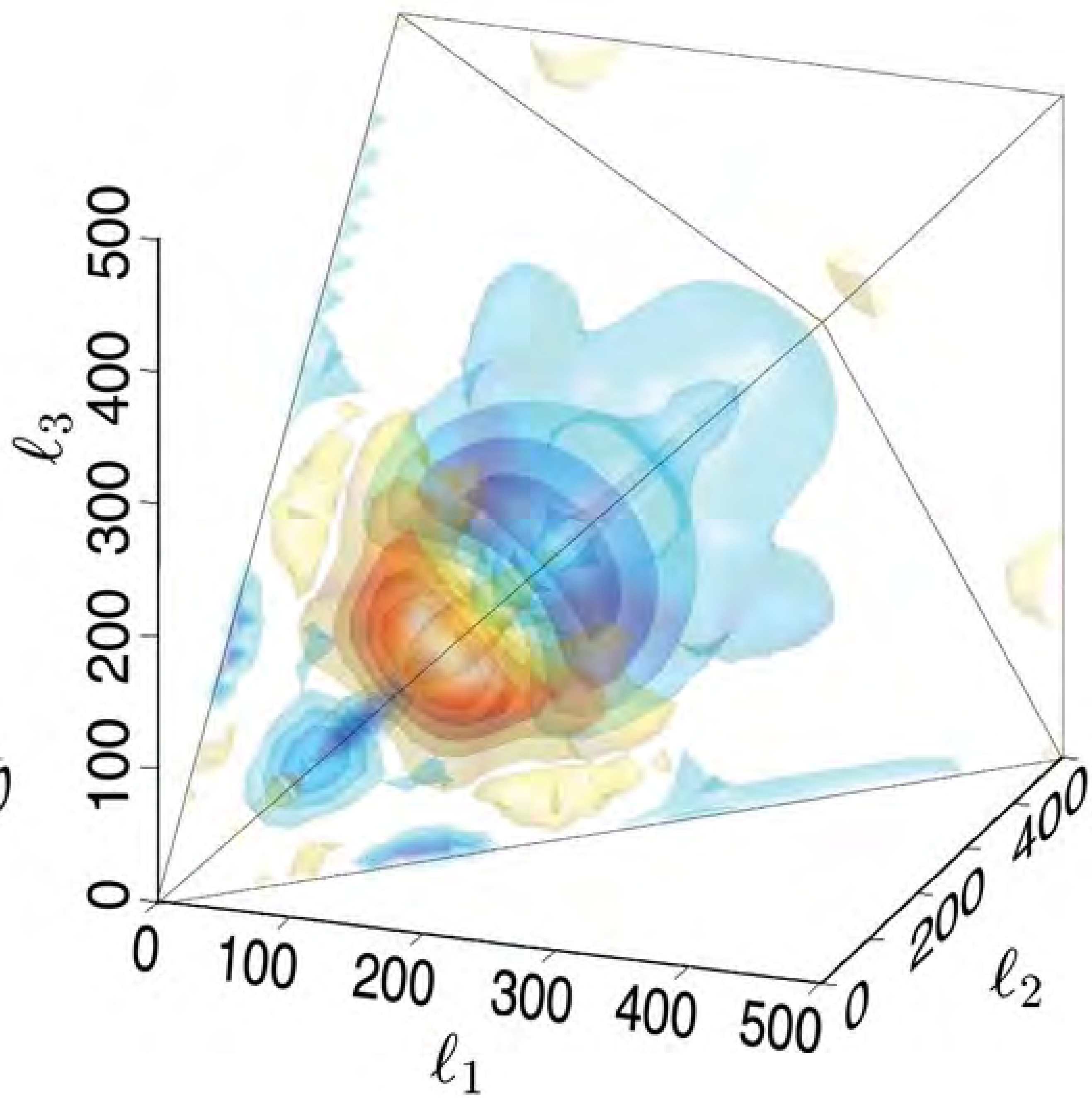


WMAP vs Planck

WMAP 7 year



Planck SMICA

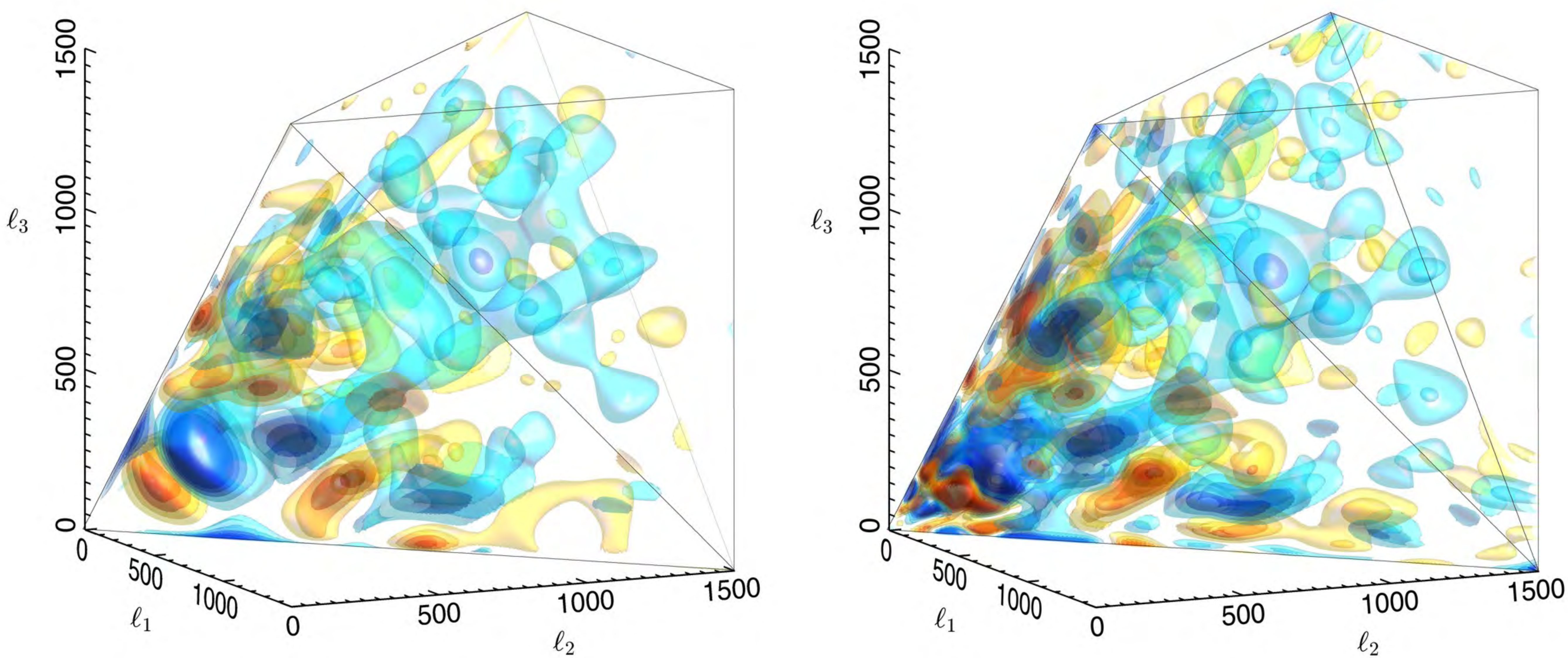


Fergusson, Liguori and EPS, 2010

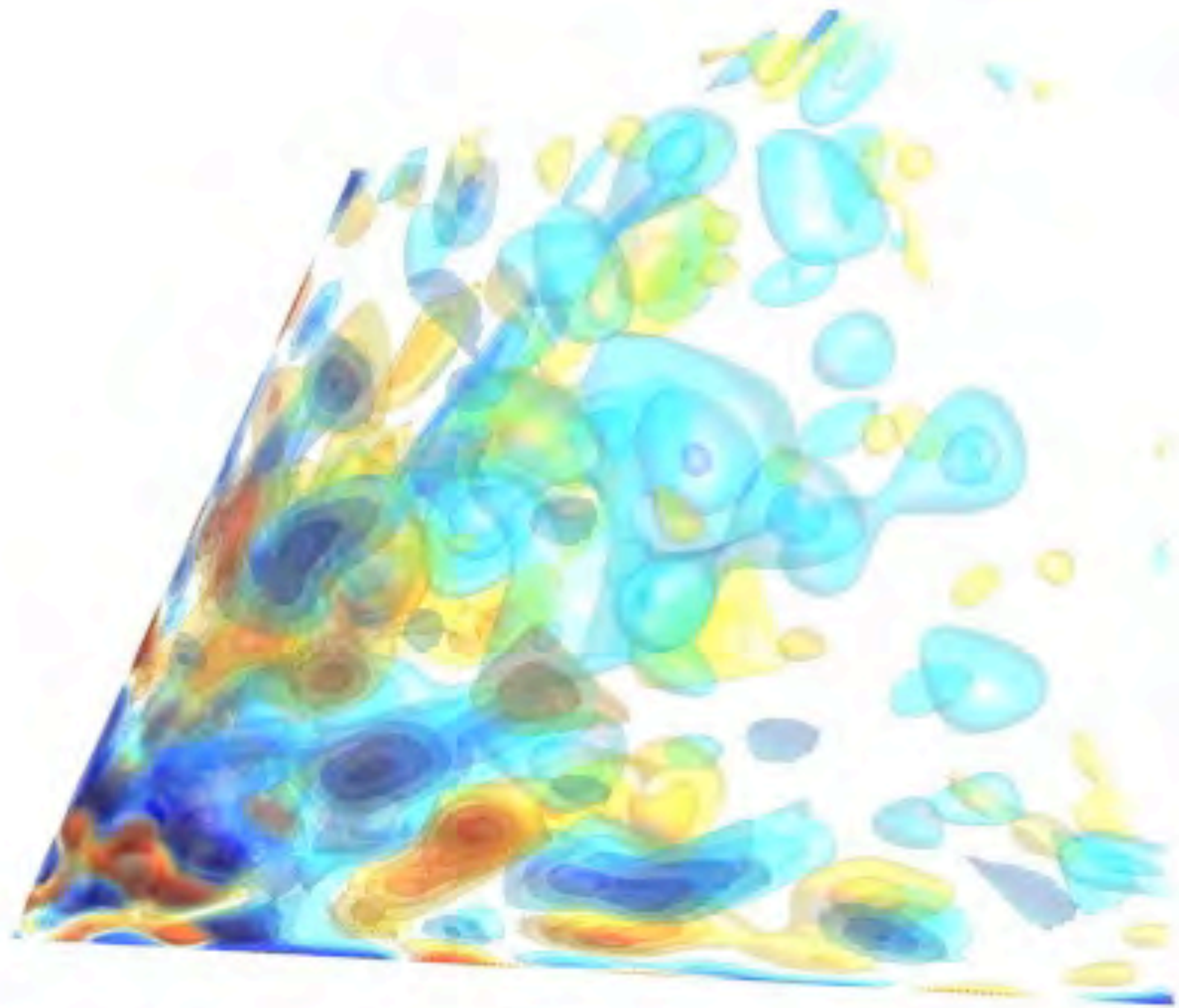
Paper XXIII. Isotropy and statistics of the CMB

The Planck Bispectrum

Modal reconstruction of the full 3D Planck bispectrum



Fourier modes vs Polynomials

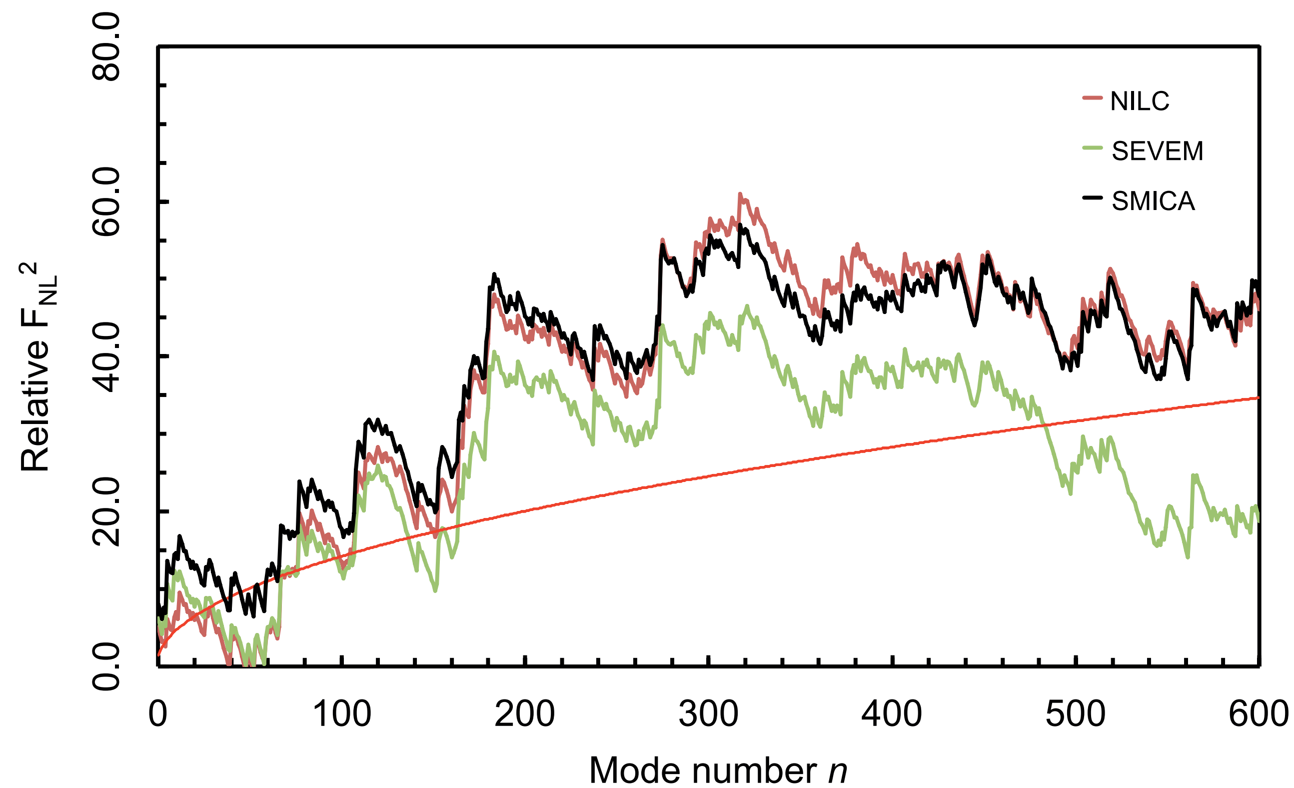


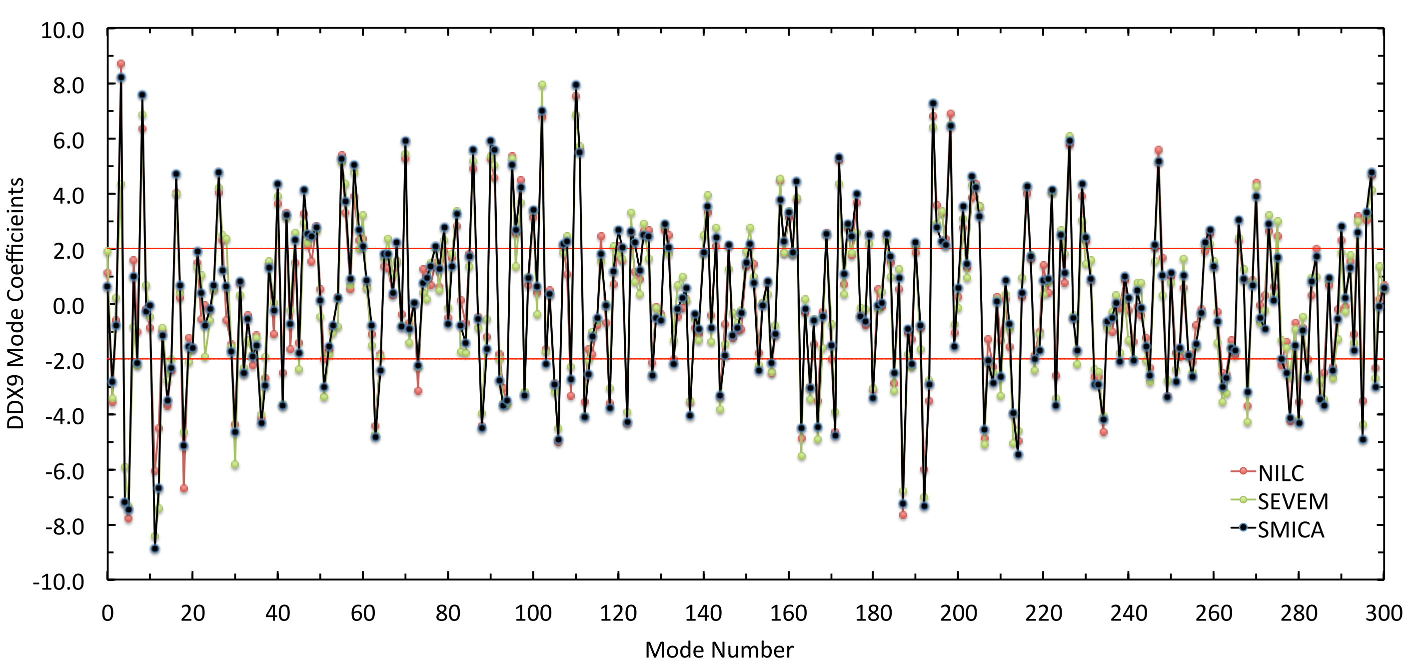
Modal FLS Bispectrum Reconstruction (Planck Collaboration 2013)

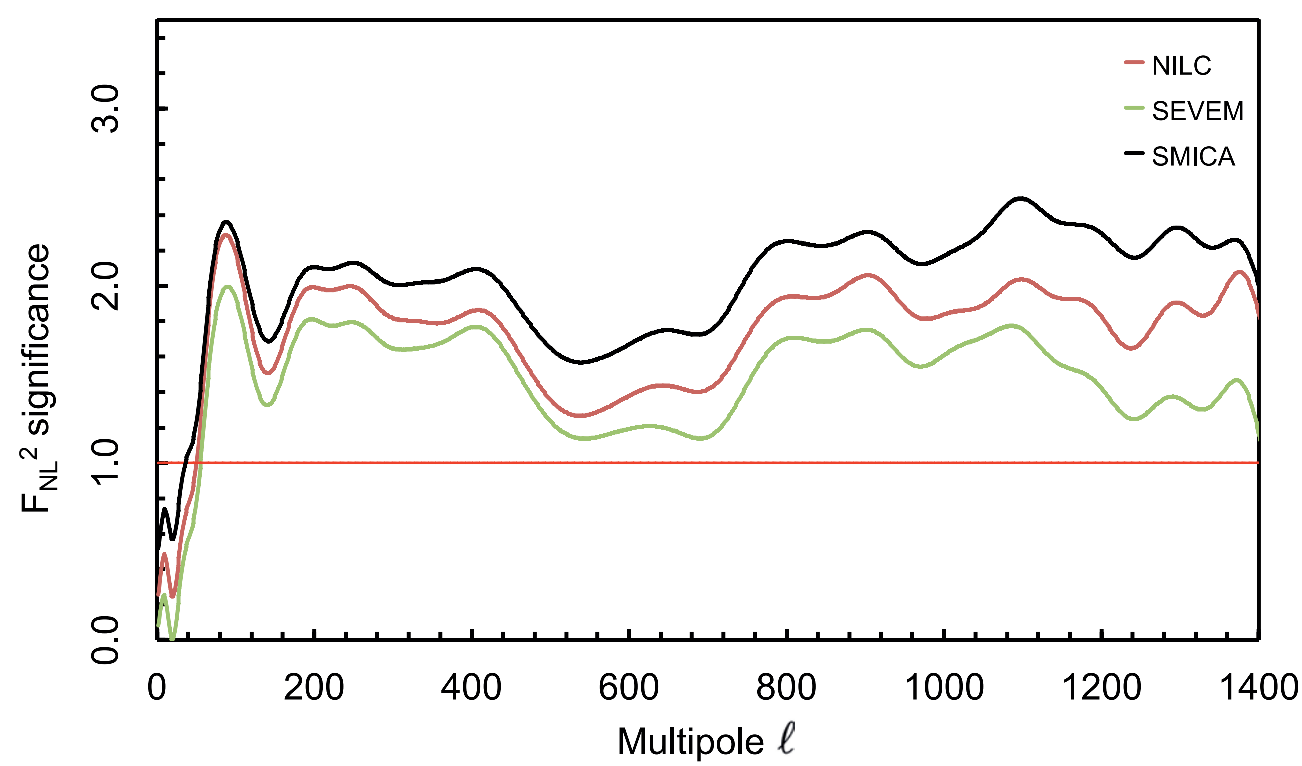
High bispectrum signal

χ^2-tests for integrated bispectrum consistent with Gaussianity, but signal always high.

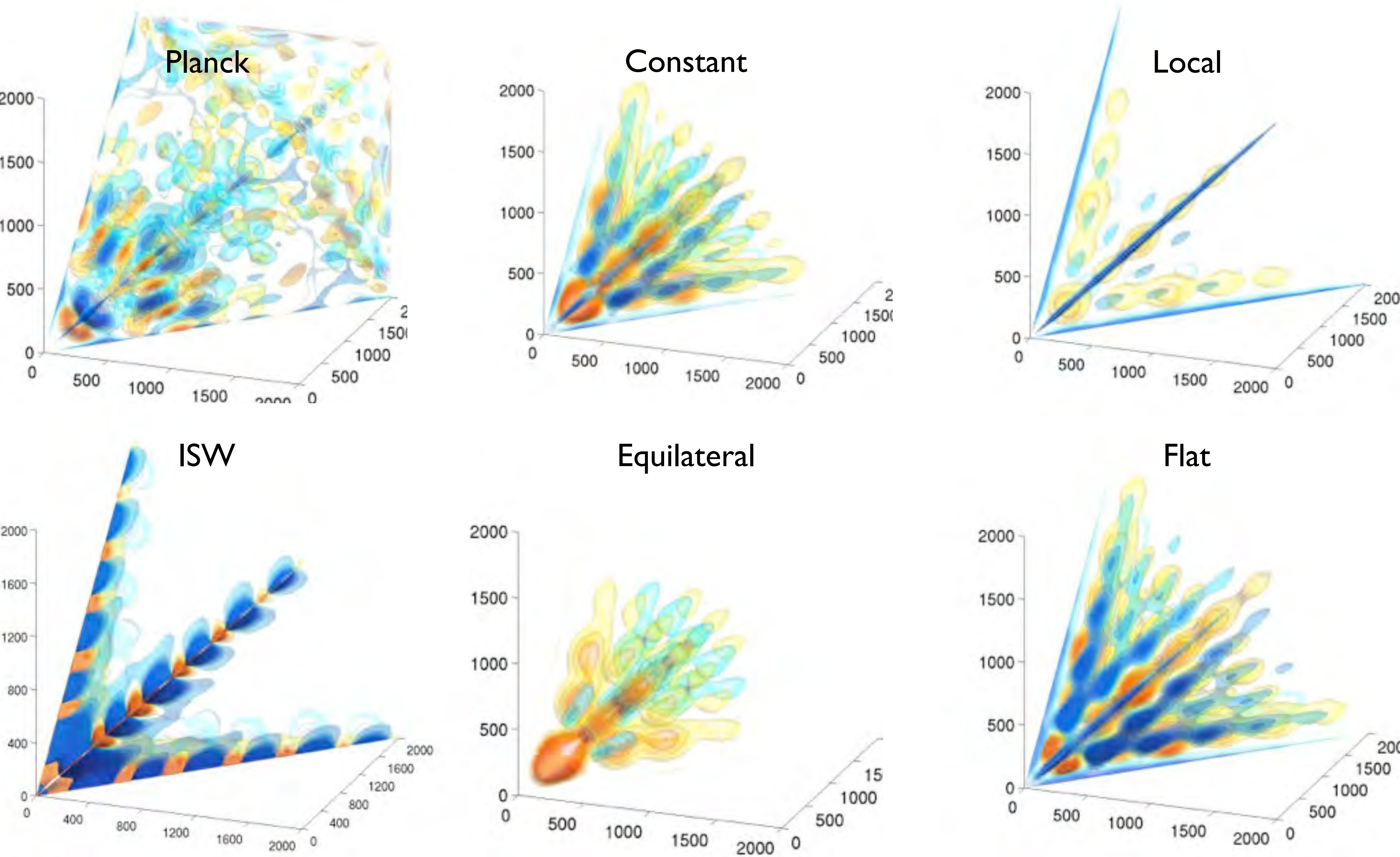
Comparison with 200 lensed CMB Gaussian maps with Planck noise.



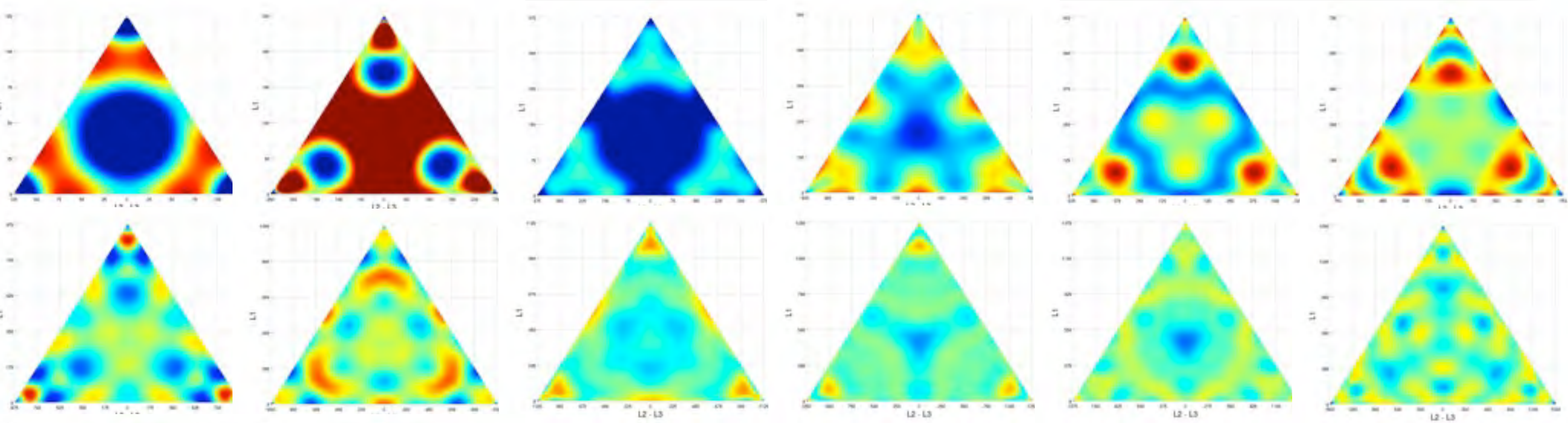


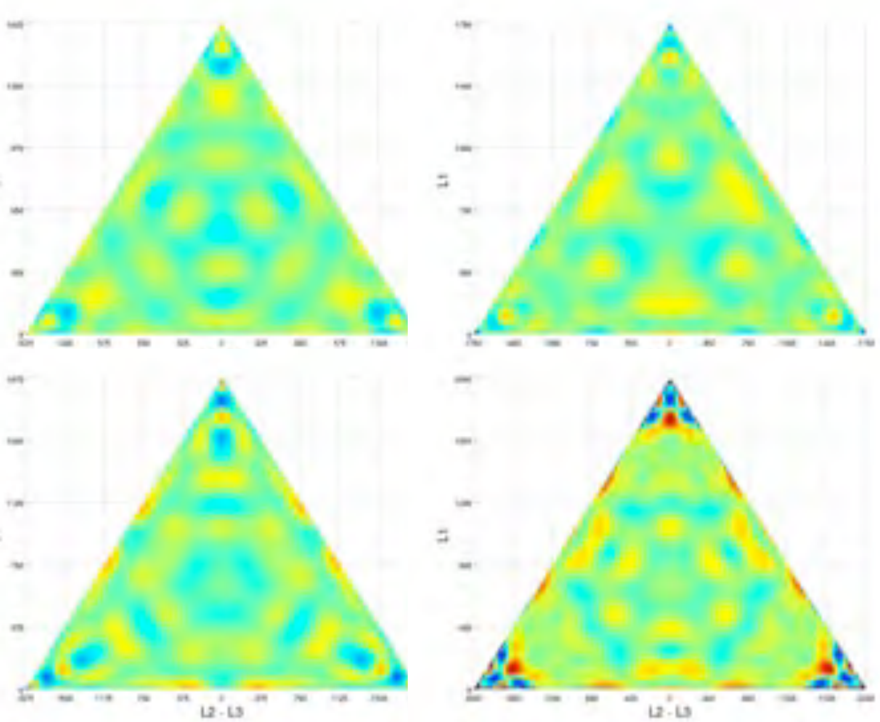


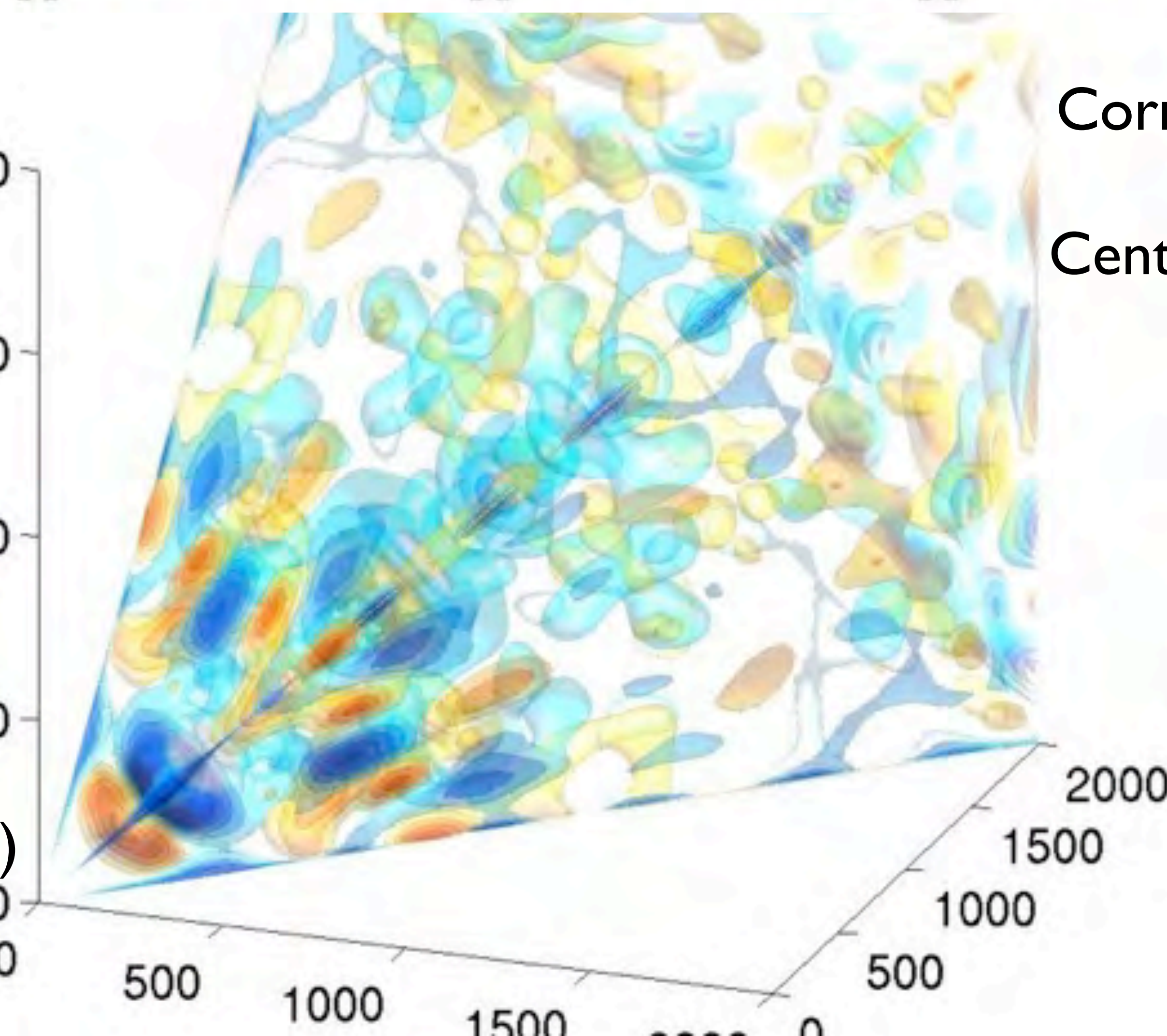
Bispectrum in detail



Bispectrum in detail





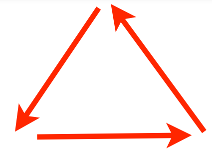


Corner=Squeezed
Edge=Flat
Centre=Equilateral

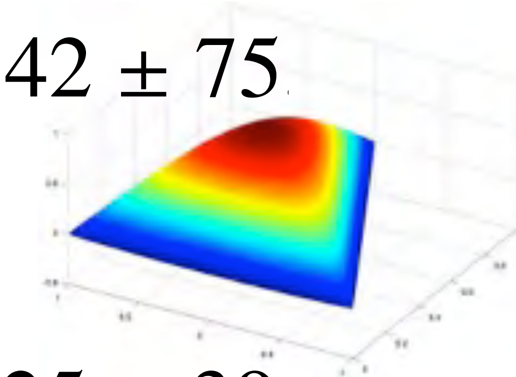
Cross sections where L1+L2+L3=const
Vertical axis is L1 (increasing downwards)
Horizontal is L2-L3

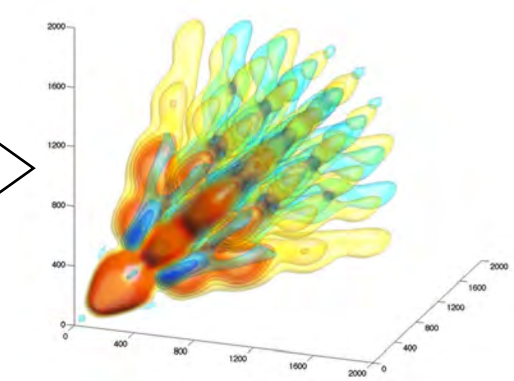
Standard Bispectra

Equilateral bispectra



$$f_{NL}^{equil} = -42 \pm 75$$

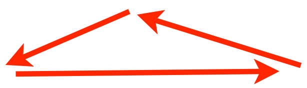




Inflation from higher dimensions

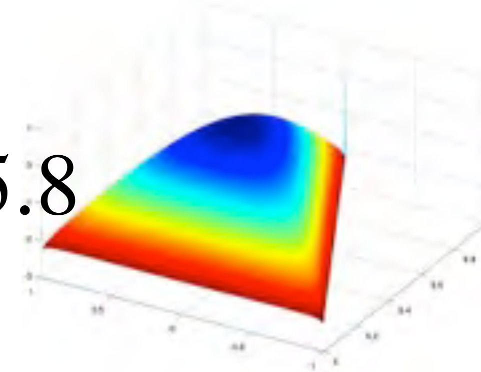
Single-field - sound speed $c_s << c$

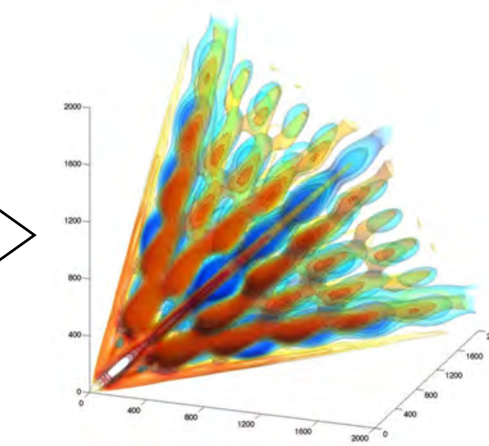
Orthogonal bispectra



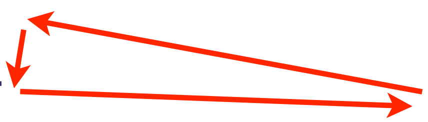
$$f_{NL}^{ortho} = -25 \pm 39$$

Single-field, complement of equilateral





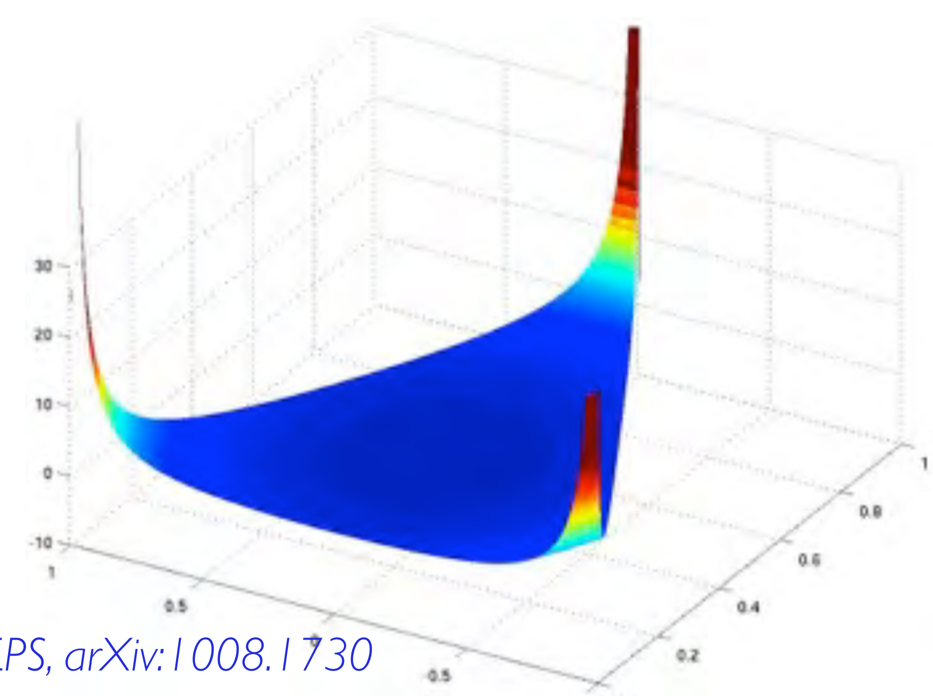
Local bispectra

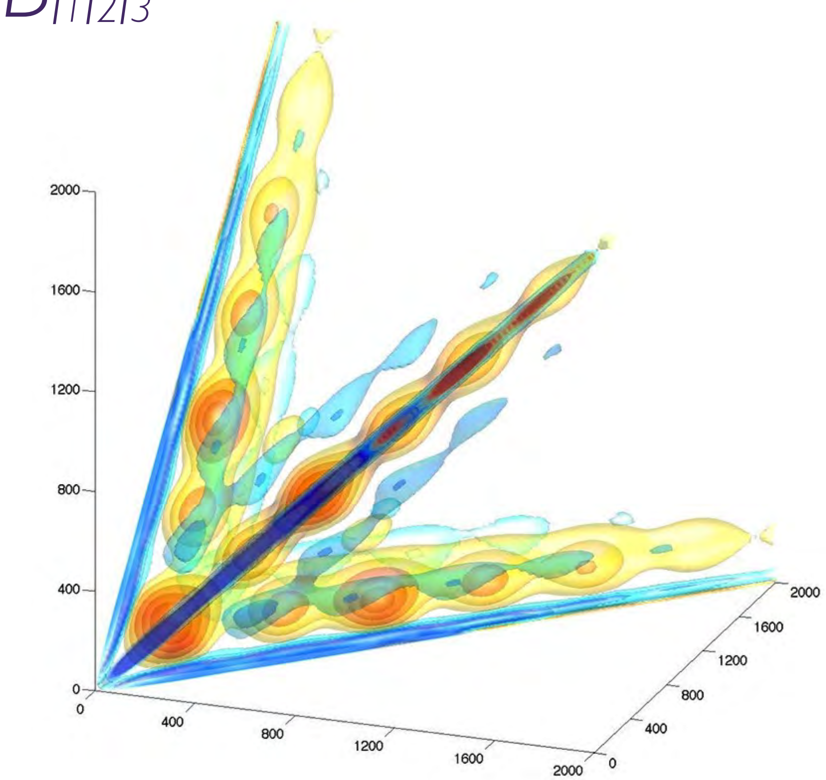


$$f_{NL}^{local} = 2.7 \pm 5.8$$

Multifield inflation, curvaton etc.

Primordial $B(k_1,k_2,k_3)$ CMB $B_{l_1 l_2 l_3}$





Fergusson & EPS, arXiv:1008.1730

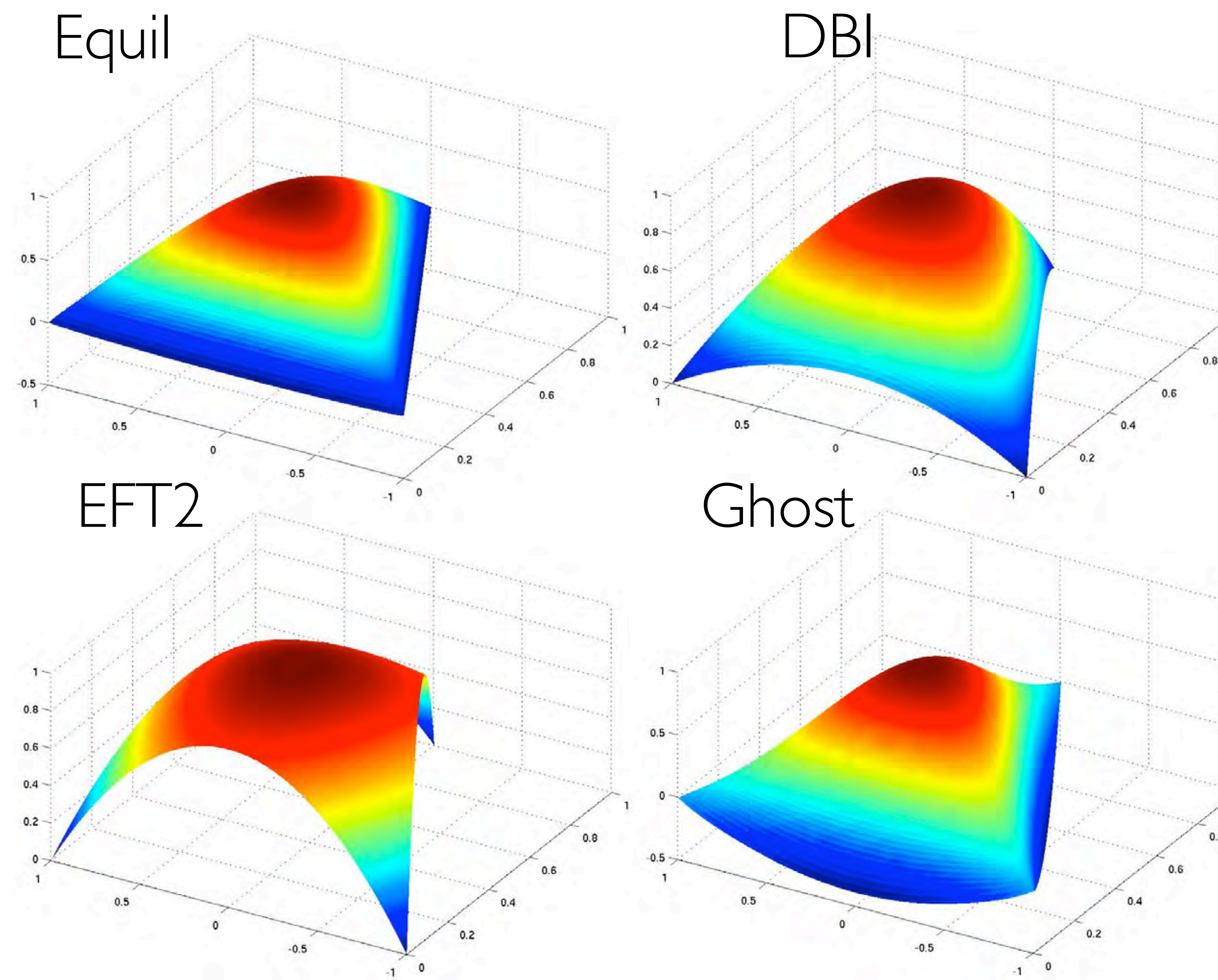
Non-separable bispectra

Specific *key* single-field models constrained

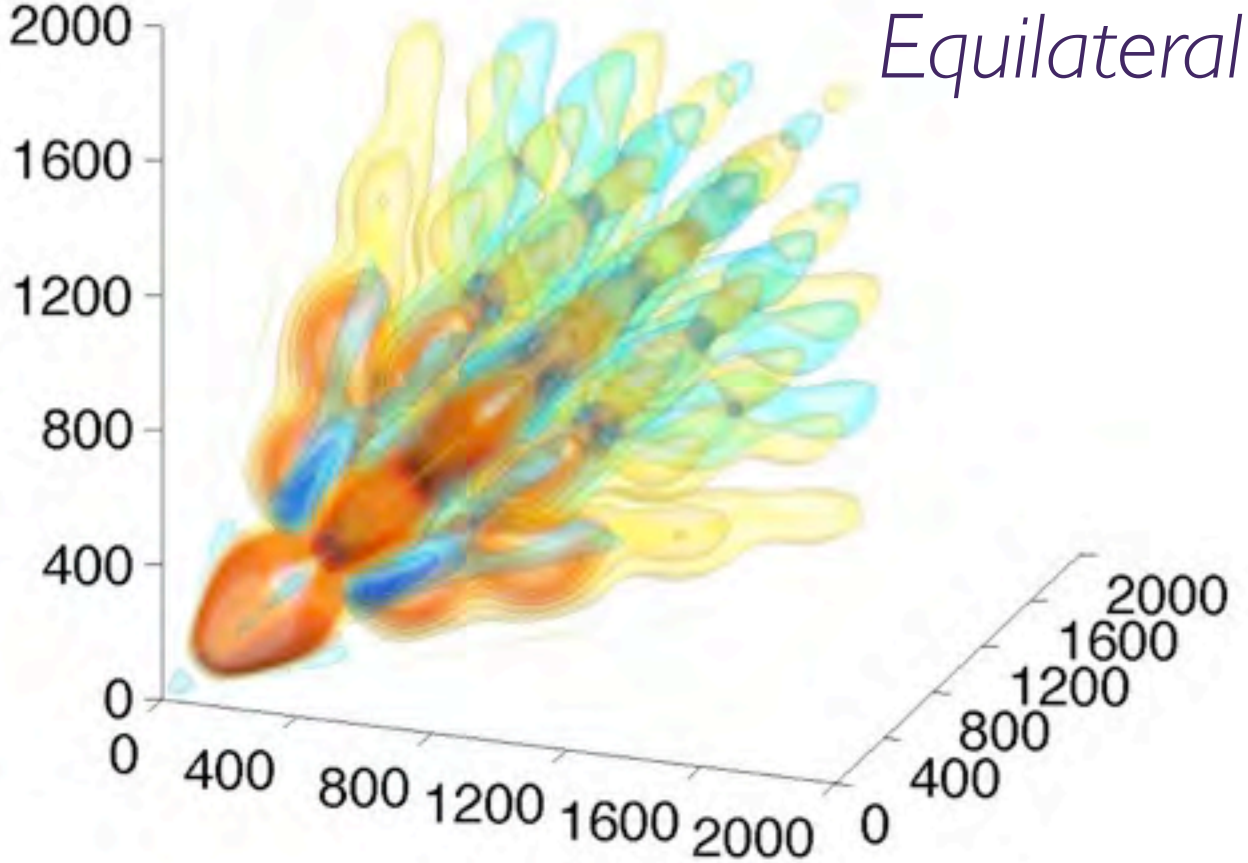
DBI inflation, effective field theory and higher derivative models ...

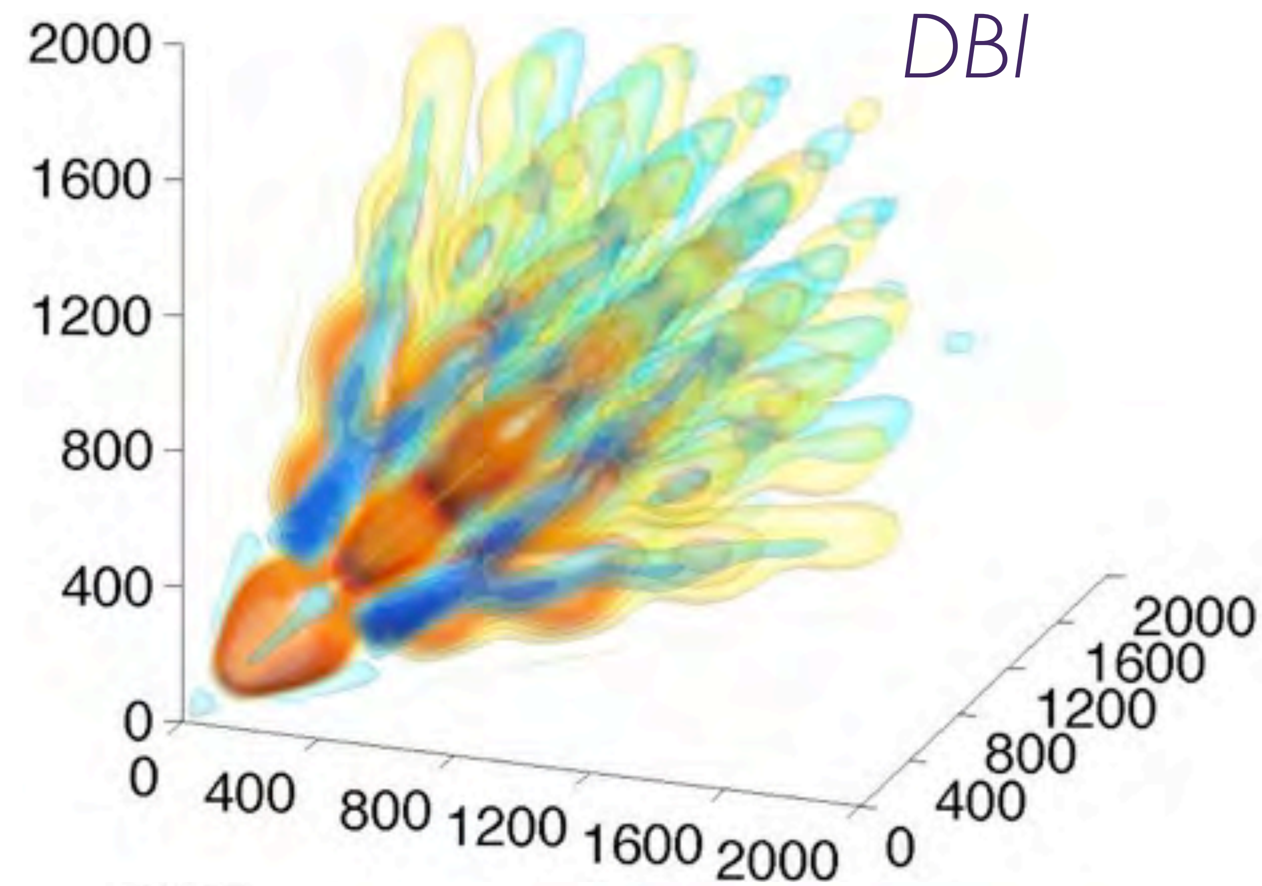
$$
\begin{aligned}
f_{\rm NL}^{\rm DBI} &= 11 \pm 69 \\
f_{\rm NL}^{\rm EFT1} &= 8 \pm 73 \\
f_{\rm NL}^{\rm EFT2} &= 19 \pm 57 \\
f_{\rm NL}^{\rm Ghost} &= -23 \pm 88
\end{aligned}
$$

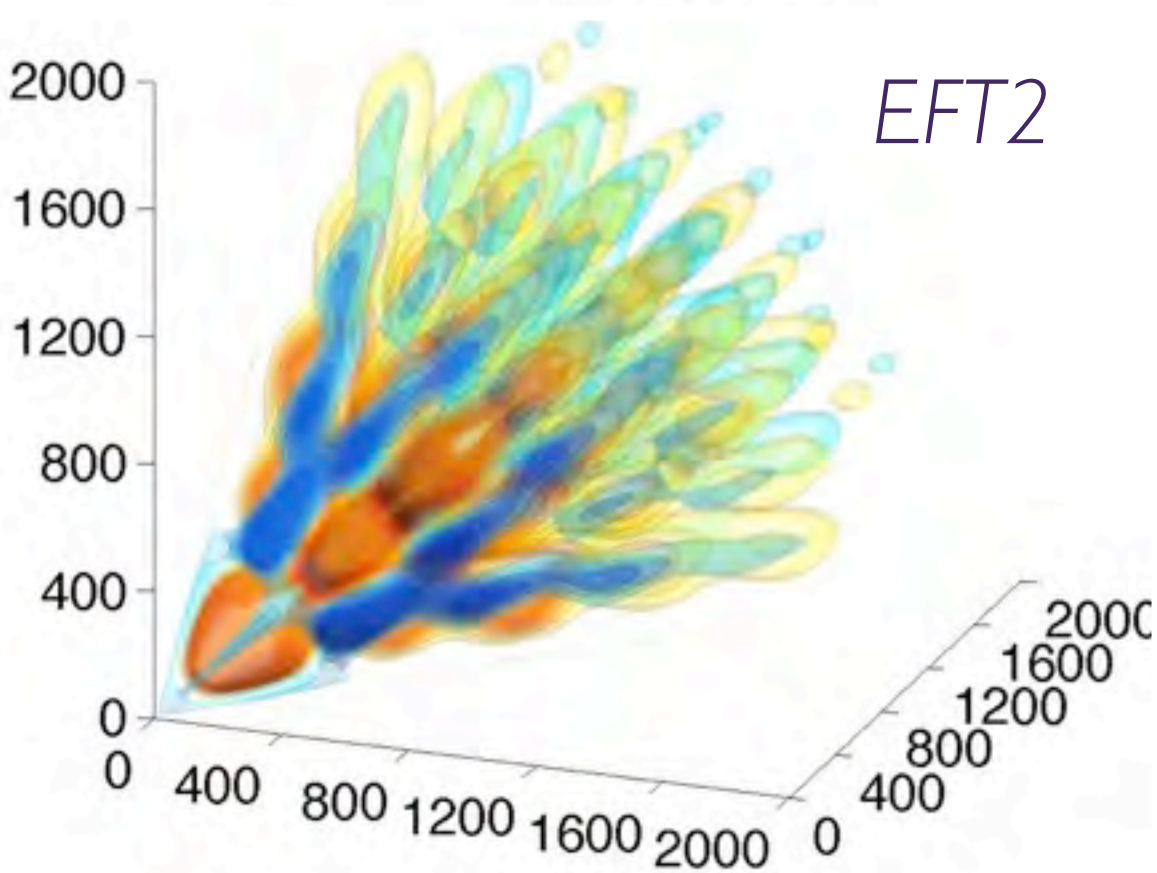
Equilateral/orthogonal constraint on sound speed $c_s > 0.02$.

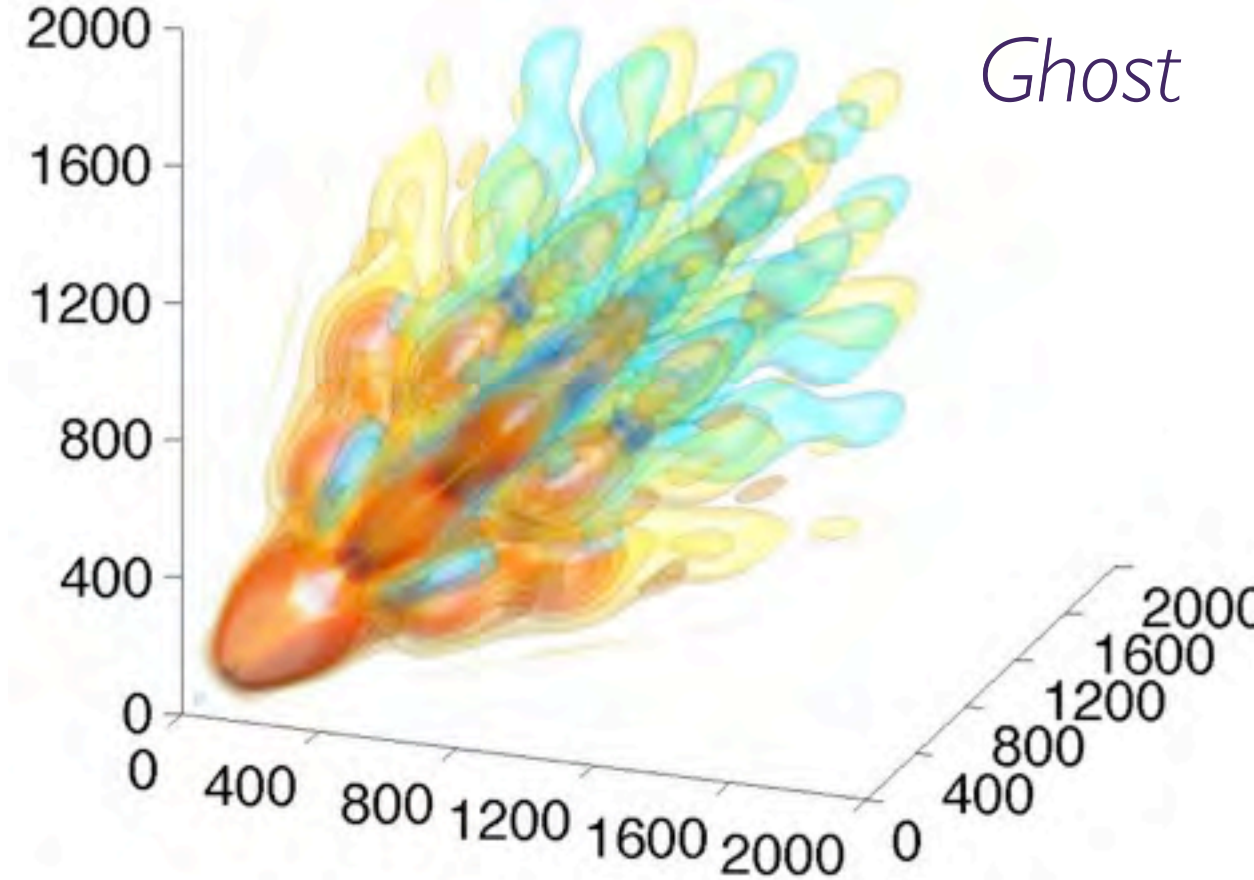


Equilateral shapes







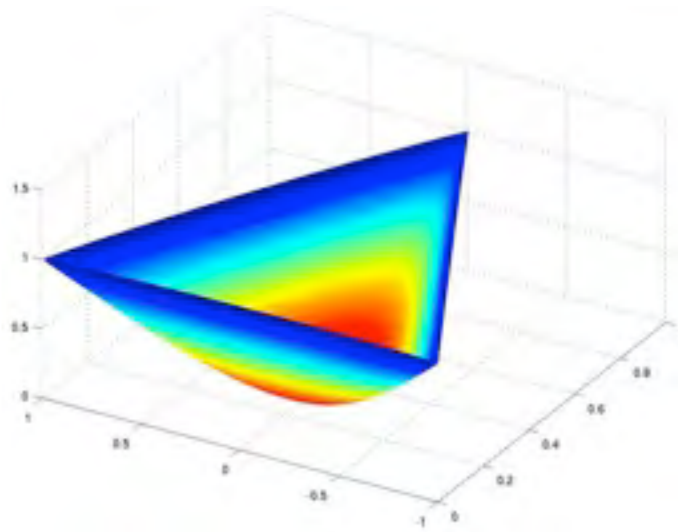


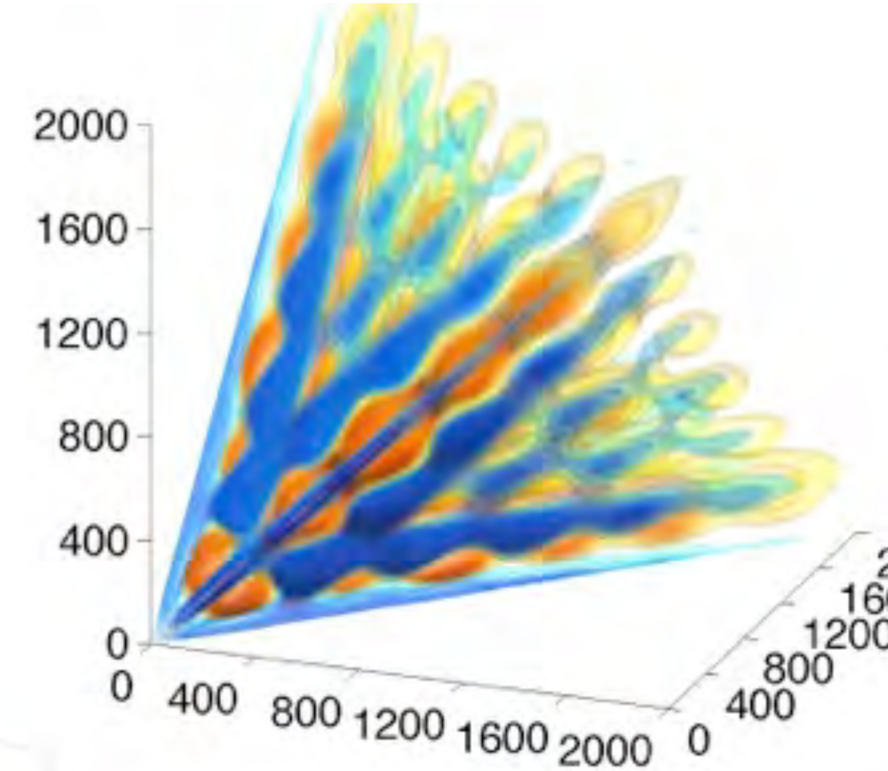
Flattened Models

Flat model

$$B_{\Phi}^{\text{flat}}(k_1,k_2,k_3) = 6A^2 f_{\text{NL}}^{\text{flat}}$$
$$\times \left\{ \frac{1}{k_1^{4-n_s}k_2^{4-n_s}} + \frac{1}{k_2^{4-n_s}k_3^{4-n_s}} + \frac{1}{k_3^{4-n_s}k_1^{4-n_s}} \right.$$
$$+ \frac{3}{(k_1k_2k_3)^{2(4-n_s)/3}} - \left[\frac{1}{k_1^{(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{4-n_s}} \right.$$
$$\left.\left. +(5\text{ perm.})\right]\right\}.$$

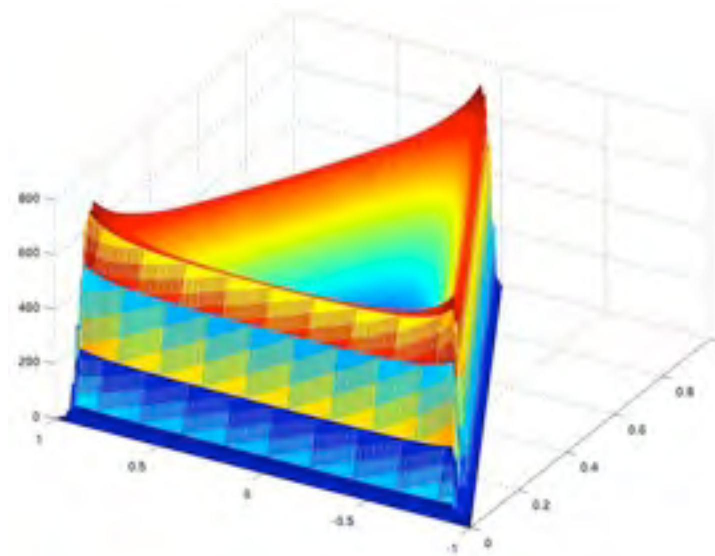
(Meerburg et al, 2009)





NBD model

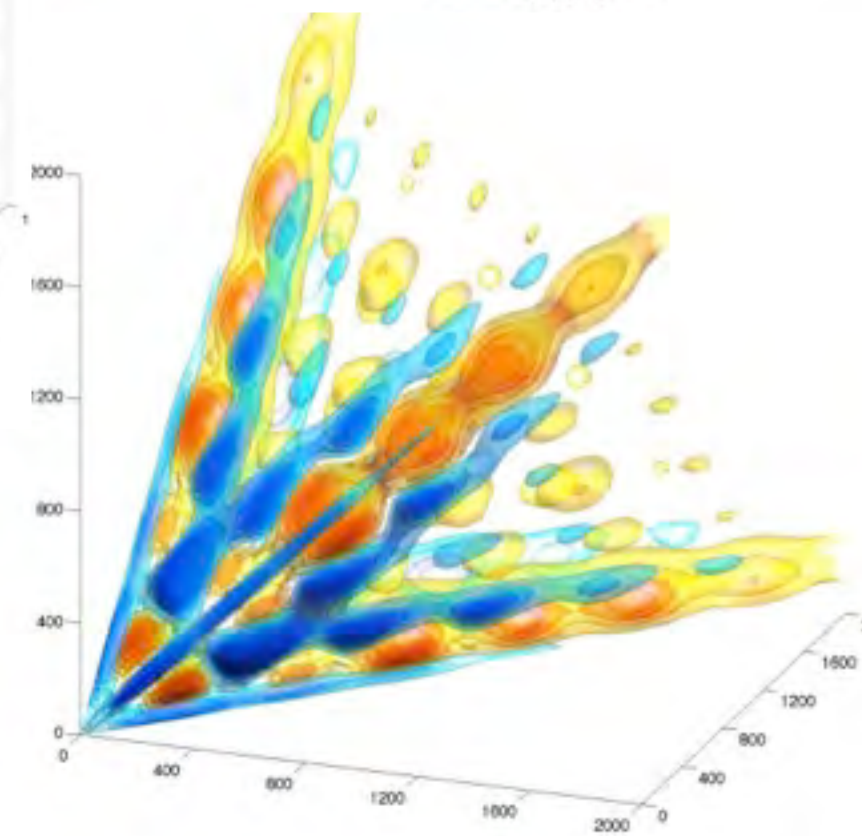
Original excited *(Chen et al, 2007)*



NBD1 and NBD2 models

$$B_{\Phi}^{\text{NBD}i} = \frac{2A^2 f_{\text{NL}}^{\text{NBD}i}}{(k_1k_2k_3)^3} \left\{ f_i(k_1,k_2,k_3) \times \frac{1-\cos[(k_2+k_3-k_1)/k_c]}{k_2+k_3-k_1} + 2\text{ perm.} \right\}$$

(Agullo & Parker, 2011)



NBD3 model

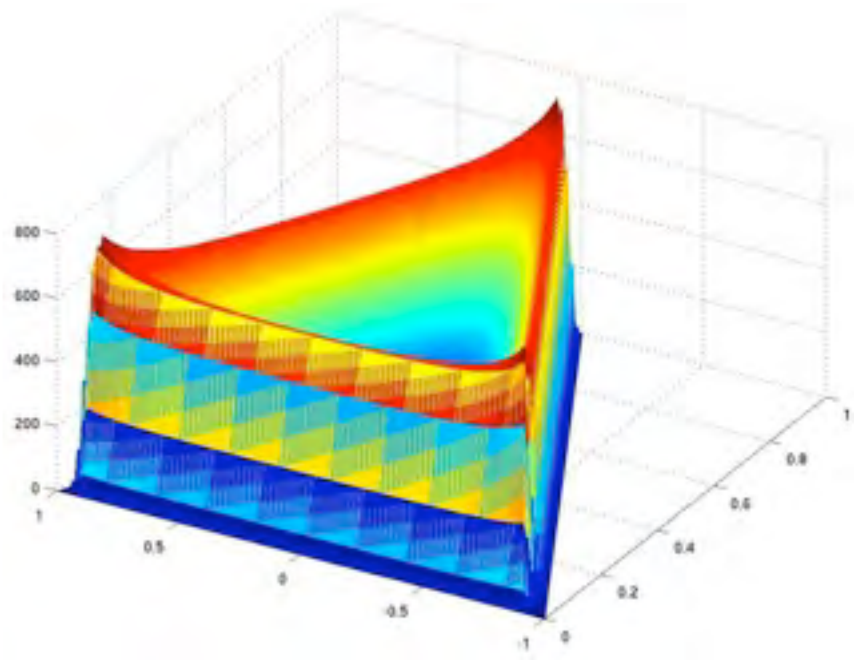
(Chen et al, 2010)

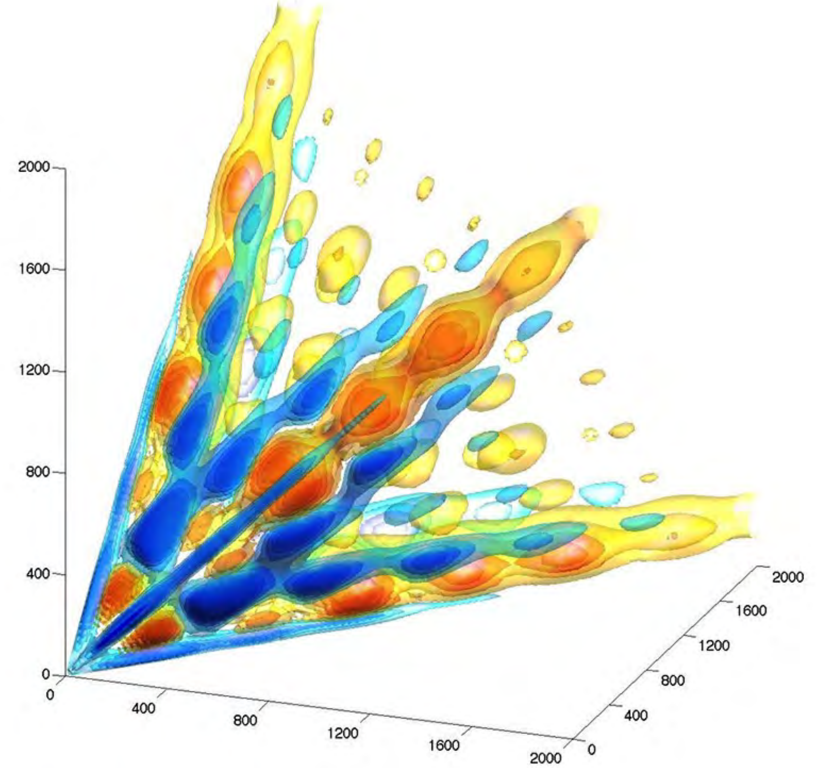
$$B_{\Phi}^{\text{NBD3}} = \frac{2A^2 f_{\text{NL}}^{\text{NBD3}}}{k_1k_2k_3} \left[\frac{k_1+k_2-k_3}{(k_c+k_1+k_2-k_3)^4} + 2\text{ perm.} \right]$$

Highest results from most flattened
- modal estimator resolution limited

Excited Initial States

Non-Bunch-Davies vacua from trans-Planckian effects or features



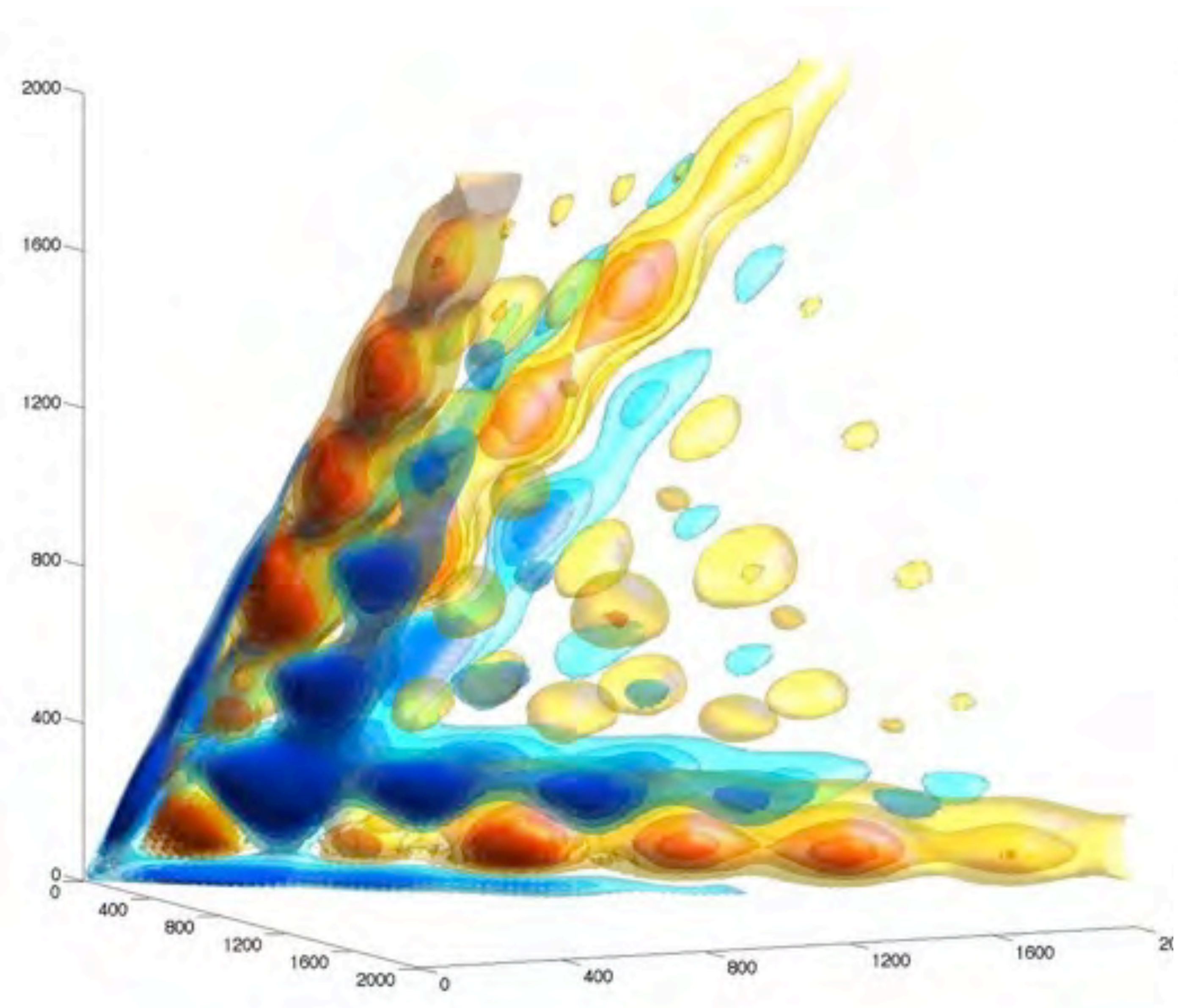


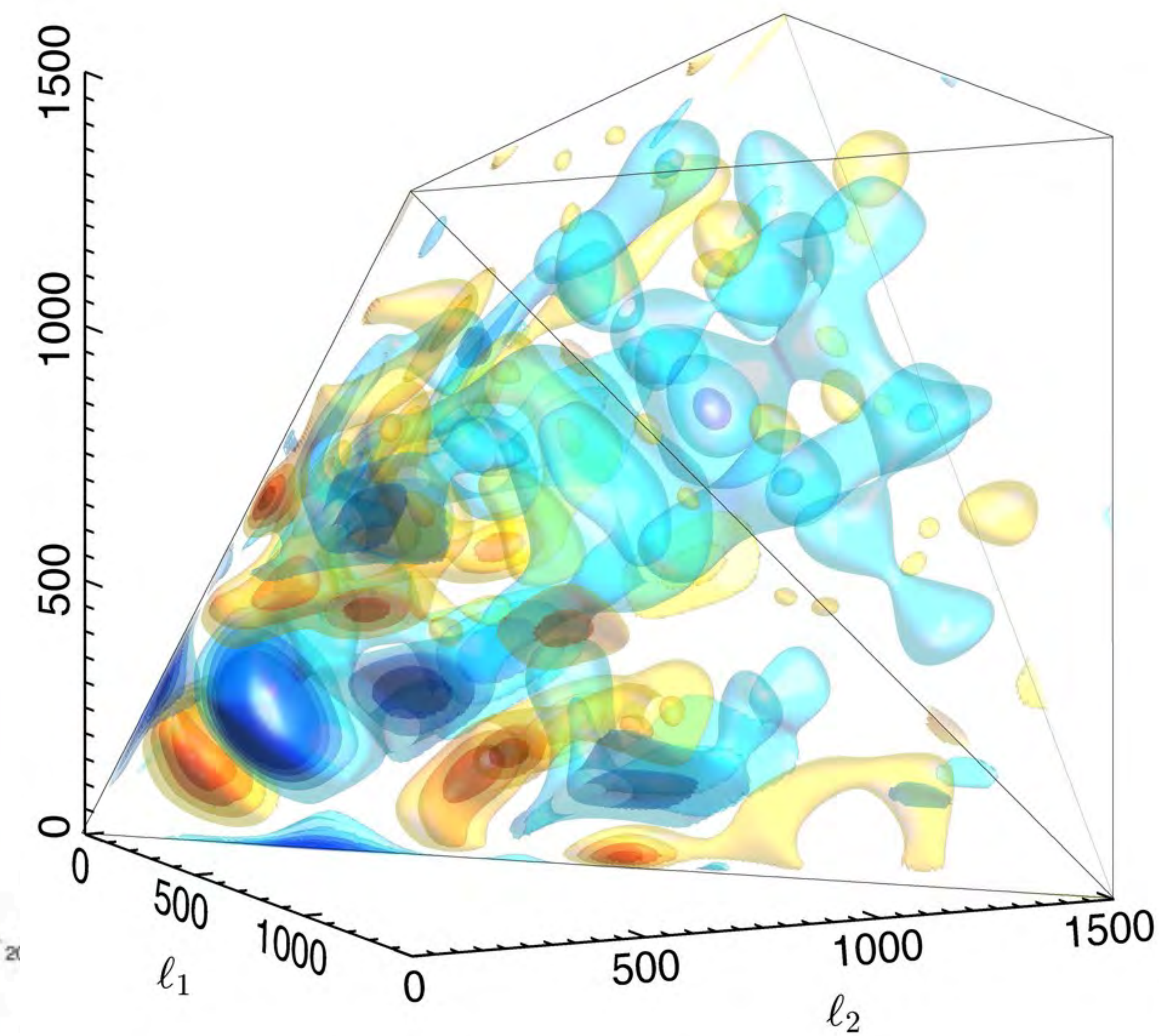
Five exemplar flattened models constrained (plus vector models)

Flattened model (Eq. number)	Raw $f_{\rm NL}$	Clean $f_{\rm NL}$	$\Delta f_{\rm NL}$	σ	Clean σ
Flat model (13)	70	37	77	0.9	0.5
Non-Bunch-Davies (NBD)	178	155	78	2.2	2.0
Single-field NBD1 flattened (14)	31	19	13	2.4	1.4
Single-field NBD2 squeezed (14)	0.8	0.2	0.4	1.8	0.5
Non-canonical NBD3 (15)	13	9.6	9.7	1.3	1.0
Vector model $L = 1$ (19)	−18	−4.6	47	−0.4	−0.1
Vector model $L = 2$ (19)	2.8	−0.4	2.9	1.0	−0.1

NBD vs Planck

Comparison/similarities of non-Bunch-Davies and Planck bispectra





Vector Inflation/Warm inflation

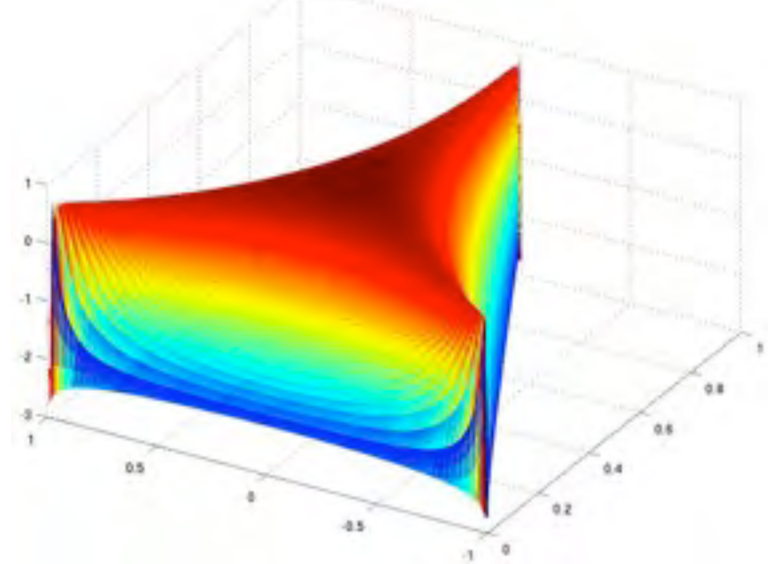
Inflation with gauge/vector fields can have non-trivial directional dependencies

$$B_\Phi(k_1, k_2, k_3) = \sum_L c_L[P_L(\mu_{12})P_\Phi(k_1)P_\Phi(k_2) + 2\,\mathrm{perm}].$$

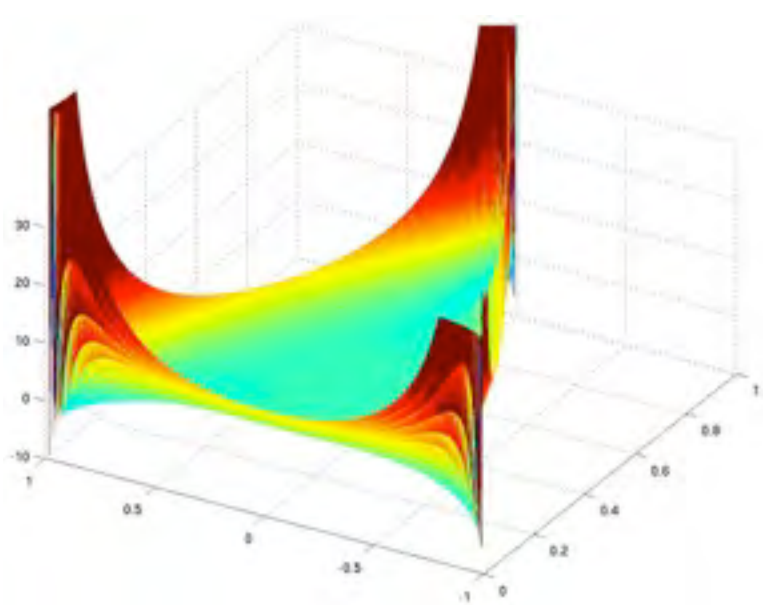
(see e.g. Shiraishi et al, 2012)

Similarly 'twisted' bispectrum for warm inflation

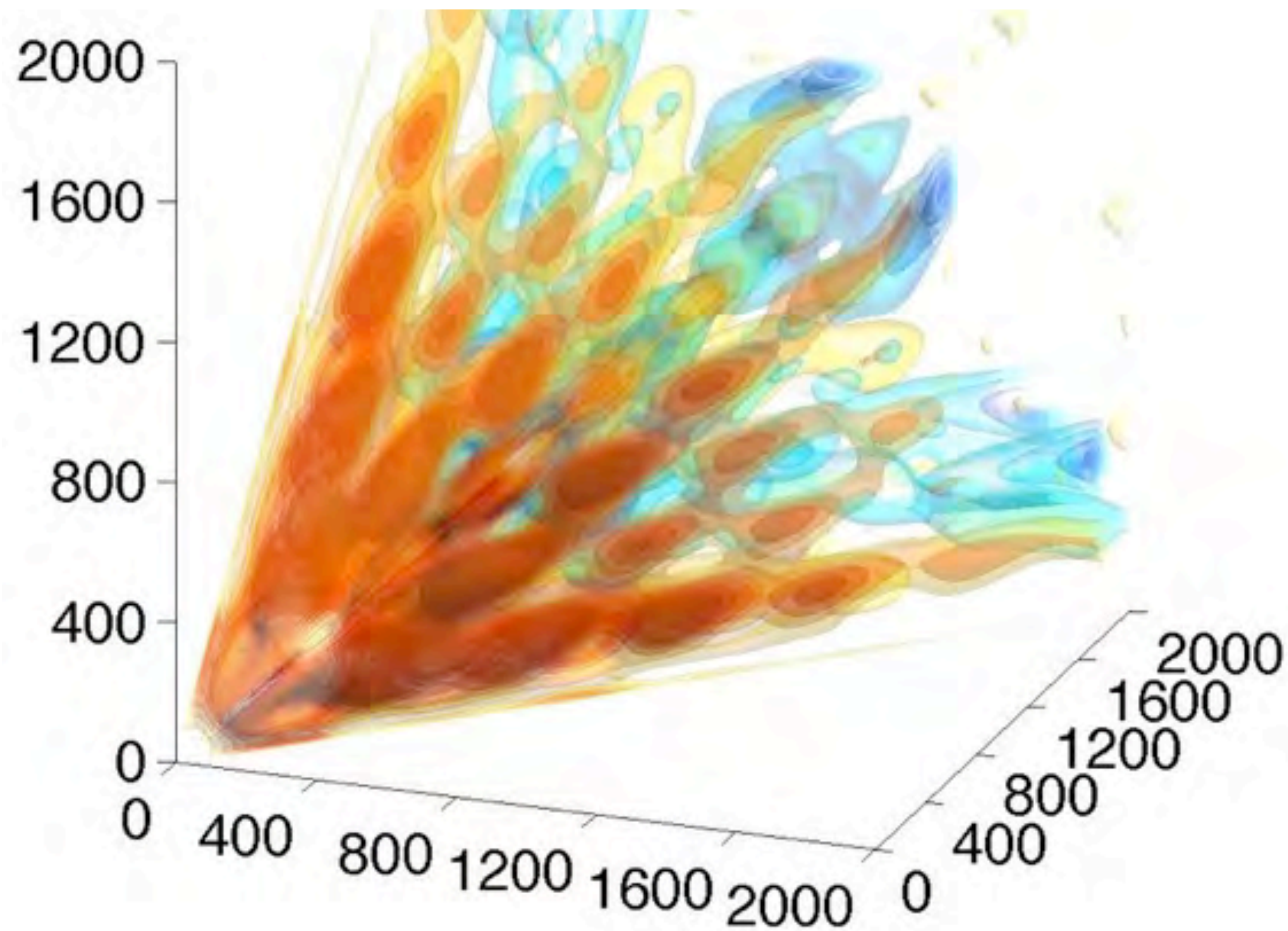
No directional evidence but modal correlation could be improved ...



$L=1$



$L=2$



Feature models

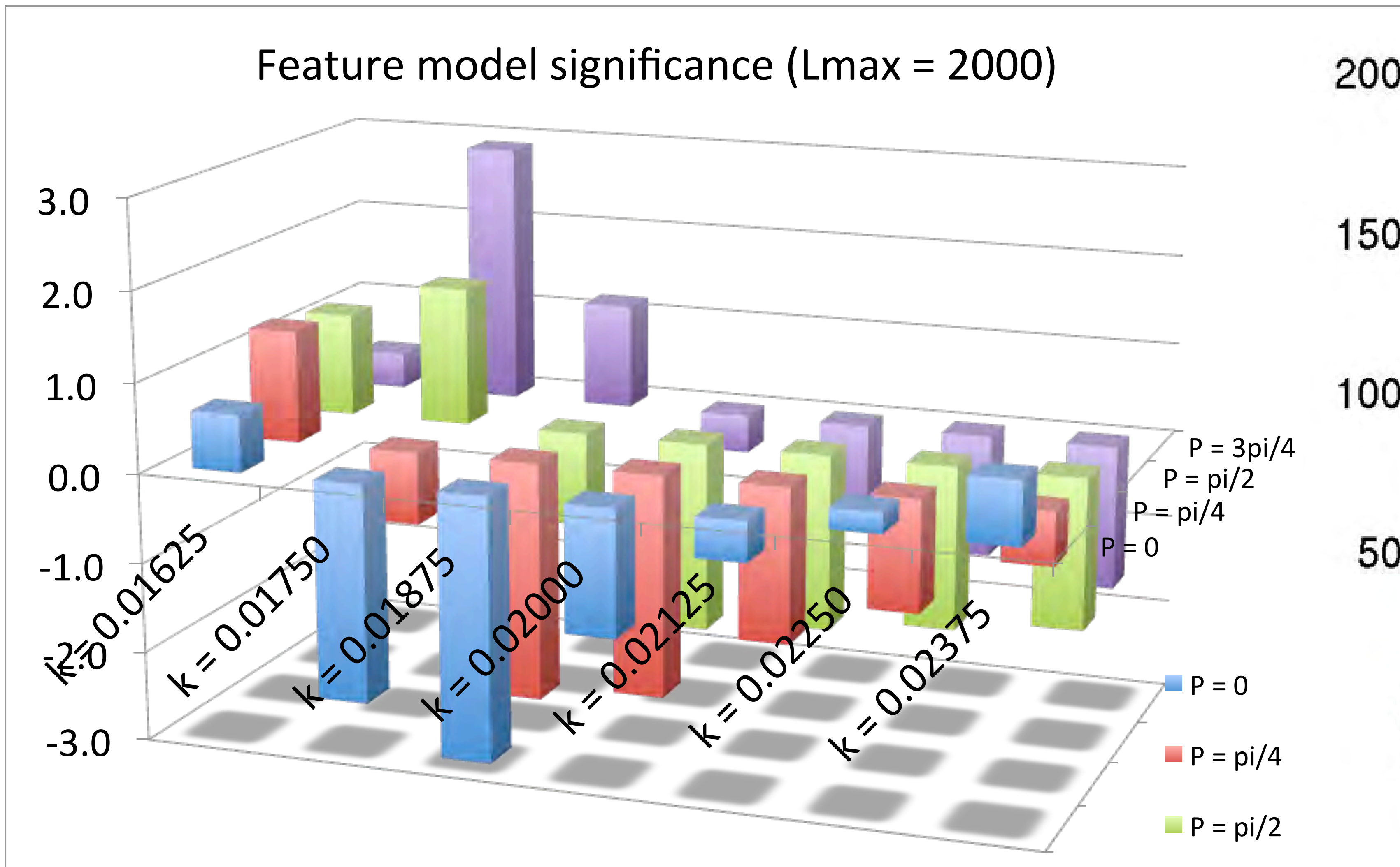
Inflaton potential can have a feature which disturbs slow-roll:

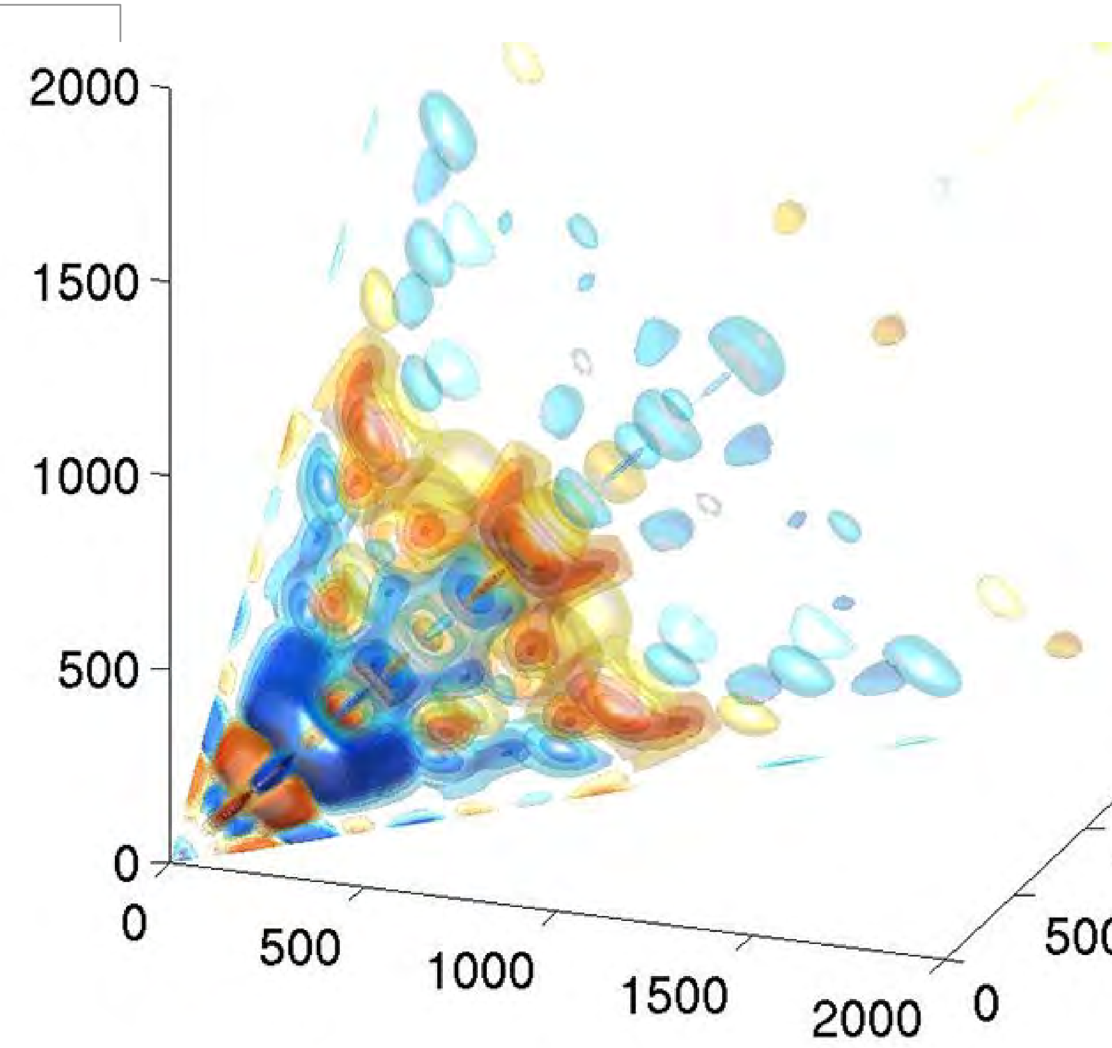
$$B_\Phi^{\rm feat}(k_1,k_2,k_3) = \frac{6A^2 f_{\rm NL}^{\rm feat}}{(k_1k_2k_3)^2} \sin\left[\frac{2\pi(k_1+k_2+k_3)}{3k_{\rm c}} + \phi\right]$$

(Chen et al, 2007)

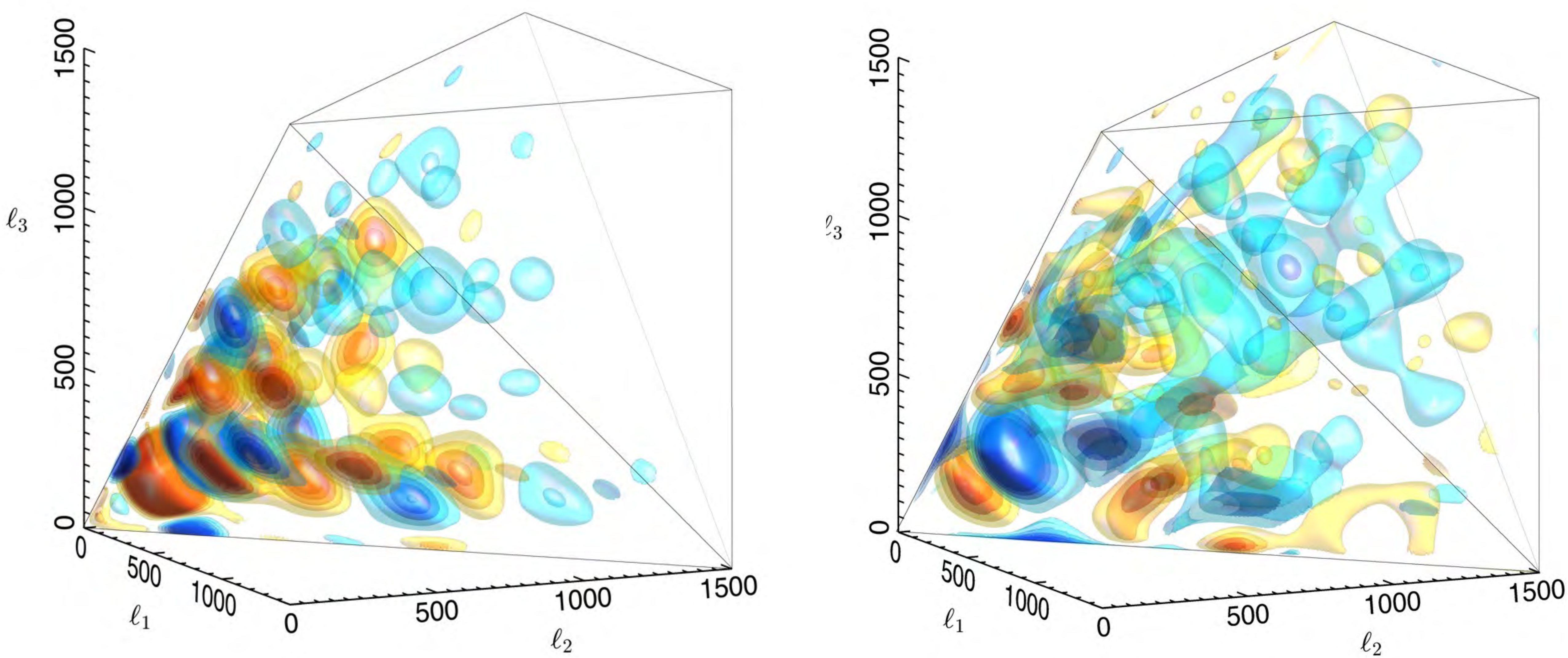
Can match the observed "oscillatory" signal in the Planck bispectrum (consistent with WMAP results)

Initial two-parameter survey only (k_c, Φ) ..





Feature envelope best-fit

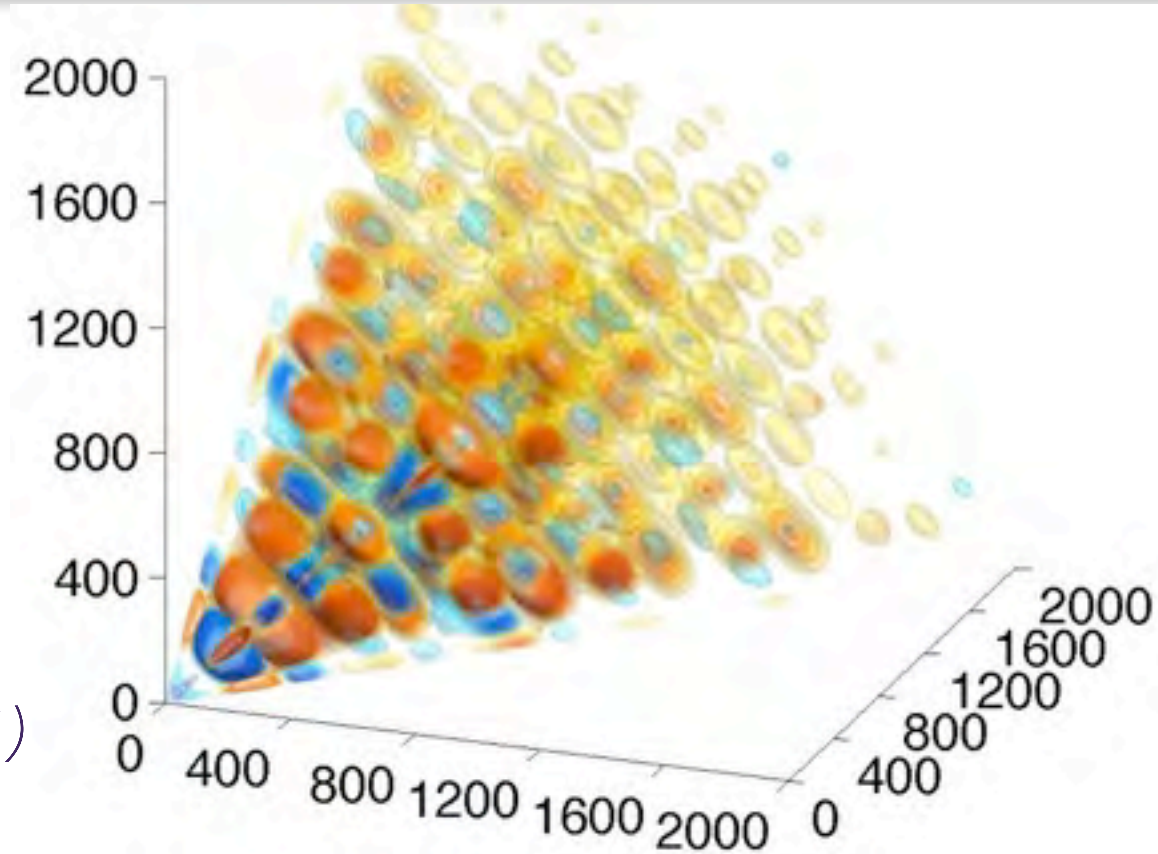


Model	Width $\Delta k = 0.015$ $f_{NL} \pm \Delta f_{NL}$ (σ)	$\Delta k = 0.03$ $f_{NL} \pm \Delta f_{NL}$ (σ)	$\Delta k = 0.045$ $f_{NL} \pm \Delta f_{NL}$ (σ)	Full $f_{NL} \pm \Delta f_{NL}$ (σ)
$k_c = 0.01125;\ \phi = 0$.	765 ± 275 (2.8)	703 ± 241 (2.9)	648 ± 218 (3.0)	434 ± 170 (2.6)
$k_c = 0.01750;\ \phi = 0$.	-661 ± 234 (-2.8)	-494 ± 192 (-2.6)	-425 ± 171 (-2.5)	-335 ± 137 (-2.4)
$k_c = 0.01750;\ \phi = 3\pi/4$	399 ± 207 (1.9)	438 ± 183 (2.4)	442 ± 165 (2.7)	366 ± 126 (2.9)
$k_c = 0.01875;\ \phi = 0$.	-562 ± 211 (-2.7)	-559 ± 180 (-3.1)	-515 ± 159 (-3.2)	-348 ± 118 (-3.0)
$k_c = 0.01875;\ \phi = \pi/4$	-646 ± 240 (-2.7)	-525 ± 189 (-2.8)	-468 ± 164 (-2.9)	-323 ± 120 (-2.7)
$k_c = 0.02000;\ \phi = \pi/4$	-665 ± 229 (-2.9)	-593 ± 185 (-3.2)	-500 ± 160 (-3.1)	-298 ± 119 (-2.5)

Resonance and NBD Features

Feature models

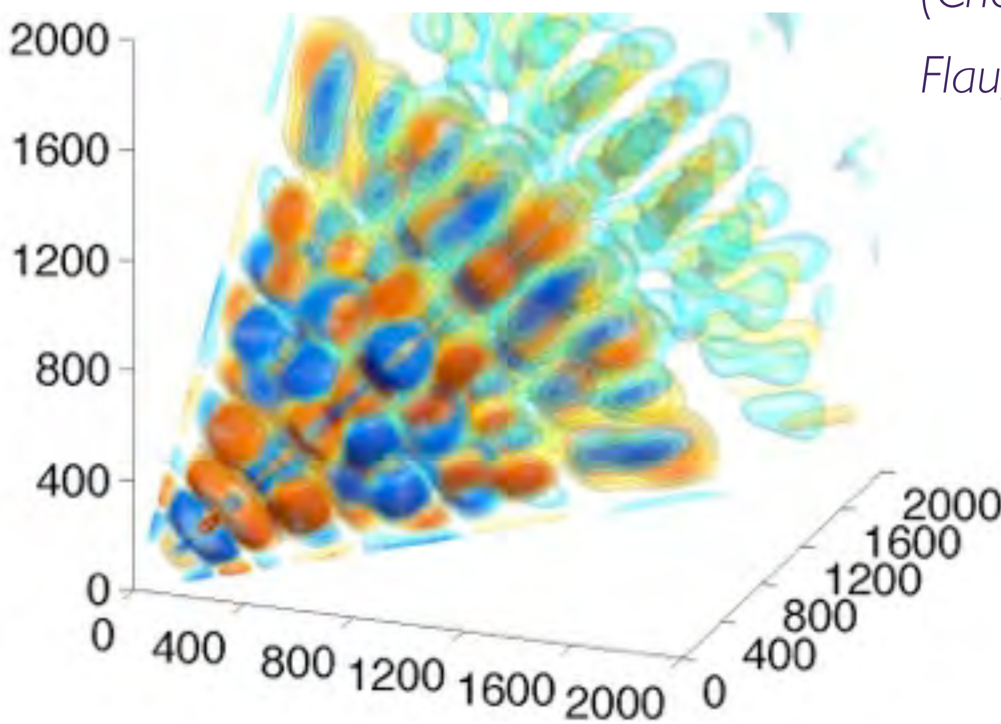
$$B_\Phi^{\rm feat}(k_1,k_2,k_3) = \frac{6A^2 f_{\rm NL}^{\rm feat}}{(k_1k_2k_3)^2}\sin\left[\frac{2\pi(k_1+k_2+k_3)}{3k_{\rm c}}+\phi\right],$$



Resonance models (e.g. axion monodromy)

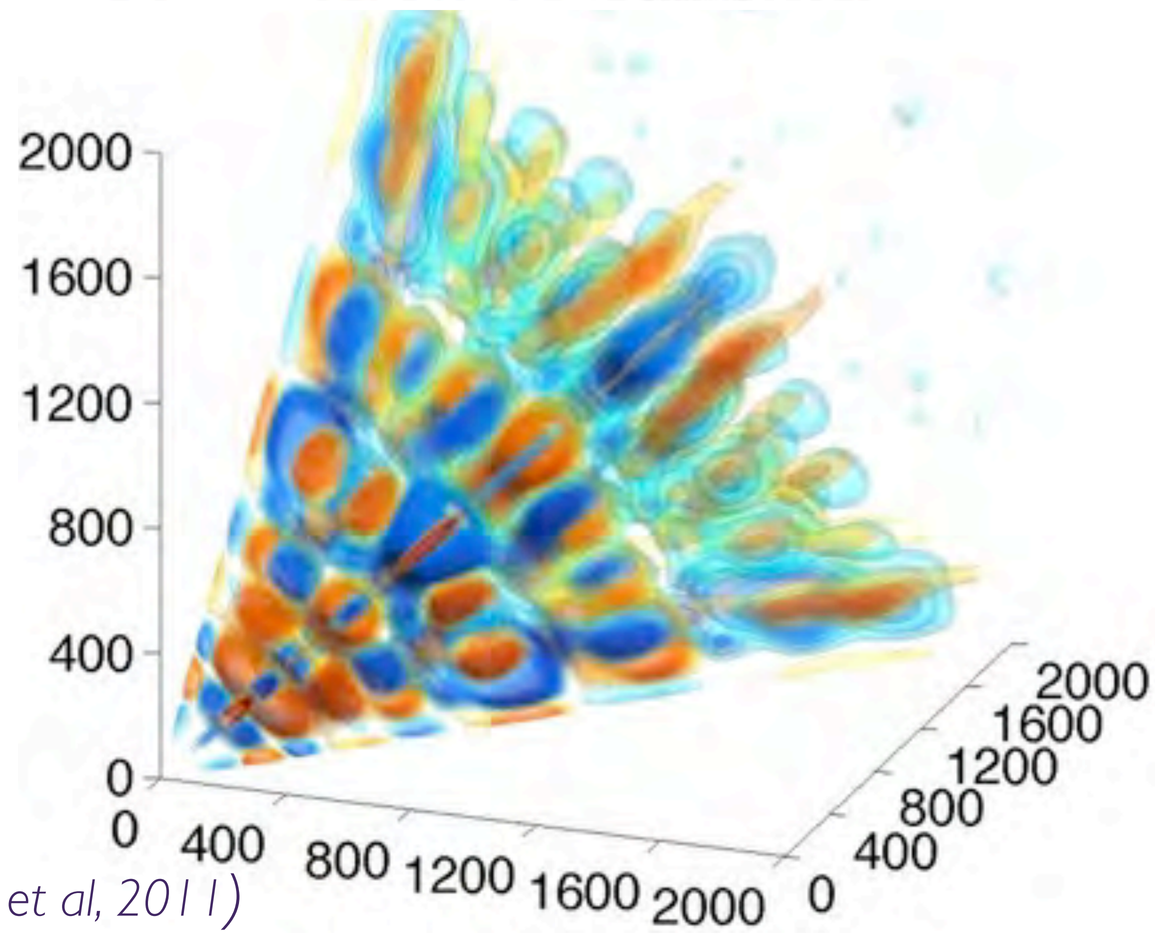
$$B_\Phi^{\rm res}(k_1,k_2,k_3) = \frac{6A^2 f_{\rm NL}^{\rm res}}{(k_1k_2k_3)^2}\sin\left[C\ln(k_1+k_2+k_3)+\phi\right],$$

(Chen et al, 2008, Flauger & Pajer 2011)



Enfolded resonance models

$$B_\Phi^{\rm resNBD}(k_1,k_2,k_3) = \frac{2A^2 f_{\rm NL}^{\rm resNBD}}{(k_1k_2k_3)^2}\Big\{\exp(-k_{\rm c}^{3/5}(k_2+k_3-k_1)/2k_1)$$
$$\times\sin[k_{\rm c}((k_2+k_3-k_1)/2k_1+\ln k_1)+\phi]+2\text{ perm.}\Big\}. \quad (18)$$



(Chen et al, 2011)

Resonance and NBD Features

Table B.1. Results from a limited f_{NL} survey of resonance models of Eq. (17) with $0.25 \le k_c \le 0.5$ using the SMICA component-separated map. These models have a large-ℓ periodicity similar to the feature models in Table 12.

Phase Wavenumber	$\phi = 0$ $f_{NL} \pm \Delta f_{NL}$	$\phi = \pi/5$ $f_{NL} \pm \Delta f_{NL}$	$\phi = 2\pi/5$ $f_{NL} \pm \Delta f_{NL}$	$\phi = 3\pi/5$ $f_{NL} \pm \Delta f_{NL}$	$\phi = 4\pi/5$ $f_{NL} \pm \Delta f_{NL}$	$\phi = \pi$ $f_{NL} \pm \Delta f_{NL}$
$k_c = 0.25$	-16 ± 57	6 ± 63	19 ± 67	31 ± 69	38 ± 68	-6 ± 60
$k_c = 0.30$	-66 ± 73	-57 ± 74	-44 ± 73	-26 ± 72	-7 ± 71	-65 ± 73
$k_c = 0.40$	5 ± 57	40 ± 66	55 ± 71	63 ± 73	63 ± 71	22 ± 61
$k_c = 0.45$	25 ± 56	34 ± 59	36 ± 62	34 ± 67	27 ± 69	30 ± 56
$k_c = 0.50$	-2 ± 65	-13 ± 72	-16 ± 69	-16 ± 60	-14 ± 55	-7 ± 71

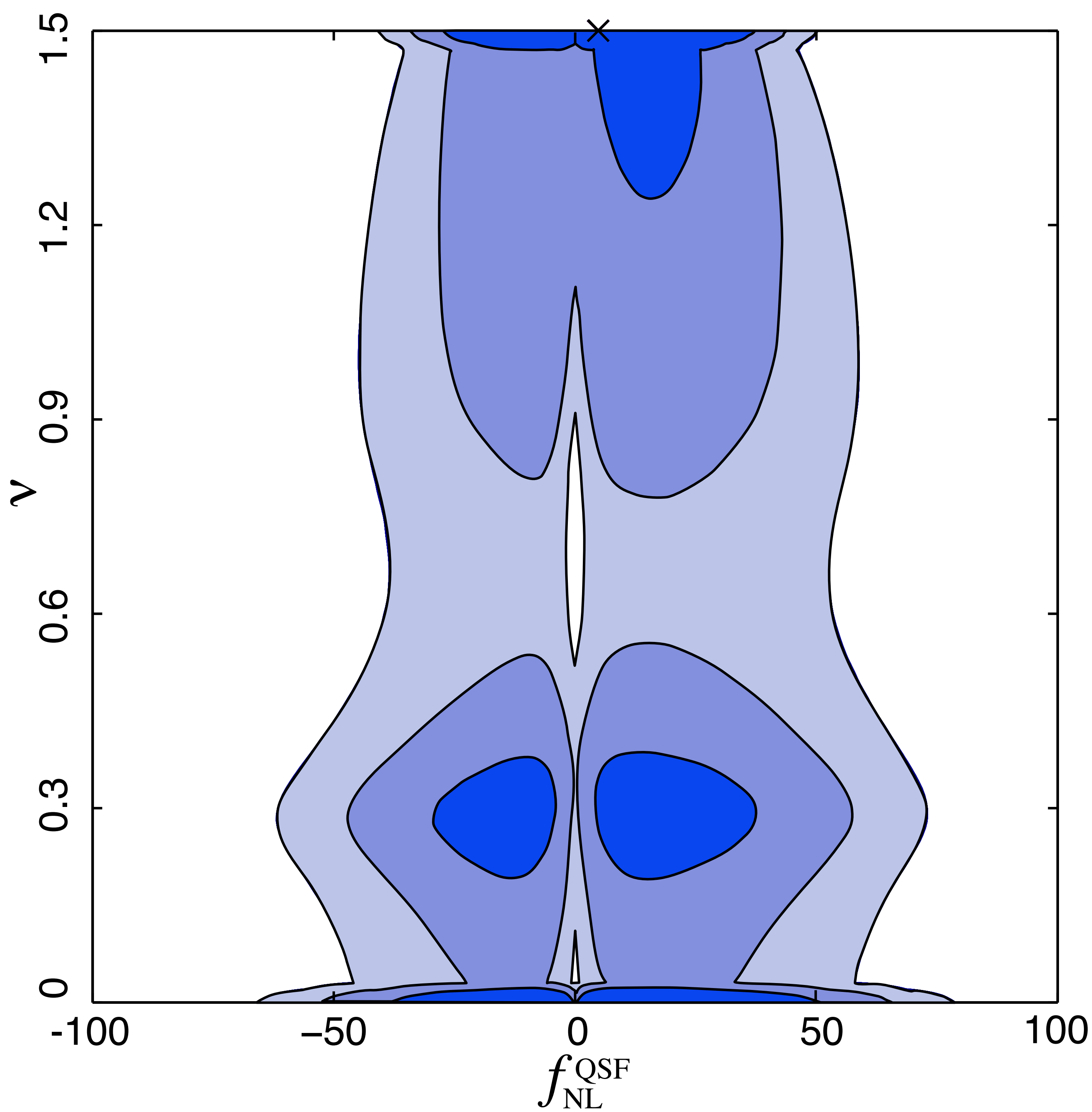
Table B.2. Results from a limited f_{NL} survey of non-Bunch-Davies feature models (or enfolded resonance models) of Eq. (18) with $4 \le k_c \le 12$, again overlapping in periodicity with the feature model survey.

Phase Wavenumber	$\phi = 0$ $f_{NL} \pm \Delta f_{NL}$ (σ)	$\phi = \pi/4$ $f_{NL} \pm \Delta f_{NL}$ (σ)	$\phi = \pi/2$ $f_{NL} \pm \Delta f_{NL}$ (σ)	$\phi = 3\pi/4$ $f_{NL} \pm \Delta f_{NL}$ (σ)
$k_c = 4$	11 ± 146 (0.1)	2 ± 145 (0.0)	-7 ± 143 (0.0)	-15 ± 142 (−0.1)
$k_c = 6$	52 ± 202 (0.3)	63 ± 203 (0.3)	72 ± 201 (0.4)	80 ± 197 (0.4)
$k_c = 8$	100 ± 190 (0.5)	130 ± 189 (0.7)	158 ± 189 (0.8)	183 ± 190 (1.0)
$k_c = 10$	188 ± 241 (0.8)	210 ± 242 (0.9)	230 ± 242 (1.0)	248 ± 243 (1.0)
$k_c = 12$	180 ± 307 (0.6)	171 ± 310 (0.6)	158 ± 312 (0.5)	142 ± 314 (0.5)

Quasi-Single field

$$B_{\Phi}^{\mathrm{QSI}}(k_1,k_2,k_3) = \frac{6A^2 f_{\mathrm{NL}}^{\mathrm{QSI}}}{(k_1k_2k_3)^{3/2}} \frac{3^{3/2}N_\nu[8k_1k_2k_3/(k_1+k_2+k_3)^3]}{N_\nu[8/27](k_1+k_2+k_3)^{3/2}}($$

Alpha were calculated for 150 values of ν and the Beta covariance matrix was used to produce 2 billion simulations around the measured value of ν and f_{NL} which were used to produce the likelihood plot



Conclusions

Local, equilateral and orthogonal shapes constrained $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$

All bispectrum paradigms investigated - squeezed, equil, flat, oscillatory

Some implications for fundamental cosmology:

- Effective field theory sound speed $c_s > 0.02$
- For DBI inflation sound speed $c_s > 0.07$
- Power law K-inflation ruled out (cf power spectrum)
- Curvaton model constraint on "decay fraction" r_D
- Ekpyrotic/cyclic "conversion mechanism" ruled out
- Excited initial states and vector inflation constrained
- Feature model results not significant - interesting 'hints'

Planck bispectrum reconstruction "patterns" appear to have high NG signal

Further investigation warranted at higher resolution ... we've only just begun