From the QCD Flux Tubes to the Hierarchy Problem

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Three parts of the story:

- ✳ Dynamics of QCD flux tubes
- ✳ Integrable quantum gravity

SD, Raphael Flauger, Victor Gorbenko,
1203.1054, 1205.6805, 1301.2325
+more to appear

- ✳ (Bad) idea for "solving" the EW hierarchy problem

SD, Victor Gorbenko, Mehrdad Mirbabayi
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Sanity

BORING

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CRAZY

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Hierarchy problem: first iteration

- ✶We saw the 125 GeV Higgs
- ✶Quadratic divergencies indicate that for a *generic* new physics

$$\delta m_H^2 = \left(\frac{g^2}{16\pi^2}\right)^{\#} \Lambda_{NP}^2$$

- ✶Gravity attests the presence of new physics coupled to the Standard Model at least at the Planck scale

(“Good”) Conservative ideas:

- Quadratic divergencies are cancelled by new TeV scale physics
- Electroweak scale is tuned as a consequence of anthropic selection

(“Bad”) Radical idea: *NATURAL TUNING*

- Nature does not calculate in the Wilsonian way

Which way it calculates???

Proof of Concept

NB: The construction will be in (1+1)d. 2d theories are special in many respects, but not as far as the hierarchy problem goes

Start with a UV complete natural QFT $\mathcal{L}(\psi, H)$
Non-protected scalars are allowed as soon as they are heavy

↓

Calculate S-matrix $S_n(p_i)$

↓

"Gravitational dressing" gives $\hat{S}_n(p_i, \ell)$

Properties of gravitational dressing

- ✶Results in a well-to-do S-matrix
- ✶Physical spectrum remains the same
- ✶Low energy EFT description, tuned for $m\ell \ll 1$

$$\mathcal{L}(\psi, H) + \sum_{\Delta_i > 2} \ell^{\Delta_i - 2} \mathcal{O}_i$$

free massive scalar:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{\ell^2}{8}\left((\partial\phi)^4 - m^4\phi^4\right) + \ldots$$

- ✶Never had to tune anything

Am I cheating?

We never see fine-tuning at the S-matrix level...

I feel the construction is interesting:

✶Normally, one has to go through the Lagrangian to construct the theory, that's where the fine-tuning enters. Here we escaped this path.

✶Even stronger: we are not aware of the Wilsonian path to define the theory at all energies, and it appears very likely that it does not exist.

✶New asymptotic behavior at large energies. No mass thresholds at the scale ℓ^{-1}.

Hierarchy problem: second iteration

Directly in terms of properties of the RG flow,
without ever mentioning quadratic divergencies

For concreteness, let us place the discussion in the context of non-SUSY GUTs

$$m_H \ll E \ll m_{GUT} \;:\; \mathcal{L} = CFT_{321} + \underbrace{m_H^2 H^2}_{\text{relevant}} + \underbrace{\sum_i \frac{\mathcal{O}_i}{M_{GUT}^{\Delta_i - 4}}}_{\text{irrelevant}}$$

How comes $m_H \ll m_{GUT}$ given no symmetry?

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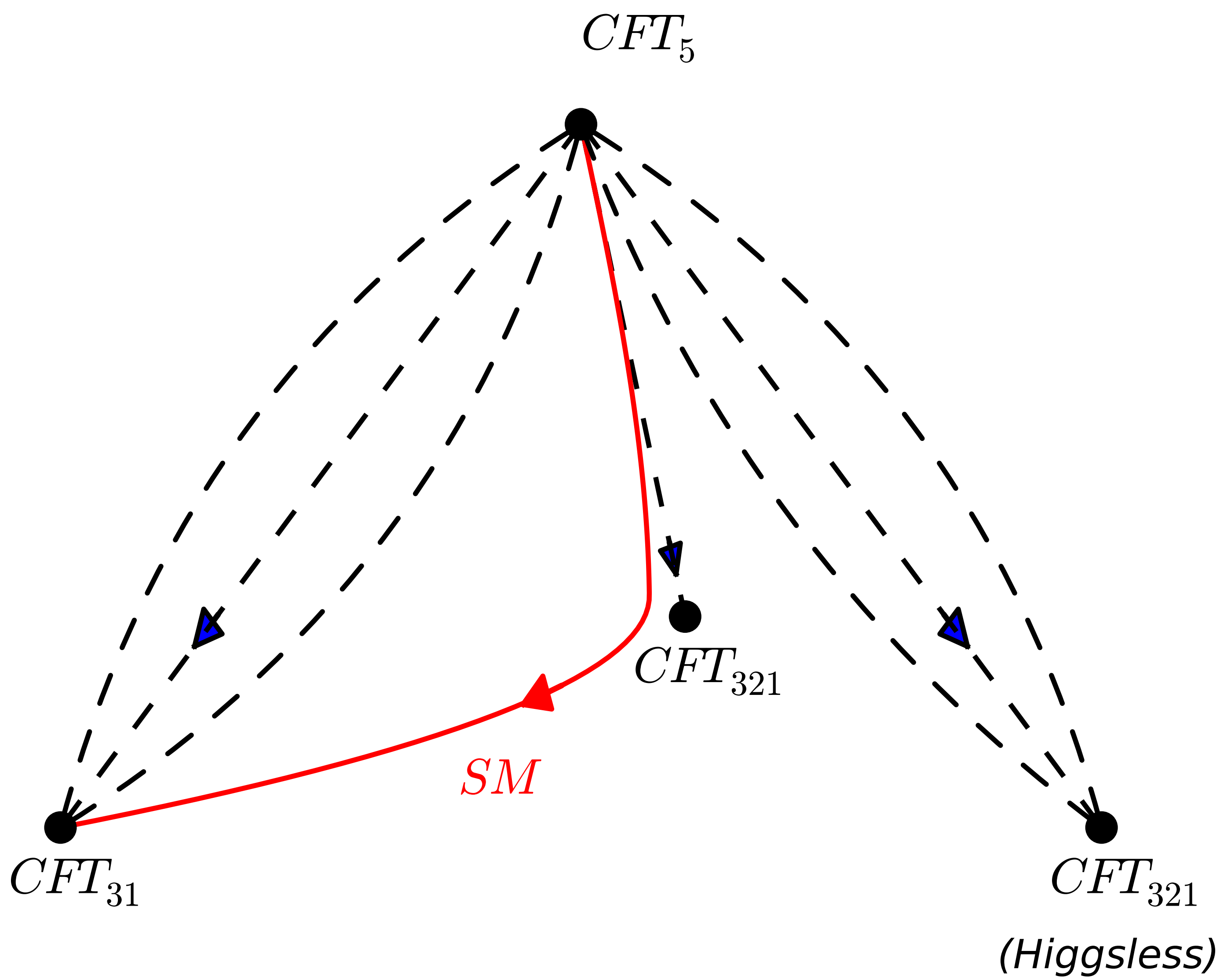
However, fine-tuning is truly manifest only as seen from higher energies:

$$m_{GUT} \ll E : \mathcal{L} = CFT_5 + \underbrace{g_h m_{GUT}^2 H^2}_{\text{relevant}} + \underbrace{g_\Sigma m_{GUT}^2 \Sigma^2}_{\text{relevant}} + \ldots$$


CFT_5
CFT_{321}
SM
CFT_{31}
CFT_{321}
(Higgsless)

Two notions of a naturalness:

1) *If a natural theory possesses unprotected relevant operators (scalar masses), the corresponding energy scale should be the highest*

2) *Among all possible scales set by relevant operators unprotected operators should correspond to the highest scale*

✶Agree for QFT = UV CFT perturbed by relevant operators.

✶May disagree in the presence of gravity.

Indeed, disagree in the gravitational dressing construction.

Let us see how it works...

This is how these theories should have been found:

What are the possible integrable reflectionless massless theories in two dimensions?

Everything is determined by a two-particle phase shift:

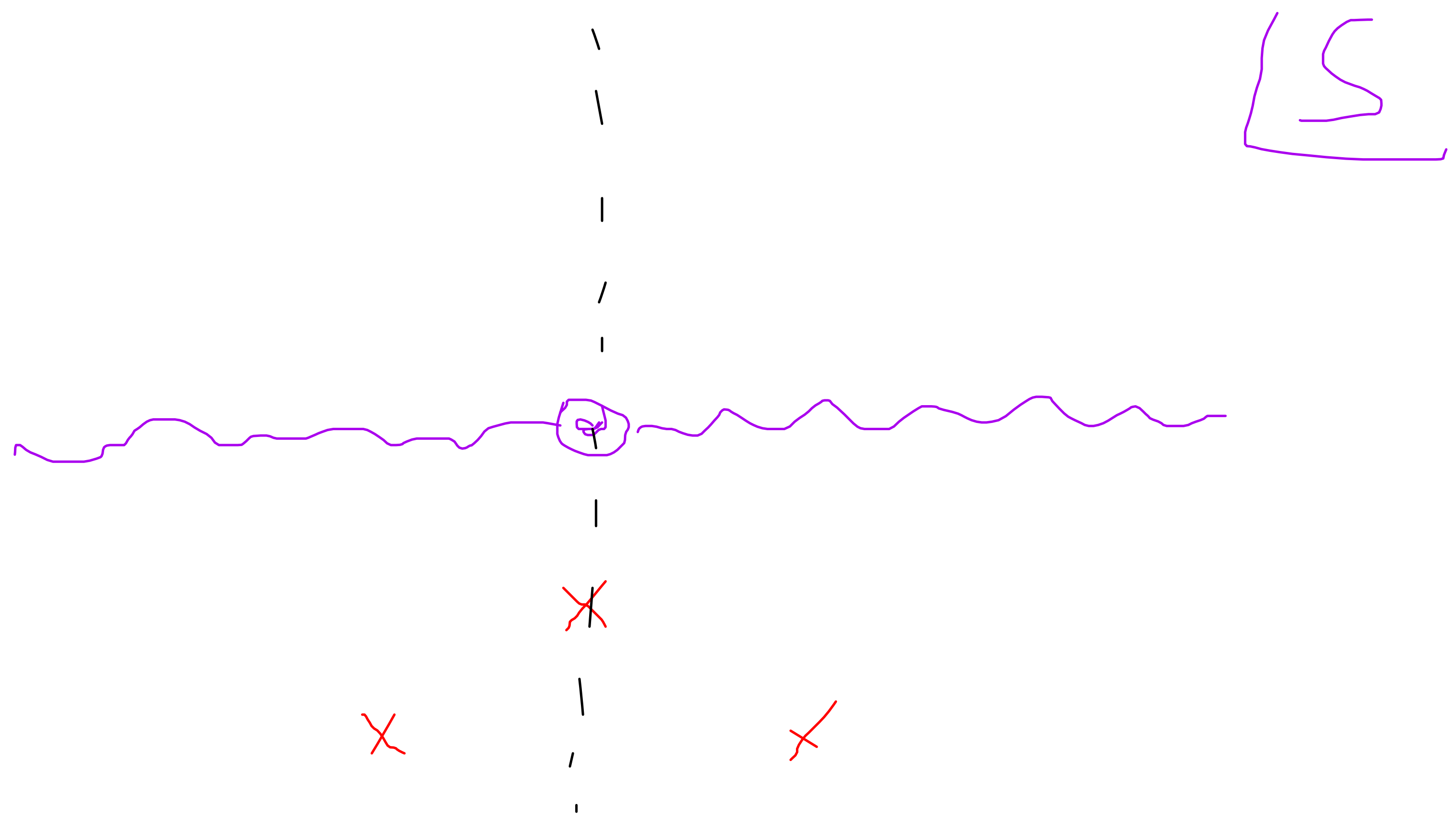
$$\boxed{S = e^{2i\delta(s)}\mathbf{1}}$$

Unitarity+Analyticity+Crossing:

Zamolodchikov '91

$$e^{2i\delta(s)} = \prod_j \frac{\mu_j + s}{\mu_j - s} e^{iP(s)}$$

$\mathrm{Im}\ s > 0$



Expectation from Locality: $P(s) = 0$

Goldstino (Volkov-Akulov) Theory

$$\mathcal{L} = \psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi} - \frac{1}{M^2}(\psi\partial\psi)(\bar{\psi}\bar{\partial}\bar{\psi}) + \ldots$$

$$\boxed{\mathrm{e}^{2i\delta_{\mathrm{Gold}}(s)} = \frac{iM^2 - s}{iM^2 + s}}$$

A simple example of "Asymptotic Safety":

naively non-renormalizable theory flows into a strongly coupled UV fixed point, no new stuff added

Corresponds to integrable RG flow between tricritical Ising model in the UV and Ising model in the IR

(equivalently, N=1 Wess-Zumino model in the UV and free fermion in the IR)

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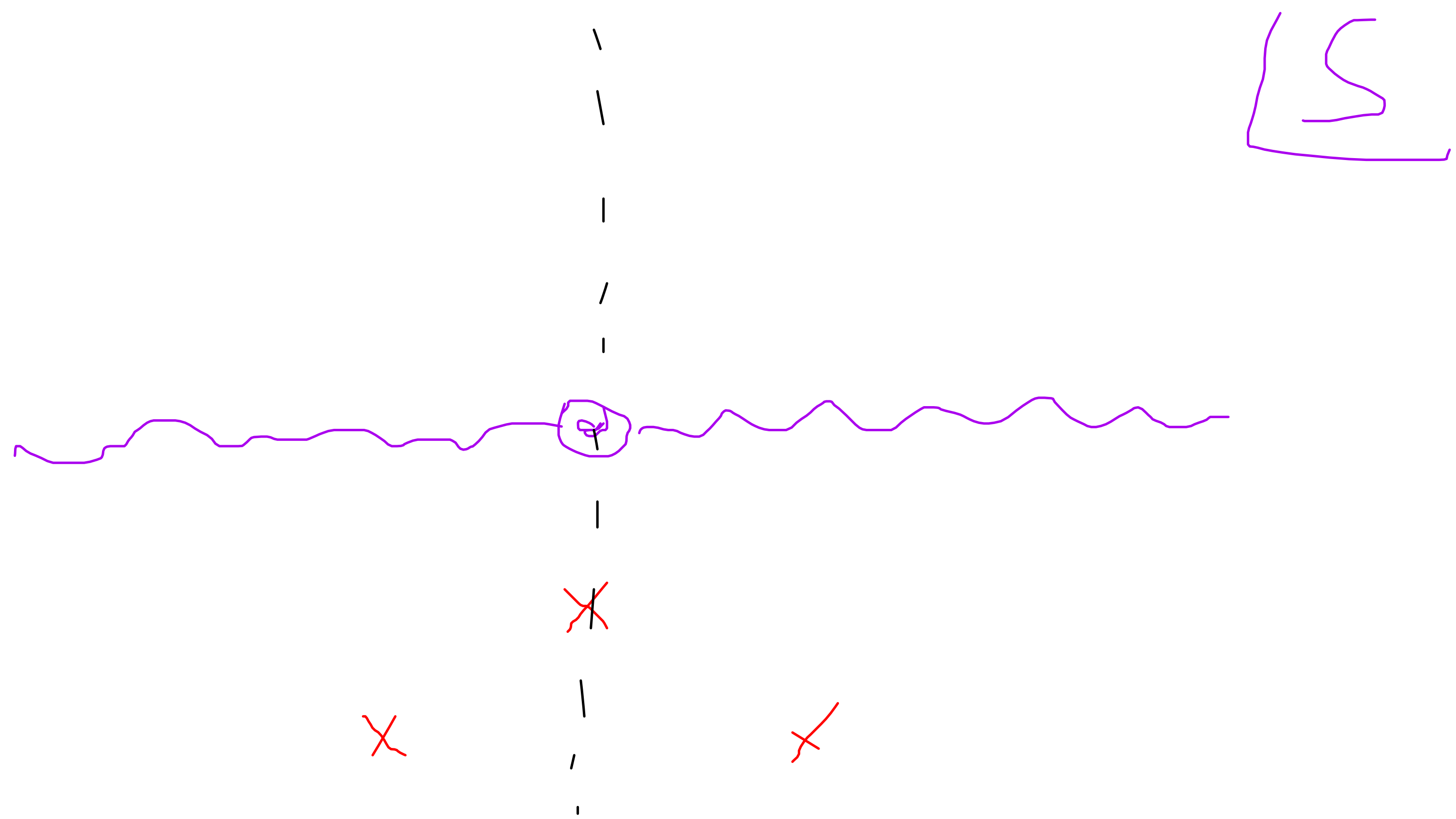

Expectation from Locality: $P(s) = 0 + \ell^2 s$

Let us look at
at (D-2) bosons with

$$e^{2i\delta(s)} = e^{is\ell^2/4}$$

- ✶Polynomially bounded on the physical sheet
- ✶No poles anywhere. A cut all the way to infinity with an infinite number of broad resonances
- ✶One can reconstruct the entire finite volume spectrum using Thermodynamic Bethe Ansatz

$$E(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell^4} + \frac{4\pi}{\ell^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

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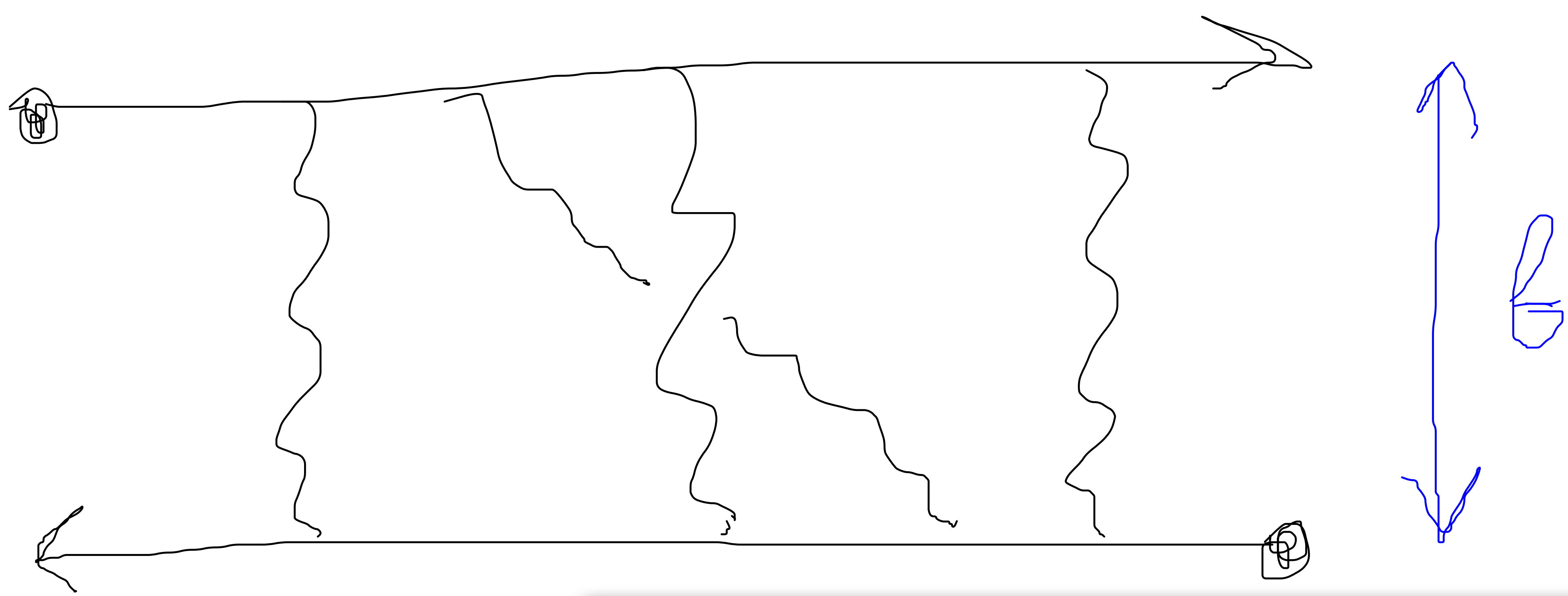
This is a light-cone quantized bosonic string

Integrable QG rather than QFT

Gravitational shock waves:

Dray, 't Hooft '85
Amati, Ciafaloni, Veneziano '88

$s >> M_{pl}^2$
$b >> R_s$



Eikonal phase shift:

$$e^{i2\delta_{eik}(s)} = e^{i\ell^2 s/4}$$

$$\ell^2 \propto G_N b^{4-d}$$

Some properties of the theory

classical action:

$$S_{NG} = -\ell^2 \int d^2\sigma \sqrt{-\det\left(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i\right)}$$

- ✶Theory of gravitational shock waves.
- ✶No UV fixed point and central charge.
- ✶No local off-shell observables.
- ✶Maximal achievable (Hagedorn) temperature.
- ✶Integrable cousins of black holes.
- ✶Minimal length.

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A new type of RG flow behavior:

Asymptotic Fragility

Integrable theory of gravity

Hagedorn transition and absence of $T_{\mu\nu}$

UV central charge from

$$T^{-1} f(T) = E_0(T^{-1}) \simeq \frac{\pi c_{UV}}{6} T \quad \text{at} \quad T \to \infty$$

we have

$$\boxed{f(T) = \frac{1}{\ell_s^2}\left(\sqrt{1 - T^2/T_H^2} - 1\right)} \qquad T_H = \frac{1}{\ell_s}\sqrt{\frac{3}{\pi(D-2)}}$$

$$c_v \simeq (T_H - T)^{-3/2}$$

Chaplygin gas $\quad \rho = \dfrac{p}{1 - \ell_s^2 p} \qquad c_s = \left(\dfrac{\partial \rho}{\partial p}\right)^{-1/2} = 1 - \ell_s^2 p$

Integrable Black Hole Precursors

Time Delay

$$\boxed{\Delta t_{cms} = \frac{1}{2}\ell_s^2 E_{cms}}$$

c.f. $\Delta t_H = \ell_{Pl}^4 E_{cms}^3$ for Hawking evaporation in 4d

Equivalence Principle at work

Δt is the same for a single hard particle and for a bunch of soft ones

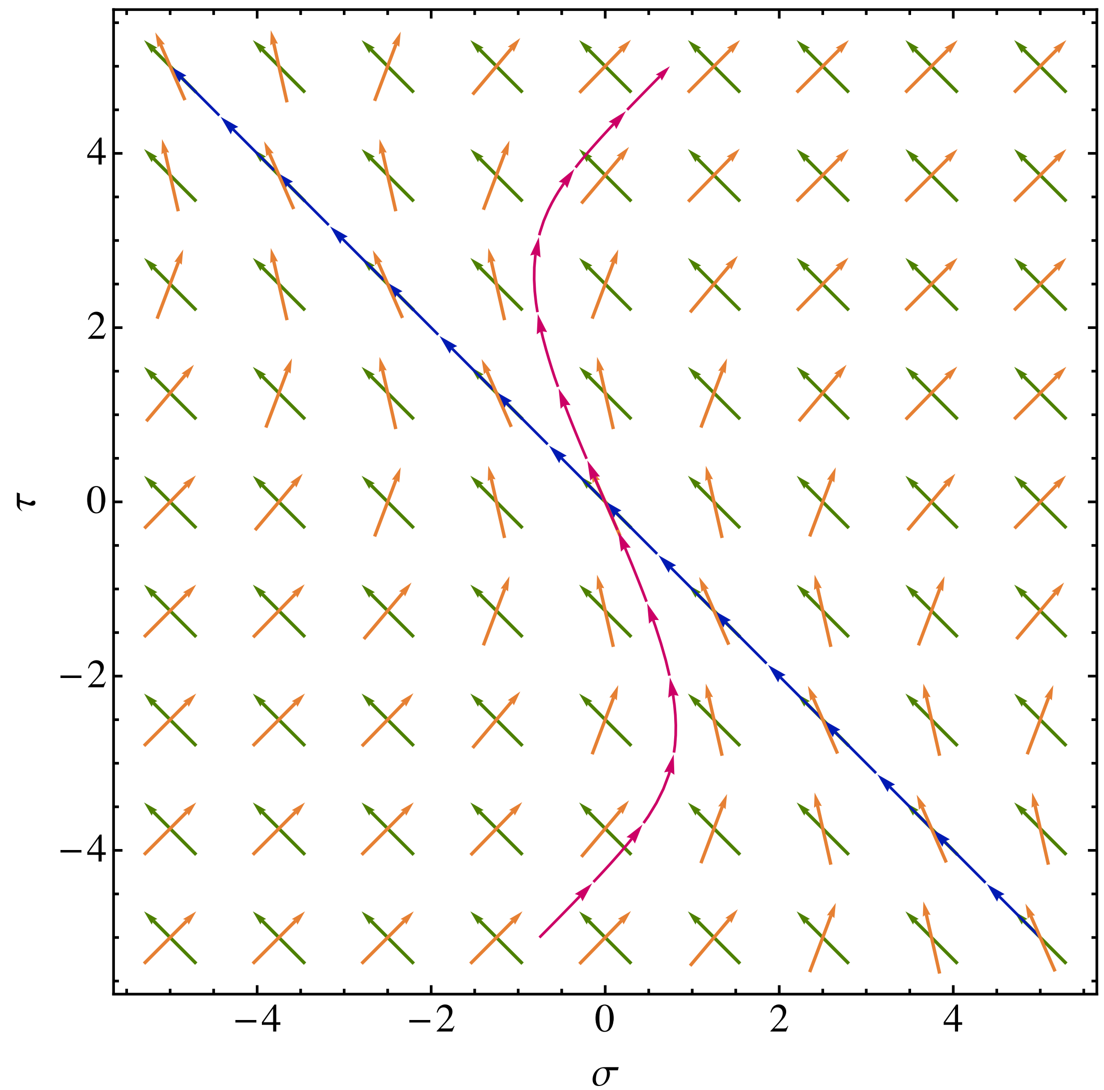
String uncertainty principle

$$\boxed{\Delta x_L \Delta x_R \geq \ell_s^2}$$

for identical packets $\Delta x_{out}^2 = \Delta x_{in}^2 + \frac{\ell_s^4}{\Delta x_{in}^2}$

Classical Origin of the Time Delay

$X^i_{cl}(\tau + \sigma)$ is a solution



$$\Delta t = \int_{-\infty}^{\infty} dz X'^2_{cl} = \ell^2 E$$

exactly reproduces the quantum answer

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How to dress more general theories?

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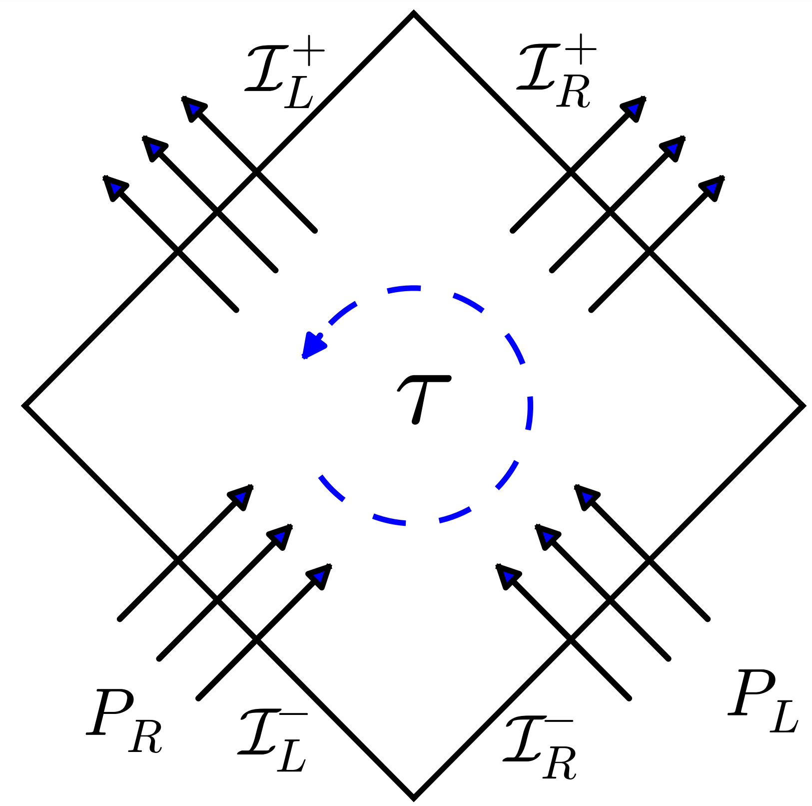
Eikonal Scattering From Boundary Quantum Mechanics

Verlinde & Verlinde '91

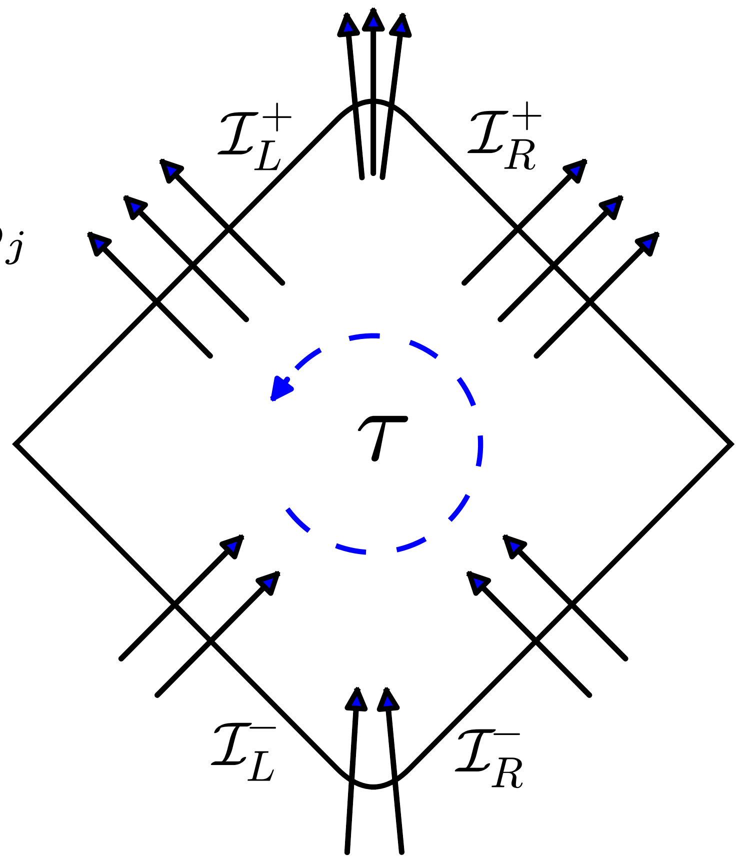
$$g_{\mu\nu} = \begin{pmatrix} e^{\phi}\partial_\alpha X^a \partial_\beta X^b \eta_{ab} & 0 \\ 0 & h_{ij} \end{pmatrix}$$



$$\boxed{S_{CS}[X] = \ell^{-2} \oint d\tau \epsilon_{\alpha\beta} X^\alpha \partial_\tau X^\beta}$$

$$S_{eik} = \int \mathcal{D}X e^{iS_{CS}[X]+i\left(\sum_i p^\alpha_{iR} X_\alpha(\tau_i)+\sum_j p^\alpha_{jL} X_\alpha(\tau_j)+\sum_i \bar{p}^\alpha_{iR} X_\alpha(\bar{\tau}_i)+\sum_j \bar{p}^\alpha_{jL} X_\alpha(\bar{\tau}_j)\right)}$$

Most simple-minded generalization:



$$\mathcal{D}(p_i) = \int \mathcal{D}X e^{iS_{CS}[X]+i\sum_i p_i^\alpha X_\alpha(\tau_i)} = e^{i\ell^2/4\sum_{i<j} p_i * p_j}$$

$$p_i * p_j = \epsilon_{\alpha\beta} p_i^\alpha p_j^\beta$$

does not produce a consistent S-matrix,

but allows to dress:

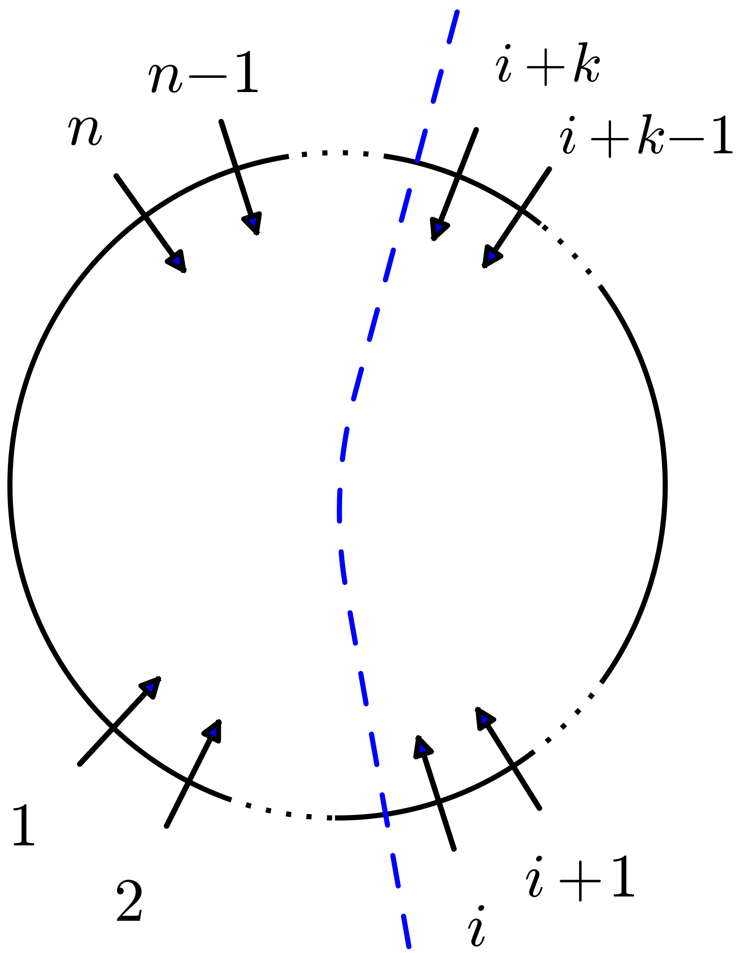
$$\boxed{\hat{S}_n(p_i) = e^{i\ell^2/4\sum_{i<j} p_i * p_j} S_n(p_i)}$$

✓Crossing Symmetry

✓Analyticity

? Unitarity

✓ Unitarity from Factorization



$$\mathcal{D}(p_i) = e^{i\ell^2/4\sum_{a<a'} k_a * k_{a'}} e^{i\ell^2/4\sum_{b<b'} q_b * q_{b'}}$$

$$\hat{S} = U S U$$

$$U|\{k_a\}\rangle = e^{i\ell^2/4\sum_{a<a'} k_a * k_{a'}} |\{k_a\}\rangle$$

The whole story is a bit similar to non-commutativity.

Two crucial differences:

✳Dressing of the full S-matrix, rather than of the tree amplitudes.

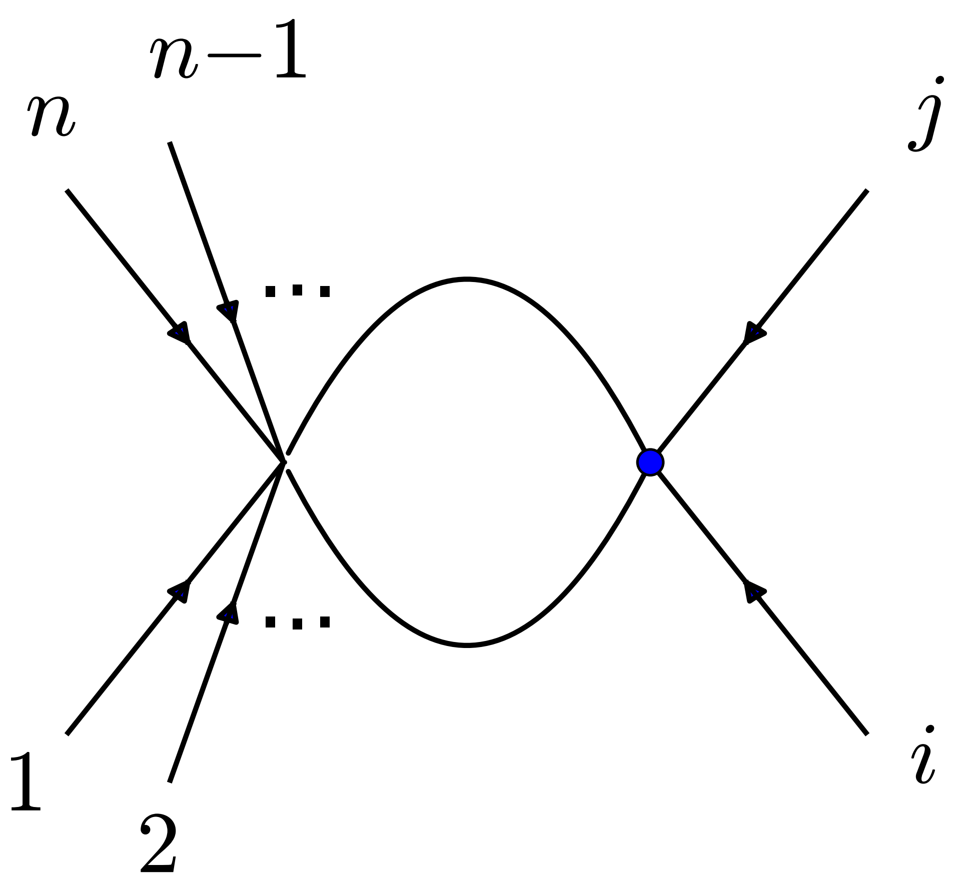
✳No summation over different cycling orderings. Preserves causality.

Perturbative Check

$$\mathcal{L}_{QFT} = \frac{1}{2}\left(\sum_i \partial\phi_i\partial\phi_i - m_i^2\phi_i^2\right) - \lambda\phi_1\phi_2\ldots\phi_n$$

Free Field Dressing:

$$\Delta\mathcal{L}_2 = -\frac{\ell^2}{8}\left((\partial_\alpha\phi_i\partial^\alpha\phi_i)^2 - 2(\partial_\alpha\phi_i\partial^\alpha\phi_j)^2 + m_i^2 m_j^2\phi_i^2\phi_j^2\right)$$



Reproduces $\mathcal{O}(\lambda\ell^2)$-dressing up to local polynomial terms

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* Observation of additional unprotected scalars (non-SUSY 2-Higgs doublet,...) would strongly push in this direction

* We had to start with a UV complete QFT. The SM is not like that (Landau poles). TeV-scale unification is needed?

One new pseudoscalar particle has already been discovered following these ideas... :)

Data from Athenodorou,Bringoltz,Teper
'1007.4720